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Quantum Mechanics as a Theory of Relativistic Brownian Motion

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Abstract

It is shown that non-relativistic quantum mechanics can be treated as a kind of relativistic statistical theory which describes a random motion of a classical particle. The theory is relativistic in the sense that for the description of the particle behaviour the relativistic notion of the state is used. This is very important because a statistics is a state calculus and the result depends on the definition of the notion of state.

Attempts by different authors [1—14][1]) to treat quantum mechanics from the position of classical theory did not lead to full success[2]). Though it may seem paradoxical, the reason there of, in our opinion, lies in the fact that the attempts to understand non-relativistic quantum mechanics were based on non-relativistic classical mechanics.

In this paper we shall show that quantum mechanics is a variety of the relativistic theory of Brownian motion[3]). The difference of our approach from approaches of others lies in the fact that we treat non-relativistic quantum mechanics from the point of view of relativistic classical mechanics. New principles are not needed for understanding the quantum mechanics of a single particle in this approach. In particular, such specific quantum mechanical principles as the uncertainty and the correspondence principle may be understood from the classical position. The clue to such an approach is the relativistic notion of system state.

1. The notion of state

In non-relativistic physics the state of a physical system is defined[4]) as a set of quantities which are given at a certain moment of time and determine these quantities at any subsequent moment of time. For this purpose equations of motion are used. They describe the time evolution of the system state. The state and the equations of motion describing the time evolution of the state are two essential elements of any non-relativistic physical theory.

[1]) A more comprehensive bibliography in [15].

[2]) The paper [16] is an exeption. The success here was reached, it seems, because what we call a relativistic notion of the state has been taken into account.

[3]) We call Brownian motion any indeterministic motion of particles irrespective of the causes of its indeterminism.

[4]) Sometimes the state is defined as a set of independent variables. For our purpose the independence is unessential.

As follows from the definition, the state of the system is given at a certain time moment. But in relativistic theory simultaneity is relative. Which events are synchronous and which are not depends on the choice of a frame of reference. If, for example, one knows a state of a physical system in a frame of reference K one could describe the state in a frame of reference K' moving relative to K only in the case when the equations of motion are known and can be solved. Thus, in the relativistic theory the state and the equations of motion are connected closely. Because there is no absolute simultaneity in the relativistic theory it seems more consistent to define the state of a system not for a given moment but over all space-time. In this case the conception of state will include the law of evolution of the physical system. The equations of motion are treated now as constraints imposed on the possible states.

From all possible states not all states are realized but only those which satisfy certain equations. We shall call them the constraint equations. In reality they are the same equations of motion but now they do not describe the time evolution of the state but are restrictions which choose the physically allowable states from the virtual ones.

In short, in the non-relativistic theory the unique division of the physical phenomena description into states and equations of motion corresponds to the unique division of space-time into space and time. In the relativistic theory where the division of space-time into space and time is conventional and not unique the division of the physical phenomena description into states and equations of motion is not unique either. The physical system state defined over all space-time corresponds much better to the indivisible space-time.

The manner of division of the description of a physical system into states and equations of motion is unimportant for the dynamics but is important for the statistics because statistics is the calculus of states. It is important for statistics what is understood by "state". In general, a statistics that corresponds to a different division of the description of a physical system into states and equations of motion leads to different results.

Let us describe the state of a single particle by giving its world-line over all the space time, i.e. by giving four functions $q^i = q^i(\tau)$, $i = 0, 1, 2, 3$, where τ is some parameter along the world-line. We need not give the momenta, provided the mass m of the particle and its state are known. If the world-line is known the momenta are determined by the relations

$$p_i = -mc \frac{g_{ik}\dot{q}^k}{\sqrt{\dot{q}^l g_{lj} \dot{q}^j}}, \quad \dot{q}^k \equiv \frac{dq^k}{d\tau} \tag{1.1}$$

where c is light speed, g_{ik} the metric tensor

$$g_{ik} = \begin{Vmatrix} c^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{Vmatrix}. \tag{1.2}$$

In equation (1.1) and henceforth the latin subscripts take the values $0, 1, 2, 3$ and greek ones take $1, 2, 3$. As usual summation is made on like super- and subscripts.

2. The quantum ensemble

Let us suppose that the state of a particle, i.e. its world-line, is a random quantity. Let us take for the sake of simplicity that the world-line cannot turn back in the time direction, i.e. that $dq^0/d\tau$ always keeps its sign. To describe the random states let us introduce the notion of state density. We consider at a point q^i of space-time an infinitesimal area dS_k. It is evident that the number dN of world-lines which cross the area dS_k is proportional to the value of the area, that is

$$dN = j^k dS_k$$

where j^k is a factor. The vector j^k is proportional to the density of the world-lines in the vicinity of the point q. According to definition j^k is the state density vector. This is connected with the fact that the object of statistics are one-dimensional lines and not points as in non-relativistic statistics. According to the definition of j^k the time component j^0 represents the mean density of particles and the space components j^α represent the mean density of the particle flux.

Thus the difference of principle between the non-relativistic and relativistic statistics consists in the fact that for the former the state density is a scalar while for the later it may be, for example, a vector. Because the state density is a vector we shall be able to represent quantum mechanics as a theory of the relativistic BROWNian motion. Later on we shall consider only the non-relativistic case, adopting from relativity only the relativistic notion of state density.

Let us consider now a statistical ensemble of world-lines. Suppose the case is non-relativistic, i.e. the world-lines derivate but little from some constant direction in space-time. We choose this direction as the time axis, then

$$cj^0 \gg |j^\alpha|. \tag{2.1}$$

The ensemble state is described by the state density. This means that the state of the ensemble is considered as the state of some deterministic physical system. The state of the system is determined by the vector j^k. The statistical ensemble is a deterministic system with the help of which one can describe non-deterministic ones. In the non-relativistic case the state density W is a function defined in the phase space. W is the ensemble state in the sense that beeing given at a moment of the time it can be uniquely determined for any following moment. There is another aspect. W is a non-negative quantity and with a suitable normalization $Wd\Omega$ can be interpreted as the probability to find the particle in the volume element $d\Omega$ of the phase space. Together with the fact that W is a state this fact gives us a chance to speak about the random MARKOVian process. In general, the two aspects of a statistical ensemble are independent, i. e. the state of the ensemble, being a state density, can not be a probability density.

In the relativistic case it is important that the statistical ensemble described by the vector j^k is a deterministic physical system. The fact that $j^k dS_k$ can be interpreted as a probability to find a particle in the 3-volume dS_k is valid only when the world-lines of particles do not zigzag in time. For relativistic particles when the generation of paires is possible such an interpretation is not suitable.

We shall obtain the equations for the vector j^k. In the simplest case the ensemble consists of strait world-lines which do not cross each other. It describes the motion of a gas of zero temperature. In such a gas the velocity of a single molecule coincides with the mean velocity of the gas stream. The chaotic motion of the molecules and their diffusion are absent and there is no pressure at all. In this case the equations have the form

$$\frac{\partial \varrho}{\partial t} + \frac{\partial j^\alpha}{\partial q^\alpha} = 0,\ t = q^0,\ \varrho = j^0$$
$$\frac{\partial j^\alpha}{\partial t} + \frac{\partial}{\partial q^\beta}\left(\frac{j^\alpha j^\beta}{\varrho}\right) = 0 \tag{2.2}$$

When the diffusion of world-lines is taken into account an additional term comprising the gradient of the particle density appears in the second equation.

In order to find the form of this term let us consider an example of the one-dimensional ensemble of world-lines. We shall describe it as one-dimensional continuous medium with the help of the action

$$S = \int\int \left(\frac{m\dot{q}^2}{2} - U\right) d\xi dt,\quad q = q(t, \xi), \dot{q} \equiv \frac{dq}{dt} \tag{2.3}$$

Here ξ is a LAGRANGian variable describing the position of an element of the medium. The first term describes the kinetic energy and the second one describes an interaction of the elements of the medium (the potential energy) which is connected with the fact that the motion of a single particle differs from a mean motion. If $U = U(\varrho)$ then the equation (2.3) describes some hydrodynamics where the pressure is $p = -\varrho^2 dU/d\varrho$ and $\varrho = l^{-1} = \partial\xi/\partial q$ is the medium density. To describe diffusion it is necessary to suppose that U depends on the gradient of density ϱ because diffusion is connected with the density gradient.

Let us suppose for simplicity

$$U = \frac{k}{2}\left(\frac{\partial \varrho}{\partial \xi}\right)^2 = \frac{k}{2}\left(\frac{1}{l^2}\frac{\partial l}{\partial \xi}\right)^2,\ \varrho^{-1} = l = \partial q/\partial \xi \tag{2.4}$$

where k is the constant diffusion factor. The fact that dependence on $\partial\varrho/\partial\xi$ (not on $\partial\varrho/\partial q$) is taken is explained by the demand that U be invariant with respect to the transformation

$$\xi \to \xi' = A\xi,\ A = \text{const.} \tag{2.5}$$

This invariance of U with respect to (2.5) means that the equations are independent of the number of particles in the ensemble. Variating q we get from (2.3)

$$-m\ddot{q} + \frac{\partial}{\partial \xi}\frac{\partial U}{\partial l} - \frac{\partial^2}{\partial \xi^2}\frac{\partial U}{\partial l_\xi} = 0,\quad l_\xi \equiv \frac{\partial l}{\partial \xi}\,. \tag{2.6}$$

Transforming the LAGRANian variables (t,ξ) to the EULERian variables (t, q) and using the notations

$$\varrho = \frac{\partial \xi}{\partial q},\ j = \varrho v = \varrho \dot{q} \tag{2.7}$$

we get after some calculations

$$\frac{\partial j}{\partial t} + \frac{\partial}{\partial q}\left(\frac{j^2}{\varrho}\right) = -\frac{k}{m}\frac{\partial}{\partial q}\left(\frac{\varrho'^2}{\varrho} - \varrho''\right),\ \varrho' \equiv \frac{\partial \varrho}{\partial q} \tag{2.8}$$

The continuity equation

$$\frac{\partial \varrho}{\partial t} + \frac{\partial j}{\partial q} = 0 \tag{2.9}$$

was used. In the LAGRANGian coordinates this equation is an identity.

The equation (2.8) can be extended to the three dimensional case. Let $k = \hbar^2/(4m)$ and suppose that in the non-relativistic approximation (2.1) only the ensemble states j^k satisfying the equations

$$\frac{\partial \varrho}{\partial t} + \frac{\partial j^\alpha}{\partial q^\alpha} = 0, \tag{2.10}$$

$$\frac{\partial j^\alpha}{\partial t} + \frac{\partial}{\partial q^\beta}\left(\frac{j^\alpha j^\beta}{\varrho}\right) = -\frac{\hbar^2}{4m^2}\frac{\partial}{\partial q^\beta}\left(\frac{\varrho^\alpha \varrho^\beta}{\varrho} - \delta^{\alpha\beta}\Delta\varrho\right) \tag{2.11}$$

are possible, where

$$\varrho = j^0, \quad \varrho^\alpha = -\frac{\partial \varrho}{\partial q^\alpha}, \quad \Delta = \sum_{\alpha=1}^{3} \frac{\partial^2}{\partial q^\alpha \partial q^\alpha}, \tag{2.12}$$

$\hbar$ is the Planck's constant, m — the mass of the particle of the ensemble. The statistical ensemble the state of which is described by the vector j^k satisfying equations (2.11) we shall call quantum ensemble.

From the non-relativistic point of view the four equations (2.10), (2.11) are the equations of motion of the ensemble and enable us, if a state j^k is given at a certain moment of time, to determine the state at any following moment.

Let us introduce the notation

$$v^\alpha = j^\alpha/\varrho, \tag{2.13}$$

where $v^\alpha = v^\alpha(q)$ is a mean velocity of the ensemble particles at point q, and let us suppose that the velocity has a potential

$$v^\alpha = \frac{1}{m}\varphi_\alpha, \; \varphi_\alpha \equiv \partial_\alpha \varphi, \; \partial_\alpha \equiv \partial/\partial q^\alpha. \tag{2.14}$$

It can be shown [3, 12] that under restrictions (2.14) the equations (2.10)—(2.11) are equivalent to Schrödinger's equation

$$i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m}\partial_\alpha \partial_\alpha \psi = 0 \tag{2.15}$$

where ψ is the quantity

$$\psi = \sqrt{\varrho}\exp(i\varphi/\hbar). \tag{2.16}$$

According to our definition of the ensemble of world-lines the quantity $j^k dS_k$ represents the mean number of world-lines crossing dS_k.

In the non-relativistic case (2.1) this quantity reduces to $\varrho dS_0 = \varrho dV$ and, under the normalization condition

$$\iiint_{-\infty}^{\infty} \varrho dV = 1, \tag{2.17}$$

can be interpreted as probability to find the particle in the volume dV. If the normalization condition (2.17) is satisfied at some moment of time then, due to (2.10), it will be satisfied at any time moment.

Hence, the relativistic notion of state (the vector character of the state density) permits to present Schrödinger's equation as the equation of motion of a statistical ensemble.

3. The variational principle for a quantum ensemble

The equations (2.10)—(2.11) for a quantum ensemble can be derived from a variational principle if the action is chosen as

$$S = \int L d^4 q, \quad d^4 q \equiv dq_0\, dq^1\, dq^2\, dq^3, \tag{3.1}$$

$$L = \frac{m j^\alpha j^\alpha}{2\varrho} - \frac{\hbar^2}{8m} \frac{1}{\varrho} \frac{\partial \varrho}{\partial q^\alpha} \frac{\partial \varrho}{\partial q^\alpha} + \mu(\varrho - j^0). \tag{3.2}$$

Here, vector $j^k = \{j^0, j^\alpha\}$, $\varrho = j^0$ is the state density of the ensemble where j^k are known functions of variables $\xi_i = \xi_i(q)$; m is the mass of the ensemble particles; μ is introduced as LAGRANGian multiplier and it means the energy per particle. The independent variables are coordinates q^i in space-time. The variables $\xi = (\xi_0, \xi_1, \xi_2, \xi_3)$, ϱ, μ are to be varied. If the quantum emsenble is considered as some liquid, ξ_1, ξ_2, ξ_3 are LAGRANGian coordinates of the liquid elements, i.e. the set of ξ_1, ξ_2, ξ_3 is "a label" of the liquid element. ξ_0 is a LAGRANGian time coordinate. But it is fictitious and L is independent on ξ_0. The variables enter into L through the state density j^k; and

$$j^k = \frac{\partial J}{\partial \xi_{0,k}}, \quad \xi_{i,k} \equiv \partial \xi_i / \partial q^k \tag{3.3}$$

where

$$J = \frac{\partial(\xi_0, \xi_1, \xi_2, \xi_3)}{\partial(q^0, q^1, q^2, q^3)} \tag{3.4}$$

LAGRANGian coordinates ξ_α describe not the motion of single particles of the ensemble but the m e a n motion of the ensemble particles. The first term in the LAGRANGian (3.2) is a density of the ensemble kinetic energy $m\varrho v^\alpha v^\alpha/2$. The second term is a density of the internal (potential) energy of the ensemble. This energy is connected with the chaotic motion of ensemble particles. Apparently this isn't the total energy of the chaotic motion but only the part that influences the ensemble motion. The second term is proportional to the square of the density gradient $\partial \varrho / \partial q^\alpha$. It is clear because the influence of the chaotic motion on the ordered one manifests itself in diffusion that appears in the presence of density gradient. This fact is well-known from BROWNian motion theory. The last term is a means of introducing the notation

$$\varrho = j^0 \tag{3.5}$$

by means of LAGRANGian multiplier [17]. After the notation (3.5) is introduced the LAGRANGian (3.2) has only the variables to be varied ξ_i, ϱ, μ and their first derivatives. It can be shown that the equation of motion derived from (3.1) by varying μ, ϱ, ξ, after exlusion of μ and use of the notations (3.3), coincide with (2.11). In this case the continuity equation (2.10) in terms of μ, ϱ, ξ_i is an indentity.

The existence of the LAGRANGian allows to introduce by the canonical method [18] the energy E, the momentum P_β and the angular momentum $M^{\alpha\beta}$ of the quantum ensemble

$$E = \int T_0^0\, dV, \quad P_\beta = \int T_\beta^0\, dV \tag{3.6}$$

$$M^{\alpha\beta} = \int M^{0,\alpha\beta}\, dV$$

where T_0^0, T_β^0 and $M^{0,\alpha\beta}$ are an energy density, a momentum density and a density of the angular momentum, respectively. These quantities for the LAGRANGian (3.2) are determined by the formulae

$$T_0^0 = \frac{mj^\alpha j^\alpha}{2\varrho} + \frac{\hbar^2}{8m}\frac{\varrho_\alpha\varrho_\alpha}{\varrho} = \mu\varrho + \frac{\hbar^2}{4m}\partial_\alpha\partial_\alpha\varrho, \quad \varrho_\alpha \equiv \partial_\alpha\varrho \tag{3.7}$$

$$T_\beta^0 = -mj^\beta = mj_\beta, \;\; j_\beta = g_{\beta k}j^k \tag{3.8}$$

$$M^{0,\alpha\beta} = q^\beta T^{\alpha 0} - q^\alpha T^{\beta 0} = m(q^\beta j^\alpha - q^\alpha j^\beta). \tag{3.9}$$

Let condition (2.17) be fulfilled. Then, according to the conception of a statistical ensemble, E, P_α and $M^{\alpha\beta}$ from (3.6) represent the mean energy, the mean momentum and the mean angular momentum of a quantum particle, respectively.

4. Statistical parameters of a quantum particle. Correspondence principle. Indeterminacy principle

Let us denote the mean over the ensemble by angular brackets $\langle\rangle$ and introduce the operator

$$\hat{p}_\alpha = -i\hbar\partial/\partial q^\alpha, \;\; \hat{p}^\alpha = i\hbar\partial/\partial q^\alpha \tag{4.1}$$

We shall consider the potential condition (2.14) and the normalization condition (2.17) to be fulfiled. Due to (3.6)—(3.9) we have for mean energy, mean momentum, mean angular momentum, and arbitrary function $F(\vec{q})$ of the space coordinates $\vec{q}$:

$$\langle F\rangle = \int_{V_0}\left(\frac{mv^\alpha v^\alpha}{2} + \frac{\hbar^2}{8m}\frac{\varrho_\alpha\varrho_\alpha}{\varrho^2}\right)\varrho dV = \int_{V_0}\psi^+\frac{\hat{p}_\alpha\hat{p}_\alpha}{2m}\psi\, dV, \tag{4.2}$$

$$\langle P_\beta\rangle = \int_{V_0} mv_\beta\,\varrho dV = \int_{V_0}\psi^+\,\hat{p}_\beta\,\psi\, dV, \tag{4.3}$$

$$\langle M^{\alpha\beta}\rangle = \int_{V_0} m(q^\beta v^\alpha - q^\alpha v^\beta)\,\varrho dV = \int_{V_0}\psi^+\,(q^\alpha\hat{p}^\beta - q^\beta\hat{p}^\alpha)\,\psi dV, \tag{4.4}$$

$$\langle F\rangle = \int_{V_0}\psi^+ F(\vec{q})\,\psi dV. \tag{4.5}$$

Here V_0 is the ordinary space, and ψ^+ is the complex conjugate of ψ. A quantum statistical ensemble does not involve any information about particle distribition over the momentum which is defined by (1.1). Only the mean momentum is known.

$$\langle p_\alpha\rangle = -\langle mv_\alpha\rangle = \int_{V_0}\psi^+\,\hat{p}_\alpha\psi dV. \tag{4.6}$$

A mean square value of the momentum may be defined by relation

$$\langle E\rangle = \left\langle\frac{p_\alpha p_\alpha}{2m}\right\rangle. \tag{4.7}$$

Such an assumption is natural because the particle is effected by no systematic external forces but only by a stochastic interaction with the medium (ether). We get from (4.2) and (4.7)

$$\langle p_\alpha p_\alpha\rangle = \int_{V_0}\psi^+\hat{p}_\alpha\hat{p}_\alpha\psi\, dV \tag{4.8}$$

From (4.8) and (4.2)—(4.6) we can conclude that to determine a mean value of a certain function $F(q^\alpha, p_\beta)$ of coordinates q^α and momenta p_β we need to build up an operator

$$\hat{F} = F(q^\alpha, \hat{p}_\beta) \tag{4.9}$$

and calculate the value

$$\langle F \rangle = \int\limits_{V_0} \psi^+ \hat{F} \psi dV. \tag{4.10}$$

The established rule holds for the values enumerated above: momentum, energy, angular momentum, and an arbitrary function of coordinates, i.e. for $F(q^\alpha, p_\beta)$ involving p_β in a power not above two.

Extrapolating this rule to arbitrary functions we obtain the so-called correspondence principle. Meanwhile the validity of such kind of extrapolation is not obvious. Essentially, such extrapolation is based on formal but not physical reasons. In addition, consistently using the correspondence principle [5] one concludes that the distribution function over the coordinates and momenta simultaneously does not exist but the distribution function exists either over the coordinates or over the momenta. It results that we cannot speak about the world-lines of the particles. This contradicts our main assumption that quantum mechanics is a statistical ensemble of relativistic BROWNian particles.

From our point of view the fact the quantum ensemble does not involve complete information about the momentum distribution does not mean absence of such distribution. Hence from our concept if follows that the applicability of the correspondence principle is limited.

The superposition principle and the linearity of the quantum mechanical equations are a matter of principle in the usual interpretation of quantum mechanics ([19], cpt 1, § 4).

If we consider quantum mechanics as a relativistic statistical ensemble the linearity of SCHRÖDINGER's equation, which is derived from (2.11) as a result of the restriction (2.14) and substitution of the variables (2.16), is something accidental and unimportant. The method of calculating mean values and the meaning of all the quantities are determined by the conception of a statistical ensemble and do not depend on the linearity of the equations derived from equations (2.11).

Hence, from our point of view the superposition principle is not an essential feature of quantum theory. It is possible that it is not valid in the relativistic case and is valid only in the non-relativistic approximation [20].

We consider now the indeterminacy principle. For a coordinate q^1 and the momentum p_1, it can be written in the form

$$\Delta p_1 \, \Delta q^1 \geq \hbar/2 \tag{4.11}$$

where

$$\Delta p_1 = \sqrt{\langle p_1^2 \rangle - \langle p_1 \rangle^2}, \quad \Delta q^1 = \sqrt{\langle (q^1)^2 \rangle - \langle q^1 \rangle^2}. \tag{4.12}$$

It is similarly written for the other components q^α, p_α. Formally (4.11) is a mathematical consequense of the definition of the operator $\hat{p}_\alpha$ (4.1) and the rule of calculating mean values (4.10). The similarity between the momentum operator $\hat{p}_\alpha$ and the momentum p_i defined by equation (1.1) is in that that both have the same mean values $\langle p_\alpha \rangle$ and $\langle \vec{p}^2 \rangle = \langle p_\alpha p_\alpha \rangle$. Besides, let us assume that they have the same mean values $\langle p_\alpha p_\beta \rangle$ for $\alpha, \beta = 1, 2, 3$. Then the uncertainty principle (4.11) will related to the momenta of the particles constituting the quantum ensemble. The latter assumption is arbitrary and does not follow directly from our conception of a quantum statistical ensemble.

The relation (4.11) is valid both for the usual interpretation of quantum mechanics ([21] cpt 3, § 4) and for our conception, while the origin of the uncertainty principle is different in these two cases. It is impossible for the particle to have both definite coordinate and momentum in the case of the usual interpretation. This has often been connected with the effect of a measuring device. For example, if the coordinate is measured exactly, the momentum is said to be perturbed in an inpredictable manner.

In our concept the uncertainty principle (4.11) exists without reference to the measurement procedure. It results from the fact that the energy of the quantum ensemble becomes large if it is localized in a small volume $(\Delta q)^3$. Actually, according to (4.2), (4.6) and (4.7) we have

$$\langle \vec{p}^2 \rangle - \langle \vec{p} \rangle^2 = 2m\langle E \rangle - \langle \vec{p} \rangle^2 = \frac{\hbar^2}{4} \int_{V_0} \frac{\varrho_\alpha \varrho_\alpha}{\varrho} dV. \tag{4.13}$$

As ϱ differs from zero only in the volume $(\Delta q)^3$ then $|\varrho_\alpha| \approx \varrho/\Delta q$ and $(\Delta p)^2 \approx \hbar^2 (\Delta q)^{-2}/4$.

Thus, from our view point the uncertainty principle results from the fact that any quantum emsemble even at rest has an energy. This energy is conditioned by the random motion of quantum particles and is the greater the more precisely the ensemble is localized.

Finally, the property of a measuring instrument to disturb a studied object so that it becomes impossible to measure both the coordinate and the momentum simultaneously, is from our point of view conditioned by properties of the quantum emsenble. The latters are conditioned by stochastic intereaction with the ether and therefore they are universal, i.e., true for ensemble of any particles.

5. Non-relativistic quantum ensemble in the electromagnetic field. Ensemble stationary state

Let us consider now the quantum ensemble of particles moving in a given electromagnetic field. To take into account the interaction between the particles of the quantum ensemble and the electromagnetic field we shall add to the Lagrangian (3.2) the term

$$L_{e\gamma} = \frac{e}{c} \varrho A_0 + \frac{e}{c} j^\alpha A_\alpha. \tag{5.1}$$

Here A_i is the 4-potential of the electromagnetic field, e is the charge of the particles which form the ensemble.

Variation of (5.1) with respect to μ, ϱ, ξ_i leads, instead of the equations (2.11), to the equations

$$m \left\{ \frac{\partial j^\beta}{\partial t} + \frac{\partial}{\partial q^\alpha} \left(\frac{j^\alpha j^\beta}{\varrho} \right) \right\} = -\frac{\hbar^2}{4m} \frac{\partial}{\partial q^\alpha} \left\{ \frac{\varrho_\alpha \varrho_\alpha}{\varrho} - \delta_{\alpha\beta} \partial_\gamma \partial_\gamma \varrho \right\} - \frac{e}{c} j^k F_{k\beta}$$

$$F_{ki} = \partial_k A_i - \partial_i A_k \tag{5.2}$$

which differ from the equations (2.11) multipled by m by the term $ec^{-1}j^k F_{k\beta}$ representing the LORENTZ force acting on the 4-current ej^k.

To obtain a complete set of equations, the equations (3.5) and (2.10) must be added to (5.2).

We shall assume the potential condition to be fulfilled, which, with 4-potential A_i different from zero, takes the form

$$j^\alpha/\varrho = \frac{1}{m}\left(\varphi_\alpha - \frac{e}{c} A_\alpha\right), \quad \varphi_\alpha \equiv \partial_\alpha \varphi; \tag{5.3}$$

then it is possible to show [13] that equations (5.2), (3.5), (2.10), (5.3) are equivalent to SCHRÖDINGER's equation for a spinless particle in the electromagetic field

$$i\hbar \frac{\partial \psi}{\partial t} + \frac{e}{c} A_0 \psi - \frac{1}{2m}\left(-i\hbar\, \partial_\alpha - \frac{e}{c} A_\alpha\right)\left(-\hbar \partial_\alpha - \frac{e}{c} A_\alpha\right)\psi = 0 \tag{5.4}$$

where ψ is defined by the relation (2.16).

Let us consider now the problem of stationary ensemble states in a stationary electromagnetic field. It may be easily shown that the 4-potential can always be chosen as stationary for a stationary electromagnetic field. For the stationary ensemble state, i.e. the state that satisfies equation

$$\frac{\partial j^k}{\partial t} = 0, \tag{5.5}$$

equations (5.2), (2.10), (3.5), (5.3) lead to the equation

$$\frac{1}{2m}\left(-i\hbar\partial_\alpha - \frac{e}{c} A_\alpha\right)\left(-i\hbar\partial_\alpha - \frac{e}{c} A_\alpha\right)\psi - \frac{e}{c} A_0 \psi = E\,\psi \tag{5.6}$$

where E is a real constant and ψ is a complex function (2.16). It is required that function ψ decreases rather fast at infinity in order to satisfy the condition

$$\int_{V_0} \psi^+ \psi \, dV = \int_{V_0} \varrho dV = 1. \tag{5.7}$$

This requirement results from the fact that ϱdV is the probability of particle detection in the volume dV. Hence, the consideration of stationary ensemble states has led us to the eigenvalue problem.

6. Conclusions

We have shown that non-relativistic quantum mechanics (at least for a single particle) may be considered from the view-point of the relativistic classical mechanics as some theory of relativistic BROWNian motion. The expression "BROWNian motion" denotes only that the particle motion is accidental and unpredictable. The particle has a random world-line but the motion of the particle ensemble is deterministic. The stochastic feature of the particle motion may be considered as a result of its interaction with the medium (ether). It is important that even non-relativistic quantum mechanics is in principle a relativistic theory and may be understood only from the relativistic position. From our view-point quantum mechanics is not more than relativistic statistics. We get the quantum mechanical principles; the linear superposition principle, the indeterminacy principle, the correspondence principle (the last in limited form) from the principles of relativistic mechanics and statistics. PLANCK's constant $\hbar$

is a measure of an interaction between particle and ether. Due to the fact that non-relativistic quantum mechanics is a relativistic theory the problem of constructing a relativistic quantum theory may prove to be more simple than is generally believed.

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