The present study is devoted to the set of boundary value problems in the frameworks of coupled thermoelastoplasticity under axial symmetry conditions for a composite circular cylinder. Throughout the paper the conventional Prandtl–Reuss elastic–plastic model generalised on the thermal effects is used. The yield stress is assumed by linear function of the temperature. The modern mechanical engineering deals with the additive manufacturing technologies based on the adding new material at the high temperature gradient. The process of adding new parts of the material can be considered as a process of discrete material growth used in the technology of additive manufacturing. The mechanics of growing bodies [1–4] can be considered as a theoretical basis for solving such problems. In [5,6], boundary value problems of the growth of heavy viscoelastic bodies were solved with the gravitational forces presence. The thermal state of a growing viscoelastic sphere was discussed in [7,8]. The Assemblages made by the method of hot fitting have become the most widespread in technological practice for the stressed joints [9,10]. Usually such joints are carried out on the cylindrical surfaces of the assembly elements, when the outer part is heated before fitting and the inner one is cooled or remains at room temperature. Simplicity of the method of connection is coupled with the possibility to transmit significant loading pressure with different magnitudes and directions. The effective approximate engineering approaches and numerical simulation were developed to prescribe the stress strain state and material behaviour during the fitting process in the assembled nodes. The main drawback of the existing approaches for fitting process simulations is the insufficiently consistent consideration of the plastic flow and irreversible deformations in the
assembled materials.

The solutions of the boundary value problem of stresses computations in circular compounds during the hot fitting process taking into account the plastic properties of the material was considered in [11]. The present study deals with a new solution obtained in the frameworks of plastic flow theory and under conditions of complete plasticity.

Consider the boundary value problem in the frameworks of the thermal stresses theory during an assembly of two infinitely long hollow cylinders made from the same thermoelastoplastic material. At the referential time \( t = 0 \) the inner circular cylinder with inner and outer radii \( R_0 \) and \( R_1 \) respectively is heated under referential temperature \( T_1 \). The outer circular cylinder under temperature \( T_2 > T_1 \) with inner and outer radii \( R_1 \) and \( R_2 \) respectively. The referential displacements in the material of the cylinders are assumed to be zero. The inner cold cylinder is inserted into the heated outer cylinder. The stresses are accumulated providing tightness in the assembled parts as a result of the thermal conductivity process through the contact surface \( r = R_1 \).

Deformations \( d_{ij} \) in the cylinders material are assumed infinitesimal and are separated into reversible part \( e_{ij} \) and irreversible \( p_{ij} \) one additively by formula

\[
d_{ij} = e_{ij} + p_{ij}, \quad d_{rr} = u_{r,r}, \quad d_{\varphi\varphi} = r^{-1} u_r.
\]  

The stresses are fully determined by reversible deformations according to the Duhamel–Neumann law [12]

\[
\begin{align*}
\sigma_{rr} &= (\lambda + 2\mu)e_{rr} + \lambda(e_{\varphi\varphi} + e_{zz}) - (3\lambda + 2\mu)\Delta, \\
\sigma_{\varphi\varphi} &= (\lambda + 2\mu)e_{\varphi\varphi} + \lambda(e_{rr} + e_{zz}) - (3\lambda + 2\mu)\Delta, \\
\sigma_{zz} &= (\lambda + 2\mu)e_{zz} + \lambda(e_{\varphi\varphi} + e_{rr}) - (3\lambda + 2\mu)\Delta.
\end{align*}
\]  

The equilibrium equation in the cylindrical coordinate system under conditions of axial symmetry reads as

\[
\sigma_{rr,r} + r^{-1}(\sigma_{rr} - \sigma_{\varphi\varphi}) = 0.
\]  

The plastic flow occurs on the contact surface of the outer cylinder and is propagated with the temperature field aligning inside the assembled material. We note that in some cases under a sufficiently high initial temperature decreasing a plastic flow can occur on the contact surface of the inner cylinder, but numerous simulations have shown that the level of plastic deformation in these cases is small in compared with deformations arising from the contact pressure and does not cause a noticeable influence on the distribution of residual stresses and deformations.
Let the time \( t = t_p^{(2)} \) is the time of the plastic flow beginning on the inner surface of the outer cylinder. The plastic potential in the form of Tresca yield criterion is more preferable for considered problem

\[
\sigma_{\varphi\varphi}^{(2)} - \sigma_{rr}^{(2)} = 2k(r, t). \tag{4}
\]

At a time \( t > t_p^{(2)} \) the plastic flow domain \( R_1 < r < a_2(t) \) in the outer cylinder, where \( a_2(t) \) denotes the elastic plastic border. Equation for stresses computation can be obtained by integration of the equilibrium equation (3) under condition (4).

The further temperature field rearranging in the assembled cylinders can lead to plastic flow on the inner surface of the inner cylinder at time \( t = t_p^{(1)} \) [13]. Then the stresses keep the following form of the Tresca yield criterion [14]:

\[
\sigma_{rr}^{(1)} - \sigma_{zz}^{(1)} = 2k(r, t). \tag{5}
\]

Consequently for a time \( t > t_p^{(1)} \) it is possible a plastic flow domain development \( R_0 < a_1(t) \) in the material of the inner cylinder. The elastic domain is separated from the domain of irreversible deformation by the elastic plastic border \( a_1(t) \). We can derived the plastic incompressibility conditions from the plastic flow rule associated with the yield criterion (5).

Note also that there is the possibility of the complete plasticity state in the frameworks of Tresca yield criterion [13,15,16]. For this case we have two valid plastic flow conditions

\[
\begin{align*}
\sigma_{rr}^{(1)} - \sigma_{\varphi\varphi}^{(1)} &= 2k(r, t), \\
\sigma_{rr}^{(1)} - \sigma_{zz}^{(1)} &= 2k(r, t). \tag{6}
\end{align*}
\]

The satisfaction of the present Tresca yield criterion forms (4), (5) and (6) depends on the cylinder size and referential temperature gradient. Figure 1 illustrates the thermal stresses in the assembled cylinders after full temperature equalization with the complete plasticity domain in the inner cylinder.

The sequence of the considered boundary value problems may be violated by the cylinder size, material properties and referential temperature gradient. The extremal influence on the final contact pressure is induced by the irreversible deformation rate under yield criterion in form \( \sigma_{zz} - \sigma_{rr} = 2k \). The dependence of the yield stress on temperature makes it possible to predict lower values of the contact pressure contrary to studies without the plastic properties accounting or with the constant yield stress. Therefore, to increase the value of the contact
pressure it is necessary to reduce the initial temperatures in both parts of the assembled construction. Obviously, in this case the yield stress have the greatest value and, consequently, the level of the contact pressure is higher.

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References


