Dynamics of stochastic systems and peculiarities of measurements in them

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Abstract

One calls attention to the fact that the stochastic physical systems are not random completely. They have both random and regular components of their evolution. Dynamic system is considered to be a special case of physical system with vanishing stochastic component of evolution. Mathematical technique for description of regular evolution component of physical systems (stochastic and deterministic) is constructed. The regular component of the system $\mathcal{S}$ evolution is described explicitly by the statistical average system $\langle\mathcal{S}\rangle$, which is a continuous dynamic system. The action $\mathcal{A}_{\langle\mathcal{S}\rangle}$ for $\langle\mathcal{S}\rangle$ is reduced to the form of the action $\mathcal{A}_{\mathcal{S}_{\rm red}[\mathcal{S}_{\rm d}]}$ for a set $\mathcal{S}_{\rm red}[\mathcal{S}_{\rm d}]$ of interacting identical deterministic systems $\mathcal{S}_{\rm d}$. The form of interaction of $\mathcal{S}_{\rm d}$ describes implicitly the character of the stochastic component of $\mathcal{S}$ evolution. Interplay between the physical system $\mathcal{S}$ and the statistical average system $\langle\mathcal{S}\rangle$ in the measurement process is discussed. There are at least two different kind of measurement: single measurements (S-measurements) connected with measurements in $\mathcal{S}$ and mass measurements (M-measurements) connected with measurements in $\langle\mathcal{S}\rangle$. Conventional identification of S-measurement and M-measurement is a source of many misunderstandings and paradoxes.

1 Introduction

Let us consider some physical system $\mathcal{S}$. Its time evolution may be deterministic or stochastic. In the first case the physical system $\mathcal{S}$ associates with some dynamic system $\mathcal{S}_{\rm d}$, (or deterministic physical system). Dynamic system $\mathcal{S}_{\rm d}$ is a totality of state described by dynamic variables X and dynamic equations, describing time evolution of the state X. If number of dynamic variables is finite or countable, the dynamic system $\mathcal{S}_{\rm d}$ is called discrete dynamic system. Evolution of the state $X=\{\mathbf{x},\dot{\mathbf{x}}\}, \mathbf{x}=\{x^\alpha\}, \quad \alpha=1,2,...,n$ of the discrete dynamic system $\mathcal{S}_{\rm d}$ is described by the action functional

$$\mathcal{S}_{\mathrm{d}}: \quad \mathcal{A}[\mathbf{x}] = \int L(t, \mathbf{x}, \dot{\mathbf{x}},)\, dt, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \tag{1.1}$$

Solutions of dynamic equations are extremals of the functional (1.1). Dynamic equations are obtained by variation of the action with respect to $\mathbf{x}$.

$$\frac{\delta \mathcal{A}}{\delta x^\alpha} = \frac{\partial L}{\partial x^\alpha} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}^\alpha} = 0, \qquad \alpha = 1, 2, ..., n \tag{1.2}$$

The canonical momentum $p = \{p_0, \mathbf{p}\} = \{p_i\}, \{p_i\}, \quad i = 1, 2, ...n$ is associated with the discrete dynamic system $\mathcal{S}_{\mathrm{d}}$

$$p_\alpha = \frac{\partial L}{\partial x^\alpha}, \qquad \alpha = 1, 2, ...n; \qquad p_0 = \frac{\partial L}{\partial \dot{x}^\beta}\dot{x}^\beta - L \tag{1.3}$$

If dynamic variables X describing the state of the dynamic system $\mathcal{S}_{\mathrm{d}}$ form a continuous set, $\mathcal{S}_{\mathrm{d}}$ is a continuous dynamic system. Dynamic variables $X = u = \{u_B\}, \quad B = 1, 2, ...s$ of continuous dynamic system are labelled by some quantities $\mathbf{x} = \{x^1, x^2, ..., x^n\}$. Dependent variables $u = \{u_B\}, \quad B = 1, 2, ...s$ are considered to be functions of independent variables $x = \{t, \mathbf{x}\} = \left\{x^k\right\}, \quad k = 0, 1, ...n$, where $t = x^0$ is the time.

Continuous dynamic system is described by the action functional

$$\mathcal{S}_{\mathrm{d}}: \quad \mathcal{A}[u] = \int \mathcal{L}(u, \partial_k u_B)\, d^{n+1}x, \qquad \partial_k u_B \equiv \frac{\partial u_B}{\partial x^k}, \qquad d^{n+1}x = \prod_{k=0}^{k=n} dx^k \tag{1.4}$$

The quantity $\mathcal{L}$ is called the Lagrange function density.

Dynamic equations for the dynamic system $\mathcal{S}_{\mathrm{d}}$ are obtained as equations for extremals of the action functional (1.4). They are obtained by means of variation with respect to dependent variables u

$$\frac{\delta \mathcal{A}}{\delta u_B} = \frac{\partial \mathcal{L}}{\partial u_B} - \partial_k\left(\frac{\partial \mathcal{L}}{\partial u_{B,k}}\right) = 0, \qquad B = 1, 2, ...s \tag{1.5}$$

Here and further summation over repeated Latin indices from 0 to n is assumed. The flux $j^k(x)$

$$j^k(x) = \sum_{B=1}^{B=s} \frac{\partial \mathcal{L}}{\partial u_{B,k}} u_B \tag{1.6}$$

and the energy-momentum tensor $T_l^k(x)$

$$T_l^k(x) = \sum_{B=1}^{B=s} \frac{\partial \mathcal{L}}{\partial u_{B,k}} u_{B,l} - \delta_l^k \mathcal{L} \tag{1.7}$$

are physical quantities associated with the continuous dynamic system $\mathcal{S}_{\mathrm{d}}$.

Let $\mathcal{S}_{\rm st}$ be a discrete stochastic system, i.e. such a system, whose state X is described by finite or countable number of variables, but there are no dynamic equation for $\mathcal{S}_{\rm st}$. Concept of the state for stochastic system $\mathcal{S}_{\rm st}$ is rather conditional, because there are no dynamic equations, and one can change the number of variables X, describing the state of $\mathcal{S}_{\rm st}$. We shall consider discrete stochastic system $\mathcal{S}_{\rm st}$ as a discrete dynamic system $\mathcal{S}_{\rm d}$, whose evolution is spoiled by influence of some stochastic agent. Then dynamic variables X, describing the state of stochastic system $\mathcal{S}_{\rm st}$ may be considered to be variables describing the state of corresponding dynamic system $\mathcal{S}_{\rm d}$.

Absence of dynamic equations for $\mathcal{S}_{\rm st}$ does not mean that the evolution of $\mathcal{S}_{\rm st}$ is completely random and has nothing regular. One can separate regular component $\mathcal{C}_{\rm reg}$ of the system $\mathcal{S}_{\rm st}$ evolution, if to keep track evolution of many similar independent systems $\mathcal{S}_{\rm st}$, starting from the similarly preparing states. Such a set of dynamic or stochastic systems $\mathcal{S}$ is called the statistical ensemble $\mathcal{E}[N,\mathcal{S}]$ of systems $\mathcal{S}$, where N is the number of systems $\mathcal{S}$ in the ensemble $\mathcal{E}[N,\mathcal{S}]$.

Let us consider the statistical ensemble $\mathcal{E}[N,\mathcal{S}_{\rm st}]$, consisting of many $(N\rightarrow\infty)$ stochastic systems $\mathcal{S}_{\rm st}$. Because of independence of $\mathcal{S}_{\rm st}$ the random components of the state evolution are compensated, but regular ones are accumulated. As a result at $N\rightarrow\infty$ only regular component of the state evolution remains. The state of the statistical ensemble evolves regularly (deterministically). Formally this circumstance is displayed in the fact that the statistical ensemble $\mathcal{E}[\infty,\mathcal{S}_{\rm st}]$ appears to be a dynamic system, for which there exist dynamic equations. But $\mathcal{E}[\infty,\mathcal{S}_{\rm st}]$ is a continuous dynamic system, although $\mathcal{S}_{\rm st}$ is a discrete one.

Existence of regular component in evolution of stochastic systems admits one to work with stochastic systems as with dynamic systems whose evolution is spoiled by presence of random component. But we must change our approach to description of the discrete physical system $\mathcal{S}$ evolution. Any discrete physical system ($\mathcal{S}_{\rm st}$ and $\mathcal{S}_{\rm d}$) is considered to be a stochastic system $\mathcal{S}_{\rm st}$, which contains regular evolution component $\mathcal{C}_{\rm reg}$ and stochastic evolution component $\mathcal{C}_{\rm st}$. Dynamic system $\mathcal{S}_{\rm d}$ (deterministic physical system) is a special case of $\mathcal{S}$, when the stochastic evolution component vanishes $\mathcal{C}_{\rm st}=0$. The regular evolution component $\mathcal{C}_{\rm reg}$ of the discrete physical system $\mathcal{S}$ is described as the statistical average dynamic system $\langle\mathcal{S}\rangle$. The statistical average system $\langle\mathcal{S}\rangle$ is a continuous dynamic system even in the case, when $\mathcal{S}$ is a discrete dynamic system $\mathcal{S}_{\rm d}$. In other words, $\langle\mathcal{S}_{\rm d}\rangle$ does not coincide with $\mathcal{S}_{\rm d}$, because $\langle\mathcal{S}_{\rm d}\rangle$ is a continuous dynamic system, whereas $\mathcal{S}_{\rm d}$ is a discrete one.

Such a situation arises, because description of $\mathcal{S}_{\rm d}$ as a partial case of physical system $\mathcal{S}$ is more informative, than description of $\mathcal{S}_{\rm d}$ as a discrete dynamic system. The statistical average system $\langle\mathcal{S}\rangle$ describes both evolution components $\mathcal{C}_{\rm reg}$ and $\mathcal{C}_{\rm st}$. The regular component $\mathcal{C}_{\rm reg}$ is described, when $\langle\mathcal{S}\rangle$ is considered to be a continuous dynamic system. The stochastic component $\mathcal{C}_{\rm st}$ is described by means of reduction of the action $\mathcal{A}_{\langle\mathcal{S}\rangle}$ for the system $\langle\mathcal{S}\rangle$ to the form of the action $\mathcal{A}_{\mathcal{S}_{\rm red}}$ for the continuous set $\mathcal{S}_{\rm red}\left[\mathcal{S}_{\rm d}\right]$ of identical discrete dynamic systems $\mathcal{S}_{\rm d}$, interacting between themselves. Then the form of interaction of $\mathcal{S}_{\rm d}$ labels and describes implicitly the stochastic evolution component $\mathcal{C}_{\rm st}$.

Formally the stochastic evolution component $\mathcal{C}_{\mathrm{st}}$ is described by introduction of a new dependent dynamic variable. In the simplest case, when the dynamic system $\mathcal{S}_{\mathrm{d}}$ is a free particle, this additional variable is the velocity $\mathbf{u}_{\mathrm{st}}(t,\mathbf{x})$ describing mean value of stochastic evolution component $\mathcal{C}_{\mathrm{st}}$ (random velocity). It looks as follows.

Let $\mathcal{S}_{\mathrm{st}}$ be stochastic particle, whose state X is described by variables $\left\{\mathbf{x}, \frac{d\mathbf{x}}{dt}\right\}$, where $\mathbf{x}$ is the particle position. Evolution of the particle state is stochastic, and there exist no dynamic equations for $\mathcal{S}_{\mathrm{st}}$. Evolution of the state of $\mathcal{S}_{\mathrm{st}}$ contains both regular and stochastic components. To separate the regular evolution components, we consider a set (statistical ensemble) $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ of many independent identical stochastic particles $\mathcal{S}_{\mathrm{st}}$. All stochastic particles $\mathcal{S}_{\mathrm{st}}$ start from the same initial state. It means that all $\mathcal{S}_{\mathrm{st}}$ are prepared in the same way. If the number N of $\mathcal{S}_{\mathrm{st}}$ is very large, the stochastic elements of evolution compensate each other, but regular ones are accumulated. In the limit $N\to\infty$ the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ turns to a dynamic system, whose state evolves according to some dynamic equations.

Let the statistical ensemble $\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}]$ of deterministic classical particles $\mathcal{S}_{\mathrm{d}}$ be described by the action $\mathcal{A}_{\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}(P)]}$, where P are parameters describing $\mathcal{S}_{\mathrm{d}}$ (for instance, mass, charge). Let under influence of some stochastic agent the deterministic particle $\mathcal{S}_{\mathrm{d}}$ turn to a stochastic particle $\mathcal{S}_{\mathrm{st}}$. The action $\mathcal{A}_{\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]}$ for the statistical ensemble $\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]$ is reduced to the action $\mathcal{A}_{\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}]}=\mathcal{A}_{\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]}$ for some set $\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}]$ of identical interacting deterministic particles $\mathcal{S}_{\mathrm{d}}$. The action $\mathcal{A}_{\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}]}$ as a functional of $\mathcal{S}_{\mathrm{d}}$ has the form $\mathcal{A}_{\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}(P_{\mathrm{eff}})]}$, where parameters P_{eff} are parameters P of the deterministic particle $\mathcal{S}_{\mathrm{d}}$, averaged over the statistical ensemble, and this averaging describes interaction of particles $\mathcal{S}_{\mathrm{d}}$ in the set $\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}]$. It means that

$$\mathcal{A}_{\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]}=\mathcal{A}_{\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}(P)]}=\mathcal{A}_{\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}(P_{\mathrm{eff}})]} \tag{1.8}$$

In other words, stochasticity of particles $\mathcal{S}_{\mathrm{st}}$ in the ensemble $\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]$ is replaced by interaction of $\mathcal{S}_{\mathrm{d}}$ in $\mathcal{S}_{\mathrm{red}}[\mathcal{S}_{\mathrm{d}}]$, and this interaction is described by a change

$$P\to P_{\mathrm{eff}} \tag{1.9}$$

in the action $\mathcal{A}_{\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}(P)]}$.

The action for the ensemble $\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}]$ of free deterministic particles $\mathcal{S}_{\mathrm{d}}$ has the form

$$\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}]: \qquad \mathcal{A}_{\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}(m)]}[\mathbf{x}]=\int L\left(\mathbf{x},\frac{d\mathbf{x}}{dt}\right)dt\,d\boldsymbol{\xi}, \tag{1.10}$$

where the Lagrangian function is described by the relation

$$L\left(\mathbf{x},\frac{d\mathbf{x}}{dt}\right)=-mc^2+\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 \tag{1.11}$$

$\mathbf{x}=\mathbf{x}(t,\boldsymbol{\xi})$, $\boldsymbol{\xi}=\{\xi_1,\xi_2,\xi_3\}$. Here variables $\boldsymbol{\xi}$ label the particles $\mathcal{S}_{\mathrm{d}}$ of the statistical ensemble $\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}]$. The action (1.10) describes some fluid without pressure.

The mass m is the only parameter of a free noncharged particle. If the stochasticity is the quantum one, the change (1.9) has the form

$$m\to m_{\mathrm{eff}}=m\left(1-\frac{\mathbf{u}^2}{2c^2}+\frac{\hbar}{2mc^2}\boldsymbol{\nabla}\mathbf{u}\right) \tag{1.12}$$

where $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$ is the mean value of the stochastic velocity component. Quantum constant $\hbar$ appears here as a coupling constant, describing connection between the regular and stochastic components of particle motion. The velocity $\mathbf{u}$ is supposed to be small with respect to c. Then we must make the change (1.12) only in the first term of (1.11), because the same change in the second term give the quantity of the order $O(c^{-2})$. After change (1.12) the action (1.10) turns to the action

$$\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]: \qquad \mathcal{A}_{\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}(m_{\mathrm{eff}})]}[\mathbf{x}, \mathbf{u}] = \int L\left(\mathbf{x}, \frac{d\mathbf{x}}{dt}\right) + L_{\mathrm{st}}(\mathbf{u}, \nabla\mathbf{u})\, dt d\boldsymbol{\xi}, \tag{1.13}$$

$$L_{\mathrm{st}}(\mathbf{u}, \nabla\mathbf{u}) = \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\nabla\mathbf{u} \tag{1.14}$$

where $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$, $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, but $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$ is a function of $t, \mathbf{x}$. The action (1.13) also describes a fluid, but now it is a fluid with a pressure, and its irrotational flow is described by the Schrödinger equation [5].

Thus, essentially the change (1.12) describes *procedure of quantization*, because it introduces quantum constant $\hbar$ and additional dynamic variable $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$, describing stochastic component. The action (1.13) describes the free quantum particle $\langle \mathcal{S}_{\mathrm{st}} \rangle$ completely, and other additional suppositions (for instance, quantum principles) are not necessary. In principle, the change (1.9) admits some modification or correction of the quantization procedure, provided the change (1.12) is replaced by some other proper change. In conventional quantum theory such a modification of quantization procedure is impossible, because it is determined by the quantum principles. In other words, quantum system is only one of possible stochastic systems.

Description of physical system $\mathcal{S}$ by means of the statistical average system is a statistical description of $\mathcal{S}$, because it refers to many identical systems $\mathcal{S}$. The statistical description cannot predict result of a measurement of some physical quantity $\mathcal{R}$. Statistical description can predict only probability $w(R')$ of measurement result R', but not the result R' itself. It means that any prediction of the statistical description is tested by a mass experiment (M-measurement), i.e. by a set of many similar single experiments. It is valid even in the case, when result of prediction is quite definite, for instance, if $w(R') = 1$, or $w(R') = 0$. To verify that the prediction $w(R') = 1$ is valid, it is not sufficient to produce one measurement an to obtain the result R'. It is necessary to produce many measurements and to obtain the result R' in all cases.

Description of $\mathcal{S}_{\mathrm{d}}$ as a discrete dynamic system supposes a priori knowledge, that $\mathcal{S}_{\mathrm{d}}$ is a deterministic physical system. The fact that $\mathcal{S}_{\mathrm{d}}$ is a deterministic physical system is not tested in experiment. It is supposed that repeated experiments give the same result, and there is no necessity to verify this. This approach is valid, only if our world is deterministic completely. It cannot be used, if there are stochastic physical systems, and one cannot state a priori, whether the given physical system is deterministic or not.

Describing the discrete dynamic system $\mathcal{S}_{\mathrm{d}}$ as a discrete dynamic system, one predicts result of measurement of some quantity $\mathcal{R}$, but not its probability. It is possible only, if all possible physical systems are deterministic, and one is sure a priori, that the considered physical system is a dynamic system.

Real world is not deterministic completely. There are stochastic physical systems (for instance, quantum systems), and one should describe statistically even those physical systems which are supposed to be deterministic. If the physical system $\mathcal{S}$, being in reality $\mathcal{S}_{\rm d}$, is described as $\langle\mathcal{S}\rangle$, then the deterministic character of $\mathcal{S}$ can be obtained from the form of the statistical description in terms of the statistical average system $\langle\mathcal{S}\rangle$. In other words, one should describe any physical system $\mathcal{S}$ as a stochastic system $\mathcal{S}_{\rm st}$, using the statistical average system $\langle\mathcal{S}\rangle$ as a main object of dynamics, which describes both regular and stochastic evolution components of physical system. Such an approach to the dynamics of physical systems will be referred to as dynamic conception of statistical description (DCSD).

Let $\mathcal{E}\left[N,\mathcal{S}\right]$ be a statistical ensemble, consisting of N independent identical physical systems $\mathcal{S}$. If N is large enough ($N\rightarrow\infty$), $\mathcal{E}\left[N,\mathcal{S}\right]$ is a continuous dynamic system independently of whether $\mathcal{S}$ is stochastic or deterministic. Let $\mathcal{A}_{\mathcal{E}[N,\mathcal{S}]}$ be the action for the statistical ensemble $\mathcal{E}\left[N,\mathcal{S}\right]$. As far as the statistical ensemble consists of identical independent systems, its action $\mathcal{A}_{\mathcal{E}[N,\mathcal{S}]}$ has the property

$$\mathcal{A}_{\mathcal{E}[aN,\mathcal{S}]}=a\mathcal{A}_{\mathcal{E}[N,\mathcal{S}]},\qquad a>0,\qquad a=\text{const},\qquad N,aN\gg 1 \tag{1.15}$$

Let us consider statistical ensemble, whose action $\mathcal{A}_{\langle\mathcal{S}\rangle}$ has the form

$$\mathcal{A}_{\langle\mathcal{S}\rangle}=\lim_{N\rightarrow\infty}\frac{1}{N}\mathcal{A}_{\mathcal{E}[N,\mathcal{S}]} \tag{1.16}$$

Deterministic physical system, whose action has the form (1.16), will be referred to as statistical average dynamic system $\langle\mathcal{S}\rangle$, because the action $\mathcal{A}_{\langle\mathcal{S}\rangle}$ is the mean action for the statistical ensemble $\mathcal{E}\left[N,\mathcal{S}\right]$. The system $\langle\mathcal{S}\rangle$ is the statistical ensemble $\mathcal{E}\left[N,\mathcal{S}\right]$, normalized to one system. According to definition (1.16) and the property (1.15) the action $\mathcal{A}_{\langle\mathcal{S}\rangle}$ of the statistical average system $\langle\mathcal{S}\rangle$ is invariant with respect to transformation

$$N\rightarrow aN,\qquad a>0,\qquad a=\text{const} \tag{1.17}$$

Formally the statistical average system $\langle\mathcal{S}\rangle$ may be considered as a statistical ensemble consisting of one system $\mathcal{S}$. At the same time according to (1.16) this statistical ensemble $\langle\mathcal{S}\rangle$ is a deterministic physical system, because there is an action for $\langle\mathcal{S}\rangle$ and this action is constructed of the action for the statistical ensemble $\mathcal{E}\left[N,\mathcal{S}\right]$ with very large number N of elements ($N\rightarrow\infty$).

For non-relativistic deterministic system $\mathcal{S}_{\rm d}$ the state X is a point in the phase space. The state of the ensemble $\mathcal{E}\left[\mathcal{S}_{\rm d}\right]$ is the density $d\left(X\right)$ of states X in the phase space. The quantity $dN=d(X)dX$ may be regarded as the number of the ensemble elements, whose state is found inside the interval $(X,X+dX)$. The number $d(X)dX$ of states inside the phase volume dX is always nonnegative. At the proper normalization the quantity $d\left(X\right)$ can be interpreted as a probability density. In this case of non-relativistic $\mathcal{S}_{\rm d}$ one can calculate the mean value $\langle f\left(X\right)\rangle$ over the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm d}\right]$ of any function f of the state X of $\mathcal{S}_{\rm d}$.

In the general case of relativistic stochastic system $\mathcal{S}_{\rm st}$ one uses a statistical ensemble $\mathcal{E}[N,\mathcal{S}_{\rm st}]$ for description of evolution of the stochastic system $\mathcal{S}_{\rm st}$. The state of the ensemble $\mathcal{E}[N,\mathcal{S}]$ is also determined by the quantity $d(X)$, which describes distribution (density) of states X of single elements $\mathcal{S}$ of the ensemble $\mathcal{E}[N,\mathcal{S}]$. But in the case of relativistic system $\mathcal{S}_{\rm st}$ is not a density of points in the phase space. The state $d(X)$ is rather complicated tensor quantity. Rank of tensor $d(X)$ depends on the number of particles in the physical system $\mathcal{S}_{\rm st}$.

In any case the quantity $d(X)$ is not a scalar and cannot be interpreted as a probability density, because it may be negative. It leads to the fact that the number dN of the ensemble elements with the state lying in the interval $(X, X+dX)$, may be negative. Possibility of negative number of the ensemble elements is connected with existence of dynamic systems (antisystems), evolving in the direction inverse with respect to the time increase direction. For instance, a particle is described by its world line $\mathcal{L}$ in the space-time. It is supposed that the world line has an orientation (unit vector n^i, tangent to $\mathcal{L}$). This vector describes direction of the system evolution. If this vector is directed towards the future, $\mathcal{L}$ describes evolution of a particle. If it is directed towards the past, $\mathcal{L}$ describes evolution of an antiparticle. Thus, particle and antiparticle distinguish in the direction of time evolution, i.e. in the sign of the temporal component n^0 of the orientation vector n^i.

As far as in the relativistic case one cannot introduce probability density of the state X, one cannot calculate, in general, the mean value $\langle f(X)\rangle$. But one can calculate mean values $\langle A\rangle$ of additive quantities A: number of particles n, energy E, momentum $\mathbf{P}$, angular momentum $M^{\alpha\beta}$ and their densities. It appears to be possible, because the statistical ensemble $\mathcal{E}[N,\mathcal{S}]$ is a set of independent dynamic systems $\mathcal{S}$, and, if E is the energy of $\mathcal{S}$, then the energy of $\mathcal{E}[N,\mathcal{S}]$ is a sum of energies of single systems $\mathcal{S}$, constituting $\mathcal{E}[N,\mathcal{S}]$. But the mean value $\langle f(A)\rangle$ of some function f of the additive quantities $A = n, E, \mathbf{P}, M^{\alpha\beta}$ cannot be determined, in general. For instance, $\langle E^2\rangle$ cannot be calculated, in general. Such a description of the stochastic system dynamics is less informative, than the conventional probabilistic description. The loss of description informativity is connected with the fact that the distribution $d(X)$ may be negative and cannot be regarded as the probability density of the state X.

There are several reasons, why in the relativistic case the distribution $d(X)$ may be negative . We point only one of them: description of production and annihilation of particle pairs.

Particle and antiparticle are two different states of one physical object – world line. The world line, *considered to be a physical object* (but not as a method of description of the particle evolution) will be denoted by WL. It should keep in mind that consideration of a particle and an antiparticle as two different states of WL is connected with the circumstance that consideration of evolution of the physical system in the reverse direction is necessary, if we want, that the number of physical systems conserves in the dynamic theory, and in the same time the theory could describe such processes as production and annihilation of particle pairs. If a particle and an antiparticle are considered to be two different physical systems distinguishing in value of some parameter, then in the process of particle – antiparticle annihilation the number of physical systems decreases by two units. It is unclear, how one could describe a change of number of physical systems in the classical theory. If one assumes that WL is a real physical object, then section of WL by hyperplane t =const (SWL) form two kinds of pointlike objects (particle and antiparticle). Number dN of this pointlike objects (SWL) in the 3-volume dS_k is defined by the relation

$$dN = j^k dS_k$$

where j^k is the proportionality coefficient, describing density of SWLs inside the 3-volume dS_k. The number dN of SWLs may have any sign, because component j^0 may have any sign for fixed dS_k. We assume that sign of dN is positive for particles and negative for antiparticles. The term 'SWL' is used as a collective concept with respect to concepts 'particle' and 'antiparticle'. If for instance, there are a particle and an antiparticle, the total number of SWLs is equal to zero, and this number does not change in the process of annihilation. If we connect a separate dynamic system with any SWL, then to conserve the number of dynamic systems (SWLs) in the dynamic theory, it is necessary to consider distributions $d(X)$ (in the give case $d(X) = j^k(x)$), where d may be any real number.

The fact that the distribution $d(X)$ describing the state of the statistical ensemble $\mathcal{E}[N, \mathcal{S}_{\rm st}]$ may be negative, presents some problems for interpretation. In particular, $d(X)$ cannot be regarded as the probability density. For instance, the state, where the number of physical objects is equal to 1, cannot be considered as the state with one particle. It may be the state with two particles and one antiparticle, or the state with three particles and two antiparticles, etc.. But only such a description of evolution of the statistical ensemble $\mathcal{E}[\infty, \mathcal{S}]$, where the number of physical systems does not change, is possible in classical theory. Describing relativistic stochastic systems, we are forced to use dynamic conception of statistical description (DCSD), which cannot use the probability theory for statistical description. DCSD is more general, but less informative conception of statistical description, than the conventional probabilistic statistical description.

In the case of non-relativistic deterministic systems $\mathcal{S}$ it is possible non-relativistic (probabilistic) version of the statistical ensemble $\mathcal{E}[\infty, \mathcal{S}]$ description. But in the case of non-relativistic stochastic systems $\mathcal{S}$ the statistical ensemble $\mathcal{E}[\infty, \mathcal{S}]$ should be described, using the general method of DCSD, because the stochastic component of evolution may be relativistic even in the case, when the regular component of evolution is non-relativistic.

Considering deterministic physical system $\mathcal{S}_{\rm d}$ as a special case of physical system $\mathcal{S}$, we must consider and investigate $\langle \mathcal{S}_{\rm d} \rangle$ instead of $\mathcal{S}_{\rm d}$. Only then the methods of description of $\mathcal{S}_{\rm d}$ and of $\mathcal{S}_{\rm st}$ appear to be similar. Indeed, if $\mathcal{S}_{\rm st}$ is described as $\langle \mathcal{S}_{\rm st} \rangle$, we need to describe $\mathcal{S}$ as $\langle \mathcal{S}_{\rm st} \rangle$ even in the case, when stochastic component of evolution of $\mathcal{S}$ tends to zero, and $\mathcal{S} \to \mathcal{S}_{\rm d}$. In this case description of $\mathcal{S}$ tends to description of $\langle \mathcal{S}_{\rm d} \rangle$, but not to description of $\mathcal{S}_{\rm d}$. From formal point of view it is very reasonable to describe all $\mathcal{S}$ as $\langle \mathcal{S}_{\rm st} \rangle$ to avoid a jump in description, when $\mathcal{S}$ coincide with $\mathcal{S}_{\rm d}$.

Description of $\mathcal{S}$ as $\langle\mathcal{S}\rangle$, even in the case, when $\mathcal{S} = \mathcal{S}_{\rm d}$ leads to the circumstance that the basic object of dynamics is always the statistical average system $\langle\mathcal{S}\rangle$. It means that the dynamic formalism deals only with statistical average systems $\langle\mathcal{S}\rangle$. But measurements under dynamic system $\mathcal{S}$ deal with $\mathcal{S}$ and $\langle\mathcal{S}\rangle$. It is very important one to distinguish between $\langle\mathcal{S}\rangle$ and $\mathcal{S}$ at consideration of experiments with stochastic and deterministic physical systems, because confusion of $\langle\mathcal{S}\rangle$ and $\mathcal{S}$ leads to misunderstandings and paradoxes.

Our conception of physical system dynamics may be qualified as dynamic conception of statistical description (DCSD). This conception carries out a statistical description, because its basic object: statistical average system $\langle\mathcal{S}\rangle$ is constructed on the basis of many identical objects $\mathcal{S}$. Simultaneously the statistical description of physical systems is produced in the non-probabilistic dynamic form, i.e. without a use of the probability theory.

In DCSD the statistical average system $\langle\mathcal{S}\rangle$ is considered to be a basic object of dynamics. The recent development of the space-time geometry [1, 2, 3] stimulates such an approach. According to this space-time theory the space-time geometry is a reason of the fact that the motion of particle appears to be primordially stochastic. The role of this stochasticity is the more, the less is the particle mass. It means that the deterministic motion of particles should be explained via stochastic one, but not vice versa, as one attempted to do earlier, when the particle motion in the space-time was considered to be primordially deterministic.

In the second section different methods of the statistical average system $\langle\mathcal{S}\rangle$ description are considered. In the third section peculiarities of measurement in stochastic systems are investigated.

2 Methods of description of statistical ensembles

The pure statistical ensemble $\mathcal{E}_{\rm p}[\mathcal{S}]$ of physical systems $\mathcal{S}$ is considered to be a fundamental object of the theory. In the non-relativistic theory the pure statistical ensemble $\mathcal{E}_{\rm p}[\mathcal{S}_{\rm d}]$ of deterministic physical systems $\mathcal{S}_{\rm d}$ is defined as a statistical ensemble, whose distribution function $F_{\rm p}(t,\mathbf{x},\mathbf{p})$ in the phase space of Hamiltonian variables $\mathbf{x} = \{x^\alpha\}$, $\mathbf{p} = \{p_\alpha\}$, $\alpha = 1,2,...n$ may be represented in the form

$$\mathcal{E}_{\rm p}[\mathcal{S}_{\rm d}]: \qquad F_{\rm p}(t,\mathbf{x},\mathbf{p}) = \rho(t,\mathbf{x})\,\delta(\mathbf{p} - \mathbf{P}(t,\mathbf{x})) \tag{2.1}$$

where $\rho(t,\mathbf{x})$ and $\mathbf{P}(t,\mathbf{x}) = \{P_\alpha(t,\mathbf{x})\}$, $\alpha = 1,2,...n$ are functions of only time t and generalized coordinates $\mathbf{x}$. In other words, the pure ensemble $\mathcal{E}_{\rm p}[\mathcal{S}_{\rm d}]$ is a dynamic system, considered in the configuration space V_n of coordinates $\mathbf{x}$. Relativistic generalization of the pure statistical ensemble, defined in the form (2.1) is difficult, and for such a generalization we use another forms of the pure statistical ensemble representation. There are several different forms.

The system (2.1) is a fluidlike dynamic system in n-dimensional space of V_n of coordinate $\mathbf{x}$. The action for $\mathcal{E}_{\rm p}[\mathcal{S}_{\rm d}]$ is written as a sum of actions for independent dynamic systems $\mathcal{S}_{\rm d}$, which are described by the Lagrangian function $L\left(t,\mathbf{x},\frac{d\mathbf{x}}{dt}\right)$.

The action functional for the dynamic system, described by the distribution function (2.1) can be written as follows

$$\mathcal{E}_{\mathrm{p}}\left[\mathcal{S}_{\mathrm{d}}\right]: \qquad \mathcal{A}_L[\mathbf{x}] = \int\limits_{V_\xi} \mathrm{d}^n\boldsymbol{\xi} \int L(t,\mathbf{x},\frac{d\mathbf{x}}{dt})\mathrm{d}t, \qquad \mathrm{d}^n\boldsymbol{\xi} = \prod_{\alpha=1}^{\alpha=n} \mathrm{d}\xi_\alpha, \tag{2.2}$$

where dependent variables $\mathbf{x} = \{x^\alpha\}$, $\alpha = 1,2,...n$, are considered to be functions of independent variables $\xi = \{t,\boldsymbol{\xi}\} = \{\xi_0,\boldsymbol{\xi}\} = \{\xi_k\}$, $k = 0,1,...n$. Variables $\boldsymbol{\xi}$ label elements $\mathcal{S}_{\mathrm{d}}$ of the statistical ensemble and change inside the region V_ξ.

The number N of elements $\mathcal{S}_{\mathrm{d}}$ in the statistical ensemble $\mathcal{E}_{\mathrm{p}}\left[\mathcal{S}_{\mathrm{d}}\right]$ is defined by the relation

$$N = \int\limits_{V_\xi} \mathrm{d}^n\boldsymbol{\xi} = \int\limits_{V_x} \rho \mathrm{d}^n x, \qquad \rho = \frac{\partial\left(\xi_1,\xi_2,...\xi_n\right)}{\partial\left(x^1,x^2,...x^n\right)} \tag{2.3}$$

where V_x is the region in the space V_n of coordinates $\mathbf{x}$, occupied by the fluid. The number N is supposed to be very large $N \to \infty$. Properties of the statistical ensemble do not depend on the number N of its elements, provided N is large enough. The action (2.2) is appropriate for description of relativistic dynamic systems $\mathcal{S}_{\mathrm{d}}$. Generalization on the case of stochastic systems $\mathcal{S}_{\mathrm{st}}$ is also possible, although it is not so evident.

Let us make a change of variables in (2.2) and consider independent variables $\boldsymbol{\xi}$ as functions of variables $\mathbf{x}$, or functions $\xi = \{\xi_i\}$, $i = 0,1,...n$ as functions of variables $x = \{x^i\}$, $i = 0,1,...n$. with ξ_0 being a fictitious variable, and $x^0 = t$ being the time. Then after integration of some dynamic equations the action (2.2) takes the form [4, 5]

$$\mathcal{E}_{\mathrm{p}}\left[\mathcal{S}_{\mathrm{d}}\right]: \quad \mathcal{A}_E[j,\varphi,\boldsymbol{\xi}] = \int \{L(x^0,\mathbf{x},\frac{\mathbf{j}}{j^0})j^0 - j^i p_i\}\mathrm{d}^{n+1}x, \tag{2.4}$$

$$p_i = b_0\left[\partial_i\varphi + g^\alpha(\boldsymbol{\xi})\partial_i\xi_\alpha\right], \qquad i = 0,1,...,n,, \qquad \partial_i \equiv \frac{\partial}{\partial x^i} \tag{2.5}$$

where dependent variables ρ, φ, $\boldsymbol{\xi} = \{\xi_\alpha\}$, $\alpha = 1,2,...n$ of the dynamic system $\mathcal{E}_{\mathrm{p}}\left[\mathcal{S}_{\mathrm{d}}\right]$ are functions of independent variables $t,\mathbf{x}$, where $\mathbf{x} = \{x^\alpha\}$, $\alpha = 1,2,...n$. Here and below a summation is produced over repeating Greek indices from 1 to n and over Latin ones from 0 to n. The flux of this fluid is described by the vector j^i

$$j^i = \frac{\partial J}{\partial\xi_{0,i}} = \frac{\partial\left(x^i,\xi_1,\xi_2,...\xi_n\right)}{\partial\left(x^0,x^1,...x^n\right)} = J\frac{\partial x^i}{\partial x^0}, \qquad i = 0,1,...n, \qquad \rho = j^0 = \frac{\partial J}{\partial\xi_{0,0}} \tag{2.6}$$

where the Jacobian J is considered to be multilinear function of variables $\xi_{i,k} \equiv \partial_k\xi_i$, $i,k = 0,1,...n$

$$J \equiv \frac{\partial\left(\xi_0,\xi_1,...\xi_n\right)}{\partial\left(x^0,x^1,...x^n\right)} \equiv \det\left|\left|\xi_{i,k}\right|\right|, \qquad \xi_{i,k} \equiv \frac{\partial\xi_i}{\partial x^k} \equiv \partial_k\xi_i \qquad i,k = 0,1,...n \tag{2.7}$$

The variable $\rho = j^0$ describes density of the ensemble elements (density of the fluid). The quantity b_0 is an arbitrary nonvanishing constant, the functions $\mathbf{g}\left(\boldsymbol{\xi}\right) =$

$\{g^{\alpha}(\boldsymbol{\xi})\}$, $\alpha = 1, 2, ...n$ are arbitrary functions of argument $\boldsymbol{\xi}$. Both b_0 and $\mathbf{g}$ are a result of integration of dynamic equations which was made in the process of a change of variables [4].

Attributes of dynamic system $\mathcal{E}_{\text{p}}[\mathcal{S}_{\text{d}}]$ are the number N of dynamic systems $\mathcal{S}_{\text{d}}$ (particles), flux j^k of $\mathcal{S}_{\text{d}}$ (particles), the energy-momentum tensor T_l^k

$$T_l^k = -p_l j^k - \left\{ L(x^0, \mathbf{x}, \frac{\mathbf{j}}{j^0}) j^0 - j^i p_i \right\} \delta_l^k, \tag{2.8}$$

and the angular momentum density M^{ikl}. The momentum density p_k and the particle flux j^k are expressed by equations (2.5) and (2.6) respectively. The number N of $\mathcal{S}_{\text{d}}$ in $\mathcal{E}_{\text{p}}[\mathcal{S}_{\text{d}}]$ is described by the relation (2.3). Transformation of the number N of the form (1.17)

$$N \to \tilde{N} = a^n N \tag{2.9}$$

arises at the transformation of $\boldsymbol{\xi}$ of the form

$$\boldsymbol{\xi} \to \tilde{\boldsymbol{\xi}} = a\boldsymbol{\xi} \tag{2.10}$$

At this transformation

$$j^k \to \tilde{j}^k = a^n j^k, \qquad g^{\alpha}(\boldsymbol{\xi}) \to \tilde{g}^{\alpha}(\tilde{\boldsymbol{\xi}}) = g^{\alpha}(\boldsymbol{\xi})/a. \tag{2.11}$$

Then the action (2.4) transforms in the same way as N, i.e.

$$\mathcal{A} \to a^n \mathcal{A}. \tag{2.12}$$

It means that the action (2.4) describes a statistical ensemble. The action (2.4) describes statistical ensemble of deterministic physical systems. It follows from the fact that the action (2.4) contain derivatives only in direction of the vector j^k in the form $j^k \partial_k \quad (\partial_k j^k = 0)$. In this case the system of partial differential equations reduces to a system of ordinary differential equations. For the action (2.4) to describe a statistical ensemble of stochastic systems, it is sufficient to modify it in such a way, that the action stops to depend only on $j^k \partial_k$, but at transformations (2.10), (2.11) the action (2.4) continues to transform according to (2.12).

Eliminating the variables j^k from the action (2.4) [5], one obtains instead of (2.4)

$$\mathcal{E}_{\text{p}}[\mathcal{S}_{\text{d}}]: \qquad \mathcal{A}_E[\rho, \varphi, \boldsymbol{\xi}] = \int \rho \{-H(t, \mathbf{x}, \mathbf{p}) - p_0\} \mathrm{d}^{n+1}x, \tag{2.13}$$

The quantity $H(t, \mathbf{x}, \mathbf{p}) = H(x^0, \mathbf{x}, \mathbf{p})$ is the Hamilton function of the dynamic system $\mathcal{S}_{\text{d}}$. The variable $\rho = j^0$ is defined by (2.6), and the quantities p_k are defined by relations (2.5). The dynamic variables $\boldsymbol{\xi}$ are regarded as Lagrangian coordinates, or hydrodynamic potentials (Clebsch potentials [6, 7]) which are constant along world line of any fluid particle.

All three forms (2.2), (2.13), (2.4) of the action are equivalent and describe the same variational problem [5].

By means of a change of variables the action (2.13) can be transformed to a description in terms of a wave function [4]. Let us introduce k-component complex function. $\psi = \{\psi_\alpha\}$, $\alpha = 1, 2, \ldots k$, defining it by the relations

$$\psi_\alpha = \sqrt{\rho} e^{i\varphi} u_\alpha(\boldsymbol{\xi}), \qquad \psi_\alpha^* = \sqrt{\rho} e^{-i\varphi} u_\alpha^*(\boldsymbol{\xi}), \qquad \alpha = 1, 2, \ldots k \tag{2.14}$$

$$\psi^*\psi \equiv \sum_{\alpha=1}^{k} \psi_\alpha^* \psi_\alpha$$

where (*) means the complex conjugate, $u_\alpha(\boldsymbol{\xi})$, $\alpha = 1, 2, \ldots k$ are functions of only variables $\boldsymbol{\xi}$. They satisfy the relations

$$-\frac{i}{2}\sum_{\alpha=1}^{k}(u_\alpha^* \frac{\partial u_\alpha}{\partial \xi_\beta} - \frac{\partial u_\alpha^*}{\partial \xi_\beta} u_\alpha) = g^\beta(\boldsymbol{\xi}), \qquad \beta = 1, 2, ...n, \qquad \sum_{\alpha=1}^{k} u_\alpha^* u_\alpha = 1 \tag{2.15}$$

k is such a natural number that equations (2.15) admit a solution. In general, k depends on the form of the arbitrary integration functions $\mathbf{g} = \{g^\beta(\boldsymbol{\xi})\}$, $\beta = 1, 2, ...n$.

It is easy to verify, that

$$\rho = \psi^*\psi, \qquad p_l = -\frac{ib_0}{2\psi^*\psi}(\psi^*\partial_l\psi - \partial_l\psi^* \cdot \psi), \qquad l = 0, 1, ...n \tag{2.16}$$

The variational problem with the action (2.13) appears to be equivalent [4] to the variational problem with the action functional

$$\begin{aligned} \mathcal{A}[\psi, \psi^*] \quad = \quad & \int \left\{ \frac{ib_0}{2}(\psi^*\partial_0\psi - \partial_0\psi^* \cdot \psi) \right. \\ & \left. -H\left(x, -\frac{ib_0}{2\psi^*\psi}(\psi^*\nabla\psi - \nabla\psi^* \cdot \psi)\right)\psi^*\psi \right\} \mathrm{d}^{n+1}x \end{aligned} \tag{2.17}$$

where $\nabla = \{\partial_\alpha\}$, $\alpha = 1, 2, ...n$.

At the description by means of the action (2.17) the transformation (2.9) of the number N of $\mathcal{S}_\mathrm{d}$ arises at the transformation

$$\psi \to \tilde{\psi} = \alpha^{n/2}\psi \tag{2.18}$$

when the flux $\mathbf{j}$ and density $\rho = j^0$

$$\mathbf{j} = -\frac{ib_0}{2}(\psi^*\nabla\psi - \nabla\psi^* \cdot \psi), \qquad \rho = \psi^*\psi \tag{2.19}$$

transforms according to (2.11). Then action (2.17) transforms as (2.12), and hence it is the action for a statistical ensemble. As in the case of action (2.4) the action (2.13) and (2.17) also contain derivatives only in combination $j^k\partial_k$ ($\partial_k j^k = 0$), but it is not seen explicitly. To show this for (2.13), one should use definition of the Hamilton function

$$H(\mathbf{x}, \mathbf{p}) = \dot{\mathbf{x}}\mathbf{p} - L(\mathbf{x}, \dot{\mathbf{x}}), \qquad \mathbf{p} = \frac{\partial L}{\partial \dot{\mathbf{x}}}$$

Then taking into account that $\mathbf{j} = \dot{\mathbf{x}} j^0 = \dot{\mathbf{x}}\rho$, the Lagrange function density of (2.13)

$$\rho\{-H(t,\mathbf{x},\mathbf{p}) - p_0\} = \rho L\left(\mathbf{x}, \frac{\mathbf{j}}{j^0}\right) - \mathbf{j}\mathbf{p} - \rho p_0 = \rho L\left(\mathbf{x}, \frac{\mathbf{j}}{j^0}\right) - j^k p_k$$

reduces to Lagrangian density of (2.4). The same situation with the action (2.17), whose Lagrange function density reduces to Lagrange function density of (2.13).

For the action (2.4) to describe a statistical ensemble of stochastic systems, it is sufficient that the action contains derivatives not only in direction of the flux vector j^k and in the same time the action (2.4) transforms according to (2.12) at the transformation (2.9) of the number of the ensemble elements.

Let us imagine now that the Lagrangian function in the action (2.4) depends additionally on the arguments $s_k = h_k \partial_k \log\rho = h_k \partial_k \log j^0$, $s_{kl} = h_{kl}\partial_k\partial_l \log\rho$, $k,l = 0,1,...n$, (there is no summation over k and l here), where h_k, h_{kl} are some constants. Arguments s_k and s_{kl} are invariant with respect to transformation (2.18), (2.9). If the Lagrangian function L in (2.4) depends on s_k and s_{kl}, it transforms under transformation (2.9) in the same way as if it would not depend on s_k and s_{kl}. It means that it remains to be the action for some statistical ensemble of independent physical systems. But now the action contains derivatives not only in the direction of the vector j^k. Then the system of dynamic equations is essentially such a system of partial differential equations, which cannot be reduced to a system of ordinary differential equations. Hence, the statistical ensemble cannot be considered to be constituted of dynamic systems, because in this case dynamic equations cannot be reduced to a system of ordinary differential equations. Such a situation can be interpreted in the sense that elements of the statistical ensemble are stochastic systems, and there exist no dynamic equations for them.

Of course, the same consideration is valid for other forms (2.13), (2.2), (2.17) of the action. In particular, if the Hamilton function $H(\mathbf{x},\mathbf{p})$ in (2.17) is taken in the form

$$H_{\mathrm{eff}} = mc^2 + \frac{\mathbf{p}^2}{2m} + \frac{\hbar^2}{8m}(\nabla \ln\rho)^2, \tag{2.20}$$

the dynamic equation for the statistical average system $\langle\mathcal{S}\rangle$ has the form of the free Schrödinger equation, provided besides *the arbitrary constant b_0 is chosen to be equal to quantum constant $\hbar$, and the case of the irrotational flow of the fluid* is considered. This case, when the wave function ψ can have only one component, is considered in [4, 5].

It is worth to note that consideration of the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ with the effective Hamilton function, depending on variables s_k and s_{kl} is equivalent to consideration of some set $\mathcal{S}_{\mathrm{q}}[\mathcal{S}_{\mathrm{d}}]$ of interacting identical dynamic systems $\mathcal{S}_{\mathrm{d}}$, which is described by the action (1.13). For instance, when the effective Hamilton function is written in the form (2.20), the corresponding action

$$\mathcal{S}_{\mathrm{q}}[\mathcal{S}_{\mathrm{d}}]: \qquad \mathcal{A}_{\mathrm{q}}[\mathbf{x},\mathbf{u}_{\mathrm{st}}] = \int \left\{\frac{m\dot{\mathbf{x}}^2}{2} + \frac{m\mathbf{u}_{\mathrm{st}}^2}{2} - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}_{\mathrm{st}}\right\} dt\, d^3\boldsymbol{\xi} \tag{2.21}$$

coincides with the action (2.2), obtained after the change (1.12) in the action for the ensemble of deterministic particle. Here $\mathbf{x}=\mathbf{x}(t,\boldsymbol{\xi})$, $\mathbf{u}_{\mathrm{st}}=\mathbf{u}_{\mathrm{st}}(t,\mathbf{x})$ is a new dynamic variable and $\boldsymbol{\nabla}=\left\{\frac{\partial}{\partial x^1},\frac{\partial}{\partial x^2},\frac{\partial}{\partial x^3}\right\}$, The new dynamic variable $\mathbf{u}_{\mathrm{st}}$ is a function of $\mathbf{x}$ and depends on $\boldsymbol{\xi}$ via $\mathbf{x}$. The quantity $\mathbf{u}_{\mathrm{st}}$ may be regarded as the mean velocity of stochastic component. The last term in (2.21) describes influence of the stochasticity on the regular evolution component. Dynamic equations in terms of independent variables $(t,\mathbf{x})$, generated by the action (2.21) for dependent dynamic variables ρ, $\mathbf{u}=\frac{d\mathbf{x}}{dt}$, $\mathbf{u}_{\mathrm{st}}$, $\boldsymbol{\xi}$ have the form

$$\frac{\partial\rho}{\partial t}+\boldsymbol{\nabla}(\rho\mathbf{u})=0,\qquad m\left(\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\boldsymbol{\nabla})\mathbf{u}\right)=-\boldsymbol{\nabla}U_{\mathrm{B}}(t,\mathbf{x})$$

$$\mathbf{u}_{\mathrm{st}}=-\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho,\qquad \frac{\partial\boldsymbol{\xi}}{\partial t}+(\mathbf{u}\boldsymbol{\nabla})\boldsymbol{\xi}=0$$

where

$$\rho=\frac{\partial(\xi_1,\xi_2,\xi_3)}{\partial(x^1,x^2,x^3)},\qquad \mathbf{u}=\frac{\partial(\mathbf{x},\xi_1,\xi_2,\xi_3)}{\partial(t,\xi_1,\xi_2,\xi_3)},\qquad U_{\mathrm{B}}(t,\mathbf{x})=-\frac{\hbar^2}{2m}\frac{1}{\sqrt{\rho}}\nabla^2\sqrt{\rho}$$

where $U_{\mathrm{B}}(t,\mathbf{x})$ is the Bohm potential, considered to be a function of $t,\mathbf{x}$. Introducing

$$R=\rho^{-1}=\frac{\partial(x^1,x^2,x^3)}{\partial(\xi_1,\xi_2,\xi_3)}$$

as a multilinear function of variables $x^{\alpha,\beta}=x^{\alpha,\beta}(t,\boldsymbol{\xi})\equiv\partial x^\alpha/\partial\xi_\beta$ and taking into account that

$$\frac{\partial}{\partial x^\alpha}=\frac{\partial\xi_\beta}{\partial x^\alpha}\frac{\partial}{\partial\xi_\beta}=\frac{1}{R}\frac{\partial R}{\partial x^{\alpha,\beta}}\frac{\partial}{\partial\xi_\beta},$$

one can write dynamic equations in terms of independent variables $t,\boldsymbol{\xi}$

$$m\ddot{x}^\alpha=\frac{\hbar^2}{2mR}\frac{\partial R}{\partial x^{\alpha,\beta}}\frac{\partial}{\partial\xi_\beta}\left[\frac{1}{\sqrt{R}}\frac{\partial R}{\partial x^{\mu,\nu}}\frac{\partial}{\partial\xi_\nu}\left(\frac{1}{R}\frac{\partial R}{\partial x^{\mu,\sigma}}\frac{\partial}{\partial\xi_\sigma}\frac{1}{\sqrt{R}}\right)\right]\tag{2.22}$$

$$u_{\mathrm{st}}^\alpha(t,\boldsymbol{\xi})=\frac{\hbar}{2mR}\frac{\partial R}{\partial x^{\alpha,\beta}}\frac{\partial}{\partial\xi_\beta}\frac{1}{R},$$

This example shows that the free non-relativistic quantum particle is a stochastic system $\mathcal{S}_{\mathrm{st}}$, whose regular component of evolution is described by the statistical average system $\langle\mathcal{S}_{\mathrm{st}}\rangle$. Dynamic equation for $\langle\mathcal{S}_{\mathrm{st}}\rangle$ is the free Schrödinger equation. Technique of quantum mechanics deals with $\langle\mathcal{S}_{\mathrm{st}}\rangle$ only and ignores $\mathcal{S}_{\mathrm{st}}$ completely. But at consideration of quantum measurements one deals with both physical systems $\mathcal{S}_{\mathrm{st}}$ and $\langle\mathcal{S}_{\mathrm{st}}\rangle$. Considering quantum measurements, one confuses sometimes $\mathcal{S}_{\mathrm{st}}$ and $\langle\mathcal{S}_{\mathrm{st}}\rangle$. It leads sometimes to misunderstandings and paradoxes.

Dynamic equations (2.22) may be considered as equations for continuous set of identical deterministic particles labelled by parameters $\boldsymbol{\xi}$. These particles interact between themselves via some self-consistent potential $U_{\mathrm{B}}(t,\mathbf{x})$. Thus, ensemble of independent stochastic systems $\mathcal{E}_{\rm st}[\mathcal{S}_{\rm st}]$ may be considered to be a set $\mathcal{S}_{\rm red}[\mathcal{S}_{\rm d}]$ of interacting deterministic systems $\mathcal{S}_{\rm d}$, and the form of interaction describes the character of stochasticity. At probabilistic description of stochastic systems (for instance, at conventional description of Brownian particles) the character of stochasticity is described by means of the probability of transition $w(X, X')$ from the state X to the state X'. In DCSD a character of stochasticity is described by the form of interaction between deterministic systems $\mathcal{S}_{\rm d}$ in the continuous set $\mathcal{S}_{\rm red}[\mathcal{S}_{\rm d}] = \mathcal{E}_{\rm st}[\mathcal{S}_{\rm st}]$ of deterministic systems $\mathcal{S}_{\rm d}$.

Generalization of the action to the case of stochastic systems has the simplest form in the case (2.17), when the action is described in terms of wave function. For the action to be an action of a statistical ensemble, it is sufficient that the Lagrange function density would be the first order uniform function of the combination $\psi^*\psi$.

3 Measurement in stochastic systems

Let us consider some problems of measurement in stochastic systems $\mathcal{S}_{\rm st}$ in the example of quantum mechanics which is a special case of dynamics of stochastic systems. Quantum mechanics is an axiomatic version of stochastic system dynamics, where the wave function ψ is considered to be a fundamental object of dynamics, whereas in DCSD the wave function is only a method of description of statistical ensemble. In quantum mechanics the Lagrange function density for any quantum system $\mathcal{S}_{\rm q}$ is supposed to be a linear function of ψ and linear function of ψ^*. Then at the transformation (2.8) the condition (2.12) is fulfilled, and the considered dynamic system $\mathcal{S}_{\rm q}$, described by means of the wave function, may be considered to be a pure statistical ensemble $\mathcal{S}_{\rm q} = \mathcal{E}_{\rm p}[\mathcal{S}_{\rm st}]$. If besides the wave function is normalized to unity, the statistical ensemble $\mathcal{S}_{\rm q} = \mathcal{E}_{\rm p}[\mathcal{S}_{\rm st}]$ turns to a statistical average system $\langle\mathcal{S}_{\rm st}\rangle$.

Quantum mechanics is a special case of the stochastic systems dynamics. Quantum systems $\mathcal{S}_{\rm q}$ are special well investigated kind of stochastic systems. Discussing measurement in non-deterministic system we shall keep in mind mainly theory of quantum measurements.

Mathematical side of the quantum measurement theory is developed very well [8, 9, 10]. Unfortunately, practically all authors ignore the fact that the quantum mechanics is a statistical conception, which contains two sorts of objects: individual physical systems $\mathcal{S}$, and statistical average systems $\langle\mathcal{S}\rangle$. Mathematical formalism of quantum mechanics deals only with $\langle\mathcal{S}\rangle$, describing them in terms of wave functions. All predictions of quantum mechanics are produced only for statistical average systems $\langle\mathcal{S}\rangle$, but not for individual systems $\mathcal{S}$.

In the previous section we have seen that the wave function describes the state of the statistical average system $\langle\mathcal{S}\rangle$, or statistical ensemble $\mathcal{E}[\mathcal{S}]$, but not the state of individual system $\mathcal{S}$. Producing a single measurement, we deal mainly with an individual system $\mathcal{S}$.

In fact, in quantum mechanics there are at least two different sorts of measurements: single measurement (S-measurement), i.e. measurement under individual system $\mathcal{S}$ and mass measurement (M-measurement), i.e. measurement under many single systems, constituting the statistical ensemble $\mathcal{E}[\mathcal{S}]$, or the statistical average system $\langle\mathcal{S}\rangle$. Properties of S-measurement and of M-measurement quite different and confusion of them leads to misunderstandings.

The S-measurement of the quantity $\mathcal{R}$ is produced in a single system S by means of some measuring device $\mathcal{M}$. S-measurement leads to a definite result R' and does not change the wave function ψ, describing the state of $\langle S\rangle$. As to the state X of the stochastic system S, it is random and is not considered in quantum mechanics, which describes only reproducible (regular) features of stochastic systems.

The M-measurement of the quantity $\mathcal{R}$ is a set of N ($N\rightarrow\infty$) S-measurements, produced in single systems S, constituting the system $\langle S\rangle$. Result of M -measurement is some distribution $F(R)$ of values R', because different S-measurements, constituting the M-measurement, give, in general, different values R' of the quantity $\mathcal{R}$. After M-measurement the pure state ψ of $\langle S\rangle$ turns, in general, to a mixed state, described by the matrix density $\rho=|\psi_{R'}\rangle\left|\langle\psi|\psi_R\rangle\right|^2\langle\psi_{R'}|$, where $|\psi_{R'}\rangle$ is the eigenfunction of the operator R for the quantity $\mathcal{R}$, corresponding the eigenvalue R'.

According to quantum principles the action of a measuring device $\mathcal{M}$ on the measured statistical average system $\langle\mathcal{S}\rangle$ may only change the Hamiltonian H, describing evolution of statistical average system $\langle\mathcal{S}\rangle$. However, no matter what this change of Hamiltonian may be, the state evolution happens to be a such one that the pure state ψ of statistical average system $\langle\mathcal{S}\rangle$ remains to be pure. Nevertheless, action of the measuring device $\mathcal{M}$ on $\langle\mathcal{S}\rangle$ at the state ψ leads to a passage of $\langle\mathcal{S}\rangle$ into a mixed state ρ. Qualitatively it is explained by that the measuring device $\mathcal{M}$ transforms the total Hamiltonian H into several different Hamiltonians H_1, $H_2,\ldots$, depending on the state φ_k, $k=1,2,\ldots$ of measuring device $\mathcal{M}$.

Let us consider those systems $\mathcal{S}$ of the statistical ensemble $\mathcal{E}[N,\mathcal{S}]$, which gave the value R_1 at the measurement of the quantity $\mathcal{R}$. They form subensemble $\mathcal{E}_1[N_1,\mathcal{S}]$ of the statistical ensemble $\mathcal{E}[N,\mathcal{S}]$. At the fixed value R_1 of the measured quantity $\mathcal{R}$ the measuring device $\mathcal{M}$ is found at the state φ_1. Then subensemble $\mathcal{E}_1[N_1,\mathcal{S}]$ of systems $\mathcal{S}$, which gave the result R_1 at the measurement, evolves with the Hamiltonian H_1. The subensemble $\mathcal{E}_2[N_2,\mathcal{S}]$ of systems $\mathcal{S}$, which gave the result R_2 at the measurement, evolves with the Hamiltonian H_2, because the measuring device $\mathcal{M}$ is found now in another state φ_2. Each value R_k of the measured quantity $\mathcal{R}$ associates with an evolution of the subensemble $\mathcal{E}_k[N_k,\mathcal{S}]$ of the statistical ensemble $\mathcal{E}[N,\mathcal{S}]$ with the Hamiltonian H_k.

The number N_k of systems in the corresponding subensemble $\mathcal{E}_k[N_k,\mathcal{S}]$ is proportional to the probability of the measured value R_k of the measured quantity $\mathcal{R}$. Evolution of different subensembles $\mathcal{E}_k[N_k,\mathcal{S}]$ is different. As a corollary one cannot speak about one wave function, describing the state of the whole ensemble $\mathcal{E}[N,\mathcal{S}]$. One should speak about states of subensembles $\mathcal{E}_k[N_k,\mathcal{S}]$, constituting the statistical ensemble $\mathcal{E}[N,\mathcal{S}]$. Each of subensembles $\mathcal{E}_k[N_k,\mathcal{S}]$ is described by the wave function ψ_k, And the whole ensemble $\mathcal{E}[N,\mathcal{S}]$ is described by the matrix density, i.e. by a set of wave functions ψ_k, taken with statistical weights $\left|\langle\psi|\psi_k\rangle\right|^2$.

In general, any M-measurement may be conceived as an abstract single procedure, produced on the statistical average system $\langle\mathcal{S}\rangle$. Action of M-measurement on statistical average system $\langle\mathcal{S}\rangle$ is described formally by the rule of von Neumann [8].

$$\psi \to |\psi_R\rangle \left|\langle\psi|\psi_R\rangle\right|^2 \langle\psi_R| \tag{3.1}$$

Influence (3.1) of the M-measurement on the state ψ $\langle\mathcal{S}\rangle$ is known as a reduction of the wave function. For the reduction process it is important that result R_k of measurement of the quantity $\mathcal{R}$ be fixed, i.e. that the measuring device $\mathcal{M}$ be found at the state φ_k, because only in this case one can speak on a definite Hamiltonian H_k, which determines evolution of $\mathcal{E}_k[N_k, \mathcal{S}]$. It is this procedure of measurement (M-measurement) that is considered in most of papers [8, 9].

Is it possible to speak on derivation of a definite value R_k at M-measurement of the quantity $\mathcal{R}$ for statistical average system $\langle\mathcal{S}\rangle$? It is possible, but it is a new kind of measurement, so-called selective M-measurement, or SM-measurement. SM-measurement is M-measurement, accompanied by a selection of only those systems $\mathcal{S}$, for which a single S-measurement gives the same measurement result R_k. It is of no importance, who or what carries out this selection. This may be device, human being, or environment. The selection is introduced directly in the definition of the SM-measurement, and it is to be made by anybody. The process of selection of individual systems $\mathcal{S}$ may be interpreted as a statistical influence of the measuring device $\mathcal{M}$ on the statistical average system $\langle\mathcal{S}\rangle$.

The statistical influence in itself is not a force interaction. It is an influence of the measuring device $\mathcal{M}$, leading to a selection of some and discrimination of other elements of the statistical ensemble $\mathcal{E}[\infty, \mathcal{S}]$. In general, one may say on statistical influence of the measuring device $\mathcal{M}$ on the state of the measured statistical average system $\langle\mathcal{S}\rangle$. The state of the system $\langle\mathcal{S}\rangle$ is determined by the distribution $d(X)$ of quantities X, describing the state of the system $\mathcal{S}$. Statistical influence of the measuring device $\mathcal{M}$ on $\langle\mathcal{S}\rangle$ leads to a change of this distribution.

Properties of different kinds of measurement of the quantity $\mathcal{R}$ are presented in the table

type of measur.	*object of measur.*	*state before*	*state after measurement*	*result of meas.*	*influence on the state*
S	S	ψ	ψ	R'	no
M	$\langle S\rangle$	ψ	$\|\psi_R\rangle \left\|\langle\psi\|\psi_R\rangle\right\|^2 \langle\psi_R\|$	$f(R')$	reduction
SM	$\langle S\rangle$	ψ	$\psi_{R'}$	R'	change of pure state.

Note that the quantum mechanics makes predictions, concerning only M-measurements. Any quantum mechanical prediction can be tested only by means of M-measurement. There is no quantum mechanical predictions that could be tested by means of one single measurement (S-measurement). Result of individual measurement (S-measurement) of the quantity $\mathcal{R}$ can be predicted never. Using quantum mechanical technique, one can predict only probability of the fact that the result R' will be obtained at the S-measurement of the quantity $\mathcal{R}$. But a prediction of the result probability does not mean a prediction of the result in itself.

To understand what means the prediction of the result probability from the measurement viewpoint, let us consider the following situation. Let a calculation on the basis of quantum mechanics technique gives, that the probability of obtaining the result R' at the measurement of the quantity $\mathcal{R}$ in the system $\mathcal{S}_{\mathrm{q}}$ at the state $|\psi\rangle$ is equal to 1/2. How can one test that this probability is equal to 1/2, but not, for instance, to 3/4 or 1/4? It is clear that such a test is impossible at one single measurement of the quantity $\mathcal{R}$. To test the prediction, it is necessary to carry out N, $(N \to \infty)$ individual measurements, and the part of measurements, where the value R' of the quantity $\mathcal{R}$ is obtained, gives the value of probability. It is valid even in the case, when predicted probability is equal to unity. In this case for the test of the prediction an individual experiment is also insufficient. To test the prediction, one needs to carry out a set of many individual measurements of the quantity $\mathcal{R}$, and the prediction is valid, provided the value R' is obtained in all cases.

At this point the quantum mechanics distinguishes from the classical mechanics, where results of repeated measurements coincide always. The classical mechanics accepts that two different individual measurements, produced on the systems prepared in the same way give similar results. The classical mechanics supposes that it is possible one not to test this circumstance, and it predicts the value R' of the measured quantity $\mathcal{R}$ (probability of the value R' is accepted to be equal to 1 and is not tested). The deterministic foundation of classical mechanics is manifested in this assumption.

Describing stochastic systems, the quantum mechanics admits that two different individual measurements, produced on the systems prepared in the same way, may give different results, and quantum mechanics predicts only probability of the value R' of the measured quantity $\mathcal{R}$ (but not the value R' itself). At the test only value of the probability is verified (it is tested, even if this value is equal to 1 or to 0). For such a test one needs a M-measurement. In other words, two predictions: (1) "measurement of the quantity $\mathcal{R}$ must give the value R'" and (2) "measurement of the quantity $\mathcal{R}$ must give the value R' with the probability 1" are two different predictions, tested by measurements of different kinds. Predictions of the first type can be made only by classical mechanics, which deals *only* with deterministic physical systems. If some of physical systems are stochastic, then even deterministic physical system is to be considered as a special case of (stochastic) physical system. Then predictions of the first type are impossible. The quantum mechanics deals with stochastic systems. One can make only prediction of the second type, *even in the case, when the physical system evolves as a deterministic one.* This fact is connected with the above considered statement that in the dynamics of the stochastic systems the main object of dynamics is $\langle\mathcal{S}\rangle$, and the evolution of deterministic system $\mathcal{S}_{\mathrm{d}}$ is described as $\langle\mathcal{S}_{\mathrm{d}}\rangle$.

All this means that the quantum mechanical technique and its predictions deal only with mass measurements (M-measurement) and have nothing to do with individual measurements. In general, appearance of the term "probability" in all predictions of quantum mechanics is connected with the fact that the quantum mechanical technique deals only with distributions $d(X)$ of quantities X, which are reproduced at repeated measurements, but not with the quantities X themselves, whose values are random and irreproducible. It means that the quantum mechanics technique deals with statistical average objects $\langle \mathcal{S} \rangle$ (or with statistical ensembles of single systems).

Essentially the quantum mechanics investigates dynamic characteristics of $\langle \mathcal{S} \rangle$ and claims them as mean characteristics of $\mathcal{S}$. Sometimes quantum mechanics makes bids for predictions of some distributions, which appear to be formal and cannot be tested by experiment. For instance, distribution $|\langle p|\psi\rangle|^2$ over the electron momenta at the pure state, described by the wave function $|\psi\rangle$, has a formal character, because it cannot be tested experimentally. Here $|p\rangle$ means eigenfunction of the momentum operator $p = -i\hbar\nabla$. Of course, one can obtain the distribution $|\langle p|\psi\rangle|^2$ experimentally. It is sufficient to drop a flux of electrons onto diffraction grating and to investigate the obtained diffraction picture. But the quantum mechanics technique supposes, that obtained in such a way distribution over momenta must be attributed to some state (some wave function $|\psi\rangle$). Derivation of the distribution $|\langle p|\psi\rangle|^2$ needs a long time. This time is the longer the more exact distribution is to be obtained. In this time the wave function changes, and it is not clear to what wave function the obtained distribution over momenta should be attributed. Thus, it is possible to obtain distribution $|\langle p|\psi\rangle|^2$ experimentally, but it cannot be attributed to any definite state, and it cannot be a test of the quantum mechanics prediction. The performed analysis [11] shows that the momentum distribution, obtained experimentally, cannot be attributed to any state (wave function).

We have seen in the first section that the statistical average system does not describe distribution over energies or over momenta. Then how should the experimentally obtained distribution $|\langle p|\psi\rangle|^2$ over momenta be interpreted? The answer is as follows. The distribution $|\langle p|\psi\rangle|^2$ is not a distribution over momenta. It is a distribution over *mean momenta*. Let us explain what is the mean momentum. Let the region V, occupied by the electron beam, be separated into regions V_1,V_2,...,V_n,... Each of regions V_k is small, but its linear size $l_k \gg \lambda_c$, where λ_c is the Compton wavelength. Let us calculate the mean value $\langle \mathbf{p}_k \rangle$ of the momentum $\mathbf{p}$ of an electron inside the region V_k. Such a quantity can be calculated, because the momentum density is an additive quantity, and the mean value $\langle \mathbf{p}_k \rangle$ can be calculated for any region. We refer to [11] for substantiation of this statement. Here we only illustrate the difference between the distribution over momenta and distribution over mean momenta in a simple example.

Let us consider flow of a gas in a region V. Let us separate the region V into small but macroscopic regions V_1,V_2,...,V_n,...Let us calculate the mean value $\langle \mathbf{p}_k \rangle$ of the molecule momentum $\mathbf{p}$ inside the region V_k. The set of all $\langle \mathbf{p}_k \rangle$, taken at some time moment gives distribution of the gas molecules over mean momenta. It is clear that the real distribution of gas molecules over velocities (Maxwell distribution) has nothing to do with distribution of gas particles over macroscopic mean velocities

$\langle \mathbf{v}_k \rangle = \langle \mathbf{p}_k \rangle / m$. Let us imagine that we measure the molecule positions $\mathbf{x}$ and $\mathbf{x}'$, separated by the time interval Δt and determine the molecule velocity by means of relation $\mathbf{v} = (\mathbf{x}-\mathbf{x}')/\Delta t$. If Δt much less than, the mean time τ between the molecule collisions, the obtained distribution over velocities is the Maxwell distribution. Vice versa, if $\Delta t \gg \tau$, the obtained velocity is the macroscopic mean gas velocity, and the obtained distribution is a distribution over mean gas velocities in different regions V_k.

Something like that we obtain in the case of the electron beam. A use of the diffraction grating is equivalent to measurement of the electron positions separated by the time interval $\Delta t \to \infty$. It leads to distribution over mean momenta. If Δt is small enough, and the distance $\Delta \mathbf{x} = \mathbf{x} - \mathbf{x}'$ between the subsequent electron positions $\mathbf{x}$ and $\mathbf{x}'$ is small, then the electron momentum increases according to the indeterminacy principle, and we begin to measure the stochastic component of the electron motion. At very small Δt we measure only stochastic component of motion, which depends only on Δt and does not depend practically on the wave function of the electron beam.

Let us consider now well known paradox of "Schrödinger cat". At first, let us present it in the conventional manner. There is a cat in a closed chamber. The cat's life is determined by the state of a radioactive atom, placed in this chamber. While the atom is not decayed, the cat is alive. As soon as the atom decays, the cat becomes dead. The state of the atom is a linear superposition of states of the indecomposed atom and decomposed atom. Respectively the state of the cat is a linear superposition of the alive cat and of the dead one. Paradox consists in the simultaneous existence of the dead cat and of the alive cat. If one opens the chamber and observes the cat, the cat passes instantly from the state, where the cat is neither dead, nor alive, to the definite state, where the cat is either dead, or alive.

This paradox is a result of simple misunderstanding, when one identifies two different objects: the real individual Cat and an abstract statistical average $\langle \text{cat} \rangle$. The wave function describes the state of the abstract statistical average $\langle \text{atom} \rangle$, and the state of the abstract statistical average $\langle \text{atom} \rangle$ determines the state of the statistical average $\langle \text{cat} \rangle$. The wave function has nothing to do with the real Cat in the chamber. The statistical average $\langle \text{cat} \rangle$ bears on the real Cat the same relation as the statistical average inhabitant of Moscow bears on a real Ivan Sidorov, living somewhere in Leninsky avenue. If we produce S-observation, i.e. we open one definite chamber and found there an alive cat, we have no reasons for the statement, that opening the chamber, we change the state of $\langle \text{cat} \rangle$. No paradox appears, when the chamber is open. If we produce M-observation, i.e. if we consider N, $(N \gg 1)$ chambers with cats, then opening them all simultaneously, we do not discover a definite result. In some chambers one discovers alive cats, in other ones the cats are dead. In this case observation of the state of the abstract statistical average $\langle \text{cat} \rangle$ leads to a change of the state in the sense that the state turns from pure to mixed. But there is no paradox.

Finally, if we carry out a selective mass measurement (SM-measurement), i.e. one opens simultaneously N, $(N \gg 1)$ chambers and chooses those of them, where there are alive cats, then, on one hand, one obtains a definite result (alive cats), but on the other hand, there is many alive cats, and they form a statistical ensemble, whose state is described by a certain definite wave function. But any paradox does not appears, as far as the reason of a change of the wave function of statistical average $\langle \text{cat} \rangle$ is evident. It is a selection of alive cats from the total set, consisting of alive and dead cats.

The Schrödinger cat paradox is a special case of the paradox, which is discerned sometimes in the wave function reduction, appearing as a result of a measurement. To avoid this paradox, it is sufficient to follow a simple logic rule, which asserts: "One may not use the same term for notation of different objects. (But it is admissible to use several different terms for notation of the same object.)". Usually in quantum mechanics this rule is violated. One uses the same term for the individual object and for the statistical average object. As a corollary one uses the same term for two different measurement processes.

Note that the formal representation of M-measurement as a single act of influence on the statistical average system $\langle \mathcal{S} \rangle$ makes a large contribution into identification of S-measurement with M-measurement. This representation removes distinction between the S-measurement and the M-measurement and gives the impression that S-measurement and the M-measurement are identical procedures. After such an identification the measurement process acquires contradictory properties. On one hand, such a measurement leads to a definite result (S-measurement), on the other hand, its result is a distribution of the measured quantity (M-measurement). On one hand, the measurement (S-measurement) is a single act, produced on $\mathcal{S}$, and the measurement does not bear on the wave function, and on the statistical average system $\langle \mathcal{S} \rangle$. On the other hand, the measurement (M-measurement) leads to a reduction of the state of the system $\langle \mathcal{S} \rangle$.

In general, violation of the formal logic rule cited above is an origin for appearance of paradoxes and contradictions. Some exotic interpretation of quantum mechanics were removed by themselves (for instance, [12, 13]), provided one follows this logic rule and distinguish between individual object and statistically average one.

It should note that sometimes it is difficult to determine whether some property is a property of an individual object or that of a statistical average one. For instance, whether the half-integer spin of an electron is a property of a single electron or a property of statistical average electron (statistical ensemble). At the conventional approach, when one does not distinguish between an individual electron and a statistical average electron, such a problem does not exist at all. At a more careful approach we are forced to state that properties of an individual electron have been investigated very slightly. In most of cases we deal with measurements, produced on many electrons, as far as only such experiments are reproducible. As a corollary many conclusions on properties of a single electron are unreliable.

For instance, it is a common practice to consider the Stern – Gerlach experiment (when the electron beam, passing the region with inhomogeneous magnetic field, splits into two beams) to be an evidence of the statement that each single electron has a definite spin and corresponding projection of magnetic moment onto the magnetic field direction. In reality, the Stern – Gerlach experiment shows only that the Hamiltonian, describing a motion of statistical average electron, has two discrete eigenstates, labelled by the magnetic quantum number and distinguishing by their energy. The question, whether discreteness of the magnetic quantum number is connected with a discreteness in properties of a single electron, remains open, because a discreteness of energy levels is not connected directly with the discrete character of interaction. For instance, the energy levels of a spinless charged particle in the Coulomb electric field are discrete, although there is no discreteness in the properties of a single particle. It is doubtless that the beam split is connected with 'the electron magnetic moment', because it is proportional to the magnetic field gradient. But the question remains open, whether the 'electron magnetic moment' is a property of the individual electron $\mathcal{S}$ or a collective property of the statistical average electron $\langle\mathcal{S}\rangle$ (see for details [14]). Note that at identification of $\langle\mathcal{S}\rangle$ and $\mathcal{S}$ the problem does not appear.

Sometimes one considers, that the spectrum of electromagnetic radiation, emanated by an excited atom is a property of individual (but not statistical average) atom. The argument is adduced that the modern technique admits one to confine a single atom in a trap. Then one can investigate its energy levels and spectrum of radiation. But one overlooks that the spectrum of the atom radiation cannot be measured as a result of a single measurement (S -measurement). The atom radiation spectrum is obtained as a result of many measurements of radiation of the same atom, whose state is prepared by the same way, i.e. that essentially one measures radiation of a statistical average atom. To carry out such a M-measurement, it is of no importance, whether one produces one measurement with many similarly prepared atoms, or one produces many measurements with a single atom, preparing its state many times by the same way. In both cases one deals with the statistical average atom.

4 Concluding remarks

Thus, discrete physical system $\mathcal{S}$ has a regular component of its evolution and a stochastic one. The regular component is described explicitly by statistical average system $\langle\mathcal{S}\rangle$, which is a continuous dynamic system. The stochastic component of system $\mathcal{S}$ evolution is also described by $\langle\mathcal{S}\rangle$. This description is implicit. The action of the system $\langle\mathcal{S}\rangle$ is reduced to the form of the action for dynamic system $\mathcal{S}_{\mathrm{red}}\left[\mathcal{S}_{\mathrm{d}}\right]$, which is a set of interacting identical discrete dynamic systems $\mathcal{S}_{\mathrm{d}}$. The term in the action, responsible for interaction of $\mathcal{S}_{\mathrm{d}}$, describes implicitly the character of stochastic component of the system $\mathcal{S}$ evolution. The system $\mathcal{S}$ has finite number of the freedom degrees, whereas the system $\langle\mathcal{S}\rangle$, describing its dynamics, has infinite number of the freedom degrees. From informative viewpoint the system $\langle\mathcal{S}\rangle$ is more complicated, than the system $\mathcal{S}$, but we are forced to investigate $\langle\mathcal{S}\rangle$, if we want to study the stochastic system dynamics.

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