Nowadays importance of AM-fabricated solids studying increases [1–3]. There are a lot of work which analyse the field of science. But problems of growing solids mechanics often do not have exact analytical solution or it is very difficult to obtain it. So numerical methods are used to estimate and characterize behaviour of growing solids in such cases, for example, the finite element method, the boundary element method or the finite difference method. The finite element method (FEM) is the most attractive and commonly used method, which allow to find approximate values of displacements, stresses and strains in finite set of points, but values in other points of the solid may be evaluated with approximation. The points where the approximate solution is obtained are nodes of the finite elements mesh, which has to be constructed before numerical solution process is started. The construction problem is quite complicated, because the investigated solid can have a multiply connected boundary with a complex geometry. Also it is desirable to split the body into quadrilaterals, since the grid gives a more accurate numerical solution.

It is necessary to take into consideration changing of the solid’s boundary because of its growth in the problems of growing bodies. As a result, a grid created at some point in time may not reflect the relevant geometry of the body. Thus the grid is modified at each step in time. Most of the methods described in articles [4–8] can only make a mesh but not modify. It is important to have a method which can modify the mesh to decrease evaluation time. The method based on replacing all grid edges with springs with some rigidity is used for this purpose. It reduces the time for the reorganization of the grid to $O(n^2)$, where $n$ is the number of grid nodes. This time is sufficiently small in comparison with the time of making the grid from scratch by other methods [9].

Algorithms for using this method for numerical modelling of growing bodies, optimal tuning of the method for various problems, using the method in conjunction with others and accuracy of the generated grid for some growing solids problems of cylindrical structures under the influence of gravity are examined in this paper. Especially it is worth noticing that when the grid is modified by the method of spring system, the size of each element increases. Otherwise, the adding of new grid elements on the border of solid growth allows keeping the
size of the elements small enough that the error in numerical calculations also remains small.

The method is based on the optimal solution of the next functional

\[ E = \sum_{i \in I} \frac{k_i \Delta l_i^2}{2} + \sum_{i \in J} \lambda_i (\vec{u}_i - \vec{b}_i), \]

where \( E \) — potential energy of the spring system, \( I \) — a set of all the springs, \( k_i \) — stiffness of the spring \( i \), \( \Delta l_i \) — a difference between the initial and the final length of spring \( i \), \( J \) — a set of the boundary nodes, \( \lambda_i \) — Lagrange coefficient for node \( i \), \( \vec{u}_i \) — displacement of node \( i \), \( \vec{b}_i \) — a known vector. This non-linear functional can be replaced by a linear one and its optimal solution is satisfied the next system of equations in the case when every inner node has only four neighbours

\[
(A_1 + A_2 + A_3 + A_4)\vec{u}_i = A_1\vec{u}_{j(i)} + A_2\vec{u}_{k(i)} + A_3\vec{u}_{m(i)} + A_4\vec{u}_{p(i)}, i \in I \setminus J
\]

where \( A_i \) — a matrix associated with one of four springs \( \vec{u}_i \) — displacement of node \( i \), \( j(i), k(i), m(i), p(i) \) — numbers of the nodes next to node \( i \). Matrices \( A_i \) have the next structure

\[
A_i = \begin{pmatrix}
(x_i - x_{j(i)})^2 & (x_i - x_{j(i)})(y_i - y_{j(i)}) & (x_i - x_{j(i)})(y_i - y_{j(i)}) & (y_i - y_{j(i)})^2
\end{pmatrix},
\]

where \( x_i, y_i \) — coordinates of node \( i \).

The next figures represent the initial and target domains and mesh in them.

Figure 1. The initial domain  Figure 2. The target domain
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References


