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About damage and localization of strains

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The study of damage processes of inelastic materials is based on modified constitutive equations, which represent the damage as a loss of ability of the materials to resist the deformation under some damage criteria. The numerical implementation of such models does not mean the explicit consideration of new surfaces in damageable materials but the use of continuous modeling. The development of this approach started simultaneously with appearance of computers and numerical methods. (see the pioneer work [1]).

From the beginning the works were directed mainly to the investigation of an influence of a porosity as a damage parameter on effective elastic modules under uniaxial stress-strain state [2]. Thereafter the continuous approach was extended to the fatigue phenomena of elastic [3] and elastic plastic [4] materials by introducing a special structural parameter of damage – the damage parameter. In order to get the mathematically correct description of damage processes in many modern studies the constitutive equations are supplied with various regularizations, provided by terms with higher order derivatives, which are similar to those, presented by the artificial viscosity, gradient plasticity, non-local deformation measures and so on (see reviews in [5,6]).

In most of used numerical models of continuous damage (see for instance [1-14]) the calculated zones of damaged material are unrealistically wide while the localization of strains are weak. Therefore the interpretation of the numerical results depends often on the imagination of a researcher. If the influence of regularising terms is decreased in order to provide the more strong localization of strains then the abnormal dependence of damage zones on the size and the shape of grid cells appear. It means finally the loss of the convergence of the numerical solutions.

The reasons of such the behaviour of the numerical models of continuous damage are not clearly understandable and the question about necessary properties of such models for effective description of damage and localization of strains is still opened and actual.

Suggested below the theoretical model and the numerical method are directed to the overcoming of pointed above drawbacks in order to recognise the evolution of "macro-cracks" as some narrow bands of damaged material, representing the jumps of velocity and displacement as well as high peaks of deformations. In this theoretical model the deformations are assumed to be large (they reach the values of hundreds percents in the zones of damaged material). Besides, it is taken into account, that the damage is accompanied by the change of the process rate from quasistatical while the damage is absent to highly dynamical during the development of the damage. The numerical method, which takes these features into account, is implemented in a frame of computer code "ASTRA".

The results of parametric calculations of the damage processes forced by an extension and a heating of standard specimens are presented. Several damage criteria and damage parameter kinetic equations are tested and the influence of plasticity and local heating on damage processes is highlighted. Some basic recommendations concerning the most important properties of theoretical models of continuous damage are worked out.

1. **Formulation of problem.** The system of equations, which describes the behavior of thermo-elastic-plastic damageable media, has been under development in many studies (see, for instance [1-14]) and is used here in the form presented in [15]. The system of equations consists of conservation laws for the mass, momentum and energy, and it contains also the kinematic equations

$$\frac{d\rho}{dt}+\rho\mathbf{e}:\mathbf{I}=0\ ,\quad \rho\frac{d\mathbf{u}}{dt}=\nabla\cdot\boldsymbol{\sigma},\quad \rho\frac{dU}{dt}=\boldsymbol{\sigma}:\mathbf{e}+\rho r+\nabla\cdot\mathbf{q},\quad \mathbf{F}^{-T}=\nabla\otimes\overset{o}{\mathbf{x}}\ ,$$

$$\boldsymbol{\varepsilon}=\frac{1}{2}\left(\mathbf{I}-\mathbf{F}^{-T}\cdot\mathbf{F}^{-1}\right),\quad \mathbf{e}=\frac{1}{2}\left(\mathbf{L}+\mathbf{L}^{T}\right),\quad \mathbf{e}=\frac{d\boldsymbol{\varepsilon}}{dt}+\boldsymbol{\varepsilon}\cdot\mathbf{L}+\mathbf{L}^{T}\cdot\boldsymbol{\varepsilon},\quad \mathbf{L}=\nabla\otimes\mathbf{u}\ ,\quad \mathbf{u}=\frac{d\mathbf{x}}{dt} \tag{1}$$

and constitutive equations, which are considered below in details. Here the traditional notation is used: ρ - density, $\mathbf{u}$ - a velocity of the material medium, t - time, $\mathbf{x}$ - Eulerian radius-vector (actual configuration), $\overset{o}{\mathbf{x}}$ - Lagrangian radius-vector (initiall configuration), $\mathbf{F}$ - strain gradient tensor, $\mathbf{L}$ - a velocity gradient tensor, $\boldsymbol{\varepsilon}$ - Almansi strain tensor, $\mathbf{e}$ - Eulerian strain rate tensor, $\boldsymbol{\sigma}$ - Cauchy stress tensor, U - an internal energy per unit of mass, $\mathbf{q}$ - a heat flux vector, T - a temperature, r - heat source, d/dt - material time derivative, ∇ - spatial differential operator in actual settings, $\mathbf{I}$ - unity tensor.

The constitutive equations represent the dependencies between the characteristic values of an infinitesimal volume of the continuous media, enforced by the thermodynamic laws. The minimal set of mutually independent constitutive parameters for an infinitesimal volume is $\pi=(T,\overset{0}{\boldsymbol{\varepsilon}},\overset{0}{\boldsymbol{\chi}},\frac{dT}{dt},\overset{0}{\mathbf{e}},\frac{d\overset{0}{\boldsymbol{\chi}}}{dt},\nabla T)$, where $\overset{o}{\boldsymbol{\chi}}=(\overset{o}{\boldsymbol{\varepsilon}}_p,\theta)$ - the structural parameters: plastic strain tensor $\overset{o}{\boldsymbol{\varepsilon}}_p$ and damage parameter θ , which are defined later and responsible for the structural rebuilding of the continuous media, which takes place due to the development of dislocations and the appearance of microcracks respectively. Sign "zero" above a value marks the material tensors, which relate to the spatial tensors as follows

$$\overset{0}{\boldsymbol{\varepsilon}}=\mathbf{F}^{T}\cdot\boldsymbol{\varepsilon}\cdot\mathbf{F},\quad \overset{0}{\mathbf{e}}=\mathbf{F}^{T}\cdot\mathbf{e}\cdot\mathbf{F},\quad \overset{0}{\boldsymbol{\sigma}}=\mathbf{F}^{-1}\cdot\boldsymbol{\sigma}\cdot\mathbf{F}^{-T},\quad \frac{d\overset{0}{\boldsymbol{\varepsilon}}}{dt}=\overset{0}{\mathbf{e}},\quad \overset{0}{\boldsymbol{\sigma}}\cdot\overset{0}{\mathbf{e}}=\boldsymbol{\sigma}\cdot\mathbf{e}$$

From the first thermodynamic law, which expresses the conservation of the energy, and from the second thermodynamic law, the law of entropy (η) growth

$$\rho T\frac{d\eta}{dt}-T\nabla\cdot\left(\frac{\mathbf{q}}{T}\right)-\rho r\geq 0$$

the inequality of the rate of dissipation (D) follows

$$D=-\rho\left(\eta+\frac{\partial\varphi}{\partial T}\right)\frac{dT}{dt}+\left(\overset{0}{\boldsymbol{\sigma}}-\rho_0\frac{\partial\varphi}{\partial\overset{0}{\boldsymbol{\varepsilon}}}\right):\overset{0}{\mathbf{e}}-\rho_0\frac{\partial\varphi}{\partial\overset{0}{\boldsymbol{\chi}}}:\frac{d\overset{0}{\boldsymbol{\chi}}}{dt}+\frac{1}{T}\mathbf{q}\cdot\nabla T\geq 0$$

Here $\varphi=U-T\eta$ - a free energy per unit of mass.

The free energy and rate of dissipation functions read:

$$\varphi = \frac{K}{2\rho_0}\left(\ln\frac{\rho}{\rho_0} + \beta(T - T_0)\right)^2 + \frac{\mu}{2\rho}(\boldsymbol{\varepsilon}' - \boldsymbol{\varepsilon}'_p)^2 : \mathbf{I}$$

$$D = H(\Phi_p)k_p\sqrt{\mathbf{e}_p' : \mathbf{e}_p'} + H(\Phi_\theta)k_\theta\left(\frac{d\theta}{dt}\right)^2 + \frac{k_q}{T}\nabla T \cdot \nabla T$$

It is assumed, that elastic parts $(\boldsymbol{\varepsilon}' - \boldsymbol{\varepsilon}'_p)$ of strain deviator are much less compared to unity. The effect of temperature extension is taken into account by the term with the thermal expansion factor β. The term in the expression of the rate of dissipation, which is responsible for the plastic flow, is a first order homogeneous function of the plastic strain rate, which the case corresponds to elastic plastic material. The plastic strain grows when the active plastic straining condition is fulfilled $\Phi_p(T, \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}_p, \theta, \mathbf{e}) \geq 0$. It is assumed also that the material is plastically incompressible (the rate of dissipation depends only on the deviators of plastic strain rate, which is common assumption for metals. The resistance of the media, represented by elastic modules and by the yield limit depends on the temperature, the strain the plastic strain, and also on the additional structural parameter θ, named damage:

$$\mu = \mu_0(T)g_\mu(\theta), \quad K = K_0(T)g_K(\theta), \quad k_p = k_{p0}(T)g_p(\theta)$$

where $\hat{E}_0 = \hat{E}_0(T)$, $\mu_0 = \mu_0(T)$ - elstic moduli, $k_{\sigma 0} = k_{\sigma 0}(T, \varepsilon, \varepsilon_p)$ - yield limit for undamaged material. Varying from 1 to 0 functions g_μ, g_K, g_σ provide the decrease of strain resistance of the media with growth of damage, which occurs if the damage condition is fulfilled

$$\Phi_\theta(T, e, e_p, \theta) \geq 0$$

The kinetics of damage processes is defined by the dependence of rate of dissipation on rate of damage parameter.

The constitutive equations read:

$$\eta = -\frac{\partial\varphi}{\partial T}, \; U = \varphi - T\frac{\partial\varphi}{\partial T}, \; \mathbf{q} = k_q\nabla T, \; \boldsymbol{\sigma} = -p\mathbf{I} + \boldsymbol{\sigma}', \; \sigma' = 2\mu(\varepsilon' - \varepsilon'_p),$$

$$p = K\frac{\rho}{\rho_0}\left(\ln\frac{\rho}{\rho_0} + \beta(\grave{O} - \grave{O}_0)\right), \; e'_p = H(\Phi_p)\sqrt{e'_p : e'_p}\,\sigma'/k_p, \; \frac{d\theta}{dt} = H(\Phi_\theta)k_\theta^{-1}\frac{\partial\varphi}{\partial\theta}, \quad (2)$$

Boundary conditions read:

$$\mathbf{x} \in S_{un}, \quad t \geq 0 : \mathbf{u} \cdot \mathbf{n} = u_n^*$$

$$\mathbf{x} \in S \setminus S_{un}, \quad t \geq 0 : (\boldsymbol{\sigma} \cdot \mathbf{n}) \cdot \mathbf{n} = P_n^*$$

$$\mathbf{x} \in S_{u\alpha}, \quad t \geq 0 : \mathbf{u} \cdot \tau_\alpha = u_{\tau\alpha}^*$$

$$\mathbf{x} \in S \setminus S_{u\alpha}, \; t \geq 0 : (\boldsymbol{\sigma} \cdot \mathbf{n}) \cdot \tau_\alpha = P_\alpha^*$$

$$\mathbf{x} \in S_T, \qquad t \geq 0 : T = T^*$$

$$\mathbf{x} \in S \setminus S_T, \quad t \geq 0 : \mathbf{q} \cdot \mathbf{n} = q_n^* \tag{3}$$

where $\alpha = 1, 2$. Initial conditions read:

$$t=0: \ \mathbf{x} = \overset{o}{\mathbf{x}}, \ \mathbf{u} = \mathbf{u}_o^*, \ T = T_0, \ \varepsilon_p = 0, \ \square = 0 \tag{4}$$

So, it needs to solve the initial boundary value problem for the system of equations (1)-(2) under boundary (3) and initial (4) conditions.

2. **Numerical method.** The solution algorithm is based on modified implicit finite element scheme, proposed in [16], and it is implemented in a frame of code "ASTRA". Major features of this algorithm are as follows. The unsteady Bubnov - Galerkin formulation and the finite element spatial approximations of the major unknown functions (velocity, displacement, temperature, plastic strain and damage) are used. More definitely the linear triangular and bilinear quadrilateral finite elements are implemented and integrated numerically. Quadrilateral elements are regularized by small artificial viscosity to prevent the hour glass instability. The numerical integration points coincide with grid nodes, so the matrix of mass is diagonal. Within each time step the nonlinearities are linearized by Newton's method. The auxiliary linear algebraic problems are solved by the iterative conjugate gradient method, which is implemented without matrix operations in a way usual for explicit schemes. The algebraic problems are preconditioned by using the diagonal approximations of the stiffness

3. **Results.** The solution region for the problem about damage of extending standard specimen can be seen in Fig. 1. Initial length of solution region is 3.0, height is 2.0. The left and the bottom boundaries are the axes of symmetry, the right boundary moves to the right with permanent rate of speed V_o, upper boundaries are free. Input data read:

$$K_0 = 975, \ \mu_0 = 369, \ k_{p0} = 1, \ c_o = \frac{K_o + 4/3\mu_o}{\rho_o} = 1,$$

$$k_\theta = 10^3, \ \Phi_\sigma = \sigma©:\sigma© - k_p^2, \ \Phi_\theta = \varepsilon_{\max} - 10^{-2},$$

$$c_V = 1, \ k_q = 1, \ \beta = 0.0001,$$

where $\varepsilon_{\max}$ - maximal principal strain, c_o - sound velocity, $V_o = 10^{-3} c_o$ - right boundary velocity. At the initial instant $t = 0$ the specimen is undeformed and has zero values of major unknowns under constant (spatially) dimensionless value of the temperature $T_0 = 100$. Dimensionless mass heat source $r = \pm 0.1$, acted in the narrow rectangular zone (1.9, 0, 2.1, 1). The boundary heat fluxes were of zero value. The moving Lagrangian grid consisted primarily of identical (quadrilateral or left/right-oriented triangular) cells of the following sizes: 1/15, 1/30 and 1/60 for various runs.

The development of narrow zones of the localization of strains can be seen in Fig. 1. for the three cases: damage of elastic specimen (a), elastic plastic specimen (b), elastic plastic specimen under joint action of extension and heating (c). The Figures indicate that the plasticity forces its own preferable direction of propagation of strain localization zone.

Under additional rather intensive heating the damage of elastic plastic specimen is developing in the same way as in the case of elastic specimen (narrow zone of heating is situated especially along the track of damage zone, developed in case of elastic specimen). In the case of moderate heating the two narrow zones of damage ("macrocracks") can be observed: firstly slanting ("plastic") "crack" develops, then as the effect of heating becomes stronger it stops and finally the vertical "thermal crack" completes the damage of the specimen. The graphs of horizontal displacement, mean stress and maximal principal strain along the horizontal line, which intersects the narrow zones of damaged material, are presented in Fig. 2. The behaviour of these functions are typical for internal contact boundaries and imitates the macrocracks in a frame of continuous model.

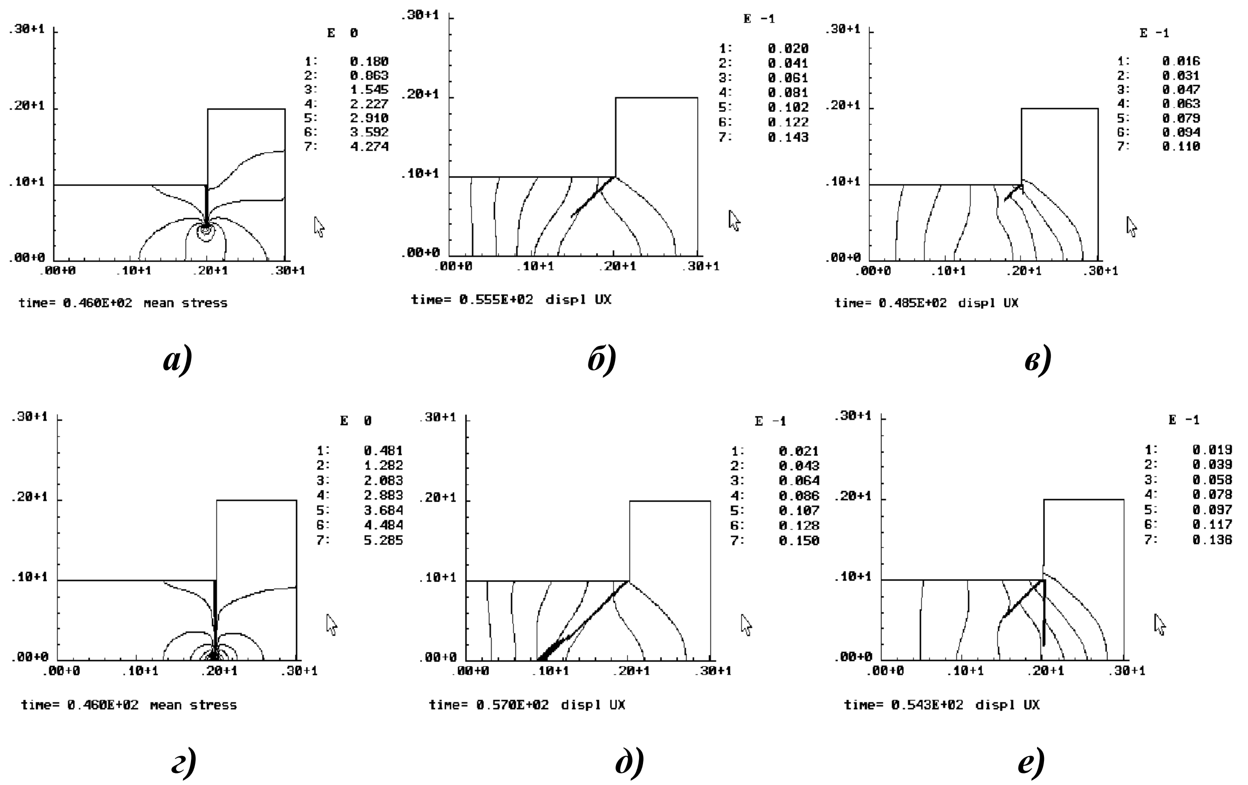
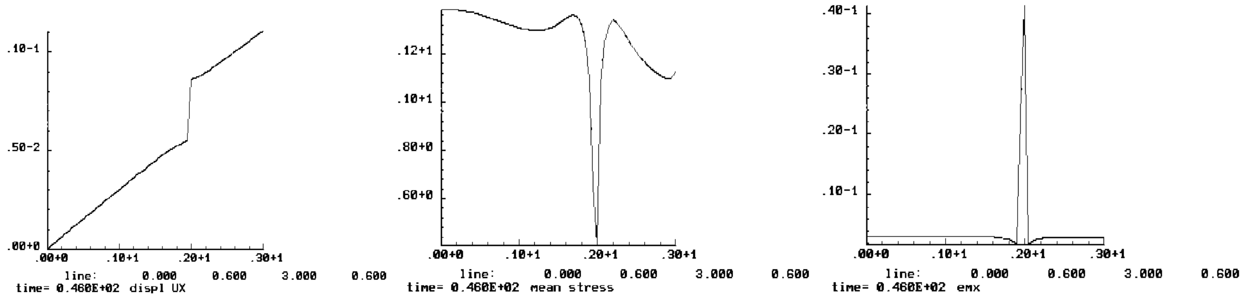


Рис. 1. Моды процесса разрушения для случаев упругого материала (а,г), упругопластического материала (б,д), упругопластического материала при совместном действии растяжения и нагрева узкой вертикальной зоны под концентратором.

The damage and the deformation in the zone of damaged material indicate very big splash of delta-function type, the values of stresses fall down to zero, and the displacements as well as velocities are undergone the strong growing jumps like those in shock waves in gas dynamics.
The intensive heating accelerates the damage process while the local cooling of the anticipating zone of damage delays the development of damage. When the damage process starts the time step is dramatically tends to its dynamical value of Courant's time step because of accuracy restriction, which is set for the strain increment. The primarily quasistatical damage process becomes dynamical. The modes of damage stay the same under variations of spatial resolution and cell shape.



а) ***б)*** ***в)***

Рис. 2. Графики горизонтального смещения (а), среднего напряжения (б) и максимальной деформации (в) вдоль горизонтальной линии (0,0.6,3,0.6) для случая разрушения упругого материала, показанного на Рис. 1а. В остальных случаях качественное поведение такое же.

4. **Conclusion.** The numerical experiments show, that

- the damage criteria based on maximal strains are preferable compared to those based on maximal stresses, at least for elastic plastic materials.
- In order to get the strong strain localization and narrow damage zones like "cracks" it is important to satisfy the following conditions in the theoretical models:
- To provide the keen decrease of the material resistance to the deformation with growth of damage parameter;
- To control the accuracy restricting the value of maximal strain increment within each time step;
- To take into account the inertia terms, which give natural regularization of the problem.
- To minimize the smoothing procedures (especially near the zones of damaged material);
- To provide the ability of material to resist the compression even in the damaged state (To prevent the violation of one to one mapping between the actual and initial configurations).

It is demonstrated, that due to pointed above special features, the proposed model is good in description of keen strain localization along narrow bands of large gradients of velocities/displacements and high intensity splashes of deformations. It supports the convergence of the numerical solutions in spite of presence of narrow zones of damaged material.

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