Quantum mechanics as a dynamic construction

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Abstract

The wave function and spin are shown to be attributes of the dynamics which is a dominant structure of the quantum mechanics. A self-consistent force field κ (not the quantum axiomatics) appears to be responsible for quantum effects. The field κ can escape from the matter and enables to produce pairs.

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1 Introduction

The quantum mechanics is presented in an unconventional form, when the dynamics is presented firstly. The quantum axiomatics is reserved, keeping in mind that it can be added at any time. The result of such a presentation is rather unexpected. Practically all quantum effects (diffraction, interference, tunneling, uncertainty relation, discrete energies of the atom stationary states) can be explained on the base of only dynamics, i.e. without a reference to the quantum axiomatics. Such specific quantum concepts as the wave function and the spin appear to be purely dynamic constructions which can be introduced without a reference to the quantum axiomatics.

Some self-consistent force field κ^l (but not the quantum axiomatics) appears to be a vehicle of all quantum properties. The κ-field is responsible for quantum effects, can escape from the matter and enables to produce pairs.

Considering the quantum mechanics as a totality of two structures: (i) quantum axiomatics (QM principles) and (ii) dynamics, one can formulate the quantum axiomatics (description in terms of wave function, linearity of dynamic equations in terms of wave functions, usage of linear operators etc.) in the form of statistical propositions (rules for calculation of mean values of observable quantities) which are formulated in the form

$$\langle R\rangle_\psi = \langle\psi^* \mid \hat{R} \mid \psi\rangle \tag{1.1}$$

where $\langle R\rangle_\psi$ denotes the mean value of a physical quantity R at the state ψ, and $\hat{R}$ means a linear operator corresponding to this quantity. J. von Neumann [1] had shown that one could derive the quantum axiomatics from the realtion (1.1) valid for all observable quantities.

Dynamics is a totality of: (i) dynamic variables, (ii) dynamic equations, (iii) expressions for the current j^k, for the energy-momentum tensor T^{kl} and, maybe, for some other conservative quantities. Dynamic equations and expressions for conservative quantities can be obtained from the expression for the action.

Formally the dynamics can be described by an expression for the action, because both dynamic equations and expressions for j^k and T^{kl} can be derived from the action. On the other side the statistical propositions (1.1) which are valid for all linear operators $\hat{R}$ can be considered as a formal representation of the quantum axiomatics [1]. Thus, if some result is obtained from the action without a usage of the statistical propositions, the result is purely dynamical one.

It should bear in mind that the dynamics and the statistical propositions are not completely independent. For instance, the expression for the current j^k can be derived both from the statistical propositions and from the action, but it does not mean that the expression for the current j^k is an attribute of the quantum axiomatics. The expressions for j^k and T^{kl} will be considered always as attributes of dynamics.

Let us consider a dynamic system $\mathcal{S}$ which can be interpreted as a set $\mathcal{S}[\mathcal{S}_{\text{cl}}, \kappa]$ of identical classical particles $\mathcal{S}_{\text{cl}}$ moving in a given electromagnetic field $A_k(x)$, $k = 0, 1, 2, 3$ and interacting between themselves via a self-consistent field κ^l, $l = 0, 1, 2, 3$, (there is no interaction between the elements $\mathcal{S}_{\text{cl}}$ via the electromagnetic field). A world line of $\mathcal{S}_{\text{cl}}$ is described by means of $x^l = x^l(\tau, \boldsymbol{\xi})$, $l = 0, 1, 2, 3$, where $\tau = \xi_0$ is a parameter along the world line. The Lagrangian coordinates $\boldsymbol{\xi} = \{\xi_\alpha\}$, $\alpha = 1, 2, 3$ label the world lines of $\mathcal{S}_{\rm cl}$. The action for the system $\mathcal{S}$ has the form

$$\mathcal{A}_L[x,\kappa] = \int (-mcK\sqrt{\dot{x}^i g_{ik}\dot{x}^k} - ec^{-1}A_k\dot{x}^k)d^4\xi, \tag{1.2}$$

$$K \equiv \sqrt{1+\lambda^2(\partial_l\kappa^l + \kappa^l\kappa_l)}, \qquad \dot{x}^k \equiv dx^k/d\tau, \qquad \partial_k \equiv \partial/\partial x^k, \tag{1.3}$$

where m, e, c are respectively the particle mass, a constant describing interaction with the electromagnetic field and the speed of the light. $\lambda \equiv \hbar/mc$ is the Compton wave length, and g_{ik} =diag$\{c^2, -1, -1, -1\}$ is the metric tensor.

$$x^k = x^k(\xi), \qquad \xi = \{\xi_k\}, \qquad k = 0, 1, 2, 3; \tag{1.4}$$

$\kappa^l = \kappa^l(x)$, $\quad x = \{x^l\}$, $\quad l = 0, 1, 2, 3$. There is a summation over repeating Latin indices (0–3), and Greek ones (1–3). The dynamic system $\mathcal{S}$ is a distributed dynamic system which can be considered as a fluid.

It will be shown that by means of a formal change of variables the $\mathcal{S}[\mathcal{S}_{\rm cl}, \kappa]$ reduces to a dynamic system $\mathcal{S}_{\rm q}[\psi]$ which implies the dynamic system $\mathcal{S}_{\rm KG}[\psi]$, described by the Klein-Gordon equation. Besides one shows that the quantum effects can be explained on the base of only $\mathcal{S}_{\rm KG}[\psi]$ without a reference to the statistical propositions (1.1). It means that the quantum mechanics is a dynamic construction which can be described in terms of a classical particle $\mathcal{S}_{\rm cl}$ and the field κ.

The same result can be obtained, considering a single quantum particle as a stochastic system $\mathcal{S}_{\rm st}$ and the quantum mechanics as a statistical description of these $\mathcal{S}_{\rm st}$. This result can be obtained by means of a reference to the general conception of a statistical description called SET (abbreviation of "statistical ensemble technique") [2]. SET can be applied for description of any stochastic system $\mathcal{S}_{\rm st}$ (including quantum electron and Brownian particle). Prescription of SET looks as follows.

For a statistical description of a stochastic system $\mathcal{S}_{\rm st}$ it is sufficient to construct a statistical ensemble $\mathcal{E}[\mathcal{S}_{\rm st}]$ (a set of many independent identical stochastic systems $\mathcal{S}_{\rm st}$) and to determine the *form* of $\mathcal{E}[\mathcal{S}_{\rm st}]$ as a *dynamic system*.

Stochasticity of $\mathcal{S}_{\rm st}$ means by definition that results of single experiments with $\mathcal{S}_{\rm st}$ are irreproducible, whereas measurements of distributions of any quantities relating to $\mathcal{S}_{\rm st}$ are reproducible. (For instance, an electron flying through a narrow slit strikes the screen; the impackt point position is irreproducible, whereas the diffraction picture formed by impackt points of many independent electrons is reproducible). It means that results of experiments with the set (statistical ensemble) $\mathcal{E}[\mathcal{S}_{\rm st}]$ of many identical independent $\mathcal{S}_{\rm st}$ are reproducible, and $\mathcal{E}[\mathcal{S}_{\rm st}]$ may be regarded as a dynamic system. It is difficult to say, why a set of *stochastic* $\mathcal{S}_{\rm st}$ is a *deterministic dynamic system*, but there is no doubt that it is so, and this property of the statistical ensemble can be used for a statistical description of $\mathcal{S}_{\rm st}$.

Determination of the form of $\mathcal{E}[\mathcal{S}_{\rm st}]$ means a detrmination of Lagrangian (if it exists), dynamic equations and other attributes of dynamics of $\mathcal{E}[\mathcal{S}_{\rm st}]$. There is no prescriptions how to determine the form of $\mathcal{E}[\mathcal{S}_{\rm st}]$, but if it has been determined, the statistical description of $\mathcal{S}_{\rm st}$ is carried out and completed. All reproducible results of experimernts with $\mathcal{E}[\mathcal{S}_{\rm st}]$ and, hence, with $\mathcal{S}_{\rm st}$ can be calculated on the base of dynamics of $\mathcal{E}[\mathcal{S}_{\rm st}]$.

Note that the statistical description does not contain any reference to probability density, probability amplitude and other probabilistic constructions. All this is not necessary for the statistical description regarded as a calculation of reproducible results of experiments. The term "statistical descriptions" means simply that instead of one object $\mathcal{S}_{\rm st}$ a set $\mathcal{E}[\mathcal{S}_{\rm st}]$ of many identical independent objects $\mathcal{S}_{\rm st}$ is considered and nothing more. In this paper this simple idea of the statistical description is illustrated by the example of a quantum particle. The quantum particle is considered as $\mathcal{S}_{\rm st}$, and the system $\mathcal{S} = \mathcal{E}[\mathcal{S}_{\rm st}] = \mathcal{S}[\mathcal{S}_{\rm cl}, \kappa]$ is described by the action (1.2), (1.3), where $\mathcal{S}_{\rm cl}$ are classical particles, interacting between themselves via some self-consistent field κ.

There is no necessity in the statistical propositions (1.1), as long as one is interested only in calculation of reproducible experiments. But, if the reproducible experiments are desired to be described in terms of an average behaviour of $\mathcal{S}_{\rm st}$, it is necessary to interpret $\mathcal{E}[\mathcal{S}_{\rm st}]$ either in terms of statistical propositions (1.1), or in terms of a set of classical $\mathcal{S}_{\rm cl}$, interacting between themselves via the self-consistent field κ, or in some other way. Such an interpretation is not sole. It depends essentially on a choice of dynamical variables describing $\mathcal{E}[\mathcal{S}_{\rm st}] = \mathcal{S}[\mathcal{S}_{\rm cl}, \kappa] = \mathcal{S}_{\rm q}[\psi]$.

Note that the dynamic system $\mathcal{S} = \mathcal{S}[\mathcal{S}_{\rm cl}, \kappa]$ is not a statistical ensemble of $\mathcal{S}_{\rm cl}$, because $\mathcal{S}_{\rm cl}$ are not independent. Nevertheless, the relation $\mathcal{E}[\mathcal{S}_{\rm st}] = \mathcal{S}[\mathcal{S}_{\rm cl}, \kappa]$ can be used for interpretation of *mean* behaviour of $\mathcal{S}_{\rm st}$ in terms of $\mathcal{S}_{\rm cl}$. For instance, a stochastic system $\mathcal{S}_{\rm Bst}$ (Brownian particle) can be described by world lines $\mathcal{L}$, arising as a result of integration of the equations $d\mathbf{x}/dt = -D\nabla w(t, \mathbf{x})$, where w is the probability density of detecting the Brownian particle. The $\mathcal{L}$ describe mean motion of Brownian particles, whereas the real motion of the Brownian particles is stochastic and has nothing to do with $\mathcal{L}$.

It is worth to note that there is no conceptual difference between the statistical description of the Brownian particle $\mathcal{S}_{\rm Bst}$ by means of $\mathcal{E}_{\rm B}[\mathcal{S}_{\rm Bst}]$ and the statistical description of the quantum paticle $\mathcal{S}_{\rm st}$ by means of $\mathcal{E}[\mathcal{S}_{\rm st}]$. The two statistical ensembles are distinguished only by the form of dynamic systems $\mathcal{E}_{\rm B}[\mathcal{S}_{\rm Bst}]$ and $\mathcal{E}[\mathcal{S}_{\rm st}]$. The statistical propositions are to be agreed with the form of the statistical ensemble. In the case of quantum particles the world lines (1.4) describe the mean motion of a quantum particle. The real motion of a quantum particle does not coincide with the mean motion. Appearance of the κ-field is not connected with so called problem of hidden variables [3]-[8], because the κ-field is an attribute of the statistical description, whereas the hidden variables are attributes of more detailed description, than the statistical one. Problems of interpretation connected with the statistical propositions used either in the form (1.1), or in some alternative form are discussed in [9].

The actions of the kind (1.2), (1.3) associate sometimes with an attempt of introduction of hidden variables and classical interpretation of quantum mechanics [3]-[8]. Such associations, leading usually to fruitless discussions, where classical and quantum properties are contrasted, are not adequate to the considered problem. Here two opposite concepts "quantum effect" and "quantum-axiomatical effect" are adequate.

Definition 1. Quantum effect is an observable macroscopic effect conditioned by a quantum stochasticity.

Quantum stochasticity is a set of physical reasons giving rise to irreproducibility of experiments with microparticles.

Definition 2. Quantum-axiomatic effect is an effect conditioned by a usage of the quantum axiomatics (QM principles).

The quantum-axiomatic effect arises at the transiton from the classical description to the quantum one (usage of the wave function and linear operators, linearity of dynamic equations in terms of the wave function etc.)

According to the definition the quantum-axiomatic effect is an effect of a description, whereas the quantum effect is a physical effect, i.e. effect which does not depend on a way of the description. Usually one does not see a necessity to distinguish between these effects and does not distinguish between them, using for both effects the same term "quantum effect". Two concepts instead of one term "quantum" are introduced in order to answer the question: "To what extent the quantum effects are dynamic effects and to what extent they are quantum-axiomatical ones?" From the conventional viewpoint such a question looks as an empty one, because under the conventional presentation of the quantum mechanics the dynamics is always presented in terms of the quantum axiomatics, and it is impossible to separate dynamics from the quantum axiomatics. Under the conventional presentation any quantum effect is a quantum-axiomatic effect simultaneously. One cannot distinguish between them.

All results are obtained formally on the base of the action (1.2), (1.3), i.e. without an explicit reference to SET. In the second section one considers a dynamic system which describes a quantum relativistic particle without a usage of the quantum axiomatics. The third section is devoted to investigation of the spin. In the fourth section the κ-field properties are investigated. In the fifth section it is shown that practically all quantum effects have a dynamic nature.

2 Dynamics

In the non-relativistic approximation the action (1.2) have the form

$$\mathcal{A}_L[x,\boldsymbol{\kappa}] = \int \{-mc^2 + m\dot{\mathbf{x}}^2/2 - \hbar^2(\nabla\boldsymbol{\kappa} - \boldsymbol{\kappa}^2)/(2m) - \frac{e}{c}(A_0 - \mathbf{A}\dot{\mathbf{x}})\} dt d\boldsymbol{\xi} + O(c^{-2}) \quad (2.1)$$

where

$$\tau = t, \qquad \dot{\mathbf{x}} \equiv d\mathbf{x}/dt, \qquad \kappa_l = \{\kappa_0, -\boldsymbol{\kappa}\}, \qquad \kappa^l = \{c^{-2}\kappa_0, \boldsymbol{\kappa}\}$$

Another form of the non-relativistical action can be obtained by means of a change of variables in Eq.(2.1)

$$\mathbf{u} = -\hbar\boldsymbol{\kappa}/m \quad (2.2)$$

One obtains

$$\mathcal{A}_L[\mathbf{x},\mathbf{u}] = \int \{\frac{m}{2}(\frac{d\mathbf{x}}{dt})^2 + \frac{m}{2}\mathbf{u}^2 + \frac{\hbar}{2m}\nabla\mathbf{u} - \frac{e}{c}(A_0 - \mathbf{A}\dot{\mathbf{x}})\} dt d\boldsymbol{\xi} + O(c^{-2}), \quad (2.3)$$

where $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$, $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$ are dynamic variables to be varied. The first constant term of Eq.(2.1) is omitted. The variables $\mathbf{u}$ can be regarded as a mean square velocity of a quantum stochastic motion, with the term $\hbar(2m)^{-1}\nabla\mathbf{u}$ describing an influence of the chaotic motion on the ordered one. In the non-relativistical case the κ-field associates with the Bohm potential [5].

Let us derive dynamic equations for the system $\mathcal{S}$. Varying the action (1.2) with respect to x^l, one obtains

$$\delta\mathcal{A}_L/\delta x^l(\xi) = -dp_l/d\tau - mc\sqrt{\dot{x}^s\dot{x}_s}\partial_l K - ec^{-1}\partial_l A_s\dot{x}^s = 0, \qquad (2.4)$$

$$\partial_l \equiv \partial/\partial x^l, \qquad p_l = -mcK\dot{x}_l(\dot{x}^s\dot{x}_s)^{-1/2} - ec^{-1}A_l, \qquad l = 0,1,2,3 \qquad (2.5)$$

Varying with respect to κ^l, one should take into account that κ^l is a function of x. Let us rewrite the action (1.2) in the form of an integral over x

$$\int(.)d^4\xi = \int(.)Jd^4x, \qquad J = \det \parallel \xi_{i,k} \parallel, \qquad \xi_{i,k} \equiv \partial_k\xi_i, \qquad i,k = 0,1,2,3 \quad (2.6)$$

where J is the Jacobian of the transformation from the Eulerian coordinates x to the Lagrangian coordinates ξ. Then varying with respect to κ^l, one obtains

$$\delta\mathcal{A}_L/\delta\kappa^l(x) = -\hbar^2(\rho\kappa_l - \partial_l\rho/2) = 0, \qquad l = 0,1,2,3 \qquad (2.7)$$

where the following designations are used

$$j^k = J\dot{x}^k \equiv \frac{\partial J}{\partial\xi_{0,k}} \equiv \frac{\partial(x^k, \xi_1, \xi_2, \xi_3)}{\partial(x^0, x^1, x^2, x^3)}, \qquad k = 0,1,2,3 \qquad (2.8)$$

$$\rho = \frac{\sqrt{\dot{x}^s\dot{x}_s}J}{mcK} = \frac{\sqrt{j^s j_s}}{mcK} \equiv \frac{\sqrt{j^s j_s}}{mc\sqrt{1 + \lambda^2(\partial_k\kappa^k + \kappa_k\kappa^k)}}. \qquad (2.9)$$

It follows from Eq.(2.7) that the field κ^l has a potential $\kappa = \frac{1}{2}\log(\rho/a^2)$, where a =const. Thus,

$$\rho = a^2 e^{2\kappa}, \qquad \kappa_l \equiv \partial_l\kappa \qquad (2.10)$$

Eqs.(2.9), (2.10) determine a dynamic equation for the potential κ, which can be written also in the form

$$\lambda^2\partial_l\partial^l e^\kappa = (a^{-4}m^{-2}c^{-2}e^{-4\kappa}j^s j_s - 1)e^\kappa. \qquad (2.11)$$

Replacing $\partial J/\partial\xi_{0,k}$ by j^k, the identity

$$\partial_k(\partial J/\partial\xi_{0,k}) \equiv 0 \qquad (2.12)$$

turns in virtue of Eq.(2.8) into the continuity equation

$$\partial_k j^k = 0 \qquad (2.13)$$

Let multiply Eq.(2.4) by J, use the designation (2.11) and the relation

$$dp_l/d\tau = \dot{x}^k\partial_k p_l, \qquad (2.14)$$

which is valid, if p_l is considered as a function of only x. After some transformations one obtains

$$j^k(\partial_k p_l - \partial_l p_k) = 0, \qquad l = 0,1,2,3 \tag{2.15}$$

$$p_l = -a^{-2}e^{-2\kappa}j_l - ec^{-1}A_l, \qquad l = 0,1,2,3 \tag{2.16}$$

Only three among four equations (2.15) are independent, as far as a convolution of Eq.(2.15) with j^l gives an identity. Thus, equations (2.11), (2.13), (2.15), (2.16) form a system of the hydrodynamical type equations for the current j^l, $l = 0,1,2,3$ of the fluid and for the potential κ of the self-consistent κ-field. The possibility of writing quantum dynamic equations in the hydrodynamical form is well known [10]. Here one uses more developed technique which permits to integrate kinematic part of hydrodynamic equations and describe them in terms of hydrodynamic potentials [11].

Equations (2.13), (2.15), (2.16) can be integrated in a general form

$$p_l = -a^{-2}e^{-2\kappa}g_{lk}\partial J/\partial\xi_{0,k} - ec^{-1}A_l = b\partial_l\varphi + bg^\alpha(\boldsymbol{\xi})\xi_{\alpha,l}, \qquad l = 0,1,2,3 \tag{2.17}$$

$$j^l = \partial J/\partial\xi_{0,l}, \qquad l = 0,1,2,3 \tag{2.18}$$

where $\partial J/\partial\xi_{0,k}$ is a function of the first derivatives $\xi_{\alpha,k} \equiv \partial_k\xi_\alpha$, $k = 0,1,2,3$, $\alpha = 1,2,3$, determined by the relation (2.8), $g^\alpha(\boldsymbol{\xi})$, $\alpha = 1,2,3$ are arbitrary functions $\boldsymbol{\xi}$, b is a constant factor $[b] = [action]$, which is used here to make the functions $g^\alpha(\boldsymbol{\xi})$ dimensionless (the quantities $\boldsymbol{\xi}$ are supposed to be dimensionless). The constant b will be referred to as a scaling constant, or scaling factor. As far as b is an arbitrary constant, it can be identified with the Planck constant $\hbar$. Substituting Eqs.(2.17), (2.18) into Eqs.(2.13), (2.15) and using identities (2.12) and

$$(\partial J/\partial\xi_{0,l})\xi_{\alpha,l} \equiv 0, \qquad \alpha = 1,2,3 \tag{2.19}$$

one can verify that the relations (2.17), (2.18) satisfy the equations (2.13), (2.15) at any choice of functions $g^\alpha(\boldsymbol{\xi})$. Equations (2.17), (2.11) present four dynamic equations of the first order for four hydrodynamic potentials $\varphi = \xi_0$, ξ_α, $\alpha = 1,2,3$, and a second order equation for the potential κ of the κ-field.

The description in terms of hydrodynamic equations (2.11), (2.13), (2.15), (2.16) contains information only on the particle velocity in the sense that, if $\kappa(x)$, $j^k(x)$ is a solution of equations (2.11), (2.13), (2.15), the velocity is determined by the kinematic equations

$$dx^\alpha/dx^0 = j^\alpha/j^0, \qquad \alpha = 1,2,3 \tag{2.20}$$

and one needs to integrate equations (2.20), in order to determine world lines of a particle.

The description (2.11), (2.17) in terms of potentials is more informative in the sense that it contains information about both position and velocity of a fluid particle. If $\boldsymbol{\xi} = \boldsymbol{\xi}(x)$ is a solution of Eqs.(2.17), the world lines of particles are determined in an implicit way by finite relations

$$\boldsymbol{\xi}(t,\mathbf{x}) = \boldsymbol{\xi}_0 = \text{const}, \tag{2.21}$$

which do not need integration. Essentially, a derivation of the integrals (2.17) is an implicit integration of the kinematic equations (2.20) without a preliminary determination of the current j^k from the hydrodynamic equtions (2.13), (2.15).

On the other side a description in terms of potentials is indefinite in the sense that there is no one-to-one correspondence between the fluid flow and potentials φ, ξ. The same flow can be described by different set of potentials $\varphi = \xi_0$, $\boldsymbol{\xi}$. It is displayed mathematically in the invariancy of the action (1.2) and dynamic equations (2.17) with respect to a group of transformations of Lagrangian coordinates.

$$\xi_\alpha \to \tilde{\xi}_\alpha = f_\alpha(\boldsymbol{\xi}), \qquad \det \parallel \partial f_\alpha/\partial\xi_\beta \parallel = 1, \qquad \alpha,\beta = 1,2,3 \tag{2.22}$$

$$\xi_0 \to \tilde{\xi}_0 = f_0(\xi_0, \boldsymbol{\xi}), \qquad \partial f_0/\partial\xi_0 > 0 \tag{2.23}$$

where $f_\alpha(\boldsymbol{\xi})$, $f_0(\xi)$ are arbitrary functions of their arguments restricted only by the second relations (2.22), (2.23). In virtue of Eq.(2.8) the equation (2.17) is invariant with respect to transformations (2.22), (2.23). Arbitrary functions $g^\alpha(\boldsymbol{\xi})$ transform as vectors in the space of Lagrangian coordinates $\boldsymbol{\xi}$.

$$g^\alpha(\boldsymbol{\xi}) \to \tilde{g}^\alpha(\tilde{\boldsymbol{\xi}}) = g^\beta(\boldsymbol{\xi})\partial\xi_\beta/\partial\tilde{\xi}_\alpha, \qquad \alpha = 1,2,3 \tag{2.24}$$

World lines of the fluid particles are labelled by Lagrangian coordinates $\boldsymbol{\xi}$. The labelling is arbitrary and does not influence on the dynamics of the system. The transformation (2.22) describes relabelling of world lines.

A description in terms of a wave function is a kind of a discription in terms of hydrodynamic potentials [11]. To obtain this description it should remove bulky expression for $j^k = \partial J/\partial\xi_{0,k}$, appearing in equations (2.17) (2.11). Let us convolute equations (2.17) with $\partial^l g^\alpha(\boldsymbol{\xi})$. In virtue of identities (2.19) one obtains three first order equations for the potentials $\boldsymbol{\xi}$

$$\pi^l(\xi)\partial_l g^\alpha(\boldsymbol{\xi}) = 0, \qquad \alpha = 1,2,3 \tag{2.25}$$

where

$$\pi^l(\xi) \equiv b\partial^l\varphi + bg^\beta(\boldsymbol{\xi})\partial^l\xi_\beta + ec^{-1}A^l, \qquad l = 0,1,2,3 \tag{2.26}$$

To obtain the equation for the potential φ let us substitute $\partial J/\partial\xi_{0,k}$ from Eq.(2.17) into Eq.(2.12). One obtains the second order equation for φ in the form

$$\partial_k[e^{2\kappa}\pi^k(\xi)] = 0, \tag{2.27}$$

Finally, substituting $\partial J/\partial\xi_{0,k}$ from Eq.(2.17) into Eq.(2.11), one obtains the generalized Hamilton-Jacobi equation

$$\hbar^2(\kappa^k\kappa_k + \partial_k\kappa^k) = \pi_l(\xi)\pi^l(\xi) - m^2c^2 \tag{2.28}$$

which should be considered as a dynamic equation for κ. The system of equations (2.25)–(2.28) has the seventh order with respect to time derivatives. It is one unit more, than the system of equations (2.11), (2.17). One has such an increase, because the continuity equation (2.13) is used instead of its integral (2.18).

Equations (2.25), (2.27), (2.28) can be obtained as a result of variation of the action

$$\mathcal{A}_E[\varphi,\boldsymbol{\xi},\kappa]=\frac{a^2}{2}\int e^{2\kappa}[\pi_l(\xi)\pi^l(\xi)+\hbar^2(\partial_l\kappa)(\partial^l\kappa)-m^2c^2]d^4x \qquad (2.29)$$

respectively with respect to $\boldsymbol{\xi}$, φ and κ.

Let $\psi=\{\sqrt{\rho}e^{i\varphi}v_\alpha(\boldsymbol{\xi})\}$, $\quad \alpha=1,2,\ldots n$, $\rho=a^2e^{2\kappa}$ be n-component complex function satisfying the condition $\sum_{\alpha=1}^n v_\alpha^* v_\alpha=1$, and v_α are functions of only $\boldsymbol{\xi}$. $(^*)$ denotes a complex conjugation. Then

$$\rho=\psi^*\psi, \qquad j_k+\frac{e}{c}A_k\rho\equiv\frac{ib}{2}(\psi^*\partial_k\psi-\partial_k\psi^*\psi)=-b\rho[\partial_k\varphi+f^\beta(\boldsymbol{\xi})\partial_k\xi_\beta] \qquad (2.30)$$

where

$$f^\beta(\boldsymbol{\xi})=-\frac{i}{2}\sum_{\alpha=1}^n(v_\alpha^*\frac{\partial v_\alpha}{\partial\xi_\beta}-\frac{\partial v_\alpha^*}{\partial\xi_\beta}v_\alpha), \qquad \beta=1,2,3 \qquad (2.31)$$

(2.30) (2.17) coincide, if $f^\alpha=g^\alpha$, $\alpha=1,2,3$. One suggests that five real dynamic variables $\rho,\varphi,\boldsymbol{\xi}$ can be substituted by a n-component complex function ψ, described by $2n$ real variables. To make such a change of variables, it is sufficient to find a solution of the system of equations

$$-\frac{i}{2}\sum_{\alpha=1}^n(v_\alpha^*\frac{\partial v_\alpha}{\partial\xi_\beta}-\frac{\partial v_\alpha^*}{\partial\xi_\beta}v_\alpha)=g^\beta(\boldsymbol{\xi}), \qquad \beta=1,2,3, \qquad \sum_{\alpha=1}^n v_\alpha^*v_\alpha=1, \qquad (2.32)$$

the form of the solution depending essentially on the form of functions $g^\alpha(\boldsymbol{\xi})$. Then

$$\psi=\{\psi_\alpha\}=\{\sqrt{\rho}e^{i\varphi}v_\alpha(\boldsymbol{\xi})\}, \qquad \alpha=1,2,\ldots n \qquad \psi^*\psi\equiv\sum_{\alpha=1}^n\psi_\alpha^*\psi_\alpha \qquad (2.33)$$

and the action (2.29) takes the form

$$\mathcal{A}_E[\psi,\psi^*]=\frac{1}{2}\int\Big\{\frac{1}{\psi^*\psi}[\frac{ib}{2}(\psi^*\psi_k-\psi_k^*\psi)-\frac{e}{c}A_k\psi^*\psi][\frac{ib}{2}(\psi^*\psi^k-\psi^{*k}\psi)-\frac{e}{c}A^k\psi^*\psi]+$$

$$+\frac{\hbar^2\partial_k(\psi^*\psi)\partial^k(\psi^*\psi)}{4\psi^*\psi}-m^2c^2\psi^*\psi\Big\}d^4x, \qquad \psi_k\equiv\partial_k\psi \qquad (2.34)$$

In fact there is no necessity to solve the system (2.32). It is sufficient to be sure that it has a solution for any functions $g^\alpha(\boldsymbol{\xi})$. In general case the dynamic equation for ψ-function is nonlinear. One distinguishes between the ψ-function and the wave function. By definition the wave function is a ψ-function, satisfying a linear dynamic equation and other restrictions of the quantum axiomatics.

Let the ψ-function have n components. Regrouping components of the ψ-function in the action (2.34), one obtains the action in the form

$$\mathcal{A}_E[\psi,\psi^*]=\frac{1}{2}\int[(ib\partial_k+ec^{-1}A_k)\psi^*(-ib\partial^k+ec^{-1}A^k)\psi-$$

$$-(m^2c^2+\frac{b^2}{2}\sum_{\alpha,\beta=1}^n Q_{\alpha\beta,k}Q_{\alpha\beta}^{*\ k})\rho$$

$$+(\hbar^2-b^2)(4\rho)^{-1}\partial_k\rho\partial^k\rho]d^4x, \qquad \rho\equiv\psi^*\psi \tag{2.35}$$

where

$$Q_{\alpha\beta,k}=\frac{1}{\psi^*\psi}\begin{vmatrix}\psi_\alpha & \psi_\beta\\ \partial_k\psi_\alpha & \partial_k\psi_\beta\end{vmatrix}, \qquad Q_{\alpha\beta}{}^k=g^{kl}Q_{\alpha\beta,l} \qquad \alpha,\beta=1,2,\dots n \tag{2.36}$$

If $g^\alpha(\boldsymbol{\xi})\equiv 0$, the equations (2.32) are satisfied by a one-component ψ-function $\psi=ae^{\kappa+i\varphi}v_1$, $v_1=1$, the Q_{11}^k vanishes, and the action (2.35) turns to the action

$$\mathcal{A}_{KG}[\psi,\psi^*]=\frac{1}{2}\int[(ib\partial_k+\frac{e}{c}A_k)\psi^*(-ib\partial^k+\frac{e}{c}A^k)\psi-m^2c^2\rho$$

$$+(\hbar^2-b^2)(4\rho)^{-1}\partial_k\rho\partial^k\rho]d^4x, \qquad \rho\equiv\psi^*\psi \tag{2.37}$$

This action associates with the Klein-Gordon dynamic system $\mathcal{S}_{KG}$, described by the Klein-Gordon equation

$$(-i\hbar\partial_k+ec^{-1}A_k)(-i\hbar\partial^k+ec^{-1}A^k)\psi-m^2c^2\psi=0 \tag{2.38}$$

where the ψ-function can be considered as a wave function in the sense that it satisfies a linear equation and is compatible with the quantum axiomatics. This equation describes a potential flow of the fluid ($p_l=b\partial_l\varphi$).

Indeed, choosing the scaling constant $b=\hbar$ in Eq.(2.37), one obtains instead of the action (2.37)

$$\mathcal{A}_{KG}[\psi,\psi^*]=\frac{1}{2}\int[(i\hbar\partial_k+\frac{e}{c}A_k)\psi^*(-i\hbar\partial^k+\frac{e}{c}A^k)\psi-m^2c^2\psi^*\psi]d^4x \tag{2.39}$$

which generates the Klein-Gordon equation (2.38) as a dynamic equation and associates with the following expessions for the current j^k and the canonical energy-momentum tensor T_k^l

$$j^k=\frac{i\hbar}{2}(\psi^*\psi^k-\psi^{*k}\psi)-\frac{e}{c}A^k\psi^*\psi, \qquad k=0,1,2,3; \qquad \psi^k\equiv\partial^k\psi \tag{2.40}$$

$$T_k^l=2^{-1}\hbar^2(\psi^{*l}\psi_k+\psi_k^*\psi^l-\delta_k^l\psi^{*s}\psi_s)-ec^{-1}(A_kj^l-\delta_k^lA_sj^s)+2^{-1}\delta_k^lm^2c^2\psi^*\psi, \tag{2.41}$$
$$k,l=0,1,2,3$$

The current density $j^k=\dot{x}^kJ$ and the tensor energy-momentum T_k^l, considered as sources of the electromagnetic and gravitational fields respectively, are attributes of the dynamic system $\mathcal{S}$ (but not attributes of the statistical propositions (1.1)). The canonical energy-momentum tensor associated with the action (1.2) has the form

$$T^l{}_k=-p_kj^l-2^{-1}\hbar^2a^2e^{2\kappa}\partial_k\kappa^l=\frac{mcKj^lj_k}{\sqrt{j^sj_s}}+\frac{e}{c}A_kj^l-\frac{1}{2}\hbar^2a^2e^{2\kappa}\partial_k\kappa^l. \tag{2.42}$$

In the potential case j^k and T_k^l can be expressed through the wave function ψ and coincide with expressions (2.40) and (2.41) respectively. The potential flow in the dynamic system $\mathcal{S}$ is described completely in terms of the dynamic system $\mathcal{S}_{KG}$.

Thus, *the dynamic system $\mathcal{S}$ is some kind of a dynamic system with the κ-field being responsible for quantum effects.*

Note that the action (2.37) describes the same dynamic system $\mathcal{S}_{KG}$ at any value of the scaling constant b. The ψ-functions in the actions (2.37) and (2.39) have different scales of the ψ-function phases. Making a change of variables

$$\psi \to \tilde{\psi} = \mid \psi \mid \exp\left[b^{-1}\hbar \log\left(\psi \mid \psi \mid^{-1}\right)\right] \tag{2.43}$$

in the action (2.37), one obtains the action (2.39). This change of variables can be made most simply in the action (2.34).

The action (2.37) generates a nonlinear dynamic equation

$$(-ib\partial_k + ec^{-1}A_k)(-ib\partial^k + ec^{-1}A^k)\psi$$

$$-[m^2c^2 + 2^{-1}(\hbar^2 - b^2)\partial_k\partial^k \log(\psi^*\psi)]\psi = 0 \tag{2.44}$$

The change of variables (2.43) transforms the nonlinear equation (2.44) into the linear Klein-Gordon equation (2.38).

The equation (2.44) can be made linear by other means. Setting $b = \hbar$ in Eq.(2.44), one obtains the linear equation (2.38). On one hand, this choice, connecting the arbitrary scaling constant with the quantum constant $\hbar$, removes nonlinear term in the equation (2.44) (and in the action (2.37)). On the other hand it turns "classical terms", containing the constant b , into "quantum terms", containing the quantum constant. As a result the action (2.39) and the dynamic equation (2.38) look as completely quantum, whereas the equivalent action (2.37) and the dynamic equation (2.44) have only *one quantum term*, containing $\hbar$.

From the practical viewpoint linear dynamic equations are very convenient. From the conceptual dynamical viewpoint the linearity means nothing. The quantum axiomatics considers the linearity as a conceptual constraint of possible dynamic equations. It is this constraint of the quantum axiomatics, that dictates the choice of the scaling factor $b = \hbar$. It is this constraint that removes non-potential flows, because they are not described by linear dynamic equations.

3 Spin as a kinematic invariant

The number n of the ψ-function components in the actions (2.34) and (2.35) is arbitrary. A formal variation of the action with respect to ψ_α and ψ_α^*, $\alpha = 1, 2, \dots n$ leads to $2n$ dynamic equaitons, but not all of them are independent. There are such combinations of variations $\delta\psi_\alpha$, $\delta\psi_\alpha^*$, $\alpha = 1, 2, \dots n$ which do not change expressions (2.30). Such combinations of variations $\delta\psi_\alpha$, $\delta\psi_\alpha^*$, $\alpha = 1, 2, \dots n$ do not change the action (2.34), and corresponding combinations of dynamic equations $\delta\mathcal{A}_E/\delta\psi_\alpha = 0$, $\delta\mathcal{A}_E/\delta\psi_\alpha^* = 0$ are identities that associates with a dependence between dynamic equations.

Thus, increasing the number n, one increases the number of dynamic equations, but the number of independent dynamic equations remains the same.

In such a situation it is important to determine the minimal number n of the ψ-function components, sufficient for a description of the given vector field $g^\beta(\boldsymbol{\xi})$ in the space $V_{\boldsymbol{\xi}}$ of the Lagrangian coordinates $\boldsymbol{\xi}$.

Let G be a set of all vector fields $g^\beta(\boldsymbol{\xi})$ in $V_{\boldsymbol{\xi}}$, and G_n be a set of such vector fields $g^\beta(\boldsymbol{\xi})$ in $V_{\boldsymbol{\xi}}$ which can be presented in the form

$$g^\beta(\boldsymbol{\xi}) = \sum_{k=1}^{n} \eta_k^2(\boldsymbol{\xi})\partial\zeta_k(\boldsymbol{\xi})/\partial\xi_\beta, \qquad \eta_1 \equiv 1 \tag{3.1}$$

where the functions η_k, ζ_k, $k = 1,2,\ldots n$ are scalars in $V_{\boldsymbol{\xi}}$. Under the relabelling transformation (2.22) the functions (3.1) transform as follows

$$\eta_k(\boldsymbol{\xi}) \to \tilde{\eta}_k(\tilde{\boldsymbol{\xi}}) = \eta_k(\boldsymbol{\xi}), \qquad \zeta_k(\boldsymbol{\xi}) \to \tilde{\zeta}_k(\tilde{\boldsymbol{\xi}}) = \zeta_k(\boldsymbol{\xi}), \qquad k = 1,2,\ldots n$$

$$g^\beta(\boldsymbol{\xi}) \to \tilde{g}^\beta(\tilde{\boldsymbol{\xi}}) = \frac{\partial\xi_\alpha}{\partial\tilde{\xi}_\beta}g^\alpha(\boldsymbol{\xi}) = \frac{\partial\xi_\alpha}{\partial\tilde{\boldsymbol{\xi}}_\beta}\sum_{k=1}^{n}\eta_k^2(\boldsymbol{\xi})\frac{\partial\zeta_k(\boldsymbol{\xi})}{\partial\xi_\alpha} = \sum_{k=1}^{n}\tilde{\eta}_k^2(\tilde{\boldsymbol{\xi}})\frac{\partial\tilde{\zeta}_k(\tilde{\boldsymbol{\xi}})}{\partial\tilde{\xi}_\alpha} \tag{3.2}$$

In other words, a vector field $g^\beta(\boldsymbol{\xi})$ of the form (3.1) transforms into the vector field $\tilde{g}^\beta(\tilde{\boldsymbol{\xi}})$ of the same form (3.1), and the set G_n is invariant with respect to the group (2.22) of the relabelling transformations.

It is easy to see that

$$G_{n-1} \subseteq G_n, \qquad G_0 = \emptyset, \qquad n = 1,2,\ldots \tag{3.3}$$

because the nth term of the sum (3.1) can be combined with the first one, if ζ_n is a function of η_n. Let

$$S_n = G_n \backslash G_{n-1}, \qquad n = 1,2,\ldots \tag{3.4}$$

Then

$$G = \bigcup_{s=1}^{s=k} S_s, \qquad S_l = \emptyset, \qquad l = k+1, k+2, \ldots \tag{3.5}$$

where k is the number of nonempty invariant subsets of the set G. Each subset S_k contains only such vector fields $g^\beta(\boldsymbol{\xi})$ which associate with the k-component ψ-function $\psi = \{\psi_\alpha\}$, $\alpha = 1,2,\ldots k$, having the components

$$\psi_1 = \left\{(1 - \sum_{\alpha=2}^{k}\eta_\alpha^2)\rho\right\}^{1/2}\exp[i(\varphi+\zeta_1)],$$

$$\psi_\alpha = \eta_\alpha\sqrt{\rho}\exp[i(\varphi+\zeta_\alpha+\zeta_1)], \qquad \alpha = 2,3,\ldots k \tag{3.6}$$

In particular, the set S_1 associates with a potential flow, described by a one-component ψ-function determined by one scalar ζ_1; and the set S_2 associates with a vortical flow described by a two-component ψ-function, determined by three scalar functions ζ_1, η_2, ζ_2.

One should expect that in the three-dimensional space $V_{\boldsymbol{\xi}}$ any three-component vector $g^\beta(\boldsymbol{\xi})$ can be expressed through three scalar fields

$$g^\beta(\boldsymbol{\xi}) = \partial\zeta_1/\partial\xi_\beta + \eta_2^2\partial\zeta_2/\partial\xi_\beta, \qquad \beta = 1,2,3 \tag{3.7}$$

But strictly, this statement is to be proved.

If, indeed, the relation (3.7) takes place for any field $g^\beta(\boldsymbol{\xi})$, there are only two nonempty invariant subsets S_1, S_2 of the set G

$$S_1 \cup S_2 = G, \qquad S_l = \emptyset, \qquad l = 3, 4, \ldots \tag{3.8}$$

In the conventional quantum mechanics the number n of the wave function components is connected with the spin s of the particle, described by the ψ, by means of the relation $s = (n-1)/2$. The spin is considered as an internal property of a quantum particle. Particles with different spins are considered as different physical objects, described by different dynamic equations.

In a like manner the potential flow, when $g^\beta(\boldsymbol{\xi})$ is described by one function ζ_1, associates with the kinematic spin $s = 0$, whereas the vortical flow, when $g^\beta(\boldsymbol{\xi})$ is described by three scalar functions ζ_1, η_2, ζ_2, associates with the kinematic spin $s = 1/2$. The term "kinematic spin" (instead of "spin" simply) is used for the following reasons.

First, the kinematic spin is determined by the form of the vector field $g^\beta(\boldsymbol{\xi})$, which arises essentially as a result of an integration of the kinematic equations (2.20). The vector field $g^\beta(\boldsymbol{\xi})$ is a kinematic structure (not a dynamic one), because it does not depend on forces (on the external electromagnetic field and on the self-consistent "quantum" κ-field). At the same time the kinematic spin is not an internal property of a particle in itself, because the action (2.34) describes, at least, two different kinematic spins ($s = 0$ and $s = 1/2$) simultaneously, and the kinematic spin looks rather as an integration constant, than a property of single particles, forming the dynamic system $\mathcal{S}$.

Second, there is a distinction between the transformation properties of the spin and those of the kinematic spin under the space rotation group. Components of the n-component ψ-function are scalars under the space rotation group for any value of n (and for any value of the kinematic spin s). In the conventional quantum mechanics a one-component wave function ψ ($s = 0$) is a scalar, whereas the two-component wave function ψ ($s = 1/2$) transforms as a spinor under the rotation group.

Taking into account the indefiniteness of the ψ-function, it is possible to change transformation properties of the ψ-function, attending any spatial rotation by a proper transformation of the group (2.22). The additional transformations (2.22) can be chosen in such a way, that the two-component ψ-function becomes a spinor under spatial rotations. Then the formal distinction between the "kinematic spin" and the "spin" vanishes, and one can identify them.

For instance, let the two-component ψ-function have the form

$$\psi = \sqrt{\rho}\exp[i(\varphi + \boldsymbol{\sigma\xi})]\chi, \tag{3.9}$$

where ρ, φ, $\boldsymbol{\xi}$ are scalar functions of x, $\boldsymbol{\sigma} = \{\sigma_\alpha\}$, $\alpha = 1, 2, 3$ are the Pauli matrices, and χ is a two-component constant column ($\chi^*\chi = 1$).

Let any infintesimal spatial rotation

$$x^0 \to \tilde{x}^0 = x^0, \qquad \mathbf{x} \to \tilde{\mathbf{x}} = \mathbf{x} + \boldsymbol{\omega} \times \mathbf{x} + O(\boldsymbol{\omega}^2), \qquad |\, \omega \,| \ll 1 \tag{3.10}$$

be attended by an infinitesimal transformation

$$\boldsymbol{\xi} \to \tilde{\boldsymbol{\xi}} = \boldsymbol{\xi} - \boldsymbol{\omega}/2 \tag{3.11}$$

Then the ψ-function transforms as a spinor with respect to the combined transformation (3.10), (3.11)

$$\psi(x) \to \tilde{\psi}(\tilde{x}) = \sqrt{\tilde{\rho}(\tilde{x})}\exp[i(\tilde{\varphi}(\tilde{x}) + \boldsymbol{\sigma}\tilde{\boldsymbol{\xi}}(\tilde{x}))]\chi = e^{-i\boldsymbol{\omega}\boldsymbol{\sigma}/2}\psi(x) + O(\boldsymbol{\omega}^2) \tag{3.12}$$

and as a scalar with respect to the space rotation (3.10) alone.

If the dynamic system is described in terms of a two-component ψ-function, and the Lagrangian coordinates $\boldsymbol{\xi}$ are not mentioned at all, the ψ-function can be considered equally readily as a scalar and as a spinor.

However, let us note that such combinations as $\psi^*\boldsymbol{\sigma}\nabla\psi$ and $\psi^*\boldsymbol{\sigma}\psi\mathbf{A}$ are scalars with respect to the space rotations, if ψ is a spinor, and they are vectors, if ψ is a scalar. Such combinations appear in the action for the dynamic systems $\mathcal{S}_D$ and $\mathcal{S}_P$, described respectively by the Dirac equation and by the Pauli equation. They are considered conventionally as scalars. It does not lead to any problems, provided the description is carried out in terms of the wave function. But if the description is produced in terms of such physical quantities as current, spin and κ-field, some problems arise with the relativistical invariance in $\mathcal{S}_D$ [12] and with the rotational invariance in $\mathcal{S}_P$ [13].

As for the actions (2.34) and (2.35), they do not contain such convolution as $\boldsymbol{\sigma}\nabla$, or $\boldsymbol{\sigma}\mathbf{A}$. For instance, in virtue of the identity

$$\partial_k\rho\partial^k\rho - (\psi^*\psi_k - \psi_k^*\psi)(\psi^*\psi^k - \psi^{*k}\psi) \equiv 4\rho\partial_k\psi^*\partial^k\psi - b^{-2}\rho^2 s_\alpha s^\alpha \tag{3.13}$$

$$\rho \equiv \psi^*\psi, \qquad s_\alpha \equiv b\psi^*\sigma_\alpha\psi/(2\rho), \qquad \alpha = 1,2,3 \tag{3.14}$$

valid for the two component ψ-function, the action (2.34) reduces to the form

$$\mathcal{A}_E[\psi,\psi^*] = \frac{1}{2}\int[(ib\partial_k + ec^{-1}A_k)\psi^*(-ib\partial^k + ec^{-1}A^k)\psi -$$

$$-(m^2c^2 + \partial_k s_\alpha \partial^k s_\alpha)\rho + (\hbar^2 - b^2)(4\rho)^{-1}\partial_k\rho\partial^k\rho]d^4x, \tag{3.15}$$

The quantity $\mathbf{s} = \{s_\alpha\}, \quad \alpha = 1,2,3$ associates with the mean spin (especially, if $b = \hbar$), because it is constructed on the base of the Pauli matrices. As one can see in Eq.(3.15) s_α convolutes only with s_α, but not with ∇, or $\mathbf{A}$. As a result the action (3.15) is invariant with respect to space-time rotations and relabelling transformations (2.22) separately.

Thus, the spin is an integral connected with kinematic properties of a dynamic system. Dynamics admits the dynamic system $\mathcal{S}$ to have a few different spins, but the quantum axiomatics forbids. For instance, applying QM principles (linearity of dynamic equation in terms of the wave function) to the action (2.35), one removes all vortical flows ($s = 1/2$). As a result the action (2.35) together with the quantum axiomatics describes only spinless particles (the dynamic system $\mathcal{S}_{KG}$ (2.37), described by the Klein-Gordon equation).

Spin and a description in terms of the ψ-function are not specific quantum properties. Setting $\kappa \equiv 0$ in the action (1.2), (1.3), one obtains a statistical ensemble of classical particles. Repeating all calculations, leading from Eq.(1.2) to (2.37), one obtains instead of Eqs.(2.34) and (2.35)

$$\mathcal{A}_{Ecl}[\psi,\psi^*] = \frac{1}{2}\int\Big\{\frac{1}{\psi^*\psi}[\frac{ib}{2}(\psi^*\psi_k - \psi_k^*\psi) - \frac{e}{c}A_k\psi^*\psi][\frac{ib}{2}(\psi^*\psi^k - \psi^{*k}\psi) - \frac{e}{c}A^k\psi^*\psi] +$$

$$-m^2c^2\psi^*\psi\Big\}d^4x, \qquad \psi_k \equiv \partial_k\psi \tag{3.16}$$

$$\mathcal{A}_{Ecl}[\psi,\psi^*] = \frac{1}{2}\int[(ib\partial_k + \frac{e}{c}A_k)\psi^*(-ib\partial^k + \frac{e}{c}A^k)\psi -$$

$$-(m^2c^2 + \frac{b^2}{2}\sum_{\alpha,\beta=1}^{n} Q_{\alpha\beta,k}Q_{\alpha\beta}^{*}{}^{k})\psi^*\psi - \frac{b^2\partial_k(\psi^*\psi)\partial^k(\psi^*\psi)}{4\psi^*\psi}]d^4x, \tag{3.17}$$

Both actions (3.16) and (3.17) describe a classical dynamic system in terms of the ψ-function. The variable ρ and the current j^k are connected with ψ by means of Eq.(2.30). The actions (3.16) and (3.17) do not contain the Planck constant $\hbar$. At the same time nothing prevents from an identification of the integration constant b with $\hbar$.

4 Properties of the κ-field

Let us list the principal properties of the κ-field, defined by the relations (1.2), (1.3).
(1) κ-field is responsible for quantum effects
(2) κ-field can escape from the matter and exist separately in the "empty" space-time
(3) κ-field enables to generate pairs.

The first property follows from the relations (1.2), (1.3). After the substitution $\kappa^l \equiv 0$, $l = 0,1,2,3$, the action (1.2), turns to the statistical ensemble of independent classical particles.

The second property follows from the dynamic equation (2.11). At $j^l \equiv 0$, $l = 0,1,2,3$, the equations (2.13), (2.15) are satisfied identically, and Eq.(2.11) turns to the free Klein-Gordon equation for e^κ.

$$\lambda^2\partial_l\partial^l e^\kappa + e^\kappa = 0 \tag{4.1}$$

In the case $j^l = 0$, $l = 0,1,2,3$ the wave function $\psi = a\exp\kappa$ is real. In other words, the real wave function ψ, satisfying the free Klein-Gordon equation, describes a free κ-field without a matter. According to Eq.(2.41) the energy-momentum tensor of a free κ-field is determined by the relation

$$T_k^l = \frac{1}{2}m^2c^2a^2e^{2\kappa}[\delta_k^l + \lambda^2(2\kappa^l\kappa_k - \delta_k^l\kappa^s\kappa_s)], \qquad k,l = 0,1,2,3 \tag{4.2}$$

$$T_0^0 = \frac{1}{2}m^2c^2a^2e^{2\kappa}(1 + \lambda^2\sum_{l=0}^{3} \mid \kappa_l \mid^2) \geq 0 \tag{4.3}$$

If j^l changes slowly on the distance of the order of the Compton wave length λ, the dynamic equation (2.11) for the κ-field is a differential equation with a small parameter before the highest derivative. In this case rhs of Eq.(2.11) vanishes and its solution has the form

$$\kappa \simeq \frac{1}{2}\log\frac{\sqrt{j^s j_s}}{a^2 mc}, \qquad K \simeq 1, \qquad \lambda \mid \partial_l j^s \mid \ll j^s \tag{4.4}$$

In this case the κ-field is connected with the matter (in the same way as the Coulomb electric field is connected with a charge) and has no proper degrees of freedom. In the non-relativistic case a variation of the proper energy $mKc^2 - mc^2 = -\frac{1}{2}\lambda^2 mc^2 (j_0)^{-1/2}\nabla^2\sqrt{j_0}$ associates with the Bohm potential $V_{\mathrm{B}} = [\hbar\nabla\log(\psi^*\psi)]^2/8m$ [5].

Before consideration of the problem of the pair production let us consider a few conceptual points, connected with the classical description of particles and antiparticles. First about terminology. The terms "world line of a particle" and "world line of an antipaticle" are essentially non-relativistic. Using these terms, one supposes that the world line is an *attribute* (history) of a pointlike physical object (a particle or an antiparticle). This physical object is a point in the three-dimensional space, and its trajectory in the space-time is its world line. From non-relativistical viewpoint a particle and an antiparticle are different physical objects.

From the relativistical point of view a world line (WL) is a physical object. This physical object is a one-dimensional line in the four-dimensional space-time. WL has two different components of orientation, which can be considered as two different directions of motion along the WL. [14, 15]. Everywhere further, when the world line is considered as a physical object (not as a history of a particle) the term "WL" will be used (instead of "world line"). Particles and antiparticles are derivative physical objects which arise as a result of intersection of WL with the hyperplane t=const at some coordinate system. One WL can describe a few particles and antiparticles.

A particle and an antiparticle is distinguished by their orientation of WL at intersection with the hyperplane t=const. From the relativistical viewpoint the particle and the antiparticle are different states (attributes) of WL, whereas from the non-relativistic viewpoint they are different physical objects. The term "SWL" (abbreviation of "section of world line") will be used as a common concept with respect to concepts "particle' and "antiparticle". SWL means any point of intersection between WL and the hyperplane t=const independently of an orientation of WL at this point.

Describing particles and antiparticles, it should distinguish between the energy-momentum vector $P_l = \{P_0, \mathbf{P}\}$ and the canonical momentum vector $p_l = \{p_0, \mathbf{p}\}$. Especially, it should distiguish between the energy $E = P_0$ and the time component p_0 of the canonical momentum. These quantities are defined in different way. The energy-momentum vector is defined by the relation

$$P_l = \int\limits_V T_l^0 d\mathbf{x}, \qquad l = 0,1,2,3, \qquad E = P_0 \tag{4.5}$$

where T_k^l is the energy-momentum tensor, and V is a 3-volume around SWL. The canonical momentum p_l is defined as a quantity canonically conjugate to the position coordinate x^l, $l = 0,1,2,3$ of SWL. In the classical physics it means that $p_l = \partial L/\partial \dot{x}^l$, $\dot{x}^l \equiv dx^l/d\tau$, $l = 0,1,2,3$, where L is the Lagrangian function, nd τ is a parameter along WL. In the quantum physics the canonical momentum operator is defined by the relation

$$[\hat{u}, \hat{p}_l]_- = i\hbar \frac{\partial \hat{u}}{\partial x^l} \qquad (4.6)$$

where $[\ldots]_-$ means a commutator, nd $\hat{u}$ means an operator of any dynamic variable in the Heisenberg picture.

The fact that the energy E of a free particle coincides with the time component $-p_0$ of the canonical momentum taken with the opposite sign $E = -p_0$ does not mean that the same relation is valid for a free antiparticle. Indeed, setting for simplicity $A_l \equiv 0$, $\kappa^l \equiv 0$, $l = 0,1,2,3$ in relations (2.42), (2.5), one obtains

$$T_0^0 = \frac{mcKj^0 j_0}{\sqrt{j^s j_s}} \geq 0, \qquad p_0 = -\frac{mcKj_0}{\sqrt{j^s j_s}} \qquad (4.7)$$

$\mathrm{sgn}(K\sqrt{j^s j_s})$ $=\mathrm{sgn}(K/\sqrt{j^s j_s}) = 1$ always. Then it follows from the relations (4.5) and (4.7) that the energy is non-negative always, whereas p_0 depends on the sign of j^0. It is different for a particle and for an antiparticle. If p_0 can be considered as a constant, it follows from Eqs.(4.5), (4.7) that

$$E = P_0 = -p_0 N, \qquad N = \int j^0 d\mathbf{x}, \qquad (4.8)$$

where N is the number of SWLs. N is positive for particles and negative for antiparticles. For one SWL the relation (4.8) reduces to $E = -p_0 \mathrm{sgn}(j^0) =\mid p_0 \mid$.

A like relation

$$Q = e\ \mathrm{sgn}(j^0) \qquad (4.9)$$

is valid for the connection between the electric charge Q of SWL, defined as a source of the electromagnetic field and the constant e, describing interaction of a charged WL with the electroagnetic field [15]. For instance in the action (1.2) e is a constant, and the electric charge is defined by the relation (4.9) which describes the well known fact that a particle and antiparticle have opposite electric charges.

Identification of P_l with p_l is possible [15], provided particles and antiparticles are considered as different physical objects (but not as attributes of the whole WL). In contemporary quantum field theory such an identification is carried out almost always (but see [16]). As a result one deals with indefinite number of SWLs (instead of a fixed number of WLs). In particular, one states conventionally [17] that some solutions of the Klein-Gordon equation associate with the negative energy and are non-physical. A reason of such a statement is a confusion between the energy E and the canonical momentum p_0. The energy of any solution is non-negative, as it follows from relations (4.5), (4.7), or (2.41), whereas p_0 may have any sign, but it does not mean that the solution with $-p_0 < 0$ is non-physical. It means simply that antiparticles dominate in the region, where $-p_0 < 0$.

The problem of the pair production is connected with a possibility of a change of the sign of p_0, or j^0. To change $\mathrm{sgn}(j^0)$, the vector j^0 is to become spacelike

$j^k j_k < 0$ inside some region. It is a necessary condition of a change of the sign of j^0. The first term in the action (1.2) is to be real. It is possible even in the case, when $\dot{x}^k \dot{x}_k J^2 = j^k j_k < 0$, provided simultaneously

$$K^2 = 1 + \lambda^2 (\kappa_k \kappa^k + \partial_k \kappa^k) < 0 \tag{4.10}$$

To satisfy the last condition, the κ-field is to be large enough and to change rapidly enough.

Let us consider for simplicity the potential case. Let us set $\hbar = c = 1$ and choose such a gauge of the electromagnetic field, where $A_0 = 0$. Under this gauge the Klein-Gordon equation takes the form

$$\partial_0^2 \psi + (-i\nabla + e\mathbf{A})^2 \psi + m^2 \psi = 0 \tag{4.11}$$

Let $\tilde{\psi}$, $\tilde{u}$ and $\tilde{\mathbf{A}}$ means spatial Fourier components of variables $\psi, \partial_0 \psi$, $\mathbf{A}$ respectively.

$$\tilde{\psi}(t, \mathbf{q}) = (2\pi)^{-3/2} \int_{-\infty}^{\infty} e^{-i\mathbf{q}\mathbf{x}} \psi(t, \mathbf{x}) d\mathbf{x} \tag{4.12}$$

$$\tilde{u}(t, \mathbf{q}) = (2\pi)^{-3/2} \int_{-\infty}^{\infty} e^{-i\mathbf{q}\mathbf{x}} \partial_0 \psi(t, \mathbf{x}) d\mathbf{x} \tag{4.13}$$

$$\tilde{\mathbf{A}}(t, \mathbf{q}) = (2\pi)^{-3/2} \int_{-\infty}^{\infty} e^{-i\mathbf{q}\mathbf{x}} \mathbf{A}(t, \mathbf{x}) d\mathbf{x} \tag{4.14}$$

Let $b(t, Q)$ be a function of the argument $Q = \{\varepsilon, \mathbf{q}\}$, $\varepsilon = \pm 1$, where $\varepsilon = \mathrm{sgn}(-p_0)$ describes an orientation of WL [15] ($\varepsilon = 1$ for particles, and $\varepsilon = -1$ for antiparticles). Let us define the variable $b(t, Q)$ by means of the relation

$$b(t, Q) = E^{1/2}(\mathbf{q}) \tilde{\psi}(t, \mathbf{q}) + i\varepsilon E^{-1/2}(\mathbf{q}) \tilde{u}(t, \mathbf{q}) \tag{4.15}$$

where $E(\mathbf{q}) \equiv \mid \sqrt{m^2 + \mathbf{q}^2} \mid$. If ψ satisfies Eq.(4.11), $b(t, Q)$ satisfies the dynamic equation

$$\partial_0 b(t, Q) = -i\varepsilon E(\mathbf{q}) b(t, Q) - \frac{e}{(2\pi)^{3/2}} \int F(\mathbf{q}, \mathbf{q}') b(t, Q') dQ', \tag{4.16}$$

$$F(\mathbf{q}, \mathbf{q}') = (\mathbf{q} + \mathbf{q}') \tilde{\mathbf{A}}(\mathbf{q} - \mathbf{q}') + e(2\pi)^{-3/2} \int \tilde{\mathbf{A}}(\mathbf{q} - \mathbf{q}'') \tilde{\mathbf{A}}(\mathbf{q}'' - \mathbf{q}') d\mathbf{q}'', \tag{4.17}$$

where $\int (.) dQ \equiv \sum_{\varepsilon = \pm 1} \int (.) d\mathbf{q}$. At $A_0 = 0$ the energy $\mathcal{E}$ and the number $\mathcal{N}$ of SWL can be presented in the following form

$$\mathcal{E} \equiv \int T^0{}_0 d\mathbf{x} = \int E(\mathbf{q}) n(Q) dQ + \int e\mathbf{j}\mathbf{A} d\mathbf{x} \tag{4.18}$$

$$\mathcal{N} \equiv \int j^0 d\mathbf{x} = \int \varepsilon n(Q) dQ, \tag{4.19}$$

where $n(Q) \equiv b^*(Q) b(Q)$, and b^* is a quantity complex conjugate to b. The nonnegetive quantity $n(Q)$ can be considered as the number of WLs having the direction of the canonical momentum vector $p_l = \{-\varepsilon E(\mathbf{q}), \mathbf{q}\}$. For $\varepsilon = 1$ $p_0 < 0$, for $\varepsilon = -1$ $p_0 > 0$. Nothing prevents from interpretating $b(1,\mathbf{q})$ and $b(-1,\mathbf{q})$ as spectra of particles and antiparticles respectively. The total number $\mathcal{N}$ of SWLs can have any sign and does not depend on the time t, but the energy $\mathcal{E}$ depends on the time, in general.

Let at $t = -\infty$ $b(-1,\mathbf{q}) = 0$, i.e. there are no antiparticles. After interaction with a strong electromagnetic field the spectum changes, and it is possible that $t = +\infty$ $b(-1,\mathbf{q}) \neq 0$. It means that the external electromagnetic field generates pairs particle-antiparticle. Then $\mathcal{N} = \mathcal{N}_p + \mathcal{N}_a =| \mathcal{N}_p | - | \mathcal{N}_a |$, where $| \mathcal{N}_a |$ is the total number of generated pairs. In general, such a situation is possible, when $| \mathcal{N}_a | \gg \mathcal{N}$.

A reason of pair production (a turn of WLs in time) is purely dynamical. κ-field changes the effective mass $m_{\text{eff}} = mK = m\sqrt{1 + \lambda^2(\kappa_k\kappa^k + \partial_k\kappa^k)}$, making it imaginary that affords a possibility of a turn of WL. The quantum axiomatics is not used, and the κ-field appears simply as a "quantum force field".

Under the conventional description of the pair production effect one uses always the Heisenberg representation which cannot be understood outside the scope of quantum axiomatics. As a result the pair production process is described in terms of virtual particles for which the relation $p^2 = m^2$ is violated (and the effective mass m_{eff} is distinguished from the mass m).

In both cases a reason of the pair production is similar: a change of the effective mass followed by a transition through the imaginary value. In the first case this change has a dynamical reason, in the other one the reason looks as quantum-axiomatical.

5 Dynamical interpretation of quantum effects

Let us study to what extent the quantum effects can be considered as dynamical ones. To explain a diffraction or interference picture of the electron distribution over the screen, it is sufficient to calculate the flux of particles through the screen surface, using dynamic equations and the expression (2.40) for the particle flux. The dynamic equations and the expression for the particle current are attributes of dynamics. The result does not depend on that which form of dynamic equations is used. Of course, the dynamic equation linear in terms of the wave function is most convenient from the practical viewpoint. But it is not conceptual and does not mean a usage of the quantum axiomatics. The tunneling effect is explained also by means of a solution of dynamic equations and usage of the expression for the current of particles. It is also fully dynamic effect which does not need a reference to the quantum axiomatics. Thus, effects of interference, diffraction and tunneling can be interpreted as purely dynamic effects.

Uncertainty relation. Let a flux of particles with the momentum $\mathbf{p} = \{0, 0, p_3\}$ falls onto diaphragm, placed in the coordinate plane x^1, x^2. Let in the diaphragm there be a slit along the axis x^2. The width $\Delta x \ll \hbar/p_3$ of the slit is much more narrow, than the width of the beam in the direction x^1 $L_1 \gg \Delta x$. Then only small part of the beam particles passes through the slit. A passage of particles through the slit can be considered as a measurement of the position coordinate x^1 of the beam particles. Before the passage through the slit the energy of the κ-field in the beam is very small. The energy increases inside the slit and becomes of the order $U_\kappa = (2m)^{-1}(\hbar\partial_1\rho/\rho)^2\rho \simeq (2m)^{-1}(\hbar/2\Delta x)^2\rho$. After the passage of the slit the potential energy U_κ of the κ-field turns to the energy of the particles passing the slit. As it follows from the dynamic equations, only the component p_1 of the momentum changes. It is added to the term $p_1^2/2m$ of the kinetic energy. The mean value of this term corresponds to $\langle p_1^2\rangle \simeq (\hbar/2\Delta x)^2$. It can be interpreted as a result of the measurement of the particle cordinate x^1. Usually it is interpreted as a corollary of the uncertainty principle, but now it is simply a dynamical effect.

Let us put the question. Where do the particles passing through the slit take the additional energy? A formal ansewer is as follows. It follows from the dynamic equations for the fluid. The observable effect reminds the cumulative effect well known in hydrodynamics. Another more physcal answer is such one. Real particles move stochastically, and the slit selects only those of them which have large enough energy. In other words, the particles of the passing beam obtain an additional energy from the particles absorbed by the diaphragm. The narrow the slit, the less the number of particles in the passing beam and the larger energy is added to the passing particles.

Discrete quantum particles. Conventionally the discrete quantum numbers are considered as eigenvalues of linear operators of corresponding physical quantities. It seems impossible to explain them without a reference to the statistical propositions (quantum axiomatics).

Let us note that the Hamiltonian $\hat{H}$, discribing the dynamic system evolution can be obtained directly from the dynamic equations written in terms of the wave function ψ. Thus, the linear operator $\hat{H}$ can be obtained from only dynamics. The eigenvalues of the Hamiltonian are stationary states which also can be obtained directly from dynamics (without a reference to the quantum axiomatics). Although the dynamic equations of the quantum mechanics admit existence of atoms at non-stationary states, nevertheless atoms and molecules exist practically always at the stationary states and are stable. Indeed, being at a non-stationary state, the atom emanates the electromagnetic emission untill it passes to a stationary state [18]. As far as in the non-relativistic case the Hamiltonian coincides with the energy, the eigenvalues of the Hamiltonian can be measured by the spectroscopic technique, measuring the frequency ω (and the energy $\hbar\omega$) of the electromagnetic emission. In other words, for measuring the Hamiltonian eigenvlues one uses only dynamics (dynamic equations and the energy conservation law, which is also a dynamic relation).

Only the particle position is measured instantly. Other quantities such as the momentum need a rather long time for a precise measurement. Under such conditions a physical quantity can be measured, provided its operator commutes with the Hamiltonian [19]. Thus, if the operator $\hat{S}$ commutes with the operator $\hat{H}$, the eigenvalues of $\hat{S}$ label the stationary states and can be measured as a result of an identification of stationary states that can be made on the base of only dynamics. If $\hat{S}$ does not commute with $\hat{H}$, the eigenvalues $\hat{S}$ cannot be measured, and the quantity

$\hat{S}$ cannot be measured.

The pair production effect is a dynamic effect also, because it is explained as an ifluence of the κ-field on WL. It is described by the dynamic equations completely.

In frames of the conventional approach the quantum effects are explained on the base of both quantum axiomatics and dynamics simultaneously. For instance, an explanation of diffraction effect is impossible without a usage of the dynamic equations. Thus, explaining the quantum effects, the dynamics is present in both approaches (conventional and dynamical), whereas the quantum axiomatics is present only in the conventional approach. It means that the quantum effects have the *dynamical nature*, (not the quantum-axiomatical one).

6 Concluding remarks

It is a common practice to believe that the quantum axiomatics plays a constructive role, that it introduces such specific non-dynamic concept as the wave function and spin, extending capacities of the dynamics. The above consideration results that the dynamics is the most important and constructive structure of the quantum mechanics, whereas the quantum axiomatics is not so constructive. It imposes only some restrictions upon possible forms of the action and dynamic equations. Such specific concepts of the quantum axiomatics as wave function and spin appear to be kinematic elements of a proper dynamic description. This dynamic description arises as a result of a mathematical development of the dynamics technique. A crucial point of this development is a general integration of the kinematic equations. This integration leads to appearance of the hydrodynamic potentials $\boldsymbol{\xi}$, to arbitrary functions $g^{\beta}(\boldsymbol{\xi})$ and to an active usage of the relabelling transformation group. Dynamic (more exactly kinematic) interpretation of the wave function and of the spin is impossible without the mathematical technique, generated by this integration and the relabelling group.

The dynamic conception of the quantum mechanics supposes that the κ-field is a vehicle of all quantum properties. In this connection a question about a nature of the κ-field arises. Such a question does not arise in the conventional presentation of the quantum mechanics, where the quantum axiomatics is considered as vehicle of all quantum properties. The most simple and reasonable explanation of the κ-field nature is a reference to the space-time properties [20, 21]. According to these papers the real space-time is a space-time, obtained from the Minkowski space-time by means of a special deformation, called distortion [22]. The distortion transforms timelike world line into timelike world tubes, with the Planck constant $\hbar$ being a measure of thickness of the tubes. Besides, the world tubes of free particles appear to be stochastic (at the deterministic space-time). The κ-field appears to be a way of a statistical description of the stochastic world tubes. All this is merely a result of a more general metric approach to geometry [23] which does not need any additional hypothesis. It is an attractive feature of this approach.

If the dynamics is indeed a principal structure of the quantum theory, it should construct and develop the quantum field theory (QFT) primarily on the base of the dynamics and dynamic properties, but not on the base of the quantum axiomatics, as it takes place in the contemporary QFT. Indeed, the mordern QFT uses mainly Heisenberg representation, or its modifications which can be formulated only on the base of the quantum axiomatics, whereas the Schrödinger representation which is less sensitive to the quantum axiomatics is not used practically.

The most serious argument against the Schrödinger representition is based on the statement that the relativistic Schrödinger description contains non-physical states with negative energy. As we have seen in Sec.4 this statement is conditioned by the identification of the energy-momentum with the canonical momentum. This identification leads to non-relativistic description in terms of particles and antiparticles, considered as physical objects (instead of a relativaistic description in terms of WLs). In turn such a description leads to the investigation strategy "all or nothing" which cannot be considered as an optimal investigation strategy [24].

The problem of the QFT development is considered sometimes as a problem of unification of the quantum axiomatics and relativity theory. If the dynamics is indeed a dominant structure of the quantum mechanics, there is no such a problem, but there is a routine dynamic problem of a correct determination of the quantum system Lagrangian. A correct model of the quantum stochasticity is very important for solving this problem.

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