Generalization of particle dynamics on the case of arbitrary space-time geometry

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Abstract

Taking into account a progress of a geometry [1] and introducing adequate relativistic concepts, the elementary particle dynamics is generalized on the case of arbitrary space-time geometry. A use of adequate relativistic concepts admits one to formulate the simple demonstrable particle dynamics.

The contemporary elementary particle theory (EPT) is qualified usually as the elementary particle physics (EPP). However, it should be qualified more correctly as a elementary particle chemistry (EPC). The fact is that, the structure of the elementary particle theory reminds the periodical system of chemical elements. Both conceptions classify elementary particles (and chemical elements). On the basis of the classification both conceptions predict successfully new particles (and chemical elements). Both conceptions are axiomatic (but not model) constructions.

The periodical system of chemical elements has given nothing new for investigation of the atomic structure of chemical elements. One should not expect any information about elementary particle structure from contemporary EPT. For this purpose a model approach to EPT is necessary.

The simplest particle is considered usually as a point in usual 3D-space. This point is equipped by a mass and by a momentum 4-vector. One may to prescribe an electric charge and some other characteristics to the point. The aggregate of this information forms a nonrelativistic concept of a particle.

In the consecutive relativistic theory one should use another concept of a particle. The simplest particle is defined by two points P, P' in the space-time. The vector $\mathbf{PP'}$, formed by the two points, is a geometric momentum of the particle. Its length $\mu = |\mathbf{PP'}|$ is the geometric mass of the particle. The geometric mass μ and geometric momentum $\mathbf{PP}'$ are connected with conventional mass m and 4-momentum $\mathbf{p}$ by means of relations

$$m = b\mu, \qquad \mathbf{p} = bc\mathbf{PP}' \tag{1}$$

where b is some universal constant, and c is the speed of the light. The electric charge appears in the 5D-geometry of Kaluza-Klein as a projection of 5-momentum on the additional fifth dimension, which is a chosen direction. Projection on this direction is invariant, because the direction is chosen. As a result all parameters of a particle appear to be geometrized. A free motion of the simplest particle in the properly chosen 5D-geometry of the space-time is equivalent to motion of a charged particle in the given gravitational and electromagnetic fields of the Minkowskian space-time geometry.

A particle may have a complicated structure. In this case the particle is described by its skeleton $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$, consisting of $n+1$ space-time points $n = 1, 2, ...$The question: "What does unite the skeleton points in a particle" is relevant only in the space-time geometry with unlimited divisibility. In the physical geometry [2, 3, 4] the skeleton points may be connected between themselves simply as points of a geometry with a limited divisibility.

The particle evolution is described by a chain $\mathcal{C}$ of connected skeletons [4, 5].

$$\mathcal{C} = \bigcup_s \mathcal{P}_n^{(s)} \tag{2}$$

Adjacent skeletons of the chain are equivalent.

$$\mathcal{P}_n^{(s+1)} \text{eqv} \mathcal{P}_n^{(s)} : \quad \mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)} \text{eqv} \mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)} \quad i, k = 0, 1, ...n, \quad s = ...0, 1, ... \tag{3}$$

Points $P_1^{(s)}$and $P_0^{(s+1)}$of the chain coincide $s = ...0, 1, ..$ Then according to (3) the leading vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)} = \mathbf{P}_0^{(s)}\mathbf{P}_0^{(s+1)}$ of skeleton $\mathcal{P}_n^{(s)}$ is equivalent to the leading vector $\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)} = \mathbf{P}_0^{(s+1)}\mathbf{P}_0^{(s+2)}$ of skeleton $\mathcal{P}_n^{(s+1)}$, i.e.

$$\mathbf{P}_0^{(s)}\mathbf{P}_0^{(s+1)} \text{eqv} \mathbf{P}_0^{(s+1)}\mathbf{P}_0^{(s+2)} \tag{4}$$

In the explicit form equations (2), (3), describing the world chain, look as follows

$$\begin{aligned}
(\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}.\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}) &= \left|\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}\right| \cdot \left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|, && (5) \\
\left|\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}\right| &= \left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|, \quad P_1^{(s)} = P_0^{(s+1)} && (6) \\
i, k &= 0, 1, ...n, \quad s = ...0, 1, ...
\end{aligned}$$

where scalar products are defined via world functions by the relation

$$\begin{aligned}
(\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}.\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}) &= \sigma\left(P_i^{(s+1)}, P_k^{(s)}\right) + \sigma\left(P_k^{(s+1)}, P_i^{(s)}\right) \\
&\quad -\sigma\left(P_i^{(s+1)}, P_i^{(s)}\right) - \sigma\left(P_i^{(s+1)}, P_i^{(s)}\right)
\end{aligned} \tag{7}$$

Rotation of skeleton is absent. The translational motion is carried out along the leading vector $\mathbf{P}_0\mathbf{P}_1$. Dynamics is described by means of finite difference equations. It is reasonable, if the space-time geometry may be discrete. The leading vector describes the evolution direction in the space-time.

The number of dynamic equations is equal to $n(n+1)$, whereas the number of variables to be determined is equal to Nn. Here N is the dimension of the space-time, and $n+1$ is the number of points in the particle skeleton. The difference between the number of equations and the number of variables, which are to be determined, may lead to different results.

1. Multivariance, i.e. ambiguity of the world chain links position, when $n(n+1) < Nn$. It is characteristic for simple skeletons, which contain small number of points. Multivariance is responsible for quantum effects [6].

2. Zero-variance, i.e. absence of solution of equations, when $n(n+1) > Nn$. It is characteristic for complicated skeletons, which contain many points. Zero-variance means a discrimination of particles with complicated skeletons. As a result there exist only particles, having only certain values of masses and other parameters.

Quantum indeterminacy and discrimination mechanism are two different sides of the particle dynamics. The conventional theory of elementary particles has not a discrimination mechanism, which could explain a discrete spectrum of masses.

There are two sorts of elementary particles: bosons and fermions. Boson has not its own angular momentum (spin). It is rather reasonable, because motion of elementary particles is translational. However, the fermions have a discrete spin, which looks rather unexpected at the translation motion. Spin of a fermion appears as a result of translation motion along a space-like helix with timelike axis [7, 8, 9]. The helix world line of a free particle is possible only for spacelike world line. It is conditioned by multivariance [10] of the space-time geometry with respect to spacelike vectors. This multivariance takes place even for space-time of Minkowski. This multivariance takes place for any space-time geometry. It does not vanish in the limit $\hbar \rightarrow 0$.

However, in the space-time geometry of Minkowski the helix world chain is impossible, because the temporal component of momentum increases infinitely. For existence of the helix world chain, the world function σ is to have the form

$$\sigma = \left\{ \begin{array}{lll} f\left(\sigma_{\mathrm{M}}\right) & \text{if} & \left|\sigma_{\mathrm{M}}\right| < \sigma_0 \\ \sigma_{\mathrm{M}} + \lambda_0^2 \operatorname{sgn}\left(\sigma_{\mathrm{M}}\right) & \text{if} & \left|\sigma_{\mathrm{M}}\right| > \sigma_0 \end{array} \right., \qquad \lambda_0^2 = \frac{\hbar}{2bc}, \qquad \sigma_0 = \text{const} \tag{8}$$

$$\left|f\left(\sigma_{\mathrm{M}}\right)\right| < \left|\sigma_{\mathrm{M}}\right| \frac{\sigma_0 + \lambda_0^2}{\sigma_0}, \qquad \left|\sigma_{\mathrm{M}}\right| < \sigma_0 \tag{9}$$

In the conventional relativity theory the helix spacelike world lines are not considered, because one assumes, that they are forbidden by the relativity principles. Fermions are described usually by means of the Dirac equation, which needs introduction of such special quantities as γ-matrices. A use of γ-matrices generates a mismatch between the particle velocity and its mean momentum. (The quantum mechanics uses the mean momentum always [11].) This enigmatic mismatch is explained easily by means of the helix world chain. The velocity is tangent to helix, whereas the mean momentum is directed along the axis of helix.

Besides, the fermion skeleton is to contain not less, than three points. It is necessary for stabilization of the helix world line [7, 8]. Existence of the fermion is possible only at certain values of its mass, which depends on the space-time geometry (the form of function f in (8)) and on a choice of the skeleton points.

Thus, the spin and magnetic moment of fermions appear to be connected with spacelike world chain and with multivariance of the space-time geometry with respect to space-like vectors. At the conventional approach to geometry the spacelike world lines are considered to be incompatible with the relativity principles. Spin is associated with existence of enigmatic γ-matrices. Multivariance with respect to timelike vectors is slight (it vanishes in the limit $\hbar = 0$). Multivariance with respect to spacelike vectors is strong (it is not connected with quantum effects)

Motion of particle is free in the properly chosen space-time geometry. However, the particle motion can be described in arbitrary geometry, given on the same point set, where true geometry is given. The world function σ of the true geometry is presented in the form

$$\sigma(P,Q) = \sigma_{K_0}(P,Q) + d(P,Q) \tag{10}$$

where $d(P,Q)$ is some addition to the world function of $\sigma_{K_0}(P,Q)$ of the space-time geometry of Kaluza-Klein, which is used in the given case as a basic geometry. In this geometry the particle motion ceases to be free. It turns into a motion in force fields, whose form is determined by the form of addition $d(P,Q)$.

Progress in the elementary particle dynamics is conditioned by a progress in geometry and by a use of adequate relativistic concepts. The suggested elementary particle dynamics is a model conception. It is demonstrable and simple. Multivariance of the geometry explains freely quantum effects. The zero-variance generates a discriminational mechanism, responsible for discrete characteristics of elementary particles. Mathematical technique is formulated in a coordinateless form, that gets rid of a necessity to investigate coordinate transformation and their invariants. Two-point technique of the dynamics and many-point skeletons contain a lot of information, which should be only ordered correctly. Simple principles of dynamics reduce a construction of the elementary particle theory to formal calculations of different skeletons dynamics at different space-time geometries. There is a hope, that true skeletons of elementary particles can be obtained by means of the discrimination mechanism of the true space-time geometry. At any rate, having been constructed in the framework of simple dynamic principles, this dynamics explains freely discrete spins and discrete masses of fermions. It explains also mismatch between the particle velocity and its mean momentum. These properties are described usually by introduction of γ-matrices, that is a kind of fitting.

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