Motion of free particle in a discrete space-time geometry

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Abstract

One considers the discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$, which is given on the set of points (events), where the geometry of Minkowski is given. This discrete geometry is not a geometry on lattice. Motion of a free particle is considered in $\mathcal{G}_{\mathrm{d}}$. Free motion in $\mathcal{G}_{\mathrm{d}}$ can be reduced to a motion in geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$ in some force field. Primordial free motion in $\mathcal{G}_{\mathrm{d}}$ appears to be stochastic. In $\mathcal{G}_{\mathrm{M}}$ it is difficult to describe the force field responsible for stochastic motion of a particle. The nature of this force field appears to be geometrical.

Key words: discrete space-time geometry; stochastic motion; reduction of discrete geometry to continuous geometry

1 Introduction

More, than hundred years ago Ludwig Boltzman suggested the method of nondeterministic particles description by means of mathematical formalism of gas dynamics. In that time only gas molecules and Brownian particles were known as stochastic (nondeterministic) particles. Quantum particles, which are stochastic particles also, were not known. At first the scientific community did not accept Boltzman's investigations. But some time ago Boltzman's kinetic equations were accepted as a method of the gas properties investigation. However, the Boltzman's investigations were not accepted as a method of the stochastic particle description.

Apparently, the reason of such a non-recognition was the fact, that the Boltzman's method cannot describe quantum particles. More exactly, one was not able to describe quantum particles by the Boltzman method. Connection between the gas dynamics and quantum mechanics was known, but it was one-sided. One can derive gas dynamics from quantum mechanics,[1], but one was not able to derive quantum mechanics from gas dynamics.

However it appears that *the classical gas dynamics can be considered as a method of a stochastical particle description.* Indeed, a gas molecule moves stochastically, because of interaction with other gas molecules. This interaction appears in the molecular collisions. If the collisions are absent, the gas molecules move deterministically. Character of stochasticity depends on the form of molecular interaction. It is turn out, that one can introduce such a molecular interaction, that the nonrotational flow of the gas with such an interaction between molecules is described by the Klein-Gordon equation. This interaction changes the molecular mass m, converting it into the effective mass M by means of the relation

$$m^2 \to M^2(x) = m^2 + \frac{\hbar^2}{c^2}\left(g_{kl}\kappa^k\kappa^l + \partial_l\kappa^l\right), \quad \partial_l \equiv \frac{\partial}{\partial x^l} \tag{1.1}$$

where κ^l, $l = 0, 1, 2, 3$ is some force field and $\hbar$ is the quantum constant. Dynamic equations for the κ-field are obtained from the corresponding action. It follows from these dynamic equations, that the κ-field has potential κ

$$\kappa_l = g_{lk}\kappa^k = \partial_l \kappa, \quad l = 0, 1, 2, 3 \tag{1.2}$$

The gas, whose molecules interact via the κ-field (1.1), is described by the action [2]

$$\mathcal{E}\left[S_{\mathrm{st}}\right]: \quad \mathcal{A}\left[x\,,\kappa\right] = \int_{\xi_0}\int_{V_\xi}\left(-mcK\sqrt{g_{lk}\dot{x}^l\dot{x}^k} - \frac{e}{c}A_l\dot{x}^l\right)d^4\xi, \quad \dot{x}^i = \frac{\partial x^i}{\partial \xi_0} \tag{1.3}$$

$$K = \frac{M}{m} = \sqrt{1 + \lambda^2\left(\kappa_l\kappa^l + \partial_l\kappa^l\right)}, \quad \lambda = \frac{\hbar}{mc}, \quad \partial_l \equiv \frac{\partial}{\partial x^l} \tag{1.4}$$

where $\xi = \{\xi_0, \boldsymbol{\xi}\}$. The variables $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_2\}$ label world line of molecules, whereas ξ_0 is a parameter along the world line. Motion of the gas molecules is stochastic. Indeed, the action for a single gas molecule is written in the form (integration over $\boldsymbol{\xi}$ is omitted)

$$S_{\mathrm{st}}: \quad \mathcal{A}\left[x\,,\kappa\right] = \int_{\xi_0}\left(-mcK\sqrt{g_{lk}\dot{x}^l\dot{x}^k} - \frac{e}{c}A_l\dot{x}^l\right)d\xi_0 \quad \dot{x}^i = \frac{\partial x^i}{\partial \xi_0}, \tag{1.5}$$

If K is defined by (1.4) and κ^l does not vanish, the action (1.5) is defined incorrectly, because $x^k = x^k(\xi_0)$ in (1.5) is one-dimensional line, whereas derivatives of κ^l in K are defined in the whole space-time.

For identification of equations of the gas dynamics with equations of quantum mechanics, it was very important, that wave function is a natural attribute of fluid dynamics [3]. Only knowing this fact, one can derive the Klein-Gordon equation, written in terms of the wave function ψ, from the classical gas dynamic equations, written in terms of hydrodynamic variables ρ,$\mathbf{v}$. There are three possible representations of gas dynamics: (1) Lagrangian representation, (2) Euler representation, and (3) reperesentation in terms of the wave function. The last representation was not known in the twentieth century.

Besides, the classical gas dynamic equations, as well as the Klein-Gordon equation describe only mean velocity and mean energy of a stochastic particle. They cannot describe distribution over velocities (for instance, Maxwell distribution). However, the gas dynamic equations can be expanded to kinetic equation, which describes the velocity distribution evolution. For quantum equation such a generalization has not been derived until now, although such a generalization is to exist, if the quantum equation is considered as the gas dynamic equations. Boltzman has derived kinetic equation, analysing the elementary act of the gas molecules collision. One should expect, that analysing interaction (1.1), one can obtain more complete information on the quantum particle motion.

In this paper we shall try to make a preliminary step for analysing the κ-field, defined by (1.1). This preliminary step is an investigation of a free particle motion in the discrete space-time geometry. The fact is that, a free motion of a particle in some exotic space-time may be equivalent to a motion in the space-time of Minkowski and some force fields in it. For instance, motion of a charged particle in a given electromagnetic and gravitational field of the four-dimensional space-time can be described as a free motion of this particle in the 5-dimensional Kaluza-Klein geometry of the space-time. In general, a boundary between the dynamics and the space-time geometry is mobile, and one can transform dynamics to the space-time geometry and vice versa. Capacities of space-time geometry are more effective, than capacities of dynamics, and we shall use this circumstance in the investigation of the κ-field, defined by (1.1).

2 Discrete geometry of space-time

All generalized geometries $\mathcal{G}$ are modifications of the the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. The discrete geometry $\mathcal{G}_{\mathrm{d}}$ by definition is such a geometry, where all distances are larger, than some minimal length λ_0. The zero length is also possible, for instance, between two coinciding points. It means mathematically, that

$$\mathcal{G}_{\mathrm{d}}: \quad |\rho_{\mathrm{d}}(P,Q)| \notin (0,\lambda_0), \ \forall P,Q \in \Omega \tag{2.1}$$

where $\rho_{\mathrm{d}}(P,Q)$ is the distance between the points P and Q in $\mathcal{G}_{\mathrm{d}}$. Ω is the set of points, where the discrete geometry $\mathcal{G}_{\mathrm{d}}$ is given.

Usually one considers the condition (2.1) as a constraint on the set Ω of points, where the discrete geometry is given. Then the constraint leads to set Ω, containing countable number of points. Such a geometry is known as a geometry on a lattice. The metric ρ (distance) is considered as a distance in the geometry of Minkowski. The geometry on a lattice has almost nothing common with the space-time geometry. In particular, one cannot construct world lines of particles in the space-time geometry on a lattice. It is not clear, how can one construct a line from points of a lattice.

We consider the condition (2.1) as a constraint on the metric ρ_{d}, whereas the set Ω is the set Ω_{M}, where the geometry of Minkowski is given. The metric ρ_{d} is chosen in such a way, that constraint (2.1) be fulfilled. For technical reason it is more convenient to use the world function $\sigma_{\mathrm{d}} = \frac{1}{2}\rho_{\mathrm{d}}^2$. The world function σ_{d} can be chosen in the form

$$\sigma_{\mathrm{d}}(P,Q) = \sigma_{\mathrm{M}}(P,Q) + \frac{\lambda_0^2}{2}\mathrm{sgn}\left(\sigma_{\mathrm{M}}(P,Q)\right) \tag{2.2}$$

where σ_{M} is the world function of the geometry of Minkowski. It easy to verify, that σ_{d} from (2.2) satisfies the constraint (2.1). At the same time the discrete geometry $\mathcal{G}_{\mathrm{d}}$, described by the world function σ_{d} is uniform and isotropic, because it is a function of σ_{M}, which is uniform and isotropic. Such a presentation of the discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ admits one to reduce a free particle motion in $\mathcal{G}_{\mathrm{d}}$ to a particle motion in geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$ with some force field. It is possible, because world line is constructed of points of the set $\Omega = \Omega_{\mathrm{M}}$.

Note, that usually researchers perceive the uniform and isotropic discrete geometry of space-time as something impossible, because usually the discrete geometry is considered as a geometry on a lattice.

There is only one quantity, which is common for Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ and the discrete geometry $\mathcal{G}_{\mathrm{d}}$. It is the distance, or world function. In order to obtain the discrete geometry $\mathcal{G}_{\mathrm{d}}$ as a modification of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$, one needs to present the Euclidian geometry $\mathcal{G}_{\mathrm{E}}$ in the monistic presentation, when all statements of $\mathcal{G}_{\mathrm{E}}$ are expressed via world function σ_{E} of $\mathcal{G}_{\mathrm{E}}$ and only via σ_{E}. It is possible [4], and we shall show, how to do this.

Let the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ be described in terms of world function σ_{E}. Vector $\mathbf{PQ} = \{P,Q\}$ is an ordered set of two points $P,Q \in \Omega$, where Ω is the set points, where $\mathcal{G}_{\mathrm{E}}$ is given. Vectors are defined by their length and mutual disposition. The length $|\mathbf{PQ}|$ of vector $\mathbf{PQ}$ is described by the relation

$$|\mathbf{PQ}| = \sqrt{2\sigma_{\mathrm{E}}(P,Q)} \tag{2.3}$$

Mutual orientation of two vectors $\mathbf{PQ}$ and $\mathbf{RS}$ is described by their scalar product $(\mathbf{PQ}.\mathbf{RS})_{\mathrm{E}}$. In terms of world function σ_{E} the scalar product is expressed by the relation

$$(\mathbf{PQ}.\mathbf{RS})_{\mathrm{E}} = \sigma_{\mathrm{E}}(P,S) + \sigma_{\mathrm{E}}(Q,R) - \sigma_{\mathrm{E}}(P,R) - \sigma_{\mathrm{E}}(Q,S) \tag{2.4}$$

The angle φ between two vectors $\mathbf{PQ}$ and $\mathbf{RS}$ is described by the relations

$$\cos\varphi = \frac{(\mathbf{PQ}.\mathbf{RS})_{\mathrm{E}}}{|\mathbf{PQ}|\,|\mathbf{RS}|} \tag{2.5}$$

Two vectors $\mathbf{PQ}$ and $\mathbf{RS}$ are equivalent (equal ($\mathbf{PQ}\mathrm{eqv}\mathbf{RS}$)), if vectors are in parallel ($\varphi = 0$) and their lengths are equal. Due to (2.4) and (2.5) the equality of vectors $\mathbf{PQ}$ and $\mathbf{RS}$ can be expressed via world function σ_{E}

$$(\mathbf{PQ}\mathrm{eqv}\mathbf{RS}): \quad (\mathbf{PQ}.\mathbf{RS})_{\mathrm{E}} = |\mathbf{PQ}| \cdot |\mathbf{RS}| \wedge |\mathbf{PQ}| = |\mathbf{RS}| \tag{2.6}$$

There is one more property of vectors, which is defined by their mutual disposition: it is the property of linear dependence of n vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$,...$\mathbf{P}_0\mathbf{P}_n$. Usually linear dependence of vectors is defined by means of linear operations over vectors. n vectors $\mathbf{u}_1 = \mathbf{P}_0\mathbf{P}_1$, $\mathbf{u}_2 = \mathbf{P}_0\mathbf{P}_2$,...$\mathbf{u}_n = \mathbf{P}_0\mathbf{P}_n$ are linear dependent, if exist such a set of real numbers $\alpha_1, \alpha_2, ...\alpha_n$, that

$$\sum_{i=1}^{i=n} \alpha_i \mathbf{u}_i = 0 \tag{2.7}$$

and not all $\alpha_i = 0$.

Although this definition is used usually, it is unsuccessful, because (1) it is nonconstructive and (2) it needs a use of linear operations on vectors. Indeed, to test whether (2.7) is valid, one needs to consider all possibilities of the choice of set $\alpha_1, \alpha_2, ...\alpha_n$. It is nonconstructive. Why one uses linear operations for definition of linear dependence, if one can define it without use of linear operations. In general, vectors u_i of linear vector space $\mathcal{L}_n$, where linear operations over vectors are defined, are another objects, which differ from vectors, defined by two points $\mathbf{PQ} = \{P, Q\}$. We shall differ vector $\mathbf{PQ} = \{P, Q\}$ from the vector $u_i \in \mathcal{L}_n$, which is an object of linear vector space $\mathcal{L}_n$. We shall refer to u_i as linear vector, or linvector [6]. The vector $\mathbf{PQ} = \{P, Q\}$ will be referred to as geometrical vector (g-vector). In the Euclidean geometry $\mathcal{G}_\mathrm{E}$ g-vector $\mathbf{PQ} = \{P, Q\}$ can be identified with linvector $u_i \in \mathcal{L}_n$. However, in the generalized geometry $\mathcal{G}$, which is obtained as a result of deformation of $\mathcal{G}_\mathrm{E}$, the linear vector space $\mathcal{L}_n$ cannot be introduced, generally speaking. In this case one should distinguish between g-vectors and linvectors.

Definition. n g-vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$,...$\mathbf{P}_0\mathbf{P}_n$ are linear dependent, if and only if the Gram's determinant

$$F_n(\mathcal{P}^n) = 0 \tag{2.8}$$

where $\mathcal{P}^n = \{P_0, P_1, ...P_n\}$

$$F_n(\mathcal{P}^n) = \det ||(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)||, \quad i, k = 1, 2, ...n \tag{2.9}$$

Definition (2.8) is constructive and it uses only information on mutual disposition of vectors. In $\mathcal{G}_\mathrm{E}$ both definitions (2.7) and (2.8) are equivalent, but in $\mathcal{G}_\mathrm{d}$ one can use only definition (2.8). It is rather unexpected, that one can speak about linear dependence of g-vectors independently of existence of linear vector space L_n, where operation over vectors are defined.

Maximally unexpected is the definition of the geometry dimension. In the Riemannian (and Euclidean) geometry the dimension (metric dimension) is introduced as independent basic quantity: "Let us consider manifold of dimension n with a smooth coordinate system on it..." In other words, the metric dimension of a geometry is defined before construction of the geometry. At such an approach it is not clear, how to introduce dimension in the discrete geometry (2.2).

Remark. In general, the metric dimension n_m is the maximal number of linear independent g-vectors in geometry $\mathcal{G}$. The coordinate dimension n_c is the number of coordinates, which are used for the geometry $\mathcal{G}$ description. In general, n_m and n_c are different quantities. However, in the Euclidean geometry and in the Riemannian geometry these dimensions coincide at the conventional description, and one does not distinguish between n_m and n_c.

In the Euclidean geometry $\mathcal{G}_\mathrm{E}$ the dimension n can be defined as a maximal number of linear independent vectors in $\mathcal{G}_\mathrm{E}$. If $\mathcal{G}_\mathrm{E}$ has dimension n, then there exist such $n+1$ points $\mathcal{P}^n = \{P_0, P_1, ...P_n\}$, that

$$\exists \mathcal{P}^n : \quad F_n(\mathcal{P}^n) \neq 0, \quad \forall \mathcal{P}^{n+k}, k \geq 1, \quad F_{n+k}\left(\mathcal{P}^{n+k}\right) = 0 \tag{2.10}$$

For the Euclidean geometry $\mathcal{G}_\mathrm{E}$ the constraints (2.10) on the world function σ_E are fulfilled. For discrete geometry $\mathcal{G}_\mathrm{d}$ with the world function σ_d defined by (2.2) the constraints (2.10) are not fulfilled, and one cannot introduce metric dimension n_m for $\mathcal{G}_\mathrm{d}$. However, describing the discrete space-time geometry $\mathcal{G}_\mathrm{d}$, we shall use four-dimensional description in the sense, that we shall use four coordinates for description of $\mathcal{G}_\mathrm{d}$, and $n_\mathrm{c} = 4$.

Constraints (2.10) mean, that introduction of the metric dimension depends on properties of the world function σ of the geometry in question. There are space-time geometries without a definite dimension. Number of space-time geometries without dimension is much greater, than the number of geometries with definite dimension (Riemannian geometries).

This fact has essential consequences. Construction of general relativity must take into account all possible space-time geometries, but not only those ones, which have a definite metric dimension. Taking into account all possible space-time geometries, one obtains extended general relativity, where existence of black holes is impossible [8] and induced antigravitation appears [9], although it is absent in conventional general relativity.

3 Motion of a particle in the discrete space-time

Smooth world lines of particles are impossible in the discrete space-time geometry. World lines are described as broken lines $\mathcal{L}_{\mathrm{br}}$. Links of the broken line are segments $T_{[P_iPi+1]}$ of straight line

$$\mathcal{L}_{\mathrm{br}} = \bigcup_s \mathcal{T}_{[P_sP_{s+1}]} \tag{3.1}$$

where segments $T_{[P_SPs+1]}$ of straight line are defined by the relation

$$T_{[P_sPs+1]} = \left\{ R | \sqrt{2\sigma(P_s, R)} + \sqrt{2\sigma(P_{s+1}, R)} - \sqrt{2\sigma(P_s, P_{s+1})} = 0 \right\} \tag{3.2}$$

This definition of the straight line segment is the same in $\mathcal{G}_{\mathrm{E}}$ and in $\mathcal{G}_{\mathrm{d}}$. But in $\mathcal{G}_{\mathrm{E}}$ $\sigma = \sigma_{\mathrm{E}}$, whereas in $\mathcal{G}_{\mathrm{d}}$ $\sigma = \sigma_{\mathrm{d}}$. Besides, in $\mathcal{G}_{\mathrm{E}}$ the length $\mu = |\mathbf{P}_s\mathbf{P}_{s+1}|$ may tend to zero, whereas in $\mathcal{G}_{\mathrm{d}}$ the link length $\mu = |\mathbf{P}_s\mathbf{P}_{s+1}| \geq \lambda_0$. Thus, in the discrete space-time geometry an additional parameter of the world line appears. The link length $\mu = |\mathbf{P}_s\mathbf{P}_{s+1}|$ is additional parameter of a world line, which is called geometrical mass, because the real particle mass m is connected with μ by means relation

$$m = b\mu \tag{3.3}$$

where b is some universal constant.

Thus, in the discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ the particle mass is a geometrical quantity with necessity. In the geometry of Minkowski, where world lines are smooth lines, the mass m may be considered as non-geometrical quantity.

In the world line of a free particle the vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$, describing adjacent links, are equal. Mathematically it means that

$$|\mathbf{P}_s\mathbf{P}_{s+1}| = |\mathbf{P}_{s+1}\mathbf{P}_{s+2}|, \quad s = ...0, 1, ... \tag{3.4}$$

$$(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2}) = |\mathbf{P}_s\mathbf{P}_{s+1}| \cdot |\mathbf{P}_{s+1}\mathbf{P}_{s+2}|, \quad s = ...0, 1, ... \tag{3.5}$$

Using for $|\mathbf{P}_s\mathbf{P}_{s+1}|$ and for $(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2})$ expressions (2.3) and (2.6) one can rewrite (3.4) and (3.5) in the form

$$\sigma_{\mathrm{d}}(P_s, P_{s+1}) = \sigma_{\mathrm{d}}(P_{s+1}, P_{s+2}) \tag{3.6}$$

$$\sigma_{\mathrm{d}}(P_s, P_{s+2}) + \sigma_{\mathrm{d}}(P_{s+1}, P_{s+1}) - \sigma_{\mathrm{d}}(P_s, P_{s+1}) - \sigma_{\mathrm{d}}(P_{s+1}, P_{s+2}) = 2\sigma_{\mathrm{d}}(P_s, P_{s+1}) \tag{3.7}$$

By means of (3.6) and $\sigma_{\mathrm{d}}(P_{s+1}, P_{s+1}) = 0$ equation (3.7) can be rewritten in the form

$$\sigma_{\mathrm{d}}(P_s, P_{s+2}) = 4\sigma_{\mathrm{d}}(P_s, P_{s+1}) \tag{3.8}$$

Two equations (3.6) and (3.8) describe world line of a free particle in the discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$.

Such a coordinate free description of a world line seems to be rather unusual. To explain situation we consider at first a coordinate free description of world line of a free particle in $\mathcal{G}_{\mathrm{M}}$. We have two equations

$$\sigma_{\mathrm{M}}(P_s, P_{s+1}) = \sigma_{\mathrm{M}}(P_{s+1}, P_{s+2}) \tag{3.9}$$

$$\sigma_{\mathrm{M}}\left(P_s, P_{s+2}\right) = 4\sigma_{\mathrm{M}}\left(P_s, P_{s+1}\right) \tag{3.10a}$$

which describe wold line of a free particle in $\mathcal{G}_{\mathrm{M}}$. It is not quite clear, how two equations (3.9) and (3.10a) describe world line, which is described usually by four space-time coordinates.

Let coordinates of points P_s, P_{s+1}, P_{s+2} look as follows

$$P_s = \left\{x^0, \mathbf{x}\right\}, \quad P_{s+1} = \left\{x^0 + p^0, \mathbf{x} + \mathbf{p}\right\}, \quad P_{s+2} = \left\{x^0 + 2p^0 + \alpha^0, \mathbf{x}+2\mathbf{p} + \boldsymbol{\alpha}\right\} \tag{3.11}$$

The quantities $x^0, \mathbf{x}, p^0, \mathbf{p}$ are known. One needs to determine $\alpha^0, \boldsymbol{\alpha}$ from two equations (3.9), (3.10a). In the coordinate form two equations (3.9), (3.10a) look as follows

$$\frac{1}{2}\left(\left(p^0\right)^2 - \mathbf{p}^2\right) = \frac{1}{2}\left(\left(p^0 + \alpha^0\right)^2 - \left(\mathbf{p} + \boldsymbol{\alpha}\right)^2\right) \tag{3.12}$$

$$\frac{1}{2}\left(\left(2p^0 + \alpha^0\right)^2 - \left(2\mathbf{p} + \boldsymbol{\alpha}\right)^2\right) = 2\left(\left(p^0\right)^2 - \mathbf{p}^2\right) \tag{3.13}$$

Resolving (3.12) with respect to α^0, one obtains

$$\alpha^0 = -p^0 \pm \sqrt{\left(p^0\right)^2 + 2\mathbf{p}\boldsymbol{\alpha} + \boldsymbol{\alpha}^2} \tag{3.14}$$

Substituting (3.14) in (3.13), one obtains equation for determination of $\boldsymbol{\alpha}$

$$\left(\left(p^0 \pm \sqrt{\left(p^0\right)^2 + 2\mathbf{p}\boldsymbol{\alpha} + \boldsymbol{\alpha}^2}\right)^2 - \left(2\mathbf{p} + \boldsymbol{\alpha}\right)^2\right) = 4\left(\left(p^0\right)^2 - \mathbf{p}^2\right) \tag{3.15}$$

After simplification it takes the form

$$\pm p^0\sqrt{\left(p^0\right)^2 + 2\mathbf{p}\boldsymbol{\alpha} + \boldsymbol{\alpha}^2} - \mathbf{p}\boldsymbol{\alpha} = \left(p^0\right)^2 \tag{3.16}$$

Eliminating radical from (3.16), one obtains after simplifications

$$\left(p^0\right)^2\left(\boldsymbol{\alpha}^2\right) = \left(\mathbf{p}\boldsymbol{\alpha}\right)^2 = \mathbf{p}^2\boldsymbol{\alpha}^2\cos^2\varphi \tag{3.17}$$

where φ is the angle between the vectors $\mathbf{p}$ and $\boldsymbol{\alpha}$**.** Expression

$$\boldsymbol{\alpha} = \left\{\alpha^1, \alpha^2, \alpha^3\right\} = 0 \tag{3.18}$$

is a solution of (3.17)

If world line is timelike, and $\left(p^0\right)^2 > \mathbf{p}^2$, then (3.18) is a unique solution of (3.17). If world line is spacelike and $\left(p^0\right)^2 < \mathbf{p}^2$, there are another solutions, when $\cos^2\varphi = \left(p^0\right)^2/\mathbf{p}^2 < 1$. In contemporary physics only tardions (particles with timelike world line) are considered. It is supposed, that tachyons (particles with spacelike world line) do not exist. Description of tachyons can find in [5]

Substituting (3.18) in (3.14), one obtains $\alpha^0 = 0$, or $\alpha^0 = -2p^0$. The value $\alpha^0 = -2p^0$ (low sign of radical) does not satisfy primary equation (3.15). Thus, for tardions one obtains $\alpha = \left\{\alpha^0, \alpha^1, \alpha^2, \alpha^3\right\} = 0$, and $\mathbf{P}_{s+1}\mathbf{P}_{s+2} = \mathbf{P}_s\mathbf{P}_{s+1} = \left\{p^0, \mathbf{p}\right\}$. As a result vectors of the broken line $\mathcal{L}_{\mathrm{br}}$ form a straight line.

Let us return to consideration of equations (3.6) and (3.8). If world line is timelike ($\sigma_{\mathrm{d}} > 0$), then using for σ_{d} expression (2.2), one can write equations (3.6) and (3.8) in the form

$$\sigma_{\mathrm{M}}\left(P_s, P_{s+1}\right) = \sigma_{\mathrm{M}}\left(P_{s+1}, P_{s+2}\right) \tag{3.19}$$

$$\sigma_{\mathrm{M}}\left(P_s, P_{s+2}\right) = 4\sigma_{\mathrm{M}}\left(P_s, P_{s+1}\right) + 3\lambda_0^2 \tag{3.20}$$

Equations (3.19) and (3.20) describe broken world line in the geometry of Minkowski. But they do not describe world line of free particle. They describe a world line of a particle, moving in the geometry of Minkowski in some force field, described by the term $3\lambda_0^2$ in (3.20).

Thus, a free motion in the discrete space-time is reduced to a motion in some force field in space-time of Minkowski. In general, coordinate free description of a world line in a physical space-time geometry $\mathcal{G}$ by means of the world function σ admits one to reduce this description to description in the geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$. At such a transformation the free motion in $\mathcal{G}$ is reduced to motion in some force field in $\mathcal{G}_{\mathrm{M}}$. For such a transformation it sufficient to represent world function σ in the form

$$\sigma = \sigma_{\mathrm{M}} + w \tag{3.21}$$

where σ_{M} is the world function of $\mathcal{G}_{\mathrm{M}}$ and the term w generates a force field in $\mathcal{G}_{\mathrm{M}}$. Such transformation is convenient in the relation that it does not need a transformation of coordinates, which is essential at a work with the Riemannian geometry. We consider example with the discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$, where $w = \frac{\lambda_0^2}{2}\operatorname{sgn}(\sigma_{\mathrm{M}})$.

4 Dynamic equations for a free particle motion in the discrete space-time geometry

We shall consider the simplest version of discrete space-time geometry (2.2), where λ_0 is a constant. If $\lambda_0 = \lambda_0(\sigma_{\mathrm{M}})$, the space-time geometry $\mathcal{G}_{\mathrm{d}}$ is also discrete. Besides, it will be uniform and isotropic. In the case, when λ_0 is a function of space-time points, $\mathcal{G}_{\mathrm{d}}$ is also discrete, but in this case $\mathcal{G}_{\mathrm{d}}$ is not uniform and isotropic, general speaking.

We shall consider the simplest case, when λ_0 =const. Besides, we shall consider the case of three dimensional space-time, in order to reduce bulky calculations. For simplicity the speed of the light is taken $c = 1$.

Let points P_s, P_{s+1}, P_{s+2} of the broken line have coordinates (3.11). Then dynamic equations (3.6) and (3.8) have the form

$$\left(p^0 + \alpha^0\right)^2 - (\mathbf{p} + \boldsymbol{\alpha})^2 = p_0^2 - \mathbf{p}^2 = \mu_0^2 \tag{4.1}$$

$$\left(2p^0 + \alpha^0\right)^2 - (2\mathbf{p} + \boldsymbol{\alpha})^2 = 4\left(p_0^2 - \mathbf{p}^2\right) + 3\lambda_0^2, \tag{4.2}$$

Taking difference of (4.2) and (4.1) one obtains

$$2p^0\alpha^0 - 2\mathbf{p}\boldsymbol{\alpha} = 3\lambda_0^2 \tag{4.3}$$

or

$$\alpha^0 = \frac{\mathbf{p}\boldsymbol{\alpha} + \frac{3}{2}\lambda_0^2}{p^0} \tag{4.4}$$

Substituting α^0 in (4.1), we obtain after simplifications

$$\left(\frac{\mathbf{p}\boldsymbol{\alpha} + \frac{3}{2}\lambda_0^2}{p^0}\right)^2 + 3\lambda_0^2 - \boldsymbol{\alpha}^2 = 0 \tag{4.5}$$

or

$$\left(p_0^2\delta^{\alpha\beta} - p^\alpha p^\beta\right)\alpha^\alpha\alpha^\beta - 3\lambda_0^2\mathbf{p}\boldsymbol{\alpha} = 3p_0^2\lambda_0^2 + \frac{9}{4}\lambda_0^4 \tag{4.6}$$

where summation is produced over repeating indices (t, x, z). Equation (4.6) can be rewritten in the form

$$\begin{aligned} &\left(p_0^2\delta^{\alpha\beta} - p^\alpha p^\beta\right)\left(\alpha^\alpha - q^\alpha\right)\left(\alpha^\beta - q^\beta\right) + 2\left(p_0^2\delta^{\alpha\beta} - p^\alpha p^\beta\right)\alpha^\alpha q^\beta - 3\lambda_0^2\mathbf{p}\boldsymbol{\alpha} \\ = \quad & 3p_0^2\lambda_0^2 + \frac{9}{4}\lambda_0^4 + \left(c^2 p_0^2\delta^{\alpha\beta} - p^\alpha p^\beta\right)q^\alpha q^\beta \end{aligned} \tag{4.7}$$

where q^α is arbitrary quantity. Let us set

$$q^\alpha = \frac{3\lambda_0^2 p^\alpha}{2\left(p_0^2 - \mathbf{p}^2\right)} = \frac{3\lambda_0^2 p^\alpha}{2\mu_0^2} \tag{4.8}$$

in (4.7). One obtains

$$\left(p_0^2\delta^{\alpha\beta} - p^\alpha p^\beta\right)\left(\alpha^\alpha - \frac{3\lambda_0^2 p^\alpha}{2\mu_0^2}\right)\left(\alpha^\beta - \frac{3\lambda_0^2 p^\alpha}{2\mu_0^2}\right) = 3p_0^2\lambda_0^2 + \frac{9}{4}\lambda_0^4 + \mathbf{p}^2\left(\frac{\frac{9}{4}\lambda_0^4}{\mu_0^2}\right) \tag{4.9}$$

Solution of equation (4.9) looks as follows

$$\boldsymbol{\alpha}_\parallel = \frac{\frac{3}{2}\lambda_0^2}{\mu_0^2}\mathbf{p} + \frac{\mathbf{p}}{|\mathbf{p}|}\frac{r}{\mu_0}\cos\theta \tag{4.10}$$

$$\boldsymbol{\alpha}_\perp = \mathbf{e}_3\frac{r}{p^0}\sin\theta \tag{4.11}$$

$$r^2 = 3\lambda_0^2 p_0^2\left(1 + \frac{3}{4}\frac{\lambda_0^2}{\mu_0^2}\right), \quad \mu_0^2 = \left(p^0\right)^2 - \mathbf{p}^2 \tag{4.12}$$

where $\boldsymbol{\alpha}_\parallel$ is component of $\boldsymbol{\alpha}$, which is in parallel with $\mathbf{p}$, whereas $\boldsymbol{\alpha}_\perp$ is component of $\boldsymbol{\alpha}$ orthogonal to $\mathbf{p}$ ($\mathbf{e}_3\mathbf{p} = 0$). The angle θ is an arbitrary quantity. Thus, solution of (4.9) is not unique even for timelike world line. Such a sitation is rather natural, because we have two dynamic equations for three variables α. It means that world line wobbles, and motion of a particle in $\mathcal{G}_\mathrm{d}$ is stochastic.

Let us introduce velocity $u = \left\{u^0, \mathbf{u}\right\}$,defined by relations

$$u^0 = \frac{cp_0 + \alpha^0}{\mu} = \frac{cp_0}{\mu}\left(1 + \frac{\frac{3}{2}\lambda_0^2}{\mu_0^2} + \frac{|\mathbf{p}|\, r}{\mu_0 c^2 p_0^2}\cos\theta\right) \tag{4.13}$$

$$\mathbf{u} = \frac{\mathbf{p} + \boldsymbol{\alpha}}{\mu} = \mathbf{u}_\parallel + \mathbf{u}_\perp \tag{4.14}$$

$$\mathbf{u}_\parallel = \frac{\mathbf{p} + \boldsymbol{\alpha}_\parallel}{\mu} = \frac{\mathbf{p}}{\mu}\left(1 + \frac{\frac{3}{2}\lambda_0^2}{\mu_0^2} + \frac{1}{|\mathbf{p}|}\frac{r}{\mu_0}\cos\theta\right) \tag{4.15}$$

$$\mathbf{u}_\perp = \frac{\boldsymbol{\alpha}_\perp}{\mu} = \mathbf{e}_3\frac{r}{\mu p^0}\sin\theta \tag{4.16}$$

were μ is a constant. The constant μ is chosen in such a way, that the length of vector $\langle u\rangle = \left\{\langle u^0\rangle, \langle\mathbf{u}\rangle\right\}$ is equal to 1. Vector $\langle u\rangle$ is the mean value of the vector $u = \left\{u^0, \mathbf{u}\right\}$. The mean value of u is defined by averaging over θ

$$\langle u\rangle = \frac{1}{2\pi}\int_0^{2\pi} u\,d\theta, \tag{4.17}$$

Averaging (4.13) - (4.16), one obtains

$$\left\langle u^0\right\rangle = \frac{cp_0}{\mu}\left(1 + \frac{\frac{3}{2}\lambda_0^2}{\mu_0^2}\right) \tag{4.18}$$

$$\left\langle \mathbf{u}_\parallel\right\rangle = \frac{\mathbf{p}}{\mu}\left(1 + \frac{\frac{3}{2}\lambda_0^2}{\mu_0^2}\right), \quad \langle\mathbf{u}_\perp\rangle = 0 \tag{4.19}$$

It follows from (4.18) and (4.19), that the length of vector $\langle u\rangle$ is described by the equation

$$\left\langle u^0\right\rangle^2 - \left\langle \mathbf{u}_\parallel\right\rangle^2 = \left(\frac{cp_0}{\mu}\right)^2\left(1 + \frac{\frac{3}{2}\lambda_0^2}{\mu_0^2}\right)^2 - \left(\frac{\mathbf{p}}{\mu}\right)^2\left(1 + \frac{3}{2}\frac{\lambda_0^2}{\mu_0^2}\right)^2 = \frac{\mu_0^2}{\mu^2}\left(1 + \frac{3}{2}\frac{\lambda_0^2}{\mu_0^2}\right)^2 = 1 \tag{4.20}$$

It follows from (4.20), that

$$\mu = \mu_0 \left(1 + \frac{3}{2}\frac{\lambda_0^2}{\mu_0^2}\right) \tag{4.21}$$

μ is minimal, if $\partial\mu/\partial\mu_0 = 0$

$$1 - \frac{3}{2}\frac{\lambda_0^2}{\mu_0^2} = 0, \quad \mu_0 = \sqrt{\frac{3}{2}}\lambda_0, \quad \mu_{\min} = 2\mu_0 = \sqrt{6}\lambda_0$$

5 Calculation of energy-momentum tensor

To derive dynamic equations for the particle motion we use the conservation laws for the matter and energy-momentum

$$\partial_i \left(\rho \left\langle u^i \right\rangle\right) = 0 \tag{5.1}$$

$$\partial_k T^{ik} = 0, \quad i = t, x, z \tag{5.2}$$

where T^{ik} is the energy-momentum tensor and ρ is the particle density. Tensor T^{ik} is expressed in terms of ρ, $\left\langle u^i \right\rangle$, and these quantities are dependent variables in dynamic equations (5.1) and (5.2).

Projections of $\mathbf{u}$ on axis OX and on axis OZ look as follows

$$u_z = u_\parallel \frac{p_z}{|\mathbf{p}|} - u_\perp \frac{p_x}{|\mathbf{p}|}, \quad u_x = u_\parallel \frac{p_x}{|\mathbf{p}|} + u_\perp \frac{p_z}{|\mathbf{p}|} \tag{5.3}$$

Using (4.15) and (4.16), the mean values of u_x and u_z can be presented in the form

$$u_x = \frac{p_x}{\mu}\left(1+\frac{\frac{3}{2}\lambda_0^2}{\mu_0^2}+\frac{1}{|\mathbf{p}|}\frac{r}{\mu_0}\cos\theta\right) + \frac{p_z}{|\mathbf{p}|}\frac{r}{\mu p^0}\sin\theta \tag{5.4}$$

$$u_z = \frac{p_z}{\mu}\left(1+\frac{\frac{3}{2}\lambda_0^2}{\mu_0^2}+\frac{1}{|\mathbf{p}|}\frac{r}{\mu_0}\cos\theta\right) - \frac{p_x}{|\mathbf{p}|}\frac{r}{\mu p^0}\sin\theta \tag{5.5}$$

Tensor energy-momentum can be obtained as follows

$$T^{ik} = \rho \left\langle u^i u^k \right\rangle, \quad i, k = t, x, z \tag{5.6}$$

Let us calculate (5.6) and express them via $\left\langle u^t \right\rangle = \left\langle u^0 \right\rangle$, $\langle u^x \rangle$, $\langle u^z \rangle$. Substituting (5.6) in (5.1) and in (5.2), one obtains dynamic equations for dependent dynamic variables $\left\langle u^0 \right\rangle$, $\langle u^x \rangle$, $\langle u^z \rangle$.

Calculation of energy-momentum tensor components gives

$$T^{00} = \rho \left\langle \left(u^0\right)^2 \right\rangle = \rho \left(\frac{cp_0}{\mu_0}\right)^2 \left(1 + \frac{3\lambda_0^2}{2\mu_0^2}\frac{|\mathbf{p}|^2}{c^2 p_0^2}\left(\frac{1+\frac{3}{4}\frac{\lambda_0^2}{\mu_0^2}}{\left(1+\frac{\frac{3}{2}\lambda_0^2}{\mu_0^2}\right)^2}\right)\right) \tag{5.7}$$

$$T^{0x} = \rho \left\langle u^0 u^x \right\rangle = \rho \frac{cp_0 p_x}{\mu_0^2}\left(1 + \frac{3\lambda_0^2}{2\mu_0^2}\frac{\left(1+\frac{3}{4}\frac{\lambda_0^2}{\mu_0^2}\right)}{\left(1+\frac{\frac{3}{2}\lambda_0^2}{\mu_0^2}\right)^2}\right) \tag{5.8}$$

$$T^{0z} = \rho \left\langle u^0 u^z \right\rangle = \rho \frac{p_0 p_z}{\mu^2}\left(1 + \frac{3\lambda_0^2}{2\mu_0^2}\frac{\left(1+\frac{3}{4}\frac{\lambda_0^2}{\mu_0^2}\right)}{\left(1+\frac{\frac{3}{2}\lambda_0^2}{\mu_0^2}\right)^2}\right) \tag{5.9}$$

$$T^{xx} = \rho\left\langle u^x u^x\right\rangle = \rho\left(\frac{p_x^2}{|\mathbf{p}|^2}\left\langle \mathbf{u}_\parallel^2\right\rangle + \frac{p_z^2}{|\mathbf{p}|^2}\left\langle \mathbf{u}_{3\perp}^2\right\rangle\right) = \rho\left(\left(\frac{p_x}{\mu}\right)^2\left(1+\frac{3}{2}\frac{\lambda_0^2}{\mu_0^2}\right)^2\right)$$
$$+\rho\left(3\frac{\lambda_0^2 c^2 p_0^2}{2\mu^2\mu_0^2}\frac{p_x^2}{|\mathbf{p}|^2}\left(1+\frac{3\lambda_0^2}{4\mu_0^2}\right)+\left(\frac{p_z}{|\mathbf{p}|}\right)^2\frac{3}{2}\frac{\lambda_0^2}{\mu^2}\left(1+\frac{3\lambda_0^2}{4\mu_0^2}\right)\right) \tag{5.10}$$

$$T^{zz} = \rho\left\langle u^z u^z\right\rangle = \rho\left(\frac{p_z^2}{|\mathbf{p}|^2}\left\langle \mathbf{u}_\parallel^2\right\rangle + \frac{p_x^2}{|\mathbf{p}|^2}\left\langle \mathbf{u}_{3\perp}^2\right\rangle\right) = \rho\left(\left(\frac{p_z}{\mu}\right)^2\left(1+\frac{3}{2}\frac{\lambda_0^2}{\mu_0^2}\right)^2\right)$$
$$+\rho\left(\left(\frac{p_z}{\mu}\right)^2\left(3\frac{\lambda_0^2 c^2 p_0^2}{2\mu^2\mu_0^2|\mathbf{p}|^2}\left(1+\frac{3\lambda_0^2}{4\mu_0^2}\right)\right)+\left(\frac{p_x}{\mu}\right)^2\frac{3}{2}\frac{\lambda_0^2}{|\mathbf{p}|^2\mu^2}\left(1+\frac{3\lambda_0^2}{4\mu_0^2}\right)\right) \tag{5.11}$$

$$T^{xz} = \rho\left\langle u^x u^z\right\rangle = \rho\left\langle u^x\right\rangle\left\langle u^z\right\rangle\left(1+\frac{3\lambda_0^2\left(\left\langle u^0\right\rangle^2-1\right)}{2\mu_0^2\left(\left\langle u_x\right\rangle^2+\left\langle u_z\right\rangle^2\right)}\frac{\left(1+\frac{3\lambda_0^2}{4\mu_0^2}\right)}{\left(1+\frac{3}{2}\frac{\lambda_0^2}{\mu_0^2}\right)^2}\right) \tag{5.12}$$

Let us express now tensor energy-momentum as a function of variables

$$v_x = \left\langle u^x\right\rangle, \quad v_z = \left\langle u^z\right\rangle \tag{5.13}$$

Eliminating

$$p_x = \mu_0\left\langle u^x\right\rangle = \mu_0 v_x, \quad p_z = \mu_0\left\langle u^z\right\rangle = \mu_0 v_z, \quad p^0 = p_0 = \mu_0\left\langle u^0\right\rangle = \mu_0 v_0 = \mu_0\sqrt{1-v_x^2-v_z^2} \tag{5.14}$$

we obtain

$$T^{00} = \rho\left(v_0^2 + B\left(v_x^2+v_z^2\right)\right) = \rho\left(v_0^2(1+B)-B\right) \tag{5.15}$$
$$T^{0x} = \rho v_0 v_x (1+B) \tag{5.16}$$
$$T^{0z} = \rho v_0 v_z (1+B) \tag{5.17}$$
$$T^{xx} = \rho\left(v_x^2(1+B)+B\right) \tag{5.18}$$
$$T^{zz} = \rho\left(v_z^2(1+B)+B\right) \tag{5.19}$$
$$T^{xz} = \rho\left\langle u_x u_z\right\rangle = \rho v_x v_z (1+B) \tag{5.20}$$

where

$$B = \frac{3\lambda_0^2}{2\mu_0^2}\frac{\left(1+\frac{3}{4}\frac{\lambda_0^2}{\mu_0^2}\right)}{\left(1+\frac{\frac{3}{2}\lambda_0^2}{\mu_0^2}\right)^2} \tag{5.21}$$

6 Dynamic equations in the space-time of Minkowski

Equation for the matter conservation

$$\partial_t(\rho v_0) + \partial_x(\rho v_x) + \partial_z(\rho v_z) = 0 \tag{6.1}$$

Energy-momentum conservation equations

$$\partial_t\left(\rho\left(v_0^2(1+B)-B\right)\right) + \partial_x(\rho v_0 v_x(1+B)) + \partial_z(\rho v_0 v_z(1+B)) = 0 \tag{6.2}$$
$$\partial_t(\rho v_0 v_x(1+B)) + \partial_x\left(\rho\left(v_x^2(1+B)+B\right)\right) + \partial_z(\rho v_x v_z(1+B)) = 0 \tag{6.3}$$
$$\partial_t(\rho v_0 v_z(1+B)) + \partial_x(\rho v_x v_z(1+B)) + \partial_z\left(\rho\left(v_z^2(1+B)+B\right)\right) = 0 \tag{6.4}$$

Taking into account last relation (5.14), one can write equation (6.1) in the form

$$\partial_t \left(\rho v_0^2\right) + \partial_x \left(\rho v_0 v_x\right) + \partial_z \left(\rho v_0 v_z\right) = \frac{B}{(1+B)} \partial_t \rho \tag{6.5}$$

Equations (6.3) and (6.4) can be written in the form

$$\partial_t \left(\rho v_0 v_x\right) + \partial_x \left(\rho v_x^2\right) + \partial_z \left(\rho v_x v_z\right) = -\frac{B}{(1+B)} \partial_x \rho \tag{6.6}$$

$$\partial_t \left(\rho v_0 v_z\right) + \partial_x \left(\rho v_x v_z\right) + \partial_z \left(\rho \left(v_z^2\right)\right) = -\frac{B}{(1+B)} \partial_z \rho \tag{6.7}$$

Differentiating the left part side of (6.5) and using (6.1), one obtains

$$\frac{dv_0}{d\tau} \equiv \left(v_0 \partial_t + v_x \partial_x + v_z \partial_z\right) v_0 = \frac{B}{(1+B)\,\rho} \partial_t \rho \tag{6.8}$$

where $d/d\tau$ is derivative with respect to the proper time. In the same way one obtains from (6.6) and (6.7)

$$\frac{dv_x}{d\tau} \equiv \left(v_0 \partial_t + v_x \partial_x + v_z \partial_z\right) v_x = -\frac{B}{(1+B)\,\rho} \partial_x \rho \tag{6.9}$$

$$\frac{dv_z}{d\tau} \equiv \left(v_0 \partial_t + v_x \partial_x + v_z \partial_z\right) v_z = -\frac{B}{(1+B)\,\rho} \partial_z \rho \tag{6.10}$$

It follows from (6.8) -(6.10) and (4.20), that

$$\frac{Bv_0}{(1+B)\,\rho} \partial_t \rho + \frac{Bv_x}{(1+B)\,\rho} \partial_x \rho + \frac{Bv_z}{(1+B)\,\rho} \partial_z \rho = 0 \tag{6.11}$$

Then it follows from (6.11) and (6.1), that

$$\partial_t v_0 + \partial_x v_x + \partial_z v_z = 0 \tag{6.12}$$

If $\lambda_0 = 0$, it follows from (5.21), that $B = 0$. Equations (a6.8) - (6.10) turns to dynamic equations for a free particle in space-time of Minkowski.

In the discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ acceleration of a free particle has the form

$$\frac{d\mathbf{v}}{d\tau} = -D \boldsymbol{\nabla} \log \rho, \quad D = \frac{B}{1+B} \tag{6.13}$$

which associates with the diffusion velocity

$$\mathbf{v}_{\mathrm{dif}} = -D \boldsymbol{\nabla} \log \rho \tag{6.14}$$

where D is the diffusion coefficient.

Thus, a free motion in the discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ is reduced to a motion in the space-time of Minkowski with some force field, which generates a diffusion. It is rather difficult to imagine a force field or its potential, which should generate stochastic (diffusion) motion of a free particle. But namely such a field is necessary, in order to generate interaction of the type (1.1).

It means, that for analysing the source of the κ-field (1.1) one needs to investigate physical geometries of space-time, and, in particular, discrete space-time geometries.

7 Concluding remarks

Consideration of discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ and of particles motion in $\mathcal{G}_{\mathrm{d}}$ is possible, only if one knows physical geometry, i.e. geometry, which is obtained from the Euclidean geometry by means of its deformation. Usually one used geometry on a lattice, but not variant (2.2) of the constraint (2.1) resolution, because it was not clear, how to use the world function for description of a discrete geometry. The fact is that, the contemporary researchers (especially mathematicians) do not accept physical geometry on the reason, that the physical geometry is not a logical construction, generally speaking. In physical geometry the equivalence relation is intransitive. It leads to ambiguity of solution of two equations (3.6) and (3.8), which describe equality of two vectors. Stochasticity of the particle world line is a corollary of this ambiguity. This ambiguity is described by a dependence of (4.10) and (4.11) on the arbitrary angle θ. Consideration of this ambiguity in physical space-time geometry admits one to investigate the particle stochasticity more particularly, than it is possible in axiomatic quantum mechanics.

References

[1] E. Madelung, Quanten theorie in hydrodynamischer Form. *Z.Phys.* **40,** 322-326, (1926).

[2] Rylov Yu.A. Gas dynamics as a tool for description of nondeterministic particles. *International J. Theoretical physics* DOI 10.1007/s10773-015-2897-3

[3] Yu. A. Rylov, Spin and wave function as attributes of ideal fluid, *J. Math. Phys.* **40**, 256-278, (1999).

[4] Rylov Yu.A. Metrical conception of the space-time geometry *Int. J. Theor, Phys.***54**, iss.1, 334-339, (2014), DOI: 10.1007/s10773-014-2228-0.

[5] Rylov Yu. A. Dynamic equations for tachyon gas, *Int. J. Theor. Phys.* **52**, 133(10), 3683- 3695, (2013)

[6] Rylov Yu. A., Inadequacy of the linear vector space formalism at metric approach to geometry. *Hypercomplex numbers in physics and geometry* **11**, pp.88-95 (2014). (In Russian)

[7] Rylov Yu. A. Euclidean geometry as algorithm for construction of generalized geometries. *http://arXiv.org/abs/math.GM/0511575*

[8] Rylov Yu. A. General relativity extended to non-Riemannian space-time geometry. *Electronic Journal of Theoretical Physics* **11**, No. 31 (2014) pp 177-202, see also *e-print /0910.3582v7*

[9] Rylov Yu. A. Induced antigravitation in the extended general relativity . *Gravitation and Cosmology*, Vol. **18**, No. 2, pp. 107–112,(2012). DOI: 10.1134/S0202289312020089