# Investigation methods in model conception of quantum phenomena.

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**Abstract**

One can construct the model conception of quantum phenomena (MCQP) which relates to the axiomatic conception of quantum phenomena (ACQP), (i.e. to the conventional quantum mechanics) in the same way, as the statistical physics relates to thermodynamics. Such a possibility is based on a new conception of geometry, which admits one to construct such a deterministic space-time geometry, where motion of free particles is primordially stochastic. The space-time geometry can be chosen in such a way that statistical description of random particle motion coincides with the quantum description. Methods of MCQP in investigation of quantum phenomena appear to be more subtle and effective than, that of ACQP. For instance, investigation of the free Dirac equation in framework of MCQP shows that the Dirac particle is in reality a rotator, i.e. two particles rotating around their common center of inertia. In the framework of MCQP one can discover the force field, responsible for pair production, that is impossible in the framework of ACQP.

## 1 Introduction

Sometimes investigation of a new class of physical phenomena is carried out by two stages. At first, the simpler axiomatic conception based on simple empiric considerations arises. Next, the axiomatic conception is replaced by the more developed model conception, where axioms of the first stage are obtained as properties of the model. Theory of thermal phenomena was developed according to this scheme. At first, the thermodynamics (axiomatic conception of thermal phenomena, or ACTP) appeared. Next, the statistical physics (model conception of thermal phenomena, or MCTP) appeared. Axioms of thermodynamics were obtained as properties of the chaotic molecule motion.

The contemporary quantum theory is the first (axiomatic) stage in the development of the microcosm physics. Formal evidences of this is an existence of quantum principles. Appearance of the next (model) stage, where the quantum principles are consequences of the model, seems to be unavoidable. The model conception is attractive, because it uses more subtle and effective mathematical methods of investigation. Besides it gives boundaries of the axiomatic conception application. We can see this in example of statistical physics and thermodynamics.

The main difference between the axiomatic and model stages of a theory lies in mathematical methods of description and investigation. Methods of axiomatic conception are more rough are rigid. They can be changed only by a change of axiomatics. This is produced mainly by introduction of additional suppositions. Mathematical methods of model conception are more flexible and adequate, because the model parameters are usually numbers and functions, which can be changed fluently.

Let us imagine, that we do not know methods of statistical physics and try to investigate nature of crystal anisotropy, using only thermodynamical methods. Using experimental date we can calculate thermodynamical potentials and describe macroscopic properties of crystal, but we hardly can calculate anything theoretically. For description of the next crystal we are forced to use experimental data again. By means of methods of statistical physics we can explain and calculate parameters of crystal anisotropy theoretically. At any rate, methods of model conception appears to be more effective in the given case.

Something like that we observe in the theory elementary particles, when theorists use rough and rigid methods of axiomatic conception (quantum theory), which do not admit to construct a perfect theory. There is a hope that mathematical methods of the model conception of quantum phenomena (MCQP) appear to be more effective, because MCQP does not use the rigid principles of quantum mechanics. Instead quantum principles MCQP uses parameters of space-time geometry which can be changed fluently.

According to MCQP quantum phenomena is a result of stochastic behavior of microparticles. Statistical description of this stochastic motion leads to description of quantum phenomena. This idea is very old and very reasonable after the thermal phenomena have been explained by chaotic motion of molecules. There were many attempts of this idea realization, but all these attempts had failed. As a result a sceptic relation to this idea appeared. Now most of physicists believe that quantum principles describe correctly the origin of physical phenomena in microcosm.

There are three obstacles on the way of creation MCQP: (1) inadequate space-time geometry, which describes the particle motion in microcosm as a deterministic, although experiments show that this motion is random in reality, (2) inadequate statistical description, when particles and antiparticles are considered to be objects of statistical description, whereas objects of statistical description are to be world lines, which are primary physical objects in relativistic theory, (3) integration of dynamic equations for ideal fluid, which is necessary for transformation of wave function and spin (which are fundamental objects of quantum theory) in a method of description of ideal fluid, i.e. in attributes of the physical object description.

Model conception of quantum phenomena (MCQP) could be constructed only after overcoming of these three obstacles. But each of the said obstacles was a very difficult problem. Besides, it was necessary to realize that on the way to creation of MCQP there are these obstacles. To overcome the first obstacle it was necessary to construct a new conception of geometry (T-geometry), which contains such a space-time geometry, where the particle motion be random.

The true space-time geometry is such a geometry, where motion of microparticles is primordially stochastic, although the geometry in itself is not random (intervals between the events in such a geometry are deterministic, but not random). To construct such a geometry (T-geometry), one needs to go outside the framework of Riemannian geometry, which is the most general contemporary geometry fitting for the space-time description. Being flat, uniform and isotropic, the true geometry of the absolute space-time distinguishes from the Minkowski geometry only in some correction containing the quantum constant $\hbar$. This correction is essential only for short space-time intervals, i.e. only in microcosm. Formally, introduction of this correction is equivalent to introduction of some fundamental length, but it is a transverse length (thickness of world line).

Statistical description of random motion of particles generated by the space-time geometry leads to the quantum mechanical description (the quantum constant $\hbar$ appears in the theory via geometry), in the same way as the statistical description of chaotic molecule motion leads to thermodynamics. Essential difference between the two statistical descriptions lies in the difference between the statistical description for relativistic and nonrelativistic cases. In the case of the statistical physics both regular and random components of the velocity are nonrelativistic, whereas in the case of geometric stochasticity the random velocity component is relativistic, although the regular component may be nonrelativistic. As a result even the nonrelativistic quantum mechanics appears to be a hidden relativistic theory. There is essential difference between the relativistic and nonrelativistic statistical descriptions. The fact is that the nonrelativistic statistical description may be probabilistic, i.e. it can be carried out in terms of the probability density, whereas *the relativistic statistical description cannot be produced in terms of the probability theory.* The problem lies in the fact that physical objects to be statistically described are different in the relativistic and nonrelativistic theories.

In the nonrelativistic theory the physical object is a point, i.e. a pointlike (zero-dimensional) object in three-dimensional space, and the particle world line describes a history of the pointlike object. In other words, the particle is primary and its world line is secondary. In the *consequent* relativistic theory the situation is inverse. The physical object is the world line, i.e. the one-dimensional line in the space-time, whereas the particle and the antiparticle are derivative objects (intersections of the world line with the surface $t$ =const). In other words, in the consequent relativistic theory the world line is primary, whereas the particle and antiparticle are secondary. The term 'WL' will be used for the world line considered to be the primary physical object. The difference in the choice of the primary physical object is conditioned by different relation to the existence of absolute simultaneity in relativistic and nonrelativistic physics. Statistical description must be a description of primary physical objects. Foundation for such a description is the density of physical objects in the three-dimensional space (for particles) or in the space-time (for WLs). Density $\rho(\mathbf{x})$ of particles at the point $\mathbf{x}$ is defined by the relation

$$dN = \rho(\mathbf{x})dV \tag{1.1}$$

where $dN$ is the number of particles in the 3-volume $dV$. The particle density is defined as a proportionality coefficient between $dN$ and $dV$. The density $j^k(x)$ of WLs is defined by the relation

$$dN = j^k(x)\,dS_k \tag{1.2}$$

where $dN$ is the flux of WLs through three-dimensional area $dS_k$ in the space-time in vicinity of the point $x$. The quantity $j^k(x)$ is the proportionality coefficient between $dN$ and $dS_k$ in vicinity of the point $x$. The quantity $\rho(\mathbf{x})$ is a nonnegative 3-scalar. It can serve as a basis for introduction of the probability density, whereas the 4-vector $j^k(x)$ is not a nonnegative quantity, and it cannot serve as a basis for introduction of the probability density, which must be nonnegative quantity.

It is a common practice to think that terms "probabilistic description" and "statistical description" are synonyms. It is a delusion, because the probabilistic description is a description, founded on a use of the probability theory, whereas the statistical description is a description, dealing with many similar or almost similar objects. Such a set of similar objects is called statistical ensemble. Statistical description is an investigation and description of the statistical ensemble properties. Statistical description without a use of the probability density is possible. It is necessary only to investigate the statistical ensemble without a use of the probability theory. We shall consider statistical ensembles of dynamic or stochastic systems and use essentially the circumstance, that *the statistical ensemble is a dynamic system*, even if its *elements are stochastic systems*. Use of the statistical ensemble as some means for calculation of statistical averages is not necessary. Such an approach may be qualified as the dynamic conception of statistical description (DCSD). It is appropriate in any case (relativistic and nonrelativistic). Interpretation of DCSD is carried out in terms of the ideal fluid and world lines of its particles. Construction of DCSD is a result of overcoming of the second obstacle.

Between DCSD and axiomatic conception of quantum phenomena (ACQP), i.e. conventional quantum mechanics, there is a connection. To obtain this connection it was necessary to obtain fundamental objects of ACQP (wave function and spin) as attributes of ideal fluid, which appears in DCSD. Connection between the fluid and the Schrödinger equation is known since the beginning of the quantum mechanics construction [1, 2]. In after years many authors developed this interplay known as hydrodynamic interpretation of quantum mechanics [3, 4, 5, 6, 7, 8, 9, 10, 11]. But this interpretation was founded ultimately on the wave function as a fundamental object of dynamics. It cannot go outside the framework of quantum principles, because the connection between the hydrodynamic interpretation and the quantum mechanics was one-way connection. One could obtain the irrotational fluid flow from the dynamic equation for the wave function (Schrödinger equation), but one did not know how to transform dynamic equations for a fluid to the dynamic equation for a wave function. In other words, we did not know how to describe rotational fluid flow in terms of the wave function. In terms of the wave function we could describe only irrotational fluid flow.

To describe arbitrary fluid flow in terms of a wave function, one needs *to integrate conventional dynamic equations for a fluid* (Euler equations). Indeed, the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\psi=0 \tag{1.3}$$

may be reduced to the hydrodynamic equations for the variables $\rho, \mathbf{v}$, describing the fluid state. Substituting $\psi=\sqrt{\rho}\exp(i\hbar\varphi)$ in (1.3) and separating real and imaginary parts of the equation, we obtain expressions for time derivatives $\partial_0\rho$ and $\partial_0\varphi$. To obtain expression for the time derivative $\partial_0\mathbf{v}$ of the velocity $\mathbf{v}=\frac{\hbar}{m}\nabla\varphi$, we need to differentiate dynamic equation for $\partial_0\varphi$, forming combination $\partial_0\mathbf{v}=\nabla\left(\frac{\hbar}{m}\partial_0\varphi\right)$. The reverse transition from hydrodynamic equations to dynamic equations for the wave function needs a general integration of hydrodynamic equations. This integration is simple in the partial case of irrotational flow, but it is a rather complicated mathematical problem in the general case, when a result of integration has to contain three arbitrary functions of three arguments. Without producing this integration, one cannot derive description of a fluid in terms of the wave function, and one cannot manipulate dynamic equations, transforming them from representation in terms of $\rho$, $\mathbf{v}$ to representation in terms of wave function and back. This problem has not been solved for years. It had been solved in the end of eighties, and the first application of this integration can be found in [12]

Statistical ensemble of discrete dynamic or stochastic systems is a continuous dynamic system, i.e. some ideal fluid. Integration of hydrodynamic equations admits one to show that the wave function and spin is a way of description of an ideal fluid [13]. This was overcoming of the third obstacle. In other words, the wave function appears as a property of some model (but not as a fundamental object whose properties are defined by axiomatics). Under some conditions the irrotational flow of the statistical ensemble (fluid) is described by the Schrödinger equation [14, 15]. Thus, one can connect MCQP with the conventional quantum description and show that in the nonrelativistic case the description in terms of MCQP agrees with description in terms of conventional quantum mechanics.

Note that the said obstacles were overcame in other order, than they are listed above. It took about thirty years for overcoming the first obstacle and construction of T-geometry. In the first paper [16] the system of differential equations for world function of Riemannian geometry was obtained. These equations do not contain metric tensor. They contain only world function and their derivatives with respect to both arguments. These equations put the following question. Let the world function do not satisfy these equations. What is then? Do we obtain non-

Riemannian geometry, or does no geometry exist? Then we could not answer this question, because as well as other scientists, we believed that the straight line must be one-dimensional geometrical object (curve) in any geometry. Only thirty years later we succeeded to answer this question and to construct non-Riemannian geometry (T-geometry) [17]. It appeared to be possible, because we had realized that in some geometries the straight line can be a non-one-dimensional object (surface). Moreover, it is the general case of geometry, whereas the Riemannian geometry with one-dimensional straight lines (geodesic) is a special (degenerate) case. The string theory suggests this idea of non-one-dimensional straight line. The second obstacle was overcame the first [18, 19, 20, 21]. The third obstacle was overcame only in the end of eighties. Finally the first obstacle was overcame last [22].

It is worth to note that all obstacles had been ovecame on the strictly logical foundation, i.e. only logical constructions based on already known principles of classical physics. New hypotheses and principles were not used, and this is not characteristic for the microcosm physics, developed in XXth century. In other words, the possibility of the MCQP construction was contained in principles of classical physics. It was necessary only to use them consistently. Unfortunately, it was unwarranted additional suppositions and absence of logical consistency, that was the stumbling block for construction of MCQP.

In the next two sections we consider modification of the quantum phenomena theory, generated by overcoming of the said obstacles.

# 2 Geometry

There are two different approaches to geometry: mathematical and physical ones. In the mathematical approach a geometry is a construction founded on a system of axioms about points and straights. Practically any system of axioms, containing concepts of a point and of a straight, may be called a geometry. Well known mathematician Felix Klein [23] supposed that only such a construction on a point set is a geometry, where all points of the set have the same properties (uniform geometry). For instance, Felix Klein insisted that Euclidean geometry and Lobachevsky geometry are geometries, because they are uniform, whereas the Riemannian geometries are not geometries at all. As a rule the Riemannian geometries are not uniform, and their points have different properties. According to the Felix Klein viewpoint, they should be called as "Riemannian topographies" or as "Riemannian geographies". It is a matter of habit and taste how to call the geometry. But Felix Klein was quite right in the relation, that he suggested to differ between the Euclidean geometry and Riemannian one. The fact is that the principle of the Riemannian geometry construction is quite different from that of the Euclidean geometry construction. The Euclidean geometry is constructed on the basis of axioms, whereas the Riemannian geometry is constructed as a deformation of the Euclidean geometry.

At the physical approach the geometry is the science on mutual disposition of points and geometric objects in the space, or events in the space-time. The mutual disposition is described by the metric $\rho$ (distance between two points), or by the world function $\sigma = \frac{1}{2}\rho^2$ [24]. It is question of the secondary importance, whether all points have the same properties or not, and what axioms are satisfied by the metric.

The Riemannian geometry is obtained as a result of the proper Euclidean geometry deformation, when the infinitesimal Euclidean interval $ds_{\mathrm{E}}^2$ is replaced by the Riemannian interval $ds^2 = g_{ik}dx^i dx^k$. Such a change is a deformation of the Euclidean space. Such an approach to geometry, when a geometry is a result of the proper Euclidean geometry deformation will be referred to as a physical approach to geometry. The obtained geometry will be referred to as a physical geometry. The physical geometry has no own axiomatics. It uses "deformed" Euclidean axiomatics. The physical geometry describes mutual disposition of points in the space, or of events in the space-time. It is described by setting the distance between any two points. The metric $\rho$ is the only characteristic of a physical geometry. The world function $\sigma = \frac{1}{2}\rho^2$ is more convenient for description of the physical geometry, because it is real even for the space-time, where $\rho = \sqrt{2\sigma}$ may be imaginary.

Construction of any physical geometry is determined by the *deformation principle* [25]. It works as follows. The proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ can be described in terms and only in terms of the world function $\sigma_{\mathrm{E}}$, provided $\sigma_{\mathrm{E}}$ satisfies some constraints formulated in terms of $\sigma_{\mathrm{E}}$ [25]. It means that all geometric objects $\mathcal{O}_{\mathrm{E}}$ can be described $\sigma$-immanently (i.e. in terms of $\sigma_{\mathrm{E}}$ and only of $\sigma_{\mathrm{E}}$) $\mathcal{O}_{\mathrm{E}} = \mathcal{O}_{\mathrm{E}}(\sigma_{\mathrm{E}})$. Relations between geometric objects are described $\sigma$-immanently by some expressions $\mathcal{R}_{\mathrm{E}} = \mathcal{R}_{\mathrm{E}}(\sigma_{\mathrm{E}})$. Any physical geometry $\mathcal{G}_{\mathrm{A}}$ can be obtained from the proper Euclidean geometry by means of a deformation, when the Euclidean world function $\sigma_{\mathrm{E}}$ is replaced by some other world function $\sigma_{\mathrm{A}}$ in all definitions of Euclidean geometric objects $\mathcal{O}_{\mathrm{E}} = \mathcal{O}_{\mathrm{E}}(\sigma_{\mathrm{E}})$ and in all Euclidean relations $\mathcal{R}_{\mathrm{E}} = \mathcal{R}_{\mathrm{E}}(\sigma_{\mathrm{E}})$ between them. As a result we have the following change

$$\mathcal{O}_{\mathrm{E}} = \mathcal{O}_{\mathrm{E}}(\sigma_{\mathrm{E}}) \to \mathcal{O}_{\mathrm{A}} = \mathcal{O}_{\mathrm{E}}(\sigma_{\mathrm{A}}), \qquad \mathcal{R}_{\mathrm{E}} = \mathcal{R}_{\mathrm{E}}(\sigma_{\mathrm{E}}) \to \mathcal{R}_{\mathrm{A}} = \mathcal{R}_{\mathrm{E}}(\sigma_{\mathrm{A}})$$

The set of all geometric objects $\mathcal{O}_{\mathrm{A}}$ and all relations $\mathcal{R}_{\mathrm{A}}$ between them forms a physical geometry, described by the world function $\sigma_{\mathrm{A}}$. Index 'E' in the relations of physical geometry $\mathcal{G}_{\mathrm{A}}$ means that axiomatics of the proper Euclidean geometry was used for construction of geometric objects $\mathcal{O}_{\mathrm{E}} = \mathcal{O}_{\mathrm{E}}(\sigma_{\mathrm{E}})$ and of relations between them $\mathcal{R}_{\mathrm{E}} = \mathcal{R}_{\mathrm{E}}(\sigma_{\mathrm{E}})$. The same axiomatics is used for all geometric objects $\mathcal{O}_{\mathrm{A}} = \mathcal{O}_{\mathrm{E}}(\sigma_{\mathrm{A}})$ and relations between them $\mathcal{R}_{\mathrm{A}} = \mathcal{R}_{\mathrm{E}}(\sigma_{\mathrm{A}})$ in the geometry $\mathcal{G}_{\mathrm{A}}$. But now this axiomatics has another form, because of deformation $\sigma_{\mathrm{E}} \to \sigma_{\mathrm{A}}$. It means that the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ is the basic geometry for all physical geometries $\mathcal{G}$ obtained by means of a deformation of the proper Euclidean geometry. If the basic geometry is fixed (it is this case that will be considered further), the geometry on the arbitrary set $\Omega$ of points is called T-geometry (tubular geometry). The T-geometry is determined [17, 26] by setting the world function $\sigma$:

$$\sigma: \quad \Omega \times \Omega \to \mathbb{R}, \qquad \sigma(P, P) = 0, \qquad \forall P \in \Omega \tag{2.1}$$

In general, no other constraints are imposed, although one can impose any additional constraints to obtain a special class of T-geometries. T-geometry is symmetric, if in addition

$$\sigma(P,Q)=\sigma(Q,P), \qquad \forall P,Q\in\Omega \tag{2.2}$$

Consequent application of *only deformation principle* admits one to obtain any physical geometry (T-geometry), which appears to be automatically as consistent as the Euclidean geometry, which lies in its foundation. The Riemannian geometries form a special class of T-geometries, determined by the constraint, imposed on the world function

$$\sigma_{\mathrm{R}}(x,x')=\frac{1}{2}\left(\int\limits_{\mathcal{L}_{[xx']}}\sqrt{g_{ik}dx^i dx^k}\right)^2 \tag{2.3}$$

where $\sigma_{\mathrm{R}}$ is the world function of the Riemannian space, and $\mathcal{L}_{[xx']}$ means a geodesic segment between the points $x$ and $x'$. Riemannian geometry is determined by the dimension $n$ and $n(n+1)/2$ functions $g_{ik}$ of one point $x$, whereas the class of all possible T-geometries is essentially more powerful, because it is determined by one function $\sigma$ of two points $x$ and $x'$.

In general, the deformation principle admits one to obtain such geometries, where non-one-dimensional tubes play the role of the straight lines. The real space-time geometry is of such a kind. But creators of the Riemannian geometry supposed that a geometry with tubes instead of straights was impossible. The restriction (2.3) on Riemannian geometries was introduced to forbid deformation transforming one-dimensional Euclidean straights to many-dimensional tubes. But in nonuniform physical geometry one fails to suppress the tubular character of straights. In the Riemannian geometry one succeeds to make this, only refusing from the consequent application of the deformation principle and using additional means of the geometry construction. As a result the Riemannian geometry appears to be not quite consequent construction. This is displayed, in particular, in lack of absolute parallelism, whereas in any physical geometry constructed in accordance with the deformation principle the absolute parallelism takes place.(see details in [25]).

The tubular character of timelike straights in the real space-time generates the stochastic character of the free particles motion in such a space-time, because the straight (tube) $\mathcal{T}_{P_0P_1}$, passing through the points $P_0$ and $P_1$, is determined by the relation

$$\mathcal{T}_{P_0P_1}=\left\{R|\overrightarrow{P_0P_1}||\overrightarrow{P_0R}\right\}=\left\{R|S_{P_0P_1R}=0\right\} \tag{2.4}$$

where $\overrightarrow{P_0P_1}||\overrightarrow{P_0R}$ means that vectors $\overrightarrow{P_0P_1}$ and $\overrightarrow{P_0R}$ are collinear. In the proper Euclidean space the vectors $\overrightarrow{P_0P_1}$ and $\overrightarrow{P_0R}$ are linear dependent (collinear), if and only if the second order Gram determinant $F_2(P_0,P_1,R)$ vanishes.

$$\overrightarrow{P_0P_1}||\overrightarrow{P_0R}: \qquad F_2(P_0,P_1,R)=\begin{vmatrix}\left(\overrightarrow{P_0P_1}.\overrightarrow{P_0P_1}\right) & \left(\overrightarrow{P_0P_1}.\overrightarrow{P_0R}\right)\\ \left(\overrightarrow{P_0R}.\overrightarrow{P_0P_1}\right) & \left(\overrightarrow{P_0R}.\overrightarrow{P_0R}\right)\end{vmatrix}=0 \tag{2.5}$$

Here $\left(\overrightarrow{P_0P_1}.\overrightarrow{P_0R}\right)$ denotes the scalar product of two vectors $\overrightarrow{P_0P_1}$ and $\overrightarrow{P_0R}$, which is defined by the relation

$$\left(\overrightarrow{P_0P_1}.\overrightarrow{P_0R}\right) \equiv \sigma\left(P_0, P_1\right) + \sigma\left(P_0, R\right) - \sigma\left(P_1, R\right) \tag{2.6}$$

Relations (2.5), (2.6) express the collinearity condition via the world function $\sigma$ of the proper Euclidean space. In other words, the collinearity of $\overrightarrow{P_0P_1}$ , $\overrightarrow{P_0R}$ is defined $\sigma$-immanently. By definition this relation can be used in arbitrary T-geometry, i.e. for any world function $\sigma$. The quantity $S_{P_0P_1R}$ is the area of Euclidean triangle with vertices at points $P_0, P_1, R$. It is connected with the Gram determinant by the relation

$$F_2\left(P_0, P_1, R\right) = \left(2S_{P_0P_1R}\right)^2 \tag{2.7}$$

Thus, two relations (2.4) contain two equivalent conditions of collinearity.

It is worth to note that conventionally the concept of linear dependence (collinearity) is introduced in the framework of linear space. To introduce the linear space, one needs a set of restrictions (fixed dimension, continuity, coordinate system, etc.). It appears unexpectedly that all these restrictions are not necessary. The concept of linear dependence can be introduced on arbitrary set of points, where the world function (metric) is given. The concept of linear dependence appears to be independent of whether or not the linear space can be introduced on this set.

According to this definition the tube $\mathcal{T}_{P_0P_1}$ is a set of such points $R$, that vectors $\overrightarrow{P_0P_1}$ and $\overrightarrow{P_0R}$ are collinear. The tubular character of the straight (thick straight) means that there are many directions $\overrightarrow{P_0R}$, parallel to the vector $\overrightarrow{P_0P_1}$. On the other hand, the motion of a free particle in the curved space-time is described by the equation of a geodesic

$$d\dot{x}^i = -\Gamma_{kl}^i \dot{x}^k dx^l, \qquad dx^l = \dot{x}^i d\tau \tag{2.8}$$

where $\Gamma_{kl}^i$ is the Christoffel symbol. Equation (2.8) describes the parallel transport of the velocity vector $\dot{x}^i$ of the particle along the direction $dx^i = \dot{x}^i d\tau$, determined by the velocity vector $\dot{x}^i$. If there are many vectors parallel to the velocity vector $\dot{x}^i$, the parallel transport (2.8) appears to be not single-valued, and the world line becomes to be random.

The flat uniform isotropic space-time is described by the world function [22]

$$\sigma = \sigma_{\mathrm{M}} + D\left(\sigma_{\mathrm{M}}\right), \qquad D\left(\sigma_{\mathrm{M}}\right) = \frac{\hbar}{2bc} \geq 10^{-21}\mathrm{cm}^2, \quad \text{if} \quad \sigma_{\mathrm{M}} > \frac{\hbar}{2bc} \tag{2.9}$$

where $\sigma_{\mathrm{M}}$ is the world function of the Minkowski space, $c$ is the speed of the light. The distortion function $D\left(\sigma_{\mathrm{M}}\right)$ describes the character of quantum stochasticity. In the space with nonvanishing distortion $D\left(\sigma_{\mathrm{M}}\right)$ the particle mass is geometrized [22], and $b \leq 10^{-17}$g/cm is the constant, describing connection between the geometric mass $\mu$ and usual mass $m$ by means of the relation $m = b\mu$. Form of the distortion function $D\left(\sigma_{\mathrm{M}}\right)$ is determined by the demand that the stochasticity generated by distortion is the quantum stochasticity, i.e. the statistical description of the free stochastic particle motion is equivalent to the quantum description in terms of the Schrödinger equation [22].

## 3 Statistical description

Let the statistical ensemble $\mathcal{E}_{\rm d}\left[\mathcal{S}_{\rm d}\right]$ of deterministic classical particles $\mathcal{S}_{\rm d}$ be described by the action $\mathcal{A}_{\mathcal{E}_{\rm d}[\mathcal{S}_{\rm d}(P)]}$, where $P$ are parameters describing $\mathcal{S}_{\rm d}$ (for instance, mass, charge). Let under influence of some stochastic agent the deterministic particle $\mathcal{S}_{\rm d}$ turn to a stochastic particle $\mathcal{S}_{\rm st}$. The action $\mathcal{A}_{\mathcal{E}_{\rm st}[\mathcal{S}_{\rm st}]}$ for the statistical ensemble $\mathcal{E}_{\rm st}\left[\mathcal{S}_{\rm st}\right]$ of stochastic particles $\mathcal{S}_{\rm st}$ is reduced to the action $\mathcal{A}_{\mathcal{S}_{\rm red}[\mathcal{S}_{\rm d}]} = \mathcal{A}_{\mathcal{E}_{\rm st}[\mathcal{S}_{\rm st}]}$ for some set $\mathcal{S}_{\rm red}\left[\mathcal{S}_{\rm d}\right]$ of identical interacting deterministic particles $\mathcal{S}_{\rm d}$. The action $\mathcal{A}_{\mathcal{S}_{\rm red}[\mathcal{S}_{\rm d}]}$ as a functional of $\mathcal{S}_{\rm d}$ has the form $\mathcal{A}_{\mathcal{E}_{\rm d}[\mathcal{S}_{\rm d}(P_{\rm eff})]}$, where parameters $P_{\rm eff}$ are parameters $P$ of the deterministic particle $\mathcal{S}_{\rm d}$, averaged over the statistical ensemble, and this averaging describes interaction of particles $\mathcal{S}_{\rm d}$ in the set $\mathcal{S}_{\rm red}\left[\mathcal{S}_{\rm d}\right]$ [27, 28]. It means that

$$\mathcal{A}_{\mathcal{E}_{\rm st}[\mathcal{S}_{\rm st}]} = \mathcal{A}_{\mathcal{S}_{\rm red}[\mathcal{S}_{\rm d}(P)]} = \mathcal{A}_{\mathcal{E}_{\rm d}[\mathcal{S}_{\rm d}(P_{\rm eff})]} \tag{3.1}$$

In other words, stochasticity of particles $\mathcal{S}_{\rm st}$ in the ensemble $\mathcal{E}_{\rm st}\left[\mathcal{S}_{\rm st}\right]$ is replaced by interaction of $\mathcal{S}_{\rm d}$ in $\mathcal{S}_{\rm red}\left[\mathcal{S}_{\rm d}\right]$, and this interaction is described by a change

$$P \rightarrow P_{\rm eff} \tag{3.2}$$

in the action $\mathcal{A}_{\mathcal{E}_{\rm d}[\mathcal{S}_{\rm d}(P)]}$.$\vec{\xi}$

Action for the statistical ensemble of free deterministic particles has the form

$$\mathcal{A}\left[x\right] = -\int mc\sqrt{g_{ik}\dot{x}^i\dot{x}^k}d\xi_0 d\vec{\xi}, \qquad \dot{x}^i \equiv \frac{dx^i}{d\xi_0} \tag{3.3}$$

where $x = \{x^i\}$, $i = 0,1,2,3$ is a function of $\xi = \left\{\xi_0, \vec{\xi}\right\} = \{\xi_i\}$, $i = 0,1,2,3$.

The only parameter for the free particle is its mass $m$, and the change (3.2) in the nonrelativistic case has the form

$$m \rightarrow m_{\rm eff} = m\left(1 - \frac{\mathbf{u}^2}{2c^2} + \frac{\hbar}{2mc^2}\nabla\mathbf{u}\right) \tag{3.4}$$

where $\mathbf{u} = \mathbf{u}\left(t,\mathbf{x}\right)$ is the mean value of the stochastic velocity component. Quantum constant $\hbar$ appears here as coupling constant between the regular and stochastic components of the particle velocity. The velocity $\mathbf{u}$ is considered to be a new dependent variable, and dynamic equation for $\mathbf{u}$ is obtained as a result of the action variation with respect to $\mathbf{u}$ [28]. The velocity $\mathbf{u}$ is supposed to be small as compared with the speed of the light $c$.

In the relativistic case the change (3.2) takes the form

$$m^2 \rightarrow m_{\rm eff}^2 = m^2\left(1 + u_l u^l + \lambda\partial_l u^l\right), \qquad \lambda = \frac{\hbar}{mc} \tag{3.5}$$

where $u^l = \{u^0, \mathbf{u}\}$. Then the action (3.3) is transformed to the form

$$\mathcal{A}\left[x,\kappa\right] = -\int mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k}d\xi_0 d\vec{\xi}, \qquad K = \sqrt{1 + \frac{\hbar^2}{m^2c^2}\left(\kappa^l\kappa_l + \partial_l\kappa^l\right)} \tag{3.6}$$

where dependent variables $x = \{x^i\}$, $i = 0,1,2,3$ are a function of variables $\xi = \left\{\xi_0, \vec{\xi}\right\} = \{\xi_i\}$, $i = 0,1,2,3$. Dependent variables $\kappa = \{\kappa^i\}$, $i = 0,1,2,3$ are functions of $x$. The metric tensor $g_{ik}$ =diag$\{c^2,-1,-1,-1\}$. Variables $\kappa^l$ are connected with $u^l$ by means of the relation

$$u^l = \frac{\hbar}{m}\kappa^l, \qquad l = 0,1,2,3 \tag{3.7}$$

On one hand, the action (3.6) describes a set of deterministic particles interacting between themselves via self-consistent vector field $\kappa^l$. On the other hand, the action (3.6) describes a quantum fluid. Rotational flow of this fluid is described by one-component wave function $\psi$, satisfying the Klein-Gordon equation [29, 28].

In general case the fluid flow is described by two-component wave function, satisfying the dynamic equation [28]

$$-\hbar^2\partial_k\partial^k\psi - \left(m^2c^2 + \frac{\hbar^2}{4}\left(\partial_l s_\alpha\right)\left(\partial^l s_\alpha\right)\right)\psi = \hbar^2\frac{\partial_l\left(\rho\partial^l s_\alpha\right)}{2\rho}\left(\sigma_\alpha - s_\alpha\right)\psi \tag{3.8}$$

where 3-vector $\mathbf{s} = \{s_1, s_2, s_3,\}$ is determined by the relations

$$\psi = \binom{\psi_1}{\psi_2}, \qquad \psi^* = (\psi_1^*, \psi_2^*), \qquad \rho = \psi^*\psi, \qquad s_\alpha = \frac{\psi^*\sigma_\alpha\psi}{\rho}, \qquad \alpha = 1,2,3 \tag{3.9}$$

Here $\vec{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ are Pauli matrices.

From physical viewpoint the quantization procedure (3.5), when the mass $m$ is replaced by its mean value $m_{\text{eff}}$, looks rather reasonable. Indeed, the value of $m_{\text{eff}}$ depends on the state of the stochastic velocity component. Stochastic velocity component has infinite number of the freedom degrees and consideration of influence of the mean value $u^l$ on the regular component of the particle velocity appears to be very complicated. It is described by partial differential equations, whereas in absence of this influence the regular particle motion is described by the ordinary differential equations. From physical viewpoint such an interpretation of the quantization looks more reasonable, than conventional interpretation in terms of wave function and operators.

The wave function and spin appear here as a way of description of the ideal fluid [13], i.e. as fluid attributes. In other words, statistical description and hydrodynamic interpretation of the world function are primary, and wave function is secondary. Hierarchy of concepts is described by the following two schemes.

| | | | | | |
|---|---|---|---|---|---|
| ACQP: | quantum principles | $\Rightarrow$ | wave function | $\Rightarrow$ | hydrodynamic interpretation |
| MCQP: | statistical description | $\Rightarrow$ | fluid dynamics | $\Rightarrow$ | wave function |

Note that transition from ACQP to MCQP became possible only after solution of such a pure mathematical problem as integration of complete system of dynamic equations for ideal fluid. Systematical application of this integration for description of quantum phenomena began in 1995 [14, 30].

# 4 Methods and capacities of MCQP

Let us formulate the first difference between ACQP and MCQP. As any statistical theory MCQP contains two sorts of investigated objects: stochastic system $\mathcal{S}_{\mathrm{st}}$ and statistically averaged system $\langle\mathcal{S}_{\mathrm{st}}\rangle$, whereas ACQP investigates only one sort of objects: so called quantum systems $\mathcal{S}_{\mathrm{q}}$. Systems $\langle\mathcal{S}_{\mathrm{st}}\rangle$ and $\mathcal{S}_{\mathrm{q}}$ are continuous dynamic systems. They have dynamic equations and coincide practically always. The system $\mathcal{S}_{\mathrm{st}}$ is a discrete stochastic system, for which dynamic equations are absent. Dynamic system $\langle\mathcal{S}_{\mathrm{st}}\rangle$ appears as a result of statistical description of $\mathcal{S}_{\mathrm{st}}$ (consideration of statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$). The statistically averaged system $\langle\mathcal{S}_{\mathrm{st}}\rangle$ is the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$, normalized to one system, and $\langle\mathcal{S}_{\mathrm{st}}\rangle$ as a continuous dynamic system have practically all properties of statistical ensemble. Normalization to one system is possible, because properties of the statistical ensemble $\mathcal{E}\left[N,\mathcal{S}_{\mathrm{st}}\right]$ do not depend on number $N$ elements in it, if $N$ is large enough. Then one can set formally $N=1$, and $\mathcal{E}\left[N,\mathcal{S}_{\mathrm{st}}\right]$ turns to $\langle\mathcal{S}_{\mathrm{st}}\rangle$. But $\langle\mathcal{S}_{\mathrm{st}}\rangle$ remains to be continuous dynamic system and conserves all properties of the statistical ensemble $\mathcal{E}\left[\infty,\mathcal{S}_{\mathrm{st}}\right]$ [27].

Two kinds of measurement in MCQP ($S$-measurement and $M$-measurement) is another side of existence of two sorts of objects $\mathcal{S}_{\mathrm{st}}$ and $\langle\mathcal{S}_{\mathrm{st}}\rangle$. Single measurement ($S$-measurement) is produced over single stochastic system $\mathcal{S}_{\mathrm{st}}$. Result of $S$-measurement is random. It means that the result of $S$-measurement is not reproduced, in general, in other $S$-measurements on other $\mathcal{S}_{\mathrm{st}}$, prepared in the same way. $S$-measurement always leads to one definite value $R'$ of the measured quantity $\mathcal{R}$. This result does not influence on the wave function $\psi$, because wave function describes the state of $\langle\mathcal{S}_{\mathrm{st}}\rangle$ and has no relation to $\mathcal{S}_{\mathrm{st}}$. The massive measurement ($M$-measurement) is produced over many single dynamic systems $\mathcal{S}_{\mathrm{st}}$, constituting $\langle\mathcal{S}_{\mathrm{st}}\rangle$. $M$-measurement is a set of many $S$-measurements, produced over different elements of $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ or of $\langle\mathcal{S}_{\mathrm{st}}\rangle$. $M$-measurement of the quantity $\mathcal{R}$ leads always to a distribution $F\left(R'\right)$ of possible values $R'$, because $S$-measurements in different elements of $\langle\mathcal{S}_{\mathrm{st}}\rangle$ give different results, in general. Result of $M$-measurement is always a distribution, even in the case, when all $S$-measurements, constituting the $M$-measurement, give the same result. In the last case the distribution is a $\delta$-function. Distribution $F\left(R'\right)$ is reproducible. It is reproduced in other $M$-measurements, carried out in the same state $\psi$.

Thus, $S$-measurement is irreproducible, in general, and gives a definite result. The $M$-measurement is reproducible and, in general, does not give a definite result (but only a distribution of results). Nevertheless, formally the $M$-measurement of the quantity $\mathcal{R}$ may be considered to be a single act of measurement of the quantity $R'$ distribution, which is produced over the dynamic system $\langle\mathcal{S}_{\mathrm{st}}\rangle$.

Theory can predict only results of $M$-measurement, which leads to a reproducible distribution except for the case, when the measured system is an element of the statistical ensemble, which is found in such a state, where the distribution of the measured quantity is a $\delta$-function. In this case the result of $S$-measurement can be predicted, because it is determined single-valuedly by the distribution, which is a result of $M$-measurement and can be predicted by the theory.

Is it possible that $M$-measurement of the quantity $\mathcal{R}$ in the state $\psi$ lead to a definite result $R'$? Yes, it is possible, if $M$-measurement is accompanied by a selection of those $S$-measurements, constituting the $M$-measurement, which give the result $R'$. Such a measurement is called selective $M$-measurement (or $SM$-measurement). $SM$-measurement has properties of $S$-measurement (definite result $R'$) and those of $M$-measurement (it is produced in $\langle\mathcal{S}_{\rm st}\rangle$, and influences on the state of $\langle\mathcal{S}_{\rm st}\rangle$). $SM$-measurement is accompanied by a selection, and the quaestion about its reproducibility is meaningless, because it depends on the kind of selection, which depetermines the result of the $SM$-measurement.

In ACQP there is only one type of objects (quantum systems $\mathcal{S}_{\rm q}$) and only one type of measurement ($Q$-measurement). The $Q$-measurement is essentially $M$-measurement, because it always gives distribution of the measured quantity $\mathcal{R}$. Result of $Q$-measurement is determined by the rule of von Neumann. All predictions in ACQP as well as in MCQP concern only with results of $M$-measurement. In ACQP there is only one object and only one type of measurement. Nobody distinguishes in ACQP between $S-$measurement and $M$-measurement.

In ACQP the statistically averaged system $\langle\mathcal{S}_{\rm st}\rangle$ is considered to be individual quantum system $\mathcal{S}_{\rm q}$ and single measurement on $\mathcal{S}_{\rm q}=\langle\mathcal{S}_{\rm st}\rangle$ is considered to be a measurement with properties of both $M$-measurement and $S$-measurement (i.e. $SM$-measurement). In classical physics a single physical system is described as a discrete dynamic system, whereas a continuous dynamic system describes statistical ensemble of single physical systems. In ACQP the question, why the continuous dynamic system $\mathcal{S}_{\rm q}$ describes a single physical object (for instance, a single particle), is not raised usually. If nevertheless such a question is raised, one answers that quantum system is a special kind of physical system which distinguishes from classical system and whose state is described by such a misterious quantity as wave function, or something like this. The fact that the wave function is a method of the statistical ensemble description is unknown practically.

In ACQP there is only one object, and for description of a single measurement one useses $SM$-measurement (but not $S$-measurement), because $S$-measurement does not exist in framework of ACQP. $SM$-measurement is produced on the dynamic system $\mathcal{S}_{\rm q}=\langle\mathcal{S}_{\rm st}\rangle$ and influences on the wave function, describing the state of $\mathcal{S}_{\rm q}$. The $S$-measurement is produced on the system $\mathcal{S}_{\rm st}$. It does not influence on the wave function, describing the state of $\mathcal{S}_{\rm q}=\langle\mathcal{S}_{\rm st}\rangle$. Character of influence on the wave function is the main formal difference between $S$-measurement and $SM$-measurement.

Replacement of $S$-measurement by $SM$-measurement in ACQP leads to such paradoxes, as the Schrödinger cat paradox, or the EPR-paradox, which are based on the belief that a single measurement changes the wave function (i.e. that a single measurement is a $SM$-measurement). Paradoxes are eliminated, if one take into account that a single measurement is $S$-measurement, which does not change the wave function). See detail discussion in [27].

Some properties of $\langle\mathcal{S}_{\rm st}\rangle$ appear as a result of properties of $\mathcal{S}_{\rm st}$. Another ones appear as a result of statistical averaging. The last are interpreted as collective properties. For instance, in statistical physics (MCTP) the temperature is a collective property, because one molecule has no temperature. Only collective of molecules has a temperature in statistical physics. In thermodynamics (ACTP) there is no concept of collective properties, and temperature is a property of any amount of matter (one molecule, or even a half of molecule). In MCQP wave function $\psi$ is a collective quantity, which describes the state of $\langle\mathcal{S}_{\rm st}\rangle$. If we say that individual stochastic system $\mathcal{S}_{\rm st}$ is found in the state $\psi$, it means only that $\mathcal{S}_{\rm st}$ is taken from the ensemble $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$, whose state is described by the wave function $\psi$. In MCQP the spin of a particle may be a property of individual particle $\mathcal{S}_{\rm st}$, and it may be a collective property of $\langle\mathcal{S}_{\rm st}\rangle$. In different cases we have different results.

In the case of dynamic system $\mathcal{S}_{\rm P}$ described by the Pauli equation the electron spin is the collective properties [14]. It means, in particular, that the dynamic system $\mathcal{S}_{\rm S}$, described by the Schrödinger equation, and dynamic system $\mathcal{S}_{\rm P}$ consist of similar individual systems $\mathcal{S}_{\rm st}$. The difference between them is described by the type of the fluid flow. In the case of $\mathcal{S}_{\rm S}$ the fluid flow is irrotational, whereas in the case of $\mathcal{S}_{\rm P}$ the fluid flow is rotational. In other words, in $\mathcal{S}_{\rm P}$ the electron spin is conditioned by collective property (vortical flow). In the given case the electron spin is a result of the fluid flow vorticity. (Let us remember that the statistical ensemble is a fluidlike dynamic system.)

In the case of dynamic system $\mathcal{S}_{\rm D}$, described by the Dirac equation, the electron spin is the property of individual stochastic particle [31]. In this case the dynamic disquatization of Dirac equation, i.e. determination of classic analog $\mathcal{S}_{\rm Dcl}$ of the Dirac particle $\mathcal{S}_{\rm D}$ shows that $\mathcal{S}_{\rm Dcl}$ is a dynamic system, having ten degrees of freedom. It may be interpreted as two classical particles, rotating around their common center of inertia. Angular momentum of this rotation forms spin of $\mathcal{S}_{\rm Dcl}$. Thus, in the case of the Dirac electron $\mathcal{S}_{\rm D}$ spin is a property of individual stochastic system.

In ACQP there are no collective properties, because there is only one object: quantum system $\mathcal{S}_{\rm q}$. The statement of the question whether spin of the particle is a collective property is meaningless in the framework of ACQP. Is it important for investigations to distinguish between individual and collective properties? Sometimes it is of no importance, but sometimes it is important. We can investigate this question in the example of temperature in the statistical physics. Collective properties of temperature are of no importance, provided we deal with the mean values of it. If we deal with fluctuations of temperature and those of other quantities, the collective properties of temperature become to be important. In any case, method of investigation of MCQP appears to be more subtle and effective, than that of ACQP.

In general, MCQP as any model conception possesses *more subtle and flexile methods of investigation*. For instance, the change (3.5) carries out essentially the quantization procedure, i.e. transition from classical description to the quantum one. Let us imagine that the quantization (3.5) is not exactly true, and one needs to correct it slightly. In the framework of MCQP one needs only to change the form of the expression (3.5). It is not clear how such a modification can be realized in the framework of ACQP, because ACQP is connected closely with linearity of dynamic equations in terms of the wave function. One cannot imagine quantum mechanics which is nonlinear in terms of wave function, because in this case the quantum principles are violated.

Having a long history [1, 2], the hydrodynamic interpretation should rank among new methods of investigation. But in the framework of ACQP the hydrodynamic interpretation is secondary as it follows from the above mentioned schemes. In the framework of MCQP the hydrodynamic interpretation is primary, and possesses more subtle methods of investigation. In the framework of ACQP a transition to semiclassical approximation is carried out by means of transition to the limit $\hbar \to 0$ with some additional conditions. In MCQP this transition is realized by means of dynamic disquantization [31], which is a relativistic dynamic procedure. At the dynamic disquantization one removes transversal components $\partial_{\perp k} = \partial_k - j_k j^l \left(j_s j^s\right)^{-1} \partial_l$ of derivative $\partial_k$, which are orthogonal to the flux 4-vector $j^k$. After this transformation the dynamic equations contain derivatives only in direction of vector $j^k$. Such a system of partial differential equations can be reduced to a system of ordinary dynamic equations. As a result of dynamic disquantization the system of partial differential equations turns to a system of ordinary differential equations. In other words, the statistical ensemble of stochastic systems turns to a statistical ensemble of dynamic systems. As a result the continuous system can be interpreted in terms of a discrete dynamic system (with finite number of the freedom degrees). In the nonrelativistic case the dynamic disquantization is equivalent to $\hbar \to 0$. In the relativistic case the quantum constant $\hbar$ remains in the discrete dynamic system [31]. It admits one to obtain a more subtle interpretation.

This method was applied for investigation of dynamic system $\mathcal{S}_{\mathrm{D}}$, described by the Dirac equation [31]. It appears that the classical analog of the Dirac particle $\mathcal{S}_{\mathrm{D}}$ is a rotator (but not a single particle), i.e. two particles rotating around their common center of inertia. This explains freely angular and magnetic momenta of the Dirac particle. Dynamic variables describing rotation contain the quantum constant $\hbar$. If $\hbar \to 0$, degrees of freedom, connected with rotation are suppressed. Radius of rotator tends to 0, and instead of rotator we obtain pointlike particle with spin and magnetic moment. This result with $\hbar \to 0$ agrees with the results, obtained in the framework of ACQP. More soft result of rotator, when $\hbar \neq 0$, cannot be obtained by methods of ACQP. These methods are too rough. Besides, it appears (quite unexpectedly) that the internal (rotational) degrees of freedom of the dynamic system $\mathcal{S}_{\mathrm{D}}$ are described in nonrelativistic manner [31, 30, 32]. Investigation of the dynamic system $\mathcal{S}_{\mathrm{D}}$ was produced without any additional supposition. It was investigated simply as a dynamic system by means of relativistically covariant methods. These results cannot be obtained in the framework of conventional quantum mechanics (ACQP).

Methods of MCQP are consistent relativistic ones. For description of relativistic quantum phenomena ACQP uses the program of uniting of nonrelativistic quantum mechanics technique with the relativity principles. Unfortunately, this program failed. At any rate, it works unsuccessfully last fifty years, trying to construct relativistic quantum field theory and theory of elementary particles. It seems that uniting of nonrelativistic quantum mechanics technique with the relativity principles is impossible.

As another example of the MCQP methods application, we refer to the problem of the pair production, which is the central problem in the high energy physics. ACQP cannot say anything on the pair production mechanism and on the agents, responsible for this process, whereas MCQP can say something pithy on this problem. MCQP vests responsibility for the pair production on the $\kappa$-field (3.7), which is conditioned by the stochastic component of the particle motion. In MCQP the pair production is taken into account on the descriptive (before-dynamic) level, i.e. the pair production is taken in to account by consideration of WL as a primary physical object, whereas in ACQP the pair production is taken into account only on the dynamic level, i.e. by means of dynamic equations. In ACQP particles may be produced not only by pairs. The number of particles produced in an elementary act may be arbitrary. The number of produced particles depends on the form of corresponding term in Lagrangian. In MCQP the particles are produced only by pairs particle - antiparticle. This fact is fixed on the descriptive (conceptual) level, and cannot be changed on dynamical level (by a choice of Lagrangian). All this shows that the source of pair production is different in MCQP and ACQP.

Let us imagine that the particle world line turns in the time direction. Depending on situation, such a turn describes either pair production, or pair annihilation. The $\kappa$-field creates conditions for such a turn and for the pair production. The fact is that at such a turn in time the world line direction becomes spacelike ($m^2 < 0$) in the vicinity of the turning point. If one forbids the world line to be spacelike, the pair production becomes to be impossible. Such a possibility to change the particle mass and to make it imaginary is rather rare property among the force fields. For instance, the electromagnetic field of any magnitude cannot change the particle mass, and hence, to produce pairs. Pair production is a prerogative of the $\kappa$-field. According to relation (3.5) the expression containing $\kappa$-field enter in the effective squared mass as a factor. If this expression is negative, the mass becomes imaginary, and the pair production (annihilation) becomes to be possible [28].

Furthermore, pair production, obtained in ACQP at canonical quantization of nonlinear relativistic field, does not take place in reality. It was shown at canonical quantization of nonlinear complex scalar field [33], described by Lagrangian density

$$L =: \varphi_i^* \varphi^i - m^2 \varphi^* \varphi + \frac{\lambda}{2} \varphi^* \varphi^* \varphi \varphi : \tag{4.1}$$

$$\varphi = \varphi(x), \qquad \varphi_i \equiv \partial_i \varphi, \qquad , \qquad \varphi^i \equiv \partial^i \varphi, \qquad x = (t, x).$$

At canonical quantization of (4.1) the $WL$-scheme of quantization was used, when the object of quantization is WL, i.e. world line considered as the primary physical object. In the $WL$-scheme the canonical quantization is produced without imposition of additional constraint

$$[u, P_0]_- = -i\hbar \frac{\partial u}{\partial x^0}, \qquad E = P^0 = \int T^{00} d\mathbf{x} \tag{4.2}$$

where $[...]_-$ denotes commutator and $T^{ik}$ is the energy-momentum tensor. The condition (4.2) identifies the energy with the evolution operator (Hamiltonian). This identification is possible in the case, when there are only particles, or only antiparticles. It is possible also in the case, when particles and antiparticles are considered as different physical objects (but not as attributes of WL). In $WL$-scheme of quantization the number of primary physical objects (WLs) is conserved, and quantization is produced exactly (without the perturbation theory methods). In such a quantization the pair production is absent, that agrees with demands to the field producing pairs.

But then the question arises. Why does pair production appear at quantization according to $PA$-scheme [34, 35, 36, 37], when particle and antiparticle are considered as primary objects? The answer is as follows. Canonical quantization (according to $WL$-scheme) is possible without imposition of constraint (4.2). It means that the constraint (4.2) is an additional condition, and one should to verify its compatibility with dynamic equations. Unfortunately, nobody had verified this, supposing that (4.2) is a necessary condition of the second quantization and there is no necessity to verify its compatibility with dynamic equations. This test was produced in [33]. It appears that (4.2) is compatible with dynamic equations, provided $\lambda = 0$, i.e. the field is linear. In the nonlinear case $\lambda \neq 0$ imposition of the constraint (4.2) leads to overdetermination of the problem. In the overdetermined (and hence, inconsistent) problem one can obtain practically any results, which one wishes. So, authors of [34, 35, 36, 37] wanted to obtain pair production, and they had obtained it.

These examples show that MCQP and its subtle investigation methods can be useful at investigation of the microcosm phenomena properties.

Thus, MCQP makes the first successes, but not in the sense that it explains some new experiments, which could not be explained before. MCQP uses the more subtle dynamic methods of investigation (consideration of two objects $\mathcal{S}_{\rm st}$ and $\langle\mathcal{S}_{\rm st}\rangle$, hydrodynamic interpretation of relativistic processes [29, 28], dynamic quantization and disquantization [31]), which cannot be used by ACQP because of its axiomatic character. Difference between the methods of MCQP and ACQP is described by the following scheme

| ACQP | MCQP |
|---|---|
| Combination of nonrelativistic quantum technique with principles of relativity | Consequent relativistic description at all stages |
| 1. Additional hypotheses are used (QM principles) | 1. *No additional hypotheses are used* |
| 2. One kind of measurement, as far as only one statistical average object $\langle\mathcal{S}\rangle$ is considered. It is referred to as quantum system | 2. Two kinds of measurement, because two kinds of objects (individual $\mathcal{S}_{\rm st}$ and statistical average $\langle\mathcal{S}\rangle$ ) are considered |
| 3. Quantization: procedure on the conceptual level: $\mathbf{p} \to -i\hbar\nabla$ etc. | 3. Dynamic quantization: relativistic procedure on the dynamic level $m^2 \to m_{\rm eff}^2 = m^2 + \frac{\hbar^2}{c^2}\left(\kappa_l\kappa^l + \partial_l\kappa^l\right)$ |

4. Transition to classical description: procedure on conceptual level
$\hbar \rightarrow 0 \qquad \psi \rightarrow (\mathbf{x}, \mathbf{p})$

5. Interpretation in terms of wave function $\psi$

4. Dynamic disquantization: relativistic procedure on dynamic level
$\partial^k \rightarrow \frac{j^k j^l}{j_s j^s} \partial_l$

5. Interpretation in terms of statistical average world lines (WL)
$\frac{dx^i}{d\tau} = j^i(x)$,
$j^k = -\frac{i\hbar}{2}(\psi^* \partial^k \psi - \partial^k \psi^* \cdot \psi)$

MCQP is essentially more flexible conception, than ACQP, as far as all in MCQP is determined by the space-time geometry, and the set of all possible geometries is described by a function of two arguments. This is a great reserve for corrections and modifications of MCQP. At the same time all modifications of MCQP are restricted by a change of the world function, and possible modifications do not concern the structure of MCQP, which is founded on several principles, connected logically between themselves.

On the contrary ACQP is founded on a set of rigid rules, considered as principles, although there is no logical connection between them. The only foundation for application of quantum principles is the fact that they explain nonrelativistic quantum phenomena very well. They explain also relativistic quantum phenomena, when they may be considered as small correction to nonrelativistic phenomena. ACQP fails in explanation of essentially relativistic quantum phenomena (for instance, pair production). Possibility of modification of ACQP is connected mainly with application of additional principles and suppositions, which change the structure of ACQP. Possibility of dynamical modification (consideration of new dynamic systems) is also take place, but in each special case one needs to use some new ideas. ACQP in itself does not give foundation for such ideas, and this makes the further development of ACQP to be difficult.

MCQP admits one to obtain new physical object without any additional suppositions. It is sufficient to remove some constraints, imposed on the world function. The world function (2.9), determining the microcosm structure is only the first rough approximation. If it is necessary, the expression (2.9) can be modified in such a way, to take into account influence of the matter distribution in the space-time (curvature) and existence of new metric fields, generated by the possible asymmetry of the world function [38, 39].

Asymmetric world function describes the space-time, where the past and the future are unequal geometrically. One cannot imagine such a thing in the framework of Riemannian geometry. Expansion of the symmetric world function $\sigma(x, x')$ over powers of $\eta^i = x^i - x'^i$ has the form

$$\sigma(x, x') = \frac{1}{2} g_{ik}(x') \eta^i \eta^k + \frac{1}{6} \sigma_{ikl}(x') \eta^i \eta^k \eta^l + ... \qquad (4.3)$$

where $g_{ik}(x')$ describes the gravitational field, and $\sigma_{ikl}(x')$ is expressed via derivatives of metric tensor $g_{ik}(x')$. For asymmetric world function the same expansion has the form [38, 39]

$$\sigma\left(x,x'\right)=\sigma_{i}\left(x'\right)\eta^{i}+\frac{1}{2}\sigma_{ik}\left(x'\right)\eta^{i}\eta^{k}+\frac{1}{6}\sigma_{ikl}\left(x'\right)\eta^{i}\eta^{k}\eta^{l}+... \tag{4.4}$$

where three coefficients $\sigma_i\left(x'\right)$, $\sigma_{ik}\left(x'\right)$ and $\sigma_{ikl}\left(x'\right)$ are independent, and each of them is connected with some metric (geometric) field. Coefficient $\sigma_i\left(x'\right)$ describes a "vector field" which is strong and effective at small space-time intervals. Coefficient $\sigma_{ik}\left(x'\right)$ describes the second rank tensor field (gravitational field) which is strong and effective at middle space-time intervals. Finally, $\sigma_{ikl}\left(x'\right)$ is connected with the third rank tensor field, which is strong and effective at large space-time intervals. Maybe, this field is connected with astrophysical problem of dark matter, when one fails to explain observed motion of stars and galaxies by means of only gravitational field.

At construction of MCQP *one did not use any new hypotheses.* On the contrary, flexibility and subtlety of MCQP methods are connected with remove of unwarranted constraints and correction of mistakes in the approach to geometry and to statistical description. In other words, MCQP satisfies the Newton's criterion: "Hypothesis non fingo." *Only choice of true space-time geometry is determined properties of physical phenomena in microcosm.* This choice must be done in any case. But this choice may be true, or not completely true.

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