Monistic conception of geometry

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Abstract

One considers the monistic conception of a geometry, where there is only one fundamental quantity (world function). All other geometrical quantities a derivative quantities (functions of the world function). The monisitc conception of a geometry is compared with pluralistic conceptions of a geometry, where there are several independent fundamental geometrical quantities. A generalization of a pluralistic conception of the proper Euclidean geometry appears to be inconsistent, if the generalized geometry is inhomogeneous. In particular, the Riemannian geometry appears to be inconsistent, in general, if it is obtained as a generalization of the pluralistic conception of the Euclidean geometry.

1 Introduction

The term "monism" (monistic) in its application to geometry means, that the conception of a geometry uses only one fundamental quantity (distance) and all other geometrical quantities and concepts are derivatives of the fundamental quantity. The pluralistic conception is a conception, containing several fundamental quantities, which may be independent. In general, a monistic conception can be considered as a pluralistic conception. It is sufficient to declare, that some derivative quantities of monistic conception are fundamental quantities of a pluralistic conception. However, not any pluralistic conception can be considered as a monistic conception. It is necessary for such a consideration, that there are proper connections between fundamental quantities of the pluralistic conception.

If one is going to modify or to generalize a conception, it is desirable to present the conception in the form of a monistic conception. In this case it is sufficient to modify properties of the only fundamental quantity. Other (derivative) quantities will be modified automatically, because other quantities are functions of the unique fundamental quantity. If the modified conception is pluralistic, the modification becomes to be difficult, because in the pluralistic conception there may be connections between the fundamental quantities. In this case one may not modify independently properties of different fundamental quantities, which look as independent.

Usually the monistic conception is a more developed conception, than the foregoing pluralistic conception, containing several fundamental quantities. For instance, the Christianity, which contains only one god, is a more developed conception, than the heathendom, which contains many gods. Usually a monistic conception (or a less pluralistic conception) is a result of development of a pluralistic conception, which is followed by a reduction of fundamental quantities. For instance, theory of thermal phenomena developed from the axiomatic thermodynamics to the statistical physics. This development was followed by transformation of fundamental thermodynamic quantities into derivative quantities of the statistical physics. Appearance of intermediate derivative concepts of the monistic conception makes the monistic conception to be a more complicated conception, than the preceding pluralistic conception. This complexity is conditioned by a necessity of deduction of transformed quantities, which were fundamental quantities in the pluralistic conception. However, the more complicated monistic conception admits one to describe and explain such physical phenomena, which cannot be described in the framework of the pluralistic theory. For instance, the thermal fluctuations are described by the statistical physics, but they cannot be described in the framework of the axiomatic thermodynamics.

Geometry is a science on disposition and shape of geometrical objects in the space or in the event space (space-time). Any geometry is given on a set Ω of points (events). Any geometric object $\mathcal{O}$ is a set Ω' of points $P \in \Omega'$ with $(\Omega' \subset \Omega)$. The shape of the geometric object $\mathcal{O}$ is described, if the distance $\rho(P_1, P_2)$, $\forall P_1, P_2 \in \Omega'$ is given. The shape and mutual dispositions of all geometrical objects are given completely, if the distance $\rho(P_1, P_2)$, $\forall P_1, P_2 \in \Omega$ is given. In the usual space the distance ρ is a real nonnegative quantity. In the event space the distance ρ is either a real nonnegative quantity, or an imaginary quantity. In this case it is more convenient to use the quantity $\sigma = \frac{1}{2}\rho^2$, which is always real. The quantity σ is called the world function. This quantity was introduced by J.L. Synge [1] for description of the Riemannian geometry.

Thus, the geometry of the event space (space-time geometry) is described completely by one real function σ.

$$\sigma : \quad \Omega \times \Omega \to \mathbb{R}, \quad \sigma(P, Q) = \sigma(Q, P), \quad \sigma(P, P) = 0, \quad \forall P, Q \in \otimes \tag{1.1}$$

As far as a motion of physical bodies in the space is described as a geometrical object in the space-time, a construction of geometrical objects and investigation of their properties is very important in such applied sciences as physics and mechanics. Any geometrical object $\mathcal{O}$ is described by enumeration of points of the object $\mathcal{O}$. However, such a description by means of a direct enumeration of points, belonging to $\mathcal{O}$, contains too much information. One tends to simplify this procedure, constructing geometric objects from standard blocks. These blocks contain many points of the space-time, and construction of the geometric object $\mathcal{O}$ is reduced to a finite or countable number of operations with blocks.

Procedure of the geometric objects construction from blocks has been developed in the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. It can be used for description of usual space (but not for space-time), because the world function σ_{E} of the geometry $\mathcal{G}_{\mathrm{E}}$ is non-negative. The nonnegativity of σ_{E} is used essentially at construction of geometrical objects in $\mathcal{G}_{\mathrm{E}}$. A direct enumeration of points of the geometrical object may be replaced by some relations, which are fulfilled for points $P \in \mathcal{O}$ and only for them. However, such a description needs a use of some numerical functions, given on Ω, for instance, world function σ.

In reality the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ is constructed usually from three kinds of blocks (point, segment of straight, angle) without mention of the fact, that these blocks may be described in terms of the world function σ_{E} and only in terms of σ_{E} (See details in [2]). Properties of the three blocks are postulated in the form of axioms. Using the rules of construction of a geometric object from these blocks, one can formulate the proper Euclidean geometry in the form of a logical construction. Basic statements of the geometry have the form of geometrical axioms, formulated in terms of basic geometric quantities (point, segment of straight and angle). Usually the distance ρ_{E} (or world function σ_{E}) is considered to be some derivative concept, which may be not mentioned at all.

The fact, that the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ may be formulated in terms of the world function (distance) is well known. Nevertheless the distance is not used usually as a fundamental quantity of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. This quantity admits one to *formulate the geometry G_{E} as a monistic conception, based on the only fundamental concept* (world function σ_{E}). Of course, the world function σ_{E} of the geometry $\mathcal{G}_{\mathrm{E}}$ is to satisfy some conditions, formulated in terms of the world function. The necessary and sufficient conditions, that the geometry $\mathcal{G}$, described by the world function σ, is n-dimensional proper Euclidean geometry, have the form [3]

I.

$$\exists \mathcal{P}^n \subset \Omega, \qquad F_n(\mathcal{P}^n) \neq 0, \qquad F_{n+1}(\Omega^{n+2}) = 0, \tag{1.2}$$

where $\mathcal{P}^n = \{P_0, P_1, ..P_n\}$, the quantity $F_n(\mathcal{P}^n)$ is the Gramm's determinant

$$F_n(\mathcal{P}^n) = \det ||(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)||, \quad i,k = 1,2,...n \tag{1.3}$$

and $(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)$ is the scalar product of two vectors $\mathbf{P}_0\mathbf{P}_i$ and $\mathbf{P}_0\mathbf{P}_k$, which is determined in terms of the world function σ by means of the relation

$$(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k) = \sigma(P_0,P_k) + \sigma(P_0,P_i) - \sigma(P_i,P_k) \tag{1.4}$$

II.

$$\sigma(P,Q) = \frac{1}{2}\sum_{i,k=1}^{n} g^{ik}(\mathcal{P}^n)[x_i(P) - x_i(Q)][x_k(P) - x_k(Q)], \qquad \forall P,Q \in \Omega, \tag{1.5}$$

where the quantities $x_i(P)$, $x_i(Q)$ are defined by the relations

$$x_i(P) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}), \qquad x_i(Q) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{Q}), \qquad i = 1, 2, ...n \tag{1.6}$$

The contravariant components $g^{ik}(\mathcal{P}^n)$, $(i, k = 1, 2, \ldots n)$ of metric tensor are defined by its covariant components $g_{ik}(\mathcal{P}^n)$, $(i, k = 1, 2, \ldots n)$ by means of relations

$$\sum_{k=1}^{n} g_{ik}(\mathcal{P}^n)g^{kl}(\mathcal{P}^n) = \delta_i^l, \qquad i, l = 1, 2, \ldots n \tag{1.7}$$

where

$$g_{ik}(\mathcal{P}^n) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k), \qquad i, k = 1, 2, \ldots n \tag{1.8}$$

III. The relations

$$(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}) = x_i, \qquad x_i \in \mathbb{R}, \qquad i = 1, 2, \ldots n, \tag{1.9}$$

considered to be equations for determination of $P \in \Omega$ as a function of x_i, $i = 1, 2, ...n$, have always one and only one solution.

The condition I determines the dimension n of the geometry and n basic vectors $\mathbf{P}_0\mathbf{P}_i$, $i = 1, 2, ...n$ on the set Ω. The condition II determines properties of the metric tensor $g_{ik}(\mathcal{P}^n)$.

In such a representation the proper Euclidean geometry looks as a monistic conception, which is described by the only fundamental quantity: world function σ. All other geometrical quantities (concepts and geometrical objects) are derivative in the sense, that all they can be expressed in terms of points of the set Ω and in terms of world functions between them.

Perspective of constructing such a monistic conception not only for the proper Euclidean geometry seemed to be attractive. One introduced metric space and constructed so called metric geometry. However, construction of the metric geometry is possible only for the real metric (distance) $\rho \geq 0$. Such a geometry cannot be constructed in the space-time. Besides, for construction of straight lines (the shortest), one needs to impose on the metric ρ an additional condition (the triangle axiom)

$$\rho(P, Q) + \rho(P, R) \geq \rho(R, Q), \quad \forall P, Q, R \in \Omega \tag{1.10}$$

The triangle axiom (1.10) reflects our belief, that there are only such geometries, where the straight is a one-dimensional point set.

Attempts [4, 5] were made to construct so called distance geometry, which is free of the condition (1.10). Unfortunately, these attempts failed in the sense, that the obtained distance geometry was not monistic and completely metric. It contains also nonmetric procedure, which admits one to construct straight lines by means of a continuous mapping of segment [0, 1] onto the set Ω.

2 Pluralistic approach to the Riemannian geometry as a reason of its inconsistency

Usually one does not use the monistic conception of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$, where there is the only fundamental geometric quantity (world function). Instead, one uses pluralistic conception of geometry, where there are several fundamental geometric quantities (for instance, dimension, straight, angle,...), which are considered as independent geometrical quantities. In the proper Euclidean geometry one succeeds to agree properties of these different fundamental quantities and to construct a consistent geometry. However, in the non-Euclidean geometry one succeeds to agree properties of different fundamental quantities not always. Having several fundamental concepts and attributing to them some properties, one cannot be sure, that these properties may be made compatible between themselves. As a result the obtained geometry appears to be inconsistent. The problem of agreement between properties of different fundamental quantities in a pluralistic conception of geometry are formulated usually as a consistency of axioms of the geometry. Any test of the geometric axioms consistency needs a lot of labour, which can be carried out not always. As a result a pluralistic geometry appears to be inconsistent. For instance, the Riemannian geometry, which is constructed usually as a pluralistic conception, appears to be inconsistent.

Using a monistic conception, when there is only one fundamental quantity, one constructs a consistent geometry automatically. As an example, let us consider construction of a straight line in a monistic geometry $\mathcal{G}$, described by the world function σ, given on the set Ω of points P. We shall use term σ-space for the quantity $V = \{\sigma, \Omega\}$. The Euclidean space is a special case of the σ-space with $\sigma = \sigma_{\mathrm{E}}$. In the proper Euclidean geometry (Euclidean σ-space $V_{\mathrm{E}} = \{\sigma_{\mathrm{E}}, \Omega\}$), the segment $\mathcal{T}_{[P_0P_1]}$ of straight line between the points P_0 and P_1 is defined by the relation

$$\mathcal{T}_{[P_0P_1]} = \left\{R| \sqrt{2\sigma(P_0, R)} + \sqrt{2\sigma(P_1, R)} - \sqrt{2\sigma(P_0, P_1)} = 0\right\} \tag{2.1}$$

where the world function σ coincides with the Euclidean world function σ_{E}. The same form of the straight segment $\mathcal{T}_{[P_0P_1]}$ is to have in any other σ-space. In the n-dimensional σ-space the segment $\mathcal{T}_{[P_0P_1]}$ is a $(n-1)$-dimensional surface, in general. However, in the proper Euclidean geometry the $(n-1)$-dimensional surface degenerates into one-dimensional set. This fact is a corollary of special properties (1.2) - (1.9) of the Euclidean world function σ_{E}, which generates fulfillment of the triangle axiom (1.10). In the proper Riemannian geometry the triangle axiom (1.10) is also valid. It is also connected with the form of the Riemannian world function σ_{R}, although in the conventional (pluralistic) presentation of the Riemannian geometry, the world function σ_{R} is a derivative quantity, defined by the relation

$$\sigma_{\mathrm{R}}(P_0, P_1) = \frac{1}{2}\left(\int_{P_0}^{P_1} \sqrt{g_{ik}(x)\, dx^i dx^k}\right)^2 \tag{2.2}$$

where integral in (2.2) is taken along the geodesic, connecting points P_0 and P_1.

World function σ_{R}, defined by the relation (2.2) satisfies the equation [1]:

$$\frac{\partial}{\partial x^i}\sigma_{\mathrm{R}}\left(x,x'\right)g^{ik}\left(x\right)\frac{\partial}{\partial x^k}\sigma_{\mathrm{R}}\left(x,x'\right)=2\sigma_{\mathrm{R}}\left(x,x'\right) \tag{2.3}$$

which describes essentially extremal properties of the world function σ_{R}, i.e. the fact, that the world function σ_{R} generates fulfilment of condition (1.10).

One believes usually, that the space-time geometry is a Riemannian (pseudo-Riemannian) geometry, and the world function σ_{R} of the real space-time geometry satisfies the equation (2.3). What are reasons for such a statement?

The Riemannian space-time geometry is obtained usually as a generalization of the proper Euclidean geometry on the case of a curved space-time geometry. At such a generalization the original geometry is considered as a pluralistic conception of geometry, where there are several fundamental quantities: point, straight line, linear vector space, etc. The world function is considered as a derivative quantity, defined by the relation (2.2). At a generalization the properties of fundamental quantities are changed. The change of the fundamental quantities means a change of axioms, describing properties of these quantities. The change of different fundamental quantities is to be made by a consistent way. (New axioms of the generalized geometry are to be compatible).

Let us compare the monistic approach and the pluralistic one in the simple example. In the Euclidean geometry two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are collinear ($\mathbf{P}_0\mathbf{P}_1 \parallel \mathbf{Q}_0\mathbf{Q}_1$), if and only if the Gramm's determinant vanishes

$$\mathbf{P}_0\mathbf{P}_1 \parallel \mathbf{Q}_0\mathbf{Q}_1: \qquad \begin{vmatrix} (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_1) & (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) \\ (\mathbf{Q}_0\mathbf{Q}_1.\mathbf{P}_0\mathbf{P}_1) & (\mathbf{Q}_0\mathbf{Q}_1.\mathbf{Q}_0\mathbf{Q}_1) \end{vmatrix}=0 \tag{2.4}$$

or in the developed form

$$\mathbf{P}_0\mathbf{P}_1 \parallel \mathbf{Q}_0\mathbf{Q}_1: \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)^2=|\mathbf{P}_0\mathbf{P}_1|^2\cdot|\mathbf{Q}_0\mathbf{Q}_1|^2 \tag{2.5}$$

where the scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ and the module $|\mathbf{P}_0\mathbf{P}_1|$ are expressed via world function by formulas

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)=\sigma\left(P_0,Q_1\right)+\sigma\left(P_1,Q_0\right)-\sigma\left(P_0,Q_0\right)-\sigma\left(P_1,Q_1\right) \tag{2.6}$$

$$|\mathbf{P}_0\mathbf{P}_1|=\sqrt{(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_1)}=\sqrt{2\sigma\left(P_0,P_1\right)} \tag{2.7}$$

The straight line $\mathcal{T}_{P_0P_1,Q_0}$, passing through the point Q_0 collinear to the vector $\mathbf{P}_0\mathbf{P}_1$, is defined by the relation

$$\mathcal{T}_{P_0P_1,Q_0}=\{R|\mathbf{Q}_0\mathbf{R}\parallel\mathbf{P}_0\mathbf{P}_1\}=\left\{R|\left(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{R}\right)^2-|\mathbf{P}_0\mathbf{P}_1|^2\cdot|\mathbf{Q}_0\mathbf{R}|^2=0\right\} \tag{2.8}$$

If the point Q_0 coincides with the point P_0, the expression in (2.8) can be presented in the form

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{R})^2-|\mathbf{P}_0\mathbf{P}_1|^2\cdot|\mathbf{P}_0\mathbf{R}|^2=A\left(P_0,P_1,R\right)A\left(P_0,R,P_1\right)B\left(P_0,P_1,R\right) \tag{2.9}$$

where

$$A(P_0,P_1,R)=\sqrt{\sigma(P_0,R)}+\sqrt{\sigma(P_1,R)}-\sqrt{\sigma(P_1,P_0)} \tag{2.10}$$

$$A(P_0,R,P_1)=\sqrt{\sigma(P_1,P_0)}+\sqrt{\sigma(P_1,R)}-\sqrt{\sigma(P_0,R)} \tag{2.11}$$

$$B(P_0,P_1,R)=\sigma(P_1,R)-\sigma(P_0,R)-\sigma(P_1,P_0)-4\sqrt{\sigma(P_0,P_1)\sigma(P_0,R)} \tag{2.12}$$

The factor $A(P_0,P_1,R)$ is responsible for that part of the straight line, which is placed between the points P_0 and P_1, whereas the factor $A(P_0,R,P_1)$ is responsible for that part of the straight, which is placed outside the points P_0 and P_1. One can see from (2.1), that only factor $A(P_0,P_1,R)$ is used in the definition of the segment $\mathcal{T}_{[P_0P_1]}$.

Let us compare the monistic approach and the pluralistic one in the simple example. In the Euclidean geometry two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are the equivalent ($\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1$), if the two vectors are in parallel

$$\mathbf{P}_0\mathbf{P}_1\uparrow\uparrow\mathbf{Q}_0\mathbf{Q}_1: \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)=|\mathbf{P}_0\mathbf{P}_1|\cdot|\mathbf{Q}_0\mathbf{Q}_1| \tag{2.13}$$

and their modules are equal, i.e. the following conditions are fulfilled

$$(\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1): \quad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)=|\mathbf{P}_0\mathbf{P}_1|\cdot|\mathbf{Q}_0\mathbf{Q}_1|\wedge|\mathbf{P}_0\mathbf{P}_1|=|\mathbf{Q}_0\mathbf{Q}_1| \tag{2.14}$$

where the scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ and the module $|\mathbf{P}_0\mathbf{P}_1|$ are expressed via world function by formulas (2.6) and (2.7)

It is a well-wrought definition of two vectors equality, because it is formulated in terms of fundamental quantity (world function), and it does not contain a reference to means of description (coordinate system). It is reasonable to use the definition (2.14) in other geometries, and in particular, in the Riemannian geometry. In the Riemannian geometry one uses the definition (2.14) only for the case, when the points P_0 and Q_0 coincide ($P_0=Q_0$). In this case the definition (2.14) coincides with (1.4). In the case, when the points P_0 and Q_0 do not coincide, the concept of the vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ equivalence is not defined. But, why?

The answer is as follows. If in inhomogeneous geometry (in the Riemannian geometry) the two vectors equivalence is defined by the relations (2.14), the vector equivalence appears to be multivariant, in general. It means, that in the point Q_0 there are many vectors $\mathbf{Q}_0\mathbf{Q}_1,\mathbf{Q}_0\mathbf{Q}_1',\mathbf{Q}_0\mathbf{Q}_1'',...$, which are equivalent to the vector $\mathbf{P}_0\mathbf{P}_1$ and are not equivalent between themselves. Multivariance is a corollary of the fact, that to determinate the vector $\mathbf{Q}_0\mathbf{Q}_1$, one needs to solve two equations (2.14) with respect to the point Q_1 at given points P_0,P_1,Q_0. In the case of the Euclidean world function σ_{E} the solution is always unique due to properties of the Euclidean world function σ_{E}. However, in the case of arbitrary world function σ there may be many solutions, or may be no solutions at all. Multivariance of the vectors equality leads to intransitivity of the equality relation. It means, that a multivariant geometry is nonaxiomatizable, because in any axiomatizable geometry the equality relation is transitive. In general, multivariance of the equality relation leads to the fact that the straight lines appear to be not a one-dimensional sets. It follows from the fact that the condition of parallelism (2.13) is one of conditions of the vectors equivalence (2.14). The parallelism condition (2.13) is a special case of the collinearity condition (2.5). Thus, there is a connection between the multivariance and many-dimensionality of straight lines in multivariant geometry.

Multivariant nonaxiomatizable geometry seems to be inadmissible for mathematicians, who are used to consider only axiomatizable geometries. The mathematicians consider any geometry as a logical construction, and they do not know, how to work with a geometry, which is not a logical construction.

In the Riemannian geometry the multivariance of the equality of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ takes place, in general, only if $P_0 \neq Q_0$. The vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_0\mathbf{Q}_1$ equality is single-variant in the Riemannian geometry. As a result segments (2.1) of geodesics (straights) in the Riemannian geometry appear to be one-dimensional. Only straights of the form (2.8) appear to be many-dimensional in the Riemannian geometry. However, such straight lines appear practically neither in physics, nor in mathematics, and mathematicians relate with disbelief to the statement on multivariance of a geometry.

Using the pluralistic approach in the transition from the proper Euclidean geometry to the Riemannian geometry, one thinks, that the one-dimensional character of straights in the Euclidean geometry is a property of any geometry. However, this property is a special property of the proper Euclidean geometry, which is conditioned by the relations (1.2) - (1.9). In the Riemannian geometry not all properties (1.2) - (1.9) are fulfilled, and some of straights (geodesics) appear to many-dimensional.

To avoid multivariance, mathematicians decided not to consider equivalence of vectors in different points of the space. As a result in the Riemannian geometry one considers only equivalence of vectors, having common origin. Equivalence of vectors at different points is defined by means of special procedure, known as a parallel transport of a vector. Procedure of the parallel transport provides the customary single-variant equivalence of vectors at different points, although the result depends on the path of the parallel transport. (As a result essentially the equivalence of two vectors at different points appears to be multivariant). Such a solution of the vector equivalence problem corresponds to the pluralistic approach to geometry, when in the Riemannian geometry one may change properties of vectors in arbitrary way. As a result the Riemannian geometry appears to be inconsistent, although this inconsistency is well masked.

At the pluralistic approach to the Riemannian geometry the world function σ_{R}, defined by the relation (2.1), appears to be derivative quantity, which depends on the shape of geodesics. Besides, in the Riemannian geometry there may be several different geodesics, connecting points P_0 and P_1. In this case the world function $\sigma_{\mathrm{R}}(P_0, P_1)$, defined by the relation (2.2) appears to be many-valued. It is inadmissible in a monistic conception of a geometry, where the world function is a fundamental quantity of a geometry.

If nevertheless we want to introduce world function σ_{R}, obtained by means of the relation (2.2), and to consider the obtained world function σ_{R} as a fundamental quantity, we need to consider only single-valued branch of σ_{R}, removing all other branches. After such a procedure one obtains the world function as a fundamental quantity, which does not depend on choice of geodesics. Different "Riemannian" geometries correspond to different choices of geodesics, generating the world function.

Let a Riemannian geometry be given on the point set Ω. Let us cut a hole Ω_1 in the set Ω. At the pluralistic approach the geometry on the set $\Omega\backslash\Omega_1$ changes, in general, because the hole changes shape of several geodesics. As a result the nonconvex point set $\Omega\backslash\Omega_1$, cannot be embedded isometrically in the point set Ω, because the set of geodesics, determining the world function σ_{R} by means of (2.2), changes. It is also an inconsistency of the Riemannian geometry in the framework of the pluralistic approach.

Using the pluralistic approach at transition from the Euclidean geometry to the Riemannian geometry, one changes independently different fundamental quantities. It is very difficult to change them concerted and to obtain consistent conception of a Riemannian geometry. At the monistic approach there is the only fundamental quantity σ. Any change of the world function σ generates a generalized geometry. There is no problems with consistency of this generalized geometry, because this geometry is not a logical construction, in general. In the obtained generalized geometry there are no theorems and axioms, because it is a constructive geometry. All definitions and geometrical objects of the generalized geometry are obtained from corresponding definitions and geometrical objects of the proper Euclidean geometry by means of a deformation. Let a statement of the Euclidean geometry be written in terms of the Euclidean world function σ_{E}, *under condition that the special relations (1.2) - (1.9) of the proper Euclidean geometry are not used in this record.* Replacing σ_{E} by the world function σ of the generalized geometry in this record, one obtains the corresponding statement of the generalized geometry. At such a deformation of the proper Euclidean geometry one does not use the formal logic, and the problem of inconsistency of the obtained geometry cannot be stated at all.

Important remark. In applications to the relativity theory one uses practically only the relation (2.6) for scalar product of two vectors, which does not use the special relations (1.2) - (1.9).

It is of no importance, that the obtained generalized geometry is not a logical construction. The generalized geometry is a science on shape and disposition of geometrical objects. This circumstance is important in applications of the geometry to physics and mechanics. It is very important, that monistic conception of the generalized geometry does not need proofs of numerous theorems, what is important at pluralistic approach to geometry.

Inconsistency of the Riemannian geometry, obtained in the framework of the pluralistic approach, is a very serious balk for application of the Riemannian geometry in the relativistic theory of gravitation. Using monistic approach to the space-time geometry, one can generalize the general relativity theory on the case of non-Riemannian geometry [6, 7].

3 Problems of transition from the pluralistic conception of geometry to the monistic one

Although the monistic conception of geometry is more perfect, than the preceding pluralistic conception, the scientific community dislikes the new monistic conceptions as a rule. The scientific community does not acknowledge the new monistic (or less pluralistic) conception for a long time. Objections of the scientific community against a new monistic (or a less pluralistic) conception have rather a social character, than a scientific one. Indeed, reducing the number of fundamental quantities in a new monistic conception, one is forced to transform some fundamental quantities of the pluralistic conception into derivative quantities of the monistic (or less pluralistic) conception. The derivative quantities of the monistic conception look more complicated, than the aforegoing quantities of the pluralistic conception, and members of the scientific community ask themselves: "Why should we consider the new complicated quantities, if they do not give anything new?"

We had such a situation in the case of transition from the axiomatic thermodynamics to the statistical physics. Indeed, the relations of the statistical physics and those of the kinetic theory are more complicated, than the simple rules of work with the thermodynamic quantities. Really they do not give anything new in those regions of physics, where the thermodynamics works well and where most researchers work. The statistical physics gives new results only at consideration of thermal fluctuations. However, this circumstance was unclear for most physicists. The scientific community as a whole was against papers by Boltzman and Gibbs, which introduced a monistic theory of thermal phenomena, reducing thermal phenomena to mechanical ones,.

A like situation takes place in quantum mechanics. A use of nonaxiomatizable space-time geometry in the microcosm admits one to reduce the quantum effects to geometrical effects. Let in microcosm the space-time geometry be described by the world function

$$\sigma_{\mathrm{d}} = \sigma_{\mathrm{M}} + d \cdot \mathrm{sign}\sigma_{\mathrm{M}}, \qquad d = \frac{\hbar}{2bc} = \mathrm{const} \tag{3.1}$$

where σ_{M} is the world function of the Minkowski geometry. Here $\hbar$ is the quantum constant, c is the speed of the light, and b is some universal constant. World chains (lines), describing particle motion, are stochastic in this space-time geometry. Statistical description of stochastic timelike world chains (lines) is equivalent to the quantum description in terms of the Schrödinger equation [8]. In such a conception the quantum principles are corollaries of the space-time geometry parameters. The number of the fundamental quantities reduces, and the conception becomes to be less pluralistic. However, it does not obtain any new effects in the region, where the conventional quantum mechanics works. New effects may be obtained in the region of the elementary particles theory [9]. One needs to make new investigations in the theory of elementary particles, using the new less pluralistic conception. Calculations, connected with these new investigations, are rather complicated, the used formalism is new, and nobody is interested in this less pluralistic approach until one will obtain new numerical results, which could show, that the less pluralistic approach is valid.

The monistic conception of geometry is essential also in the megacosm. The contemporary general relativity theory supposes, that the space-time geometry cannot be a nonaxiomatizable geometry (a non-Riemannian geometry). Such opinion is based on results of the contemporary geometry, where any geometry is considered as a logical construction (but not as a science on the dispositions of geometrical objects). For instance, the symplectic geometry has nothing to do with properties of the space-time or of the space. But it uses a mathematical technique, which is close to the mathematical technique of the Euclidean geometry. Mathematicians use the term "geometry" for the symplectic geometry, because it is a logical construction. In other words, for mathematicians a geometry is rather a logical construction, than a science on disposition of geometrical objects in the space, or in the space-time.

Apparently, such a relation to geometry is a reason of abruption of constructive (nonaxiomatizable) geometries, which are very important in application of a geometry to physics. Mathematicians are ready to accept and to develop inconsistent Riemannian geometry [10], but they are not ready to learn monistic conception of a geometry, where everything is copacetic with consistency. For instance, when I submitted my report entitled "Nonaxiomatizable geometries and their application to physics" to one of seminars of the Steklov mathematical institute, my report was rejected. I was told only, that in the Mathematical institute there are no researchers, who are interested in such problems. Of course, it is true. However, the Steklov mathematical institute is a leading mathematical institute of Russia, and its researches prefer to develop and to use the customary inconsistent conception of a geometry, ignoring the consistent one!

Nevertheless such an approach to monistic conception of geometry is rather reasonable in the light of the general approach to transition from a pluralistic conception to a monistic one, although in the given case the monistic conception of geometry is much simpler, than, for instance, the less general Riemannian geometry.

However, there are no rules without exceptions. Such an outstanding geometer as Grisha Perelman has evaluated the situation with monistic conception of geometry correctly. I have not the pleasure of knowing him, and I estimate his viewpoint, considering his behavior. After several months (end of 2005) after publishing the paper [10], Perelman left the Mathematical institute, where he worked, and said to director of the institute, that he ceases his mathematical research. Such a decision of the prosperous geometer can be explained only by a strong disappointment in his activity. During all his life Perelman developed topological direction in geometry, where topological quantities are the fundamental quantities of geometry. When he has understood, that the topological quantities are derivative quantities, which are determined by the world function, and the Riemannian geometry, which has been used in his investigations, is inconsistent, he was shocked. Apparently, this shock determines his decision on termination of his mathematical investigations.

As concerns to his refuse from the Fields medal and from award of the Clay Mathematics Institute, it is important only from the viewpoint of mercantile grassroots, who seek material values instead of human. From viewpoint of Perelman any money are nothingness with respect to problems of mathematics. Besides, the problem, solved by Perelman, is a Millennium Prize Problems only now. After several years this problem will transform to usual problem of topology. Perelman understood this very well. He refused from the award, referring to incompetence of people, adjudging a prize to him. Indeed, one can understand behavior of Perelman, only if one does know about the Riemannian geometry inconsistency. But nobody pay attention onto this inconsistency, even if one knows about signs of this inconsistency.

I have not discussed with Perelman his relation to the Riemannian geometry. My interpretation of the Perelman's behaviour is only a hypothesis. However, it is a very verisimilar hypothesis. I do not now other reasonable hypothesis, which could explain the Perelman's behaviour. I have presented this hypothesis, to convince those researchers, who believe rather in authorities, than in logical arguments. The Riemannian geometry in its conventional presentation is inconsistent, even if my interpretation of the Perelman's behaviour is wrong.

Thus, the relation of researchers to the monistic conception of geometry develops in its natural way. I think, that those researchers, who have understood advantages of the monistic conception of geometry and will use it in their investigations, will succeed in progress of physics.

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