Classical description of pair production.

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Abstract

It is shown that the classical description of pair production effect is possible, i.e. one can describe pair production without a reference to quantum principles. Pair production appears at statistical description of stochastic relativistic particles. There is a special force field which is responsible for pair production. This field is a reasonable consequence of quantum stochasticity. Consideration of quantum systems as stochastic systems and statistical description of them generates hydrodynamic interpretation of quantum phenomena. In the collision problem the hydrodynamic interpretation appears to be alternative to the conventional interpretation, based on quantum principles and on consideration of the wave function as a principal object of dynamics. Hydrodynamic interpretation leads to such a statement of the collision problem, which is alternative to the conventional S-matrix theory.

1 Introduction

Pair production is a specific quantum effect. It is described conventionally in terms of quantum theory. We do not know how to describe the pair production effects in classical terms. But if the classical description of the pair production were possible, it would be very useful. Indeed, at high energies of particles their de Broglie wave length $\lambda_{\mathrm{B}} = \hbar/p$ becomes to be very small, and description of high energy particles becomes semiclassical or even classical. The classical description is simpler, than the quantum one, because it uses less information and admits a simpler interpretation. Unfortunately, we cannot realize advantages of classical description, because at high energies the pair production processes are dominating, but we do not know how to describe them in classical terms. In general, we know about the pair production only that it appears at collision of high energy particles, and it is described only in terms of quantum theory. But what agents are responsible for the pair production remains to be seen.

In this paper we are going to investigate what agents are responsible for the pair production and to show that a classical description of pair production effects is possible. Speaking about classical description, we mean only that *principles of quantum mechanics are not used* at such a description. Classical description does not mean a classical approximation of quantum description. Our classical description is an exact description. In particular, it describes quantum effects, using quantum constant $\hbar$, which appears as a constant describing the space-time properties (but not as an attribute of quantum principles).

In our consideration we use model conceptions of quantum phenomena (MCQP), which is a new stage in investigation of microcosm. MCQP relates to conventional quantum theory (axiomatic conception of quantum phenomena (ACQP)) in the same way, as the statistical physics relates to thermodynamics. The main difference between MCQP and ACQP in investigation methods which are used. Methods of MCQP are more subtle and effective, than those of ACQP. One can see this from the next table

ACQP	MCQP
Combination of nonrelativistic quantum technique with principles of relativity	Consequent relativistic description at all stages
1. Quantization: procedure on the conceptual level: $\mathbf{p}\rightarrow -i\hbar\boldsymbol{\nabla}$ etc.	1. Dynamic quantization: relativistic procedure on the dynamic level $m^2\rightarrow m_{\text{eff}}^2=m^2+\frac{\hbar^2}{c^2}\left(\kappa_l\kappa^l+\partial_l\kappa^l\right)$
2. Transition to classical description: procedure on conceptual level $\hbar\rightarrow 0 \qquad \psi\rightarrow(x,p)$	2. Dynamic disquantization: relativistic procedure on dynamic level $\partial^k\rightarrow\frac{j^kj^l}{j_sj^s}\partial_l$
3. Interpretation in terms of wave function ψ	3. Interpretation in terms of statistical average world lines (WL) $\frac{dx^i}{d\tau}=j^i(x)$, $j^k=-\frac{i\hbar}{2}\left(\psi^*\partial^k\psi-\partial^k\psi^*\cdot\psi\right)$
4. One kind of measurement, as far as only one statistical average object $\langle\mathcal{S}\rangle$ is considered. It is referred to as quantum system	4.Two kinds of measurement, because two kinds of objects (individual $\mathcal{S}_{\text{st}}$ and statistical average $\langle\mathcal{S}\rangle$) are considered
5. Additional hypotheses are used (QM principles)	5. *No additional hypotheses are used*

In this paper we develop technique of MCQP and apply methods of MCQP for investigation of the pair production process. This technique is based on consideration of quantum mechanics as a special case of the stochastic system dynamics [2]. The stochastic system dynamics is suitable for description of any stochastic systems (not only quantum), and because of this it cannot use quantum principles and does not use them. All description is produced in classical (non-quantum) terms, although description in terms of wave function is also possible as a special case of a description. Such an approach is very convenient, because it gives a very simple interpretation of quantum effects in terms of classical dynamics of particles and fluids.

The main distinction between the quantum mechanics and dynamics of stochastic systems lies in their relation to the wave function. In the quantum mechanics the wave function ψ is a fundamental object, whose properties are defined by quantum principles. As a result the interpretation of all objects which is constructed on the base of the wave function is rather obscure, because the meaning of the wave function itself is obscure. In the stochastic system dynamics the wave function is simply a method of description of any nondissipative fluid [3]. In this case the meaning of all objects constructed on the base of the wave function and the meaning of the wave function itself are obtained rather simply in terms of classical description of a fluid (continuous medium).

Connection between the fluid and the Schrödinger equation is known since the beginning of the quantum mechanics construction [4, 5]. In after years many authors developed this interplay known as hydrodynamic interpretation of quantum mechanics [6, 7, 8, 9, 10, 11, 12, 13, 14]. But this interpretation was founded ultimately on the wave function as a fundamental object of dynamics. It cannot go outside the framework of quantum principles, because the connection between the hydrodynamic interpretation and the quantum mechanics was one-way connection. One could obtain the irrotational fluid flow from the dynamic equation for the wave function (Schrödinger equation), but one did not know how to transform dynamic equations for a fluid to the dynamic equation for a wave function. In other words, we did not know how to describe rotational fluid flow in terms of the wave function. In terms of the wave function we could describe only irrotational fluid flow.

To describe arbitrary fluid flow in terms of a wave function, one needs to integrate conventional dynamic equations for a fluid (Euler equations). Indeed, the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi=0 \tag{1.1}$$

may be reduced to the hydrodynamic equations for the variables $\rho, \mathbf{v}$, describing the fluid state. Substituting $\psi=\sqrt{\rho}\exp(i\hbar\varphi)$ in (1.1) and separating real and imaginary parts of the equation, we obtain expressions for time derivatives $\partial_0\rho$ and $\partial_0\varphi$. To obtain expression for the time derivative $\partial_0\mathbf{v}$ of the velocity $\mathbf{v}=\frac{\hbar}{m}\boldsymbol{\nabla}\varphi$, we need to differentiate dynamic equation for $\partial_0\varphi$, forming combination $\partial_0\mathbf{v}=\boldsymbol{\nabla}\left(\frac{\hbar}{m}\partial_0\varphi\right)$. The reverse transition from hydrodynamic equations to dynamic equations for the wave function needs a general integration of hydrodynamic equations. This integration is simple in the partial case of irrotational flow, but it is a rather complicated mathematical problem in the general case, when a result of integration has to contain three arbitrary functions of three arguments. Without producing this integration one cannot derive description of a fluid in terms of the wave function, and one cannot manipulate dynamic equations, transforming them from representation in terms of ρ, $\mathbf{v}$ to representation in terms of wave function and back. This problem has not been solved for years. It had been solved in the end of eighties, and the first application of this integration can be found in [15]. Systematical application of this integration for description of quantum phenomena began in 1995 [16, 17].

The problem of general integration of four hydrodynamic Euler equations

$$\partial_0\rho + \boldsymbol{\nabla}(\rho\mathbf{v}) = 0 \tag{1.2}$$

$$\partial_0\mathbf{v} + (\mathbf{v}\boldsymbol{\nabla})\mathbf{v} = -\frac{1}{\rho}\boldsymbol{\nabla}p, \qquad p = p(\rho, \boldsymbol{\nabla}\rho) \tag{1.3}$$

seems to be hopeless. It is really so, if the Euler system (1.2), (1.3) is considered to be a complete system of dynamic equations. In fact, the Euler equations (1.2), (1.3) do not form a complete system of dynamic equations, because it does not describe motion of fluid particles along their trajectories. To obtain the complete system of dynamic equations, we should add to the Euler system so called Lin constraints [18]

$$\partial_0\boldsymbol{\xi} + (\mathbf{v}\boldsymbol{\nabla})\boldsymbol{\xi} = 0 \tag{1.4}$$

where $\boldsymbol{\xi} = \boldsymbol{\xi}(t, \mathbf{x}) = \{\xi_1, \xi_2, \xi_3\}$ are three independent integrals of dynamic equations

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(t, \mathbf{x}),$$

describing motion of fluid particles along their trajectories.

Seven equations (1.2) – (1.4) form the complete system of dynamic equations, whereas four Euler equations (1.2), (1.3) form only a closed subsystem of the complete system of dynamic equations. The wave function is expressed via hydrodynamic potentials $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, which are known also as Clebsch potentials [19, 20]. In general case of arbitrary fluid flow in three-dimensional space the complex wave function ψ has two complex components ψ_1, ψ_2 (or three independent real components)

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \sqrt{\rho}e^{i\varphi}u_1(\boldsymbol{\xi}) \\ \sqrt{\rho}e^{i\varphi}u_2(\boldsymbol{\xi}) \end{pmatrix}, \qquad |u_1|^2 + |u_2|^2 = 1 \tag{1.5}$$

It is impossible to obtain general solution of the Euler system (1.2), (1.3), but one can partially integrate the complete system (1.2) – (1.4), reducing its order to four dynamic equations for the wave function (1.5). Practically it means that one integrates dynamic equations (1.4), where the function $\mathbf{v}(t, \mathbf{x})$ is determined implicitly by equations (1.2), (1.3). Such an integration and reduction of the order of the complete system of dynamic equations appears to be possible, because the system (1.2) – (1.4) has the symmetry group, connected with transformations of the Clebsch potentials

$$\xi_\alpha \rightarrow \tilde{\xi}_\alpha = \tilde{\xi}_\alpha(\boldsymbol{\xi}), \qquad \alpha = 1, 2, 3, \qquad \frac{\partial\left(\tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3\right)}{\partial(\xi_1, \xi_2, \xi_3)} \neq 0 \tag{1.6}$$

Being a finite function of Clebsch potentials $\boldsymbol{\xi}$, the wave function contains information on motion of fluid particles along their trajectories. This information cannot be obtained from the Euler equations (1.2), (1.3) without using Lin constraints (1.4).

Dependence on $\boldsymbol{\xi}$ takes place only in the case, when the wave function has more, than one component. In the case of the Schrödinger equation for the spinless particle the wave function has only one component $\psi = \sqrt{\rho}e^{i\varphi}u_1(\boldsymbol{\xi})$, where in virtue of the second equation (1.5) $u_1(\boldsymbol{\xi}) = 1$, and the wave function does not depend on $\boldsymbol{\xi}$. See details in [21, 3].

After this integration it became clear, that the wave function is only a method of the fluid description. Application of the wave function as a fundamental object of quantum mechanics is too restrictive, and its meaning is obscure, if it is determined axiomatically by quantum principles. The meaning of the wave function becomes clear, when it is considered to be a method of the fluid description. Two possible conceptions of the quantum phenomena are described by the following scheme

$$\begin{array}{ccccccc} \text{axiomatic} & \multirow{2}{*}{:} & \text{quantum} & \multirow{2}{*}{$\Rightarrow$} & \text{wave} & \multirow{2}{*}{$\Rightarrow$} & \text{hydrodynamic} \\ \text{conception} & & \text{principles} & & \text{function} & & \text{interpretation} \end{array}$$

$$\begin{array}{ccccccc} \text{model} & : & \text{statistical} & \Rightarrow & \text{fluid} & \Rightarrow & \text{wave} \\ \text{conception} & & \text{description} & & \text{dynamics} & & \text{function} \end{array}$$

Quantum principles are foundation of the conventional quantum theory, and axiomatic properties of the wave function form a foundation of the hydrodynamic interpretation, i.e. the wave function is primary and hydrodynamic interpretation is secondary. In the model description of quantum phenomena the quantum principles are replaced by the statistical description, which is connected closely with the fluid dynamics, and fluid dynamics is a basis for introduction of the wave function, i.e. the fluid dynamics is primary and the wave function is secondary.

Let $\mathcal{S}_{\mathrm{st}}$ be stochastic particle, whose state X is described by variables $\left\{\mathbf{x}, \frac{d\mathbf{x}}{dt}\right\}$, where $\mathbf{x}$ is the particle position. Evolution of the particle state is stochastic and there exist no dynamic equations for $\mathcal{S}_{\mathrm{st}}$. Evolution of the state of $\mathcal{S}_{\mathrm{st}}$ contains both regular and stochastic components. To separate the regular evolution components, we consider a set (statistical ensemble) $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ of many independent identical stochastic particles $\mathcal{S}_{\mathrm{st}}$. All stochastic particles $\mathcal{S}_{\mathrm{st}}$ start from the same initial state. It means that all $\mathcal{S}_{\mathrm{st}}$ are prepared in the same way. If the number N of $\mathcal{S}_{\mathrm{st}}$ is very large, the stochastic elements of evolution compensate each other, but regular ones are accumulated. In the limit $N \to \infty$ the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ turns to a dynamic system, whose state evolves according to some dynamic equations.

The stochastic particle $\mathcal{S}_{\mathrm{st}}$ has six degrees of freedom, whereas the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ of stochastic particles $\mathcal{S}_{\mathrm{st}}$ has infinite number of freedom degrees. $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ is a continuous dynamic system of the type of a fluid (continuous medium). If the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ is normalized to one particle, it turns to the statistical average particle $\langle\mathcal{S}_{\mathrm{st}}\rangle$ (See details in [22]), which is a kind of a fluid. The action for $\langle\mathcal{S}_{\mathrm{st}}\rangle$, or for $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ can be reduced to the form of a continuous system $\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}]$, which consists of many deterministic particles $\mathcal{S}_{\mathrm{d}}$, interacting between themselves. Type of this interaction depends on the character of stochasticity of the particle $\mathcal{S}_{\mathrm{st}}$.

Let the statistical ensemble $\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}]$ of deterministic classical particles $\mathcal{S}_{\mathrm{d}}$ be described by the action $\mathcal{A}_{\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}(P)]}$, where P are parameters describing $\mathcal{S}_{\mathrm{d}}$ (for instance, mass, charge). Let under influence of some stochastic agent the deterministic particle $\mathcal{S}_{\mathrm{d}}$ turn to a stochastic particle $\mathcal{S}_{\mathrm{st}}$. The action $\mathcal{A}_{\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]}$ for the statistical ensemble $\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]$ is reduced to the action $\mathcal{A}_{\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}]} = \mathcal{A}_{\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]}$ for some set $\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}]$ of identical interacting deterministic particles $\mathcal{S}_{\mathrm{d}}$. The action $\mathcal{A}_{\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}]}$ as a functional of $\mathcal{S}_{\mathrm{d}}$ has the form $\mathcal{A}_{\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}(P_{\mathrm{eff}})]}$, where parameters P_{eff} are parameters P of the deterministic particle $\mathcal{S}_{\mathrm{d}}$, averaged over the statistical ensemble, and this averaging describes interaction of particles $\mathcal{S}_{\mathrm{d}}$ in the set $\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}]$. It means that

$$\mathcal{A}_{\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]} = \mathcal{A}_{\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}(P)]} = \mathcal{A}_{\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}(P_{\mathrm{eff}})]} \tag{1.7}$$

In other words, stochasticity of particles $\mathcal{S}_{\mathrm{st}}$ in the ensemble $\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]$ is replaced by interaction of $\mathcal{S}_{\mathrm{d}}$ in $\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}]$, and this interaction is described by a change

$$P \to P_{\mathrm{eff}} \tag{1.8}$$

in the action $\mathcal{A}_{\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}(P)]}$.

How to determine the change (1.8)? At this stage of investigation the change (1.8) is phenomenological. In general, the form of the change (1.8) is determined by the properties (character) of stochasticity. But we do not know how to describe the type of stochasticity and their properties. Then the form of change (1.8) labels the type of stochasticity, and the change (1.8) may be considered to be a method of the stochasticity description. Before the further discussion let us consider an example of quantum stochasticity.

Let $\mathcal{S}_{\mathrm{d}}$ be a free deterministic nonrelativistic particle of the mass m. The action for $\mathcal{S}_{\mathrm{d}}$ has the form

$$\mathcal{S}_{\mathrm{d}}: \qquad \mathcal{A}[\mathbf{x}] = \int L\left(\mathbf{x}, \frac{d\mathbf{x}}{dt}\right) dt, \tag{1.9}$$

$$L\left(\mathbf{x}, \frac{d\mathbf{x}}{dt}\right) = -mc^2 + \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 \tag{1.10}$$

where $\mathbf{x} = \mathbf{x}(t)$ and c is the speed of the light.

The action for the ensemble $\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}]$ of free deterministic particles $\mathcal{S}_{\mathrm{d}}$ has the form

$$\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}]: \qquad \mathcal{A}_{\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}(m)]}[\mathbf{x}] = \int L\left(\mathbf{x}, \frac{d\mathbf{x}}{dt}\right) dt d\boldsymbol{\xi}, \tag{1.11}$$

where the Lagrangian function has the same form (1.10), but $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$, $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$. Here variables $\boldsymbol{\xi}$ label the particles $\mathcal{S}_{\mathrm{d}}$ of the statistical ensemble $\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}]$. The action (1.11) describes some fluid without pressure.

In the case of a free noncharged particle the only parameter of the particle is its mass m. At the quantum stochasticity the change (1.8) has the form

$$m \to m_{\mathrm{eff}} = m\left(1 - \frac{\mathbf{u}^2}{2c^2} + \frac{\hbar}{2mc^2}\boldsymbol{\nabla}\mathbf{u}\right) \tag{1.12}$$

where $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$ is the mean value of the stochastic velocity component. Quantum constant $\hbar$ appears here as a coupling constant, describing connection between the regular and stochastic components of particle motion. The velocity $\mathbf{u}$ is supposed to be small with respect to c. Then we must make the change (1.12) only in the first term of (1.10), because the same change in the second term give the quantity of the order $O\left(c^{-2}\right)$. After change (1.12) the action (1.11) turns to the action

$$\mathcal{E}_{\text{st}}\left[\mathcal{S}_{\text{st}}\right]: \qquad \mathcal{A}_{\mathcal{E}_{\text{d}}\left[\mathcal{S}_{\text{d}}\left(m_{\text{eff}}\right)\right]}\left[\mathbf{x},\mathbf{u}\right]=\int L\left(\mathbf{x},\frac{d\mathbf{x}}{dt}\right)+L_{\text{st}}\left(\mathbf{u},\boldsymbol{\nabla}\mathbf{u}\right)dtd\boldsymbol{\xi}, \tag{1.13}$$

$$L_{\text{st}}\left(\mathbf{u},\boldsymbol{\nabla}\mathbf{u}\right)=\frac{m}{2}\mathbf{u}^{2}-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u} \tag{1.14}$$

where $\mathbf{x}=\mathbf{x}\left(t,\boldsymbol{\xi}\right)$, $\boldsymbol{\xi}=\left\{\xi_1,\xi_2,\xi_3\right\}$, but $\mathbf{u}=\mathbf{u}\left(t,\mathbf{x}\right)$ is a function of $t,\mathbf{x}$. The action (1.13) also describes a fluid, but now it is a fluid with a pressure and its irrotational flow is described by the Schrödinger equation [22, 23].

The action (1.13) admits a simple interpretation. The derivative $d\mathbf{x}/dt$ describes the mean velocity of a fluid particle which does not coincide with the velocity of stochastic particle $\mathcal{S}_{\text{st}}$, as well as the velocity of a gas particle does not coincide with the velocity of a gas molecule. The gas molecule has stochastic velocity component. Likewise, the stochastic particle $\mathcal{S}_{\text{st}}$ has the stochastic velocity component. The mean value of this stochastic component is described by the variable $\mathbf{u}$. For a stochastic Brownian particle the mean value of the stochastic component velocity is described by the relation

$$\mathbf{u}_{\text{Br}}=-D\boldsymbol{\nabla}\ln\rho_{\text{Br}} \tag{1.15}$$

where D is the diffusion coefficient, and ρ_{Br} is the density of randomly wandering Brownian particles. Variation of action (1.13) with respect to $\mathbf{u}$ gives the same result

$$\mathbf{u}=-\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho, \qquad \rho=\frac{\partial\left(\xi_1,\xi_2,\xi_3\right)}{\partial\left(x^1,x^2,x^3\right)} \tag{1.16}$$

where $\hbar/2m$ may be regarded as the diffusion coefficient.

To obtain the relation (1.16), we need to rewrite (1.13) in the form of integral over variables $\mathbf{x}$, because $\mathbf{u}$ is a function of $\mathbf{x}$ (but not of $\boldsymbol{\xi}$).

$$\int L_{\text{st}}\left(\mathbf{u},\boldsymbol{\nabla}\mathbf{u}\right)dtd\boldsymbol{\xi}=\int\left(\frac{m\mathbf{u}^2}{2}-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right)\rho dtd\mathbf{x}, \qquad \rho=\frac{\partial\left(\xi_1,\xi_2,\xi_3\right)}{\partial\left(x^1,x^2,x^3\right)} \tag{1.17}$$

Then variation of (1.17) with respect to $\mathbf{u}$ gives the relation (1.16). The nature of stochastic velocities $\mathbf{u}$ and $\mathbf{u}_{\text{Br}}$ is similar. Both velocities $\mathbf{u}$ and $\mathbf{u}_{\text{Br}}$ are a result of random wandering. However, dynamic equations are different, because dissipative Brownian particle obeys the Aristotelian dynamics, whereas conservative quantum particle $\mathcal{S}_{\text{st}}$ obeys the Newtonian dynamics. The state of the statistical average Brownian particle $\left\langle\mathcal{S}_{\text{Br}}\right\rangle$ is described by the variable ρ_{Br}. According to (1.15) the velocity $\mathbf{u}_{\text{Br}}$ is a function of the state ρ_{Br}, and dynamic equation for the state ρ_{Br} has the form of the continuity equation

$$\frac{d\rho_{\text{Br}}}{\partial t}+\boldsymbol{\nabla}\left(\rho_{\text{Br}}\mathbf{u}_{\text{Br}}\right)=0$$

The state of the statistical average particle $\langle\mathcal{S}_{\rm st}\rangle$ is described by the variables ρ, $\frac{d\mathbf{x}}{dt}$, $\mathbf{u}$, and the energy $m\mathbf{u}^2/2$ of the stochastic component must be taken into account in the Lagrangian function.

The first term $\frac{m}{2}\mathbf{u}^2$ of $L_{\rm st}$ is the kinetic energy of the stochastic component. But this term does not contain time derivative. It depends only on $\mathbf{x}$. It means that it acts on the particle motion as a potential energy. Indeed, variation of (1.13) with respect to $\mathbf{x}$ gives dynamic equation

$$m\frac{d^2\mathbf{x}}{dt^2}=-\boldsymbol{\nabla}U\left(\mathbf{x}\right),\qquad U\left(\mathbf{x}\right)=\left(\frac{m\mathbf{u}^2}{2}-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right)$$

The second term in (1.17) ensures connection between $\mathbf{u}$ and collective variable ρ.

For application to relativistic particle the relation (1.12) should be rewritten in the form

$$m^2\rightarrow m_{\rm eff}^2=m^2\left(1-\frac{\mathbf{u}^2}{c^2}+\frac{\hbar}{mc^2}\boldsymbol{\nabla}\mathbf{u}\right)\qquad(1.18)$$

Besides we should replace in addition nonrelativistic expressions $\mathbf{u}^2$ and $\boldsymbol{\nabla}\mathbf{u}$ by relativistic ones $-g_{ik}u^iu^k$ and ∂_iu^i respectively, where u^i is the 4-vector, describing the mean value of the stochastic component of velocity. Then the action (1.10), (1.13), (1.14) transforms to the relativistic form

$$\mathcal{A}\left[x,\kappa\right]=-\int mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k}d\xi_0d\boldsymbol{\xi},\qquad K=\sqrt{1+\frac{\hbar^2}{m^2c^2}\left(\kappa^l\kappa_l+\partial_l\kappa^l\right)}\qquad(1.19)$$

where $x=\{x^i\}$, $i=0,1,2,3$ is a function of variables $\xi=\{\xi_0,\boldsymbol{\xi}\}=\{\xi_i\}$, $i=0,1,2,3$ $\dot{x}^i\equiv dx^i/d\xi_0$. Variables $\kappa=\{\kappa^i\}$, $i=0,1,2,3$ are functions of x. The variables κ^l are connected with the variables u^l by means of relation

$$u^l=-\frac{\hbar}{m}\kappa^l,\qquad l=0,1,2,3\qquad(1.20)$$

As well as the nonrelativistic velocity $\mathbf{u}$, the vector field $\kappa_l=g_{lk}\kappa^k$ has a potential κ and can be represented in the form

$$\kappa_l=\partial_l\kappa=\frac{1}{2}\partial_l\ln\rho,\qquad(1.21)$$

where

$$\rho=\frac{J\sqrt{\dot{x}_s\dot{x}^s}}{\rho_0mcK}=\frac{\sqrt{j_sj^s}}{\rho_0mcK},\qquad j^k=J\dot{x}^k\equiv\frac{\partial J}{\partial\xi_{0,k}}\equiv\frac{\partial\left(x^k,\xi_1,\xi_2,\xi_3\right)}{\partial\left(x^0,x^1,x^2,x^3\right)},\qquad(1.22)$$

$$J=\frac{\partial\left(\xi_0,\xi_1,\xi_2,\xi_3\right)}{\partial\left(x^0,x^1,x^2,x^3\right)}=\det\left\|\xi_{i,k}\right\|,\qquad\xi_{i,k}=\partial_k\xi_i,\qquad i,k=0,1,2,3\qquad(1.23)$$

$\rho_0=$const.

Result (1.21) is obtained by variation of (1.19) with respect to κ^l in the same way, as equation (1.16) was derived from (1.13).

The action (1.19) describes some relativistic fluid, whose particles interact via the self-consistent vector field κ^l. Properties of this fluid were investigated in [24]. It has been shown that irrotational flow of this fluid is described by the Klein-Gordon equation. Deterministic relativistic particles $\mathcal{S}_{\mathrm{d}}$ of the set (fluid) $\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}]$ interact between themselves via self-consistent vector field κ^l, $i = 0, 1, 2, 3$.

We shall refer to the field κ^l and its potential κ, defined by the relation (1.20) as κ-field. We shall show that the κ-field has some unusual properties. In particular, the κ-field can produce pairs.

2 The field producing pairs

To show that the κ-field enables to produce pairs, we shall consider the expression $\frac{\hbar^2}{m^2c^2}\left(\kappa^l\kappa_l + \partial_l\kappa^l\right)$ constructed of the κ-field as some given external field f. We consider motion of a charged deterministic particle in the given electromagnetic field A_i and some given scalar field f, changing the particle mass. The action for the particle has the form of the type (1.19)

$$\mathcal{A}[q] = \int L(q, \dot{q})\, d\tau, \qquad L = -\sqrt{m^2c^2\left(1 + f(q)\right) g_{ik}\dot{q}^i\dot{q}^k} - \frac{e}{c}A_k\dot{q}^k \tag{2.1}$$

where relations $x^i = q^i(\tau)$, $i = 0, 1, 2, 3$ describe the world line of the particle, and $\dot{q}^k \equiv dq^i/d\tau$. The quantities $A_k = A_k(q)$, $k = 0, 1, 2, 3$ are given electromagnetic potentials, and $f = f(q)$ is some given field, changing the effective particle mass $m_{\mathrm{eff}} = m\sqrt{(1 + f(q))}$. The canonical momentum p_k is defined by the relation

$$p_k = \frac{\partial L}{\partial \dot{q}^k} = -\frac{mcKg_{ki}\dot{q}^i}{\sqrt{g_{ls}\dot{q}^l\dot{q}^s}} - \frac{e}{c}A_k, \qquad K = \sqrt{(1 + f(q))} \tag{2.2}$$

Dynamic equations have the form

$$\frac{dp_k}{d\tau} = -mc\sqrt{g_{ik}\dot{q}^i\dot{q}^k}\frac{\partial K}{\partial q^k} - \frac{e}{c}\frac{\partial A_i}{\partial q^k}\dot{q}^i \tag{2.3}$$

The action (2.1), as well as dynamic equations (2.3), (2.2) are invariant with respect to a transformation of the world line parametrization

$$\tau \to \tilde{\tau} = \tilde{\tau}(\tau) \tag{2.4}$$

provided $\partial\tilde{\tau}/\partial\tau > 0$. If $\partial\tilde{\tau}/\partial\tau < 0$ the dynamic equations (2.3), (2.2) stop to be invariant with respect to transformation (2.4) of the world line parametrization. The parametrization of the world line is a method of the world line description, and correctly written dynamic equations have not to depend on the method of description.

Let us modify the action (2.1), introducing orientation $\boldsymbol{\varepsilon}$ of the world line. The orientation of the world line is a unit vector $\boldsymbol{\varepsilon}$, tangent to the world line. Component ε of orientation $\boldsymbol{\varepsilon}$ is the projection of the vector $\boldsymbol{\varepsilon}$ onto the vector $dq^i/d\tau$

$$\varepsilon = \frac{g_{ik}\varepsilon^i \dot{q}^k}{\sqrt{g_{ls}\dot{q}^l \dot{q}^s}} = \pm 1 \qquad (2.5)$$

At the parametrization transformation (2.4) the component ε of orientation $\boldsymbol{\varepsilon}$ transforms as follows

$$\varepsilon \to \tilde{\varepsilon} = \varepsilon \mathrm{sgn}\left(\partial\tilde{\tau}/\partial\tau\right), \qquad \mathrm{sgn}\left(x\right) = \frac{x}{|x|} \qquad (2.6)$$

Component ε of orientation $\boldsymbol{\varepsilon}$ is invariant with respect to coordinate transformation as it follows from (2.5). Let us rewrite the relations (2.1) – (2.3) in the form, which is invariant with respect to the arbitrary transformation (2.4)

$$\mathcal{A}\left[q\right] = \int\limits_{\min(\tau',\tau'')}^{\max(\tau',\tau'')} L\left(q,\dot{q}\right) d\tau, \qquad L = -\sqrt{m^2c^2\left(1+f\left(q\right)\right) g_{ik}\dot{q}^i\dot{q}^k} - \frac{\varepsilon e}{c} A_k \dot{q}^k \qquad (2.7)$$

where τ' and τ'' are values of the parameter τ at the ends of the integration interval. The particle momentum is defined

$$p_k = \frac{\partial L}{\partial \dot{q}^k} = -\frac{mcKg_{ki}\dot{q}^i}{\sqrt{g_{ls}\dot{q}^l\dot{q}^s}} - \frac{\varepsilon e}{c}A_k, \qquad K = \sqrt{\left(1+f\left(q\right)\right)} \qquad (2.8)$$

$$\frac{dp_k}{d\tau} = -mc\sqrt{g_{ik}\dot{q}^i\dot{q}^k}\frac{\partial K}{\partial q^k} - \frac{\varepsilon e}{c}\frac{\partial A_i}{\partial q^k}\dot{q}^i \qquad (2.9)$$

Note that now the momentum (2.8) and dynamic equations (2.9) are invariant with respect to arbitrary transformation (2.4). One can see from (2.8), that the vector

$$\dot{q}_k = \sqrt{\frac{g_{ls}\dot{q}^l\dot{q}^s}{1+f\left(q\right)}}\frac{\left(p_k + \frac{\varepsilon e}{c}A_k\right)}{mc} \qquad (2.10)$$

becomes to be spacelike $\left(g_{ls}\dot{q}^l\dot{q}^s < 0\right)$, if $f\left(q\right) < -1$, because only in this case the expression under radical in (2.10) is real.

The Jacobi-Hamilton equation for the action (2.7) has the form

$$g^{ik}\left(\frac{\partial S}{\partial q^i} + \frac{\varepsilon e}{c}A_i\right)\left(\frac{\partial S}{\partial q^k} + \frac{\varepsilon e}{c}A_k\right) = m^2c^2\left(1+f\left(q\right)\right) \qquad (2.11)$$

Let us consider solution of the Hamilton-Jacobi equation for the two-dimensional space-time (t,x), when $A_i = 0$, and $f = f\left(t\right)$ is a function of only time t. In this case the solution of equation (2.11) has the form

$$S\left(t,x,p_0\right) = p_0 x + \int\limits_0^t c\sqrt{m^2c^2\left(1+f\left(t\right)\right) + p_0^2}dt + C, \qquad p_0, C = \mathrm{const} \qquad (2.12)$$

and solution of dynamic equations takes the form

$$\frac{\partial S(t,x,p_0)}{\partial p_0}-x_0=x-x_0+\int\limits_0^t\frac{p_0cdt}{\sqrt{m^2c^2(1+f(t))+p_0^2}}=0,\qquad x_0=\mathrm{const}\quad(2.13)$$

Let for example

$$f(t)=\begin{cases}0 & \text{if}\quad t<0\\ -\frac{V^2}{m^2c^4t_0^2}t(t-t_0) & \text{if}\quad 0<t<t_0\\ 0 & \text{if}\quad t_0<t\end{cases},\qquad t_0,V=\mathrm{const}\qquad(2.14)$$

The solution (2.13) takes the form

$$x=\begin{cases}x_0-\frac{p_0c^2}{E}t & \text{if}\quad t<0\\ x_0-\int\limits_0^t\frac{p_0cdt}{\sqrt{E^2-V^2t(t-t_0)/t_0^2}} & \text{if}\quad 0<t<t_0\\ x_1+\alpha\frac{p_0c^2}{E}(t-t_0) & \text{if}\quad t_0<t\end{cases},\qquad E=c\sqrt{m^2c^2+p_0^2}\qquad(2.15)$$

where $\alpha=\pm1$. Sign of α and the constant x_1 are determined from the continuity condition of the world line at $t=t_0$. The solution (2.15) has different form, depending on the sign of the constant $4E^2-V^2$.

If $4E^2>V^2$, the solution (2.15) takes the form

$$x=\begin{cases}x_0-\frac{p_0c^2}{E}t & \text{if } t<0\\ x_0-\frac{p_0c^2t_0}{V}\arcsin\frac{2V\left(\sqrt{E^2t_0^2-V^2t(t-t_0)}-E(t_0-2t)\right)}{t_0(4E^2+V^2)} & \text{if } 0<t<t_0\\ x_0-p_0c^2\frac{t_0}{V}\arcsin\frac{4EV}{4E^2+V^2}-\frac{p_0c^2}{E}(t-t_0) & \text{if } t_0<t\end{cases},\qquad E^2>V^2/4$$

(2.16)

In the case, when $4E^2<V^2$, the world line is reflected from the region Ω_{fb} of the space-time determined by the condition $0<t<t_0$, and the coordinate x is not a single-valued function of the time t. In this case we use parametric representation for the solution (2.15). We have

$$x=\begin{cases}x_0-\frac{p_0c^2t_0}{2E}(1-A\cosh\tau) & \text{if}\quad \tau<-\tau_0\\ x_0-\frac{p_0c^2t_0}{V}(\tau+\tau_0) & \text{if}\quad -\tau_0<\tau<\tau_0\\ x_0-\frac{2p_0t_0}{V}\tau_0+\frac{p_0c^2t_0}{2E}(1-A\cosh\tau) & \text{if}\quad \tau_0<\tau\end{cases}\qquad(2.17)$$

$$t=\frac{t_0}{2}(1-A\cosh\tau)\qquad(2.18)$$

where

$$A=\sqrt{1-\frac{4E^2}{V^2}},\qquad \tau_0=\operatorname{arccosh}\frac{1}{A}=\operatorname{arccosh}\frac{1}{\sqrt{1-\frac{4E^2}{V^2}}}\qquad(2.19)$$

The solution (2.17), (2.18) describes annihilation of particle and antiparticle with the energy $E < V/2$ in the region $0 < t < t_0$. Solution, describing the particle-antiparticle generation, has the form

$$x = \begin{cases} x_0 - \frac{p_0 c^2 t_0}{2E}\left(A\cosh\tau - 1\right) & \text{if} \quad \tau < -\tau_0 \\ x_0 + \frac{p_0 c^2 t_0}{V}\left(\tau + \tau_0\right) & \text{if} \quad -\tau_0 < \tau < \tau_0 \\ x_0 + \frac{2p_0 t_0}{V}\tau_0 + \frac{p_0 c^2 t_0}{2E}\left(A\cosh\tau - 1\right) & \text{if} \quad \tau_0 < \tau \end{cases} \tag{2.20}$$

$$t = \frac{t_0}{2}\left(A\cosh\tau - 1\right) \tag{2.21}$$

where parameters A, τ_0 are defined by the relation (2.19), and the relation $2E < V$ takes place.

In both cases (2.17) and (2.20) at $|t| \to \infty$ the world line has two branches, which can be approximated by the relations

$$x = x_0 + vt_1 \pm v\left(t - t_1\right), \qquad t_1 = t_0 \frac{E}{V} \tag{2.22}$$

where $v = -\frac{p_0 c^2}{E}$ is the particle velocity, and $v = \frac{p_0 c^2}{E}$ is the antiparticle velocity.

Thus, the region Ω_{fb} with the κ-field reflects world lines of particles with the energy $E < V/2$. If the particle energy $E > V/2$, the particle passes the region Ω_{fb} through. The world line penetrates into forbidden region Ω_{fb} the deeper the larger is its energy.

3 Connection between energy-momentum and canonical momentum

It is common practice to think that in the absence of electromagnetic field the canonical momentum (2.8) coincides with the particle energy-momentum P_i. Then the particle energy may be negative, as far as the canonical momentum component p_0 may be positive and negative. Existence of negative particle energy is very undesirable, because it leads to many very strange corollaries.

In reality, the energy-momentum P_i, $i = 0, 1, 2, 3$ and canonical momentum p_i, $i = 0, 1, 2, 3$ are different quantities which can coincide only in the case, when there are only particles, or only antiparticles. They are defined differently. The canonical momentum p_i is defined as the quantity canonically conjugate to coordinate x^i of the particle position by means of the relation (2.8). The energy-momentum P_i is defined by the relation

$$P_i = \int\limits_V T_i^0 d\mathbf{x}, \qquad i = 0, 1, 2, 3 \tag{3.1}$$

where $T_i^0 = g_{ik}T^{k0}$ are components of the energy-momentum tensor, and V is the 3-volume around the particle, whose energy-momentum is determined. The energy-momentum tensor T^{ik} is defined as a source of the gravitational field, i.e. by means of the relation

$$T^{ik}(x) = \frac{\delta \mathcal{A}}{\delta g_{ik(x)}} = -\frac{2}{\sqrt{-g}}\frac{\partial}{\partial g_{ik}(x)}\left(\sqrt{-g}\mathcal{L}\right) \tag{3.2}$$

where $\mathcal{A}$ is the action (2.7), written in the arbitrary curvilinear coordinate system in the form

$$\mathcal{A}\left[q^i, g_{ik}, A_i\right] = \int \mathcal{L}\left(q^i, g_{ik}, A_i\right)\sqrt{-g}d^4x \tag{3.3}$$

and $g = \det\|g_{ik}\|$.

In the same way the electric charge of a particle is defined by the relation

$$Q = \int\limits_V j_{\mathrm{c}}^0 d\mathbf{x}, \tag{3.4}$$

where j_{c}^i, $i = 0,1,2,3$ is electric current density, defined as the source of the electromagnetic field, i.e. by the relation

$$j_{\mathrm{c}}^i(x) = c\frac{\delta \mathcal{A}}{\delta A_i(x)} = \frac{c}{\sqrt{-g}}\frac{\partial}{\partial A_i(x)}\left(\sqrt{-g}\mathcal{L}\right) \tag{3.5}$$

To obtain expressions for the quantities j_{c}^l, T^{ik}, let us write the action (2.7) in the arbitrary curvilinear coordinate system and represent it in the form (3.3). One obtains

$$\mathcal{A} = \int d^4x \int\limits_{\min(\tau',\tau'')}^{\max(\tau',\tau'')} L(q,\dot{q})\,\delta^{(4)}(x - q(\tau))\,d\tau, \tag{3.6}$$

where

$$\delta^{(4)}(x - q(\tau)) = \delta\left(x^0 - q^0(\tau)\right)\delta(\mathbf{x} - \mathbf{q}(\tau)) = \prod_{i=0}^{i=3}\delta\left(x^i - q^i(\tau)\right) \tag{3.7}$$

Integrating (3.6) over τ and using δ-function $\delta(x^0 - q^0(\tau))$, one obtains

$$\mathcal{A} = \int d^4x \sum_j \frac{L(q(\tau_j), \dot{q}(\tau_j))}{|\dot{q}^0(\tau_j)|}\delta(\mathbf{x} - \mathbf{q}(\tau_j)), \tag{3.8}$$

where $\tau_j = \tau_j(x^0)$ are roots of the equation

$$x^0 - q^0(\tau_j) = 0 \tag{3.9}$$

Comparing relations (3.8) and (3.3), we conclude

$$\begin{aligned}
\mathcal{L}\sqrt{-g} &= \sum_j \frac{L(q(\tau_j), \dot{q}(\tau_j))}{|\dot{q}^0(\tau_j)|}\delta(\mathbf{x} - \mathbf{q}(\tau_j)) \\
&= \sum_j \left(-\sqrt{m^2c^2(1 + f(q(\tau_j)))\,g_{ik}\dot{q}^i(\tau_j)\,\dot{q}^k(\tau_j)} - \frac{\varepsilon e}{c}A_k\dot{q}^k(\tau_j)\right) \\
&\quad \times \frac{\delta(\mathbf{x} - \mathbf{q}(\tau_j))}{|\dot{q}^0(\tau_j)|}
\end{aligned} \tag{3.10}$$

Now let us substitute relation (3.10) in the relations (3.2), (3.5) and set $A_i = 0$, $g_{ik} =$diag$\{c^2, -1, -1, -1\}$. After calculations we obtain relations for the energy-momentum tensor and the current density

$$T^{ik}\left(x^0, \mathbf{x}\right) = \sum_j \frac{mc\sqrt{1 + f\left(q\left(\tau_j\right)\right)}\dot{q}^i\left(\tau_j\right)\dot{q}^k\left(\tau_j\right)}{c\sqrt{g_{ls}\dot{q}^l\left(\tau_j\right)\dot{q}^s\left(\tau_j\right)}} \frac{\delta\left(\mathbf{x} - \mathbf{q}\left(\tau_j\right)\right)}{\left|\dot{q}^0\left(\tau_j\right)\right|} \tag{3.11}$$

$$j_{\mathrm{c}}^k = -\sum_j \varepsilon e \dot{q}^k\left(\tau_j\right) \frac{\delta\left(\mathbf{x} - \mathbf{q}\left(\tau_j\right)\right)}{\left|\dot{q}^0\left(\tau_j\right)\right|} \tag{3.12}$$

where $\tau_j = \tau_j\left(x^0\right)$. Substituting relations (3.11), (3.12) in relations (3.1), (3.4), we obtain

$$\begin{aligned} P_i &= \sum_j \frac{mc\sqrt{1 + f\left(q\left(\tau_j\right)\right)}\dot{q}_i\left(\tau_j\right)\dot{q}^0\left(\tau_j\right)}{\left|\dot{q}^0\left(\tau_j\right)\right|\sqrt{g_{ls}\dot{q}^l\left(\tau_j\right)\dot{q}^s\left(\tau_j\right)}} \\ &= -\sum_j \frac{p_i\left(\tau_j\right)\dot{q}^0\left(\tau_j\right)}{\left|\dot{q}^0\left(\tau_j\right)\right|} = -\sum_j p_i\left(\tau_j\right)\operatorname{sgn}\left(\dot{q}^0\left(\tau_j\right)\right) \\ &= \sum_j p_i\left(\tau_j\right)\operatorname{sgn}\left(p_0\left(\tau_j\right)\right), \qquad i = 0, 1, 2, 3 \end{aligned} \tag{3.13}$$

$$Q = -\sum_j \varepsilon e \cdot \operatorname{sgn}\left(\dot{q}^0\left(\tau_j\right)\right) = \sum_j \varepsilon e \cdot \operatorname{sgn}\left(p_0\left(\tau_j\right)\right) \tag{3.14}$$

In particular, we obtain for the energy $E = P_0$

$$P_0 = E = \sum_j p_0\left(\tau_j\right)\operatorname{sgn}\left(p_0\left(\tau_j\right)\right) = \sum_j \left|p_0\left(\tau_j\right)\right| \tag{3.15}$$

In the case, when $f(q) + 1 < 0$ in some regions of the space-time, the equation (3.9) may have several roots. Each of roots corresponds to some intersection of the world line with the plane $t =$const. Any of such intersections describes either particle, when sgn$(\dot{q}^0) = -$sgn$(p_0) > 0$, or antiparticle, when sgn$(\dot{q}^0) = -$sgn$(p_0) < 0$. According to (3.14) the electric charge Q_{p} of a particle is equal to $-\varepsilon e$, whereas the electric charge Q_{a} of an antiparticle is equal to εe. Thus, the particle and antiparticle have electric charges of different signs, because the sign of p_0 is different for particle and antiparticle. According to (3.15) the energy is positive for a particle as well as for an antiparticle, although the sign of the temporal component p_0 of the canonical momentum is different for particle and antiparticle.

Such a situation seems to be very reasonable. It can be interpreted as follows. Let the term "SWL" (section of world line) mean the collective concept with respect to concepts "particle" and "antiparticle". The particle and antiparticle distinguish in the sign of the temporal component p_0 of the canonical momentum and in the sign of the temporal component $-K\dot{q}^0\left(g_{ls}\dot{q}^l\dot{q}^s\right)^{-1/2}$ of the velocity 4-vector. Flux of the world lines through some spacelike three-dimensional area dS_i for particles and the flux of the world lines through the same area dS_i for antiparticles have different signs. This fact may be interpreted in the sense that a particle is one SWL, whereas an antiparticle is minus one SWL. If SWL has the canonical momentum p_i, then the number n of SWLs is $n = -\mathrm{sgn}(p_0)$. The relations (3.13) – (3.15) can be written in the form

$$Q = -\sum_j \varepsilon e n(\tau_j), \qquad E = -\sum_j p_0(\tau_j) n(\tau_j) \tag{3.16}$$

$$P_i = -\sum_j p_i(\tau_j) n(\tau_j), \qquad i = 0, 1, 2, 3, \qquad n = -\mathrm{sgn}(p_0) \tag{3.17}$$

This is reasonable formulas for additive quantities, when, the additive quantity a of several objects (SWLs) is a sum of additive quantities a of each object (SWL) multiplied by the number n of objects (SWLs). It is unusual only that the number n of objects (SWLs) may be negative.

4 Peculiarities of description in terms of world lines

From viewpoint of the consistent relativity theory a world line is a real physical object, whereas a particle and an antiparticle (SWLs) are attributes of the world line, which appear as a result of intersection of the world line with the plane t =const. We shall use a special term 'WL' for the world line, considered to be a real physical object. In the nonrelativistic theory a real physical object is a particle (or antiparticle), whereas the world line is an attribute of a particle (its history).

Particle and antiparticle in the nonrelativistic theory (or in inconsistent relativistic theory) are two different objects, having different parameters. The particle is described by the following parameters $\{m, e\}$, where m is a mass, and e is a charge constant. The antiparticle is described by the parameters $\{m, -e\}$. Particle and antiparticle are described by different dynamic systems, because these systems have different parameters. Note that at such a description the component ε of orientation $\boldsymbol{\varepsilon}$ is not a parameter of dynamic system. It is a method of description of SWL, because ε changes at the change $\tau \rightarrow -\tau$ of the world line parametrization. At nonrelativistic approach one cannot describe pair production classically, because one cannot connect fragments of world lines of two different objects (particle and antiparticle). In the framework of classical theory consideration of a particle and an antiparticle as two different objects is incompatible. In the quantum theory, where the particle world line may not exist, it is possible to introduce special operators, describing production and annihilation of a particle (antiparticle). In quantum theory pair generation is a corollary of dynamic equations, and one can imagine such a situation (at proper choice of dynamic equations), when particles and antiparticles are generated separately. In the classical theory the pair generation of particles and antiparticles is described on the deeper (descriptive) level. Choice of dynamic equations cannot change anything.

Nevertheless, let us start from the consistent relativistic theory, where SWLs are attributes of WL, and imagine that, describing motion of SWLs classically, we divide the world line $\mathcal{L}$ into fragments $\mathcal{L}_i$, $i = 1, 2$...in such a way, that each fragment $\mathcal{L}_i$ describe either a particle, or an antiparticle. To make this, it is sufficient to make the following transformation for fragments $\mathcal{L}_i$, describing antiparticle.

$$e \to -e, \qquad \tau \to -\tau, \qquad \varepsilon \to \varepsilon \tag{4.1}$$

According to (3.13), (3.14) this transformation changes neither energy-momentum vector P_i, nor the charge Q of SWL, but transformation (4.1) changes the sign of the canonical momentum for antiparticles and does not change it for particles. As a result the temporal component p_0 of particles and antiparticles becomes negative, and one can identify the energy E of particles and antiparticles with the time component $-p_0$ of their canonical momentum, i.e. $E = -p_0$. Conventionally such an identification is always produced, to satisfy so called causality principle. Temporal component $-p_0$ of canonical momentum is considered to be an energy of both particle and antiparticle. Using such an identification, it is always supposed that $dt/d\tau > 0$, and one may ignore concept of orientation.

The action (2.7) and dynamic equations (2.9) are invariant with respect to transformation (4.1), whereas the canonical momentum (2.8) changes its sign. As a result all fragments $\mathcal{L}_i$ of $\mathcal{L}$ have the same sgn(p_0), and $E = -p_0$ along the whole $\mathcal{L}$. At the same time different fragments $\mathcal{L}_i$ of $\mathcal{L}$ (particles and antiparticles) are described as different dynamic systems with parameters $\{m, e\}$ and $\{m, -e\}$ respectively.

Then interval between any two points P and P', belonging to the same fragment $\mathcal{L}_i$, is timelike (we neglect the short spacelike segments of $\mathcal{L}_i$ near its ends). If interval between two points P_1 and P_2 is spacelike, then $P_1 \in \mathcal{L}_i$ and $P_2 \in \mathcal{L}_k$, $i \neq k$. The points belong to different dynamic systems and the Poisson brackets between dynamic variables at the points P_1 and P_2 vanish

$$\{u(P_1), v(P_2)\} = 0, \qquad \left(x_{(1)}^i - x_{(2)}^i\right)\left(x_{(1)i} - x_{(2)i}\right) < 0 \tag{4.2}$$

where $u(P_1)$ and $v(P_2)$ are any dynamic variables at the point P_1 and at the point P_2 respectively. The quantities $x_{(1)}^i$ and $x_{(2)}^i$ are coordinates of points P_1 and P_2 respectively.

In the quantum field theory, where fragmentation of the total world line $\mathcal{L}$ is used, the relation (4.2) takes the form

$$[u(x_1), v(x_2)]_- = 0, \qquad (x_1 - x_2)^2 < 0 \tag{4.3}$$

where $[...]_-$ means a commutator of two operators, describing dynamic variables u and v at points x_1 and x_2 respectively.

The relation (4.3) is known as the causality principle. Interpretation of this principle is such. Events at the points P_1 and P_2 are independent, because no interaction can propagate faster, than the speed of the light.

If the world line $\mathcal{L}$ is considered to be a whole object (WL), described by dynamic equations (2.8), (2.9) and parameter τ, changing monotonically along $\mathcal{L}$, there are points P_1 and P_2, separated by spacelike interval and belonging to $\mathcal{L}$. Of course, it is possible only, if there are pair production and pair annihilation. For instance, in the example of the second section (see (2.22)) the points with coordinates $P_1 = (t, x)$ and $P_2 = (t, 2(x_0 + vt_1) - x)$ are separated by spacelike interval $2i|x - x_0 - vt_1|$. According to (2.22) they both belong to the same world line. As far as the state (x_2, p_2) of dynamic system at the point P_2 is a function of its state (x_1, p_1) at the point P_1, the relation (4.2) cannot be fulfilled for all functions u and v of dynamic variables. It means that the causality principle in the form (4.2) is violated. Events at points P_1 and P_2 appear to be dependent, when the particle and antiparticle are described as different states of total WL (but not as separate dynamic systems).

Does it mean that the description in terms of WL is incompatible with the causality principle? No, because description in terms of one WL is only a part of the complete description of the particle-antiparticle collision. Indeed, let us imagine that we prepare high energy particle and antiparticle at the points P_1 and P_2, separated by spacelike interval. Let these SWLs move to meet each other. Then there are the following possibilities: (1) the particle and antiparticle annihilate (description in terms of one WL), (2) particle and antiparticle do not collide (description in terms of two WLs), (3) particle and antiparticle collide and generates n pairs additionally (description in terms of $n + 1$ WLs). Thus, the complete description of particle-antiparticle collision is described statistically by means of a sum of k-WL descriptions $k = 1, 2, ...$ The causality principle can be applied only to the complete description.

Description in terms of one WL is a special part of the complete description, and this partial description may contain a correlation between the points, separated by a spacelike interval, because the fact that the particle and antiparticle annihilate supposes some correlation between the states at the points P_1 and P_2, separated by a spacelike interval. This correlation for one-WL description means that not all Poison brackets (4.2) do vanish. This correlation does not mean a violation of the causality principle, because one-WL description does not pretend to a complete description of the particle-antiparticle collision. The conventional description in terms of particles and antiparticles pretend to a complete description, and fulfillment of the causality principle is essential for the complete description.

Understanding that the complete description is a very complicated problem, we divide the problem into parts (one WL, two WLs,...) and try to solve any part separately. Our strategy of investigation reminds the investigation of interaction of nonrelativistic particles, where the number of particles conserves. Then the whole investigation can be separated into one-particle problem, two-particle problem, three-particle problem,...These problems are solved in series. In the relativistic case the number of WLs is conserved, and the whole problem can be separated into parts, corresponding to fixed number of WLs. Some ideas of such an approach one can find in [25], but details need further development. In any case a description of the pair production in terms of one WL does not contradict to the causality principle, because this description does not pretend to a complete description of the collision process.

In the modern relativistic quantum field theory a description is produced in terms of particles and antiparticles (but not in terms of WLs). This statement follows from identification of the energy operator E with the Hamilton operator $H = p_0$ which is defined as an evolution operator. Formally this identification follows from relations

$$i\hbar\partial_k u = [u, P_k]_- , \qquad P_k = \int T_k^0 d\mathbf{x}, \qquad k = 0,1,2,3 \tag{4.4}$$

where u is an operator of arbitrary dynamic variable, $[...]_-$ denotes commutator, and P_k is the energy-momentum operator of the dynamic system. At such a description the causality principle is valid, because description is carried out in terms of particles and antiparticles (but not in terms of WLs).

A difficulty of such an approach is a consideration of indefinite number of non-conserved objects (particles and antiparticles). This leads a perturbative description of the pair production processes.

Conceptual defect of conventional description with a use of identification (4.4) is incompatibility of the relations (4.4) with the dynamic equations. Restrictions (4.4) are compatible with dynamic equations, obtained at the canonical secondary quantization of linear equation

$$\hbar^2\partial_i\partial^i\psi + m^2c^2\psi = 0 \tag{4.5}$$

But they stop to be compatible with dynamic equations, obtained at the canonical secondary quantization of nonlinear equation

$$\hbar^2\partial_i\partial^i\psi + m^2c^2\psi = g\psi^*\psi\psi, \qquad g = \text{const} \tag{4.6}$$

The fact is that the secondary quantization of (4.6) can be produced without imposition of constraints (4.4). Such a quantization is produced in terms of WLs [26]. It does not use the perturbation theory and admits to obtain solution of the scattering problem without cut off at $t \to \pm\infty$. If the secondary quantization can be carried out without imposition of (4.4), one should test whether the relations (4.4) are compatible with dynamic equations. The test shows that the constraints (4.4) are compatible with the dynamic equations only for linear equation (4.5), but they are incompatible with (4.6) [26]. After imposition of constraints (4.4) the problem becomes overdetermined. The overdetermined problem is inconsistent, and one can obtain practically any desired result for such a statement of the problem.

It is quite reasonable that the causality principle (4.3) is not fulfilled at the quantization in terms of WLs, because it is a corollary of the constraints (4.4). Secondary quantization in terms WLs [26] does not describe pair production. It does not describe even scattering of a particle on antiparticle. But at the same time this quantization is consistent and well defined from the formal viewpoint. What does it mean? It means only that the nonlinear self-action term in (4.6) does not describe pair production, because the pair production is apparently a more subtle effect. But why conventionally at quantization [27, 28, 29, 30] of (4.6) one obtains pair production? The answer is simple. Because of overdetermination of the problem, that makes the problem inconsistent. Practically, it looks as follows. Imposition of the constraint (4.6) leads to fragmentation of the total world line into particles and antiparticles. After scattering one needs to produce defragmentation and collect fragments into the total world lines. Process of defragmentation is imperfect, because of inconsistent perturbative description. Reminders of imperfect defragmentation imitate pair production. At the conventional approach any nonlinear term leads to pair production, whereas classical description of the second section shows that the field, producing pairs must have rather specific properties.

Failure of the paper [26] in description of pair production and inconsistency of conventional quantization in terms of particles and antiparticles show that one needs to investigate properties of the κ-field, producing pairs, more carefully, trying to understand what factor is responsible for pair production.

5 Transformation of dynamic equations

Let us add to the action (1.19) the term describing interaction with the electromagnetic field and write it in the form

$$\mathcal{A}[x,\kappa] = \int \left\{-mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k} - \frac{e}{c}A_k\dot{x}^k\right\} d^4\xi, \qquad d^4\xi = d\xi_0 d\boldsymbol{\xi}, \tag{5.1}$$

$$K = \sqrt{1+\lambda^2\left(\kappa_l\kappa^l + \partial_l\kappa^l\right)}, \qquad \lambda = \frac{\hbar}{mc} \tag{5.2}$$

where $x = \{x^i(\xi_0,\boldsymbol{\xi})\}$, $i = 0,1,2,3$ are dependent variables. $\xi = \{\xi_0,\boldsymbol{\xi}\} = \{\xi_k\}$, $k = 0,1,2,3$ are independent variables, and $\dot{x}^i \equiv dx^i/d\xi_0$. The quantities $\kappa^l = \left\{\kappa^l(x)\right\}$, $l = 0,1,2,3$ are dependent variables, describing stochastic component of the particle motion, $A_k = \{A_k(x)\}$, $k = 0,1,2,3$ is the potential of electromagnetic field. We shall refer to the dynamic system, described by the action (5.1), (5.2) as $\mathcal{S}_{\mathrm{KG}}$, because irrotational flow of $\mathcal{S}_{\mathrm{KG}}$ is described by the Klein-Gordon equation [24]. We present here this transformation to the Klein-Gordon form, because we shall use intermediate stages of this transformation for further investigations.

Let us consider variables $\xi = \xi(x)$ in (5.1) as dependent variables and variables x as independent variables. Let the Jacobian

$$J = \frac{\partial(\xi_0,\xi_1,\xi_2,\xi_3)}{\partial(x^0,x^1,x^2,x^3)} = \det\|\xi_{i,k}\|, \qquad \xi_{i,k} \equiv \partial_k\xi_i, \qquad i,k = 0,1,2,3 \tag{5.3}$$

be considered to be a multilinear function of $\xi_{i,k}$. Then

$$d^4\xi = Jd^4x, \qquad \dot{x}^i \equiv \frac{dx^i}{d\xi_0} \equiv \frac{\partial(x^i,\xi_1,\xi_2,\xi_3)}{\partial(\xi_0,\xi_1,\xi_2,\xi_3)} = J^{-1}\frac{\partial J}{\partial\xi_{0,i}} \tag{5.4}$$

After transformation to dependent variables ξ the action (5.1) takes the form

$$\mathcal{A}[\xi,\kappa] = \int \left\{-mcK\sqrt{g_{ik}\frac{\partial J}{\partial\xi_{0,i}}\frac{\partial J}{\partial\xi_{0,k}}} - \frac{e}{c}A_k\frac{\partial J}{\partial\xi_{0,k}}\right\} d^4x, \tag{5.5}$$

Let us introduce new variables

$$j^k = \frac{\partial J}{\partial \xi_{0,k}}, \qquad k = 0, 1, 2, 3 \tag{5.6}$$

by means of Lagrange multipliers p_k

$$\mathcal{A}[\xi, \kappa, j, p] = \int \left\{ -mcK\sqrt{g_{ik}j^i j^k} - \frac{e}{c}A_k j^k + p_k \left(\frac{\partial J}{\partial \xi_{0,k}} - j^k \right) \right\} d^4x, \tag{5.7}$$

Variation with respect to ξ_i gives

$$\frac{\delta \mathcal{A}}{\delta \xi_i} = -\partial_l \left(p_k \frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{i,l}} \right) = 0, \qquad i = 0, 1, 2, 3 \tag{5.8}$$

Using identities

$$\frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{i,l}} \equiv J^{-1} \left(\frac{\partial J}{\partial \xi_{0,k}} \frac{\partial J}{\partial \xi_{i,l}} - \frac{\partial J}{\partial \xi_{0,l}} \frac{\partial J}{\partial \xi_{i,k}} \right) \tag{5.9}$$

$$\frac{\partial J}{\partial \xi_{i,l}} \xi_{k,l} \equiv J \delta_k^i, \qquad \partial_l \frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{i,l}} \equiv 0 \tag{5.10}$$

one can test by direct substitution that the general solution of linear equations (5.8) has the form

$$p_k = b_0 \left(\partial_k \varphi + g^\alpha (\boldsymbol{\xi}) \partial_k \xi_\alpha \right), \qquad k = 0, 1, 2, 3 \tag{5.11}$$

where $b_0 \neq 0$ is a constant, $g^\alpha (\boldsymbol{\xi})$, $\alpha = 1, 2, 3$ are arbitrary functions of $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, and φ is the dynamic variable ξ_0, which stops to be fictitious. Let us substitute (5.11) in (5.7). The term of the form $\partial J / \partial \xi_{0,k} \partial_k \varphi$ is reduced to Jacobian and does not contribute to dynamic equation. The terms of the form $\xi_{\alpha,k} \partial J / \partial \xi_{0,k}$ vanish due to identities (5.10). We obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa, j] = \int \left\{ -mcK\sqrt{g_{ik}j^i j^k} - j^k \pi_k \right\} d^4x, \tag{5.12}$$

where quantities π_k are determined by the relations

$$\pi_k = b_0 \left(\partial_k \varphi + g^\alpha (\boldsymbol{\xi}) \partial_k \xi_\alpha \right) + \frac{e}{c} A_k, \qquad k = 0, 1, 2, 3 \tag{5.13}$$

Integration of (5.8) in the form (5.11) is that integration which was mentioned in introduction as a conceptual operation which admits to introduce a wave function. Note that coefficients in the system (5.8) of equations for p_k are constructed of minors of the Jacobian (5.3). It is the circumstance that admits to produce a general integration.

Variation of (5.12) with respect to κ^l gives

$$\frac{\delta \mathcal{A}}{\delta \kappa^l} = -\frac{\lambda^2 mc\sqrt{g_{ik}j^i j^k}}{K} \kappa_l + \partial_l \frac{\lambda^2 mc\sqrt{g_{ik}j^i j^k}}{2K} = 0 \tag{5.14}$$

It can be written in the form

$$\kappa_l = \partial_l \kappa = \frac{1}{2} \partial_l \ln \rho, \qquad e^{2\kappa} = \frac{\rho}{\rho_0} \equiv \frac{\sqrt{j_s j^s}}{\rho_0 m c K}, \tag{5.15}$$

where ρ_0 =const is the integration constant. Substituting (5.2) in (5.15), we obtain dynamic equation for κ

$$\hbar^2 \left(\partial_l \kappa \cdot \partial^l \kappa + \partial_l \partial^l \kappa \right) = \frac{e^{-4\kappa} j_s j^s}{\rho_0^2} - m^2 c^2 \tag{5.16}$$

Variation of (5.12) with respect to j^k gives

$$\pi_k = -\frac{m c K j_k}{\sqrt{g_{ls} j^l j^s}} \tag{5.17}$$

or

$$\pi_k g^{kl} \pi_l = m^2 c^2 K^2 \tag{5.18}$$

Substituting the second equation (5.15) in (5.17), we obtain

$$j_k = -\rho_0 e^{2\kappa} \pi_k, \tag{5.19}$$

Now we eliminate the variables j^k from the action (5.12), using relation (5.19) and (5.15). We obtain

$$\mathcal{A}\left[\varphi, \boldsymbol{\xi}, \kappa\right] = \int \rho_0 e^{2\kappa} \left\{ -m^2 c^2 K^2 + \pi^k \pi_k \right\} d^4 x, \tag{5.20}$$

where π_k is determined by the relation (5.13). Using expression (5.2) for K, the first term of the action (5.20) can be transformed as follows.

$$\begin{aligned} -m^2 c^2 e^{2\kappa} K^2 &= -m^2 c^2 e^{2\kappa} \left(1 + \lambda^2 \left(\partial_l \kappa \partial^l \kappa + \partial_l \partial^l \kappa \right) \right) \\ &= -m^2 c^2 e^{2\kappa} + \hbar^2 e^{2\kappa} \partial_l \kappa \partial^l \kappa - \frac{\hbar^2}{2} \partial_l \partial^l e^{2\kappa} \end{aligned}$$

Let us take into account that the last term has the form of divergence. It does not contribute to dynamic equations and can be omitted. Omitting this term, we obtain

$$\mathcal{A}\left[\varphi, \boldsymbol{\xi}, \kappa\right] = \int \rho_0 e^{2\kappa} \left\{ -m^2 c^2 + \hbar^2 \partial_l \kappa \partial^l \kappa + \pi^k \pi_k \right\} d^4 x, \tag{5.21}$$

Instead of dynamic variables $\varphi, \boldsymbol{\xi}, \kappa$ we introduce n-component complex function

$$\psi = \{\psi_\alpha\} = \left\{ \sqrt{\rho} e^{i\varphi} u_\alpha(\boldsymbol{\xi}) \right\} = \left\{ \sqrt{\rho_0} e^{\kappa + i\varphi} u_\alpha(\boldsymbol{\xi}) \right\}, \qquad \alpha = 1, 2, ...n \tag{5.22}$$

Here u_α are functions of only $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, having the following properties

$$\sum_{\alpha=1}^{\alpha=n} u_\alpha^* u_\alpha = 1, \qquad -\frac{i}{2} \sum_{\alpha=1}^{\alpha=n} \left(u_\alpha^* \frac{\partial u_\alpha}{\partial \xi_\beta} - \frac{\partial u_\alpha^*}{\partial \xi_\beta} u_\alpha \right) = g^\beta(\boldsymbol{\xi}) \tag{5.23}$$

where $(^*)$ denotes complex conjugation. The number n of components of the wave function ψ is chosen in such a way, that equations (5.23) have a solution. Then we obtain

$$\psi^*\psi \equiv \sum_{\alpha=1}^{\alpha=n} \psi_\alpha^* \psi_\alpha = \rho = \rho_0 e^{2\kappa}, \qquad \partial_l \kappa = \frac{\partial_l (\psi^*\psi)}{2\psi^*\psi} \tag{5.24}$$

$$\pi_k = -\frac{ib_0 (\psi^* \partial_k \psi - \partial_k \psi^* \cdot \psi)}{2\psi^*\psi} + \frac{e}{c} A_k, \qquad k = 0, 1, 2, 3 \tag{5.25}$$

Substituting relations (5.24), (5.25) in (5.21), we obtain the action, written in terms of the wave function ψ

$$\begin{aligned} \mathcal{A}[\psi, \psi^*] &= \int \left\{ \left[\frac{ib_0 (\psi^* \partial_k \psi - \partial_k \psi^* \cdot \psi)}{2\psi^*\psi} - \frac{e}{c} A_k \right] \left[\frac{ib_0 \left(\psi^* \partial^k \psi - \partial^k \psi^* \cdot \psi\right)}{2\psi^*\psi} - \frac{e}{c} A^k \right] \right. \\ &\quad \left. + \hbar^2 \frac{\partial_l (\psi^*\psi)\, \partial^l (\psi^*\psi)}{4 (\psi^*\psi)^2} - m^2 c^2 \right\} \psi^* \psi d^4 x \end{aligned} \tag{5.26}$$

Let us use the identity

$$\begin{aligned} &\frac{(\psi^* \partial_l \psi - \partial_l \psi^* \cdot \psi) \left(\psi^* \partial^l \psi - \partial^l \psi^* \cdot \psi\right)}{4\psi^*\psi} + \partial_l \psi^* \partial^l \psi \\ \equiv\ &\frac{\partial_l (\psi^*\psi)\, \partial^l (\psi^*\psi)}{4\psi^*\psi} + \frac{g^{ls}}{2} \psi^*\psi \sum_{\alpha,\beta=1}^{\alpha,\beta=n} Q_{\alpha\beta,l}^* Q_{\alpha\beta,s} \end{aligned} \tag{5.27}$$

where

$$Q_{\alpha\beta,l} = \frac{1}{\psi^*\psi} \begin{vmatrix} \psi_\alpha & \psi_\beta \\ \partial_l \psi_\alpha & \partial_l \psi_\beta \end{vmatrix}, \qquad Q_{\alpha\beta,l}^* = \frac{1}{\psi^*\psi} \begin{vmatrix} \psi_\alpha^* & \psi_\beta^* \\ \partial_l \psi_\alpha^* & \partial_l \psi_\beta^* \end{vmatrix} \tag{5.28}$$

Then we obtain

$$\begin{aligned} \mathcal{A}[\psi, \psi^*] &= \int \left\{ \left(ib_0 \partial_k + \frac{e}{c} A_k \right) \psi^* \left(-ib_0 \partial^k + \frac{e}{c} A^k \right) \psi + \frac{b_0^2}{2} \sum_{\alpha,\beta=1}^{\alpha,\beta=n} g^{ls} Q_{\alpha\beta,l} Q_{\alpha\beta,s}^* \psi^*\psi \right. \\ &\quad \left. - m^2 c^2 \psi^*\psi + \left(\hbar^2 - b_0^2\right) \frac{\partial_l (\psi^*\psi)\, \partial^l (\psi^*\psi)}{4\psi^*\psi} \right\} d^4 x \end{aligned} \tag{5.29}$$

Let us consider the case of irrotational flow, when $g^\alpha (\boldsymbol{\xi}) = 0$ and the function ψ has only one component. It follows from (5.28), that $Q_{\alpha\beta,l} = 0$, and only the last term in (5.29) is not bilinear with respect to ψ, ψ^*. The constant b_0 is an arbitrary integration constant. One may set $b_0 = \hbar$. Then we obtain instead of (5.29)

$$\mathcal{A}[\psi, \psi^*] = \int \left\{ \left(i\hbar \partial_k + \frac{e}{c} A_k \right) \psi^* \left(-i\hbar \partial^k + \frac{e}{c} A^k \right) \psi - m^2 c^2 \psi^*\psi \right\} d^4 x \tag{5.30}$$

Variation of the action (5.30) with respect to ψ^* generates the Klein-Gordon equation

$$\left(-i\hbar \partial_k + \frac{e}{c} A_k \right) \left(-i\hbar \partial^k + \frac{e}{c} A^k \right) \psi - m^2 c^2 \psi = 0 \tag{5.31}$$

Thus, description in terms of the Klein-Gordon equation is a special case of the stochastic system description by means of the action (5.1), (5.2).

In the case, when the fluid flow is rotational, and the wave function ψ is two-component, the identity (5.27) takes the form

$$\begin{aligned} & \frac{\left(\psi^*\partial_l\psi-\partial_l\psi^*\cdot\psi\right)\left(\psi^*\partial^l\psi-\partial^l\psi^*\cdot\psi\right)}{4\rho}-\frac{\left(\partial_l\rho\right)\left(\partial^l\rho\right)}{4\rho} \\ \equiv\ & -\partial_l\psi^*\partial^l\psi+\frac{1}{4}\left(\partial_l s_\alpha\right)\left(\partial^l s_\alpha\right)\rho \end{aligned} \tag{5.32}$$

where 3-vector $\mathbf{s}=\{s_1,s_2,s_3,\}$ is defined by the relation

$$\rho=\psi^*\psi,\qquad s_\alpha=\frac{\psi^*\sigma_\alpha\psi}{\rho},\qquad \alpha=1,2,3 \tag{5.33}$$

$$\psi=\binom{\psi_1}{\psi_2},\qquad \psi^*=\left(\psi_1^*,\psi_2^*\right), \tag{5.34}$$

and Pauli matrices $\boldsymbol{\sigma}=\{\sigma_1,\sigma_2,\sigma_3\}$ have the form

$$\sigma_1=\left(\begin{array}{cc} 0 & 1\\ 1 & 0\end{array}\right),\qquad \sigma_2=\left(\begin{array}{cc} 0 & -i\\ i & 0\end{array}\right),\qquad \sigma_3=\left(\begin{array}{cc} 1 & 0\\ 0 & -1\end{array}\right) \tag{5.35}$$

Note that 3-vectors $\mathbf{s}$ and $\boldsymbol{\sigma}$ are vectors in the space V_ξ of the Clebsch potentials $\boldsymbol{\xi}=\{\xi_1,\xi_2,\xi_3\}$ and transform as vectors at the transformations (1.6) of Clebsch potentials $\boldsymbol{\xi}$. The wave function ψ is a spinor in the space V_ξ. The quantities $\mathbf{s}$, $\boldsymbol{\sigma}$ and ψ are scalars in the 3-space V_x of usual coordinates $\mathbf{x}=\{x^1,x^2,x^3\}$.

In general, transformations of Clebsch potentials $\boldsymbol{\xi}$ and those of coordinates $\mathbf{x}$ are independent. However, the action (5.26) does not contain any reference to the Clebsch potentials $\boldsymbol{\xi}$ and transformations (1.6) of $\boldsymbol{\xi}$. If we consider only linear transformations of space coordinates $\mathbf{x}$

$$x^\alpha\rightarrow\tilde{x}^\alpha=b^\alpha+\omega_{.\beta}^\alpha x^\beta,\qquad \alpha=1,2,3 \tag{5.36}$$

nothing prevents from accompanying any transformation (5.36) with the similar transformation

$$\xi_\alpha\rightarrow\tilde{\xi}_\alpha=b^\alpha+\omega_{.\beta}^\alpha\xi_\beta,\qquad \alpha=1,2,3 \tag{5.37}$$

of Clebsch potentials $\boldsymbol{\xi}$. The formulas for linear transformation of vectors and spinors in V_x do not contain the coordinates $\mathbf{x}$ explicitly, and one can consider vectors and spinors in V_ξ as vectors and spinors in V_x, provided we consider linear transformations (5.36), (5.37) always together.

Using identity (5.32), we obtain from (5.26)

$$\mathcal{A}[\psi,\psi^*]=\int\left\{\left(i\hbar\partial_k+\frac{e}{c}A_k\right)\psi^*\left(-i\hbar\partial^k+\frac{e}{c}A^k\right)\psi-m^2c^2\rho-\frac{\hbar^2}{4}\left(\partial_l s_\alpha\right)\left(\partial^l s_\alpha\right)\rho\right\}d^4x \tag{5.38}$$

Dynamic equation, generated by the action (5.38), has the form

$$\begin{aligned}
&\left(-i\hbar\partial_k+\frac{e}{c}A_k\right)\left(-i\hbar\partial^k+\frac{e}{c}A^k\right)\psi-\left(m^2c^2+\frac{\hbar^2}{4}\left(\partial_l s_\alpha\right)\left(\partial^l s_\alpha\right)\right)\psi \\
= &\ -\hbar^2\frac{\partial_l\left(\rho\partial^l s_\alpha\right)}{2\rho}\left(\sigma_\alpha-s_\alpha\right)\psi
\end{aligned} \tag{5.39}$$

The gradient of the unit 3-vector $\mathbf{s}=\{s_1,s_2,s_3\}$ describes rotational component of the fluid flow. If $\mathbf{s}$ =const the dynamic equation (5.39) turns to the conventional Klein-Gordon equation (5.31). Curl of the vector field π_k, determined by the relation (5.25), is expressed only via derivatives of the unit 3-vector $\mathbf{s}$.

To show this, let us represent the wave function (5.22) in the form

$$\psi=\sqrt{\rho}e^{i\varphi}\left(\mathbf{n}\boldsymbol{\sigma}\right)\chi,\qquad \psi^*=\sqrt{\rho}e^{-i\varphi}\chi^*\left(\boldsymbol{\sigma}\mathbf{n}\right),\qquad \mathbf{n}^2=1,\qquad \chi^*\chi=1 \tag{5.40}$$

where $\mathbf{n}=\{n_1,n_2,n_3\}$ is some unit 3-vector, $\chi=\binom{\chi_1}{\chi_2}$, $\chi^*=(\chi_1^*,\chi_2^*)$ are constant two-component quantities, and $\boldsymbol{\sigma}=\{\sigma_1,\sigma_2,\sigma_3\}$ are Pauli matrices (5.35). The unit vector $\mathbf{s}$ and the unit vector $\mathbf{n}$ are connected by means of the relations

$$\mathbf{s}=2\mathbf{n}\left(\mathbf{nz}\right)-\mathbf{z},\qquad \mathbf{n}=\frac{\mathbf{s}+\mathbf{z}}{\sqrt{2\left(1+\left(\mathbf{sz}\right)\right)}} \tag{5.41}$$

where $\mathbf{z}$ is a constant unit vector defined by the relation

$$\mathbf{z}=\chi^*\boldsymbol{\sigma}\chi,\qquad \mathbf{z}^2=\chi^*\chi=1 \tag{5.42}$$

All 3-vectors $\mathbf{n}$, $\mathbf{s}$, $\mathbf{z}$ are vectors in V_ξ. Let us substitute the relation (5.40) into expression $\partial_l\pi_k-\partial_k\pi_l$ for the curl of the vector field π_k defined by the relation (5.25). Then gradually reducing powers of σ by means of the identity

$$\sigma_\alpha\sigma_\beta\equiv\delta_{\alpha\beta}+i\varepsilon_{\alpha\beta\gamma}\sigma_\gamma,\qquad \alpha,\beta=1,2,3 \tag{5.43}$$

where $\varepsilon_{\alpha\beta\gamma}$ is the Levi-Chivita pseudotensor ($\varepsilon_{123}=1$), we obtain after calculations

$$\begin{aligned}
\pi_k &= -\frac{ib_0\left(\psi^*\partial_k\psi-\partial_k\psi^*\cdot\psi\right)}{2\psi^*\psi}+\frac{e}{c}A_k \\
&= b_0\left(\partial_k\varphi+\varepsilon_{\alpha\beta\gamma}n_\alpha\partial_k n_\beta z_\gamma\right)+\frac{e}{c}A_k\qquad k=0,1,2,3
\end{aligned} \tag{5.44}$$

$$\partial_k\pi_l-\partial_l\pi_k=-4b_0\left[\partial_k\mathbf{n}\times\partial_l\mathbf{n}\right]\mathbf{z}+\frac{e}{c}\left(\partial_k A_l-\partial_l A_k\right),\qquad k,l=0,1,2,3 \tag{5.45}$$

The relation (5.45) may be expressed also via the 3-vector $\mathbf{s}$, provided we use the formulae (5.41).

Note that the two-component form of the wave function can describe irrotational flow. For instance, if $\psi=\binom{\psi_1}{\psi_1}$, $s_1=1$, $s_2=s_3=0$, the dynamic equation (5.39) reduces to the form (5.31), and curl of π_k, defined by (5.45) vanishes.

6 Interpretation of solutions of the free Klein-Gordon equation

Let us write the action (5.12), (5.2) in the arbitrary curvilinear coordinate system. We obtain for dynamic system $\mathcal{S}_{\text{KG}}$

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa, j] = \int \left\{ -mcK\sqrt{g_{ik}j^i j^k} - j^k \pi_k \right\} \sqrt{-g} d^4 x, \tag{6.1}$$

$$K = \sqrt{1 + \lambda^2 \left(g_{ik}\kappa^i \kappa^k + \frac{1}{\sqrt{-g}} \partial_l \left(\sqrt{-g} \kappa^l \right) \right)}, \qquad \lambda = \frac{\hbar}{mc} \tag{6.2}$$

$$g = \det ||g_{ik}|| \tag{6.3}$$

where quantities π_k are determined by the relations (5.13)

$$\pi_k = b_0 \left(\partial_k \varphi + g^\alpha (\boldsymbol{\xi}) \partial_k \xi_\alpha \right) + \frac{e}{c} A_k, \qquad k = 0, 1, 2, 3 \tag{6.4}$$

Trying to interpret dynamic system $\mathcal{S}_{\text{KG}}$ described by the action (6.1) – (6.4), we shall *use only dynamic considerations*. This means that $\mathcal{S}_{\text{KG}}$ is considered to be a fluid, and world lines of the fluid particles are interpreted as average world lines of stochastic particles. We consider $\mathcal{S}_{\text{KG}}$ as a fluid and interpret its motion in terms of hydrodynamics. We ignore interpretation, based on quantum principles, because this interpretation in terms of the wave function is purely empirical. It has been tested only for nonrelativistic quantum phenomena. In the relativistic case there are problems for interpretation of expressions constructed on the basis of the wave function.

From viewpoint of hydrodynamic interpretation the wave function is only a very special method of description of $\mathcal{S}_{\text{KG}}$. This method is very convenient for solution of dynamic equations, because they are linear in terms of wave function, but it is not the best method for interpretation of the fluid motion.

We shall interpret dynamic system, following the behaviour of such physical quantities as the current j^i and the energy-momentum tensor T^{ik}. The energy-momentum tensor for $\mathcal{S}_{\text{KG}}$ is obtained by means of relation (3.2). Applying the relation (3.2) to the action (6.1) – (6.4) and taking into account that the Lagrangian density vanishes due to (5.17), we obtain

$$\begin{aligned} T^{ik} = {} & \frac{mcKj^i j^k}{\sqrt{g_{ls}j^l j^s}} + \lambda^2 \frac{mc\sqrt{g_{ls}j^l j^s}}{K} \kappa^i \kappa^k + \lambda^2 \frac{mc\sqrt{g_{ls}j^l j^s}}{K} \left(-\frac{1}{2\left(\sqrt{-g}\right)} g^{ik} \partial_l \left(\sqrt{-g} \kappa^l \right) \right) \\ & -\lambda^2 \left(\frac{1}{2} g^{ik} \kappa^j \right) \partial_j \frac{mc\sqrt{g_{ls}j^l j^s}}{K} \end{aligned}$$

We use the relation (5.15)

$$e^{2\kappa} = \frac{\rho}{\rho_0} \equiv \frac{\sqrt{j_s j^s}}{\rho_0 mcK}. \tag{6.5}$$

Returning to the inertial coordinate system, where $\sqrt{-g}=c$, we obtain

$$T^{ik}=\frac{j^ij^k}{\rho}+\hbar^2\rho\kappa^i\kappa^k-\frac{\hbar^2}{2}g^{ik}\partial_l\left(\rho\kappa^l\right) \tag{6.6}$$

or

$$T^{ik} = \frac{j^ij^k+j_{\text{st}}^ij_{\text{st}}^k}{\rho}-\frac{\hbar}{2}g^{ik}\partial_lj_{\text{st}}^l, \tag{6.7}$$

$$j_{\text{st}}^k = \hbar\rho\kappa^k=\frac{\hbar}{2}\partial^k\rho, \qquad \rho=\rho_0e^{2\kappa}=\frac{\sqrt{j^lj_l}}{mcK}, \qquad \kappa^i=g^{ik}\partial_k\kappa \tag{6.8}$$

The energy-momentum tensor (6.7) looks as the energy-momentum tensor of two fluids. Flux of one fluid is described by the 4-vector j^i, whereas the flux of other fluid is described by the 4-vector $j_{\text{st}}^i=\hbar\rho\kappa^i=\hbar g^{ik}\rho\partial_k\kappa$. In the first fluid there is no pressure. The pressure p in the second fluid is described by the expression

$$p=\frac{\hbar}{2}\partial_lj_{\text{st}}^l$$

The two fluids interact via common factor $\rho=\rho_0e^{2\kappa}$.

Eigenvectors u_i and eigenvalues μ of the energy-momentum tensor are defined by the relations

$$T^{ik}u_k=\mu g^{ik}u_k, \qquad u_k=\alpha j_k+\beta j_{\text{st}k} \tag{6.9}$$

Calculation gives

$$\mu_{1,2}=-\frac{\hbar}{2}\partial_lj_{\text{st}}^l+\frac{1}{2\rho}\left(j_lj^l+j_{\text{st}l}j_{\text{st}}^l\pm\sqrt{\left(j^lj_l-\left(j_{\text{st}}^lj_{\text{st}l}\right)\right)^2+4\left(j_{\text{st}}^lj_l\right)^2}\right) \tag{6.10}$$

$$\beta_{1,2}=\frac{\left(j_lj^l-j_{\text{st}l}j_{\text{st}}^l\pm\sqrt{\left(j^lj_l-\left(j_{\text{st}}^lj_{\text{st}l}\right)\right)^2+4\left(j_{\text{st}}^lj_l\right)^2}\right)}{2j_lj_{\text{st}}^l}\alpha \tag{6.11}$$

There is a very simple and interesting case, when one of fluids vanishes and

$$j^i\equiv 0, \qquad i=0,1,2,3. \tag{6.12}$$

As it follows from (5.16), in this case the scalar e^κ satisfies the free Klein-Gordon equation

$$\left(\hbar^2\partial_l\partial^l+m^2c^2\right)e^\kappa=0 \tag{6.13}$$

and

$$K=\sqrt{1+\lambda^2e^{-\kappa}\partial_l\partial^le^\kappa}=0 \tag{6.14}$$

On the other side, in this case the wave function (5.22) is one-component, and it satisfies the Klein-Gordon equation (5.31). If the condition (6.12) is satisfied, the wave function has the form $\psi=e^{\kappa+i\varphi}$, $\varphi=$const. Equations (6.13) and (5.31) are compatible, provided $A_i=0$.

Under conditions (6.12) and (6.14) the density ρ (6.5) and the energy-momentum tensor (6.7) become indefinite. Besides, κ is to be real and $e^{\kappa} \geq 0$.

To obtain the canonical energy-momentum tensor for the irrotational flow, we consider the action of the form (5.30) with additional condition $A_i = 0$, where ψ is the one-component wave function. The action has the form

$$\mathcal{A}[\psi, \psi^*] = \int \left\{\hbar^2 \partial_k \psi^* \cdot \partial^k \psi - m^2 c^2 \psi^* \psi\right\} d^4 x \qquad (6.15)$$

Then canonical energy-momentum tensor

$$T_{\mathrm{c}}{}_k^{\ l} = \hbar^2 \left(\partial_k \psi^* \cdot \partial^l \psi + \partial^l \psi^* \cdot \partial_k \psi\right) - \delta_k^l \left(\hbar^2 \partial_k \psi^* \cdot \partial^k \psi - m^2 c^2 \psi^* \psi\right) \qquad (6.16)$$

coincides with the relation (6.7).

Indeed, let us substitute in (6.16) the expression

$$\partial_k \psi = \sqrt{\rho_0} e^{\kappa} \partial_k \kappa - \frac{i}{\hbar \sqrt{\rho_0}} e^{-\kappa} j_k = \sqrt{\rho} \partial_k \kappa - \frac{i}{\hbar \sqrt{\rho}} j_k \qquad (6.17)$$

which is valid for the irrotational flow.

$$T_{ckl} = \frac{j_l j_k}{\rho} + \hbar^2 \rho \partial_l \kappa \cdot \partial_k \kappa - \rho_0 e^{2\kappa} g_{kl} \left(\hbar^2 \partial_s \kappa \cdot \partial^s \kappa + \frac{1}{\rho_0^2} e^{-4\kappa} j_s j^s - m^2 c^2\right)$$

Taking into account dynamic equation (5.16), we eliminate two last terms

$$T_{ckl} = \frac{j_l j_k}{\rho} + \hbar^2 \rho \partial_l \kappa \cdot \partial_k \kappa - g_{kl} \frac{\hbar^2}{2} \partial_s \partial^s e^{2\kappa} \qquad (6.18)$$

and the relations (6.6) and (6.18) coincide.

Solution of equation (6.13), which is stationary in the whole space-time, is $e^{\kappa} = e^{\kappa(\mathbf{x})}$. It satisfies the equation

$$\left(\hbar^2 \partial_\alpha \partial_\alpha - m^2 c^2\right) e^{\kappa(\mathbf{x})} = 0$$

Solutions of this equation have the form

$$\psi_{\mathbf{k}} = e^{\kappa(\mathbf{x})} = B_{\mathbf{k}} \cos\left(\frac{\mathbf{kx}}{\hbar} + \phi_{\mathbf{k}}\right), \qquad \mathbf{k}^2 = m^2 c^2, \qquad B_{\mathbf{k}} = \mathrm{const} \qquad (6.19)$$

None of these solutions satisfies the condition $\psi \geq 0$ except for $\psi = 0$. For this solution

$$T^{00} = 0, \qquad T^{0\alpha} = 0,$$

and the solution $\psi = 0$ may be interpreted as a vacuum state. Possibility of construction of a linear combination of solutions (6.19) which would be positive in the whole space-time seems to be problematic, although it is possible in small regions Ω_{v} of the space-time. At the boundaries of Ω_{v} the solution ψ becomes negative and κ becomes to be complex. It corresponds to $j^k \neq 0$, i.e. to appearance of SWLs (particles and antiparticles). For solutions e^{κ}, which are linear combinations of solutions (6.19), $T^{00} \geq 0$, and these solutions may be interpreted as excited states of vacuum inside the region Ω_{v}. At the boundaries of Ω_{v} pairs of SWLs appear. In general, the pairs are virtual in the sense that for each of SWL $p_i p^i \neq m^2 c^2$.

Another case, when there is only one fluid, described by the vector j^k and $j^l_{\mathrm{st}} = 0$ corresponds to description of statistical ensemble of relativistic classical (deterministic) particles.

7 Interpretation of κ-field in the Klein-Gordon dynamic system

Let us consider solutions of the free Klein-Gordon equation ($A_k = 0,\ \ k = 0, 1, 2, 3$). For simplicity of calculations we set inside this paragraph $\hbar = c = 1$. The general solution of the free Klein-Gordon equation has the form

$$\psi(t, \mathbf{x}) = (2\pi)^{-3/2} \int e^{-i\varepsilon_k E(\mathbf{k})t + i\mathbf{k}\mathbf{x}} \frac{a(K)}{\sqrt{2E(\mathbf{k})}} dK, \tag{7.1}$$

$$K = \{\varepsilon_k, \mathbf{k}\}, \qquad E(\mathbf{k}) = \left|\sqrt{m^2 + \mathbf{k}^2}\right|, \qquad \mathbf{k}\mathbf{x} \equiv -k_\alpha x^\alpha \equiv k_\alpha x_\alpha \tag{7.2}$$

where $a(K)$ is a function of $K = \{\varepsilon_k, \mathbf{k}\},\ \ \mathbf{k} = \{k_1, k_2, k_3\},\ \ \varepsilon_k = \pm 1$ and

$$\int (.)\, dK \equiv \sum_{\varepsilon_k = \pm 1} \int (.)\, d\mathbf{k} \equiv \sum_{\varepsilon_k = \pm 1} \iiint_{-\infty}^{+\infty} (.)\, dk_1 dk_2 dk_3 \tag{7.3}$$

Let us set

$$a_P(K) = a_{\varepsilon_p, \mathbf{p}}(K) = A\sqrt{2E(\mathbf{k})} \exp\left(-\frac{(\mathbf{k} - \mathbf{p})^2}{2\Delta}\right) \delta_{\varepsilon_k, \varepsilon_p}, \qquad A = \mathrm{const} \tag{7.4}$$

The quantities $P = \{\varepsilon_p, \mathbf{p}\}$ and Δ are parameters of the solution. We suppose

$$\sqrt{\Delta} \ll \lambda^{-1} = m, \qquad \hbar = c = 1 \tag{7.5}$$

Under the condition (7.5) the following approximate expression for the wave function is obtained

$$\psi_P(t, \mathbf{x}) = \frac{B\Delta^{3/2}}{\left(1 + i\varepsilon_k \Delta t \left(\frac{3}{2E} + \frac{\mathbf{v}^2}{2}\right)\right)} \exp\left(-\frac{\Delta(\mathbf{x} - \mathbf{v}t)^2}{2} - i\varepsilon_p E(\mathbf{p})\, t + i\mathbf{p}\mathbf{x}\right)$$

where

$$\mathbf{v} = \frac{\varepsilon_p \mathbf{p}}{E(\mathbf{p})}, \qquad B = B^* = \mathrm{const} \tag{7.6}$$

At $|t| \ll E/\Delta$ we shall use a simpler expression for the wave function

$$\psi_P(t, \mathbf{x}) = B\Delta^{3/2} \exp\left(-\frac{\Delta(\mathbf{x} - \mathbf{v}t)^2}{2} - i\varepsilon_p E(\mathbf{p})\, t + i\mathbf{p}\mathbf{x}\right) \tag{7.7}$$

In the space-time region $|\mathbf{x}-\mathbf{v}t| < \Delta^{-1/2}$ we obtain the following values for the quantities j^k, κ^k, T_k^0

$$\begin{array}{lll} j^0 = (\varepsilon_p E(\mathbf{p}) + O(\Delta))\rho & \kappa^0 = \Delta(\mathbf{x}-\mathbf{v}t)\mathbf{v} & T_0^0 = (E^2(\mathbf{p}) + O(\Delta))\rho \\ j^\alpha = (p_\alpha + O(\Delta))\rho & \kappa^\alpha = \Delta(x^\alpha - v^\alpha t) & T_\alpha^0 = (\varepsilon_p E(\mathbf{p}) p_a + O(\Delta))\rho \\ \rho = \psi_P^* \psi_P & m_{\text{eff}}^2 = m^2 + O(\Delta) & T_\alpha^\beta = \left(p_\alpha p^\beta + O(\Delta)\right)\rho \end{array} \tag{7.8}$$

where

$$\psi_P^* \psi_P = \rho_0 e^{2\kappa} = B^2 \Delta^3 \exp\left(-\Delta(\mathbf{x}-\mathbf{v}t)^2\right) \tag{7.9}$$

Interpretation from hydrodynamic viewpoint looks as follows. We assume that the state $P = \{1, \mathbf{p}\}$ with $\varepsilon_p = 1$ corresponds to a particle, and the state $P = \{-1, \mathbf{p}\}$ with $\varepsilon_p = -1$ corresponds to an antiparticle, because as it follows from (7.8) the flux density j^0 has different sign in the two states. The total number N of SWLs is

$$N_P = \varepsilon_p E(\mathbf{p}) I, \qquad I = \int \psi_P^* \psi_P d\mathbf{x} = B^2 (\pi\Delta)^{3/2} \tag{7.10}$$

The total energy-momentum vector

$$P_k = \left\{E^2(\mathbf{p}) I, E(\mathbf{p}) \mathbf{p} I\right\} \tag{7.11}$$

does not depend on orientation ε_p, whereas the energy per one SWL

$$\frac{P_0}{N_P} = \frac{E_P}{N_P} = \varepsilon_p E(\mathbf{p}) \tag{7.12}$$

depends on ε_p. It is negative for $\varepsilon_p = -1$, because the number N_P of SWLs is negative. Thus, the state ψ_P describes a fluid, consisting of a cloud of N_P SWLs, moving with velocity $\mathbf{v} = \mathbf{p}/E(\mathbf{p}) + O(\Delta)$. The pressure in the fluid is small (of the order $O(\Delta)$), and one can speak on freely moving SWLs (particles or antiparticles).

From viewpoint of quantum principles the state $\psi_{\{1,\mathbf{p}\}}$ (for particles) is interpreted similarly. But for interpretation of the state $\psi_{\{-1,\mathbf{p}\}}$ (for antiparticles) there are problems [31], connected with negative value of the quantity E_P/N_P (energy per SWL).

Now let us consider the state $\psi = \psi_{\{1,\mathbf{p}\}} + \psi_{\{1,-\mathbf{p}\}}$. In the conventional quantum mechanics the wave function is a fundamental object of theory. The wave functions $\psi_{\{1,\mathbf{p}\}}$ and $\psi_{\{1,-\mathbf{p}\}}$ evolve independently and this fact is interpreted in the sense that the particles described by $\psi_{\{1,\mathbf{p}\}}$ and the particles described by $\psi_{\{1,-\mathbf{p}\}}$ do not interact. Thus, from viewpoint of quantum principles the state ψ describes two clouds of particles, moving with velocities $\mathbf{v}$ and $-\mathbf{v}$ one through another without interaction.

From hydrodynamic viewpoint the state $\psi = \psi_{\{1,\mathbf{p}\}} + \psi_{\{1,-\mathbf{p}\}}$ describes rather complicated picture of two colliding clouds of particles. According to the hydrodynamic interpretation the world lines of particles in the two-dimensional space-time are defined by the equation

$$\frac{dx}{dt} = \frac{j^1(t,x)}{j^0(t,x)} = \frac{(p\sinh(2\Delta xvt)) - \Delta vt(\sin(2px))}{E(\cosh(2\Delta xvt) + \cos(2px)) + \Delta xv(\sin(2px))} + O\left(\Delta^2\right) \tag{7.13}$$

Shape of world lines, obtained by numerical calculations for $-2/\sqrt{\Delta} < t < 2/\sqrt{\Delta}$ is shown in figure 1. One can see, that after collision of two clouds the particles of clouds reflect and move in the opposite direction. This situation correlates with values of the energy-momentum tensor and other hydrodynamic quantities, calculated in the space-time region $x^2, t^2 \leq \Delta^{-1}$, where the two clouds overlap. These quantities are calculated to within Δ^0

$$\begin{array}{lll} j^0 = 2CE(p)\cos^2(px) & \kappa_0 = 0 & T^{00} = 2CE^2(p)\cos^2(px) \\ j_1 = 0 & \kappa_1 = -p\tan(px) & T^{01} = 0 \\ \rho = 2C\cos^2(px) & m_{\text{eff}}^2 = E^2(p) & T^{11} = 2Cp^2\sin^2(px) \end{array} \qquad (7.14)$$

where

$$C = B^2 \exp\left(-\Delta\left(x^2 + v^2t^2\right)\right) \qquad (7.15)$$

The tensor energy-momentum component $T^{00} = j^0 j^0/\rho$ is determined by the 4-vector j^k, whereas the pressure in the fluid is determined by the component $T^{11} = \rho\kappa^1\kappa^1$.

If the particles of two clouds do not interact, the energy-momentum tensor is a sum of energy-momentum tensors for each cloud separately. It has the form

$$T^{00} = 2CE^2(p), \qquad T^{01} = 0, \qquad T^{11} = 2Cp^2 \qquad (7.16)$$

and this expression distinguishes from (7.14)

Let us consider the state $\psi = \psi_{\{\varepsilon_p,\mathbf{p}\}} + \psi_{\{-\varepsilon_p,\mathbf{p}\}}$. From viewpoint of quantum principles the state ψ describes a cloud of particles and a cloud of antiparticles, moving with velocities $\mathbf{v}$ and $-\mathbf{v}$ one through another without interaction.

From viewpoint of hydrodynamic interpretation the state $\psi_{\{1,\mathbf{p}\}} + \psi_{\{-1,\mathbf{p}\}}$ describes a complicated picture of interacting particles and antiparticles. In the process of interaction the pair production and the pair annihilation take place. The process of interaction is balanced, and finally we see a cloud of particles and a cloud of antiparticles escaping one from another, as if they have passed one through another without interaction.

In the two-dimensional space-time the mean world lines are described by the equation

$$\frac{dx}{dt} = \frac{j^1}{j^0} = \frac{p\cosh(2\Delta xvt) + p\cos(2Et) - \Delta vt\sin(2Et)}{E\sinh(2\Delta xvt) + \Delta xv\sin(2Et)}$$

Shape of world lines obtained by numerical calculations for $-2/\sqrt{\Delta} < x < 2/\sqrt{\Delta}$ is shown in figures 2. In the state $\psi_{\{1,\mathbf{p}\}} + \psi_{\{-1,\mathbf{p}\}}$ colliding particles and antiparticles annihilate, their energy transforms to energy of the κ-field. Thereafter the κ-field generate SWL pairs and energy of the κ-field turns to energy of escaping particles and antiparticles. Distribution of space-time regions, where effective mass m_{eff} is imaginary is shown in figure 3.

Described interaction of particles and antiparticles correlates with values of the energy-momentum tensor and other hydrodynamic quantities, calculated in the space-time region $x^2, t^2 \leq \Delta^{-1}$, where the two clouds overlap. These quantities are calculated to within Δ^0

$$\begin{array}{lll} j_0 = 0 & \kappa_0 = -E\tan(Et) & T^{00} = 2E^2 C \sin^2(Et) \\ j_1 = -2pC\cos^2(Et) & \kappa_1 = 0 & T^{01} = 0 \\ \rho = 2C\cos^2(Et) & m_{\text{eff}}^2 = -p^2 & T^{11} = 2p^2 C\cos^2(Et) \end{array} \tag{7.17}$$

where C is determined by the relation (7.15).

Our investigation of solutions of the free Klein-Gordon equation is a preliminary consideration, which is necessary for correct mathematical formulation of the collision problem and of the problem pair production as a special case of the collision problem. Two different interpretations of the states $\psi_{\{\varepsilon_p,\mathbf{p}\}} + \psi_{\{\varepsilon_p,-\mathbf{p}\}}$ and $\psi_{\{1,\mathbf{p}\}} + \psi_{\{-1,\mathbf{p}\}}$ generate two different statements of the collision problem. The conventional interpretation based on the wave function as a fundamental object of dynamics leads to the S-matrix theory, where detailed description of dynamic processes inside the collision space-time region is supposed to be impossible. For solution of these problems the energy-momentum conservation law is used mainly. Alternative approach based on sequential consideration of fluid dynamics and hydrodynamic quantities leads to another statement of the collision problem. Comparison of these two different statements of the collision problem is possible only at the detailed presentation of the alternative statement of the collision problem. Unfortunately, such a presentation cannot be made in the present paper.

We restrict our consideration to the problem of two elastic balls collision, which reminds to some extent the collision problem of the microparticle collision. At the head-on collision of two identical elastic balls they interchange their momenta, and the whole situation may be described as one ball passes through another one without interaction. It is a very simple interpretation, but it is not effective, because it can be used only for head-on collision. If the impact parameter does not vanish, or the balls are not identical, we are forced to introduce interaction between the balls. If we assume that the identical balls do not interact at the head-on collision, it is rather difficult to understand, why they begin to interact with non-vanishing impact parameter. One needs to invent a special interaction for explanation of non-head-on collision. Is it possible to construct such a theory of two balls collision, which is based on the supposition that the balls do not interact at the head-on collision? Yes it is possible, but such a theory is founded mainly on the energy-momentum conservation law, and it does not fit for explanation of inelastic collisions, where detailed investigation of interaction between the balls is necessary.

The approach, when the head-on collision of two identical elastic balls is considered to be a strong interaction of two balls, which leads to an interchange of momenta, seems to be more reasonable, because it explains freely, why result of collision is not reduced to the interchange of momenta at the collision with non-vanishing impact parameter. At such an approach the interchange of momenta is explained by balanced interaction at the head-on collision. If the impact parameter does not vanish, this balance is violated. Although explanation of the balanced interaction is more complicated, but application of this interaction to the case of unbalanced interaction appears to be simpler. Something like this we have in the case of two alternative approaches to the microparticle collision problem.

8 Concluding remarks

Thus, quantum system is a special case of stochastic system, and quantum phenomena can be considered to be a result of some quantum stochasticity, generated by the space-time properties [1]. Pair production is a natural result of the sequential relativistic description of this stochasticity. The statistical description of quantum phenomena generates hydrodynamic interpretation of quantum phenomena, which does not coincide, in general, with the interpretation, based on application of quantum principles, and the interpretation appears to be essential for statement of the microparticle collision problem.

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