Mathematical modeling of additive growth of solids is aimed at improving the performance of device, machine, and mechanism parts which are fabricated using modern technologies. The fundamentally new mathematical models considered in the paper describe the evolution of the end product stress-strain state in additive growth and are of general interest for modern technologies in engineering, medicine, electronics industry, aerospace industry, and other fields (see, e.g., [1]).

The description of the additive growth process involves three characteristic instants: the instant \( \tau^*_1(x) \) when the element with the position vector \( x \) is formed, the instant \( \tau_0(x) \) when a load is applied to this element, and the instant \( \tau^*(x) \) when the element is deposited on the additive manufacturing fabricated solid.

The deposition process is determined by specifying these three instants. One can suppose usually that for the additive growth process \( \tau^*_1(x) = \tau_0(x) = \tau^*(x) \), i.e. the elements are deposited at the same instant as they are formed and a load is applied to them. This is not the only case and one can consider the cases when \( \tau_0(x) = \tau^*(x) \) but the instant \( \tau^*_1(x) \) differs from them and when the deformation of elements begins as soon as they are formed and they are being added to the basic solid only over some time interval, i.e., \( \tau^*_1(x) = \tau_0(x) \neq \tau^*(x) \). Hereunder we consider the first case and the function \( \tau^*(x) \).

We suggest an approach to modeling surface growth processes in solids on the basis of the following postulates:

- Material (not reference description) is utilized.
- The additive growth of a solid is modeled by the motion of its boundary due to the influx of new material to the surface of the solid.
- The stress rate tensor, the strain rate tensor (or the stretch rate tensor), and the velocity vector to be the main variables in the system of equations describing the additive growth.
- We use new kinematic and quasistatic conditions on the moving boundary (the growth or deposition surface) which determine the conservation
law for an additive manufacturing solid composition and specific contact interaction between 3D solid and 2D deposited surfaces.

The material description of the mechanics of additive surface growth processes which differs from knows approaches in continuum mechanics is proposed. Existing approaches of material description (see, e.g., [2]) use stress tensor, strain tensor and displacement vector as basic variables of boundary value problems. We use stress rates tensor, stretch tensor and velocity vector. The relations between classical and new variables have the form

\[ T(x, t) = G(t) \left[ \frac{T(x, \tau^*(x))}{G(\tau^*(x))} + \int_{\tau^*(x)}^{t} S(x, \tau) d\tau \right], \]

\[ u(x, t) = u(x, \tau^*(x)) + \int_{\tau^*(x)}^{t} v(x, \tau) d\tau. \]

In all relations below we will use (1) to change old variables by new ones. We note that all necessary initial conditions for desired values are given and loadings change with time as the parts and structures grow.

Let us consider the general nonlinear theory of the additive growth process for a solid from hyperelastic material. For a growing solid we have the equilibrium equation

\[ \nabla \cdot S = 0, \]

the boundary conditions on the stationary part of the surface

\[ x \in S_1: n \cdot S = \frac{\partial p_0}{\partial t}, \quad x \in S_2: v = \frac{\partial u_0}{\partial t}, \]

the quasistatic condition on the surface of growth which can be obtained from the solution of the contact interaction problem between a 3D solid and 2D surface

\[ x \in S^*(t): n \cdot S = -\frac{s_n}{G}(T_s : L) n, \quad s_n = n \cdot v, \]

the kinematic boundary condition on the growth surface (condition of the velocities compatibility on the growth surface)

\[ x \in S^*(t): v = v_{\text{def}} + v_{\text{gr}}, \]

the relation between the strain rates and velocities

\[ D = \frac{1}{2} [\nabla v + (\nabla v)^T], \]
and the constitutive equation in the form

\[ \mathbf{S} = 2\mathcal{F}_t(\mathbf{D}, \mathbf{v}), \tag{7} \]

the equation of the unknown growth surface \( S^*(t) \) for an additive manufacturing solid has the form

\[ t = \tau^*(\mathbf{x}), \tag{8} \]

where \( S \) is the stress rate tensor, \( \nabla \) is the Hamilton operator, where \( \mathbf{p}_0, \mathbf{u}_0 \) are given vectors of surface forces and strains, \( \mathbf{n} \) is the unit vector normal to the solid surface, \( \mathbf{v} \) is the velocity vector, \( \mathcal{T}_s \) is the 2D tensor of the deposited elastic surface tension, \( \mathbf{L} \) is the 2D tensor of this surface curvature, \( \mathbf{D} \) is the stretch tensor, \( \mathbf{v}_{\text{def}} \) is the unknown velocity of the boundary due to deformation of a solid, \( \mathbf{v}_{\text{gr}} = -\mathbf{v}_{\text{dep}} \) is the prescribed velocity of growth which is opposite to the velocity of deposition of a new material to the surface of a solid.

Relations (2)–(8) form the general nonlinear boundary value problem for a growing solid (see also [2–4]).

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References


