The creation of new structurally sensitive materials based on nanotechnology is an important direction in the development of modern materials science. Consolidated structurally sensitive materials are usually obtained by compacting nanopowders, depositing onto a substrate, crystallizing amorphous alloys, and other methods [1–3]. Such materials, as a rule, have unique physicomechanical and thermophysical properties that allow them to be used effectively in structures subject to high-intensity thermomechanical effects [3].

An important stage in the creation and use of new structurally sensitive materials is the construction of mathematical models that allow describing their behavior in a wide range of changes in external influences. However, the general methodology for constructing such models is far from complete, since the internal structure of such materials presupposes the presence of nonlocal effects both in time and in space. This statement is primarily related to models describing the process of heat conduction in structurally sensitive materials.

In [4], thermoelasticity models were proposed using effective variables (temperature and deformation). Based on these models, the temperature and stress fields in a flat layer are analyzed for high-intensity surface heating. In this paper, the approach proposed by A.K. Eringen [4] and based on the idea of using the influence function to describe the nonlocality effect in space, which is reflected in the gradients of the unknown quantities.

Based on the proposed mathematical model of thermal conductivity, numerical solutions are obtained for determining the temperature field in the problem of high-intensity surface heating of a curved plate. We note that the influence of the curvature on the temperature distribution may be significant [6], since even with an insignificant temperature drop, temperature stresses arise which can lead to distortion of the shape of the part. In this connection, it is necessary to aim for the minimum thickness of the part of the temperature difference in the process under consideration.

The equation of thermal conductivity in a curvilinear plate has the form:

\[ \rho c \dot{T} = -\frac{\partial q_1}{\partial x_1} - 2\kappa q_1. \]  

(1)
The equation for the projection of the density vector of the heat flux (modification of the Fourier law), taking into account the structure of the material, is written as follows:

$$q_1 = -\lambda(T)p_1 \frac{\partial T}{\partial x_1} - \lambda(T)p_2 \int_V \varphi(|x'_1 - x_1|) \frac{\partial T(x', t)}{\partial x'_1} dx'_1. \quad (2)$$

where \( \varphi(|x'_1 - x_1|) \) – function describing the area of influence of structural elements, what is more

$$\int_V \varphi(|x'_1 - x_1|)dx'_1 = 1,$$

\( p_1, p_2 \in [0, 1] \) – share of influence of local and non-local effects, \( p_1 + p_2 = 1 \). A similar influence function was also used in the models proposed by A.K. Ehringen [4] and based on the idea that the long-range forces that are responsible for the nonlocal behavior of the material in a neighborhood of a given point in space \( x'_1 \) are adequately described using the distance function \( \varphi(|x'_1 - x_1|) \) decreasing with growth \( |x'_1 - x_1| \) (Fig. 1).

We choose the function \( \varphi(|x'_1 - x_1|) \) as follows [5]

$$\varphi(|x'_1 - x_1|) = \frac{1}{2a} \exp \left( -\frac{|x'_1 - x_1|}{a} \right), \quad (3)$$

where \( 2a \) - the size of the spatial influence (Figure 1).

![Figure 1. Schematic representation of the influence function](image)

Taking into account the relations (2), (3), we rewrite the heat conduction equation (1):

$$\rho c \dot{T} = \lambda(T) p_1 \frac{\partial^2 T}{\partial x_1^2} + \lambda(T) p_2 \frac{\partial}{\partial x_1} \int_V \exp \left( -\frac{|x'_1 - x_1|}{a} \right) \frac{\partial T}{\partial x'_1} dx'_1 +$$

$$+2\kappa\lambda(T)p_1 \frac{\partial T}{\partial x_1} + 2\kappa \frac{\lambda(T)p_2}{2a} \int_V \exp \left( -\frac{|x'_1 - x_1|}{a} \right) \frac{\partial T}{\partial x'_1} dx'_1. \quad (4)$$
The boundary conditions for equation (4) corresponding to the known heat flux on the left and the isolated boundary on the right are written as follows:

\[ T(x_1, 0) = T_0; \]

\[ x_1 = 0 \quad -\lambda^{(T)}p_1 \frac{\partial T}{\partial x_1} - \frac{\lambda^{(T)}p_2}{2a} \int_V \exp \left( -\frac{|x'_1 - x_1|}{a} \right) \frac{\partial T}{\partial x'_1} dx'_1 = Q_0(t), \]

\[ x_1 = L \quad \lambda^{(T)}p_1 \frac{\partial T}{\partial x_1} + \frac{\lambda^{(T)}p_2}{2a} \int_V \exp \left( -\frac{|x'_1 - x_1|}{a} \right) \frac{\partial T}{\partial x'_1} dx'_1 = 0, \]

where \( Q_0(t) = AMt^m/t_0^m \exp(-mt/t_0), \quad m \geq 1, \quad t_0 > 0, \quad A, M = \text{const}. \)

The paper considers the model of heat conductivity in a curved plate (5) and describes the results of numerical simulation.

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References


