Francfort and Marigo have formulated a variational approach to brittle fracture, see e.g. [1]. In this approach, the total energy is minimized with respect to the crack geometry and with respect to the displacement field simultaneously. The method is applicable to crack propagation, to branching of pre-existing cracks and even to crack initiation without assumption of additional criteria. The numerical treatment is not trivial. A regularized version of the model (Γ-convergence) has been presented by Bourdin in [2]. In this model, cracks are represented by an additional field variable, which is 0 if the material is cracked and 1 if it is undamaged. Due to this meaning a strong relation to damage mechanics is of course given. In extension to classical damage mechanics, the model by Bourdin [2] incorporates the surface energy by using gradients of the crack field variable. In the limiting case of steep gradients, the regularization approximates the energy considerations of fracture by Griffith.

In our work [3,4], we reinterpret the crack variable as a phase field variable or as an order parameter, and cracking is addressed as a phase transition problem. Crack growth is governed by an evolution equation of the order parameter that resembles the Ginzburg-Landau equation. The time integration of the evolution equation is done by an implicit Euler scheme. Due to the regularization in space, the numerical implementation is very sensible to the choice of the regularization parameter in conjunction with the mesh size. If a finite element discretization is used the element size has to be small enough to resolve the steep gradients in

Fig. 1: Comparison of different shape functions and different Gauss quadrature rules
the transition zone between cracked and uncracked areas. The correct approximation is important to capture the surface energy and thus the thresholds and dynamics of crack propagation correctly. This is the main computational limit and challenge of the proposed method. To overcome this limitation, a finite element method using exponential shape functions is introduced in [4]. The shape functions are constructed in such a way that they capture qualitatively the analytical solution of the regularized model. Numerical examples, show that these new shape functions perform better than standard Lagrange shape functions, see Fig. 1.

Fig. 2: Configurational forces during loading of an edge crack

The kinetics of crack propagation is analyzed by using the concept of configurational forces, (material forces). This concept, which is well established in the framework of classical thermo-mechanics, is adapted to the phase field continuum of fracture. For more details on the subject see [3]. The theory of configurational forces provides a better understanding of the kinetics in crack initiation and in crack propagation scenarios, see Fig. 2. Examples relate the phase field parameters to fracture mechanical quantities and demonstrate that the model is capable of predicting a size dependent failure of specimen with an inhomogeneous stress state.

References