Tubular geometry construction as a reason for new revision of the space-time conception.

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Abstract

The tubular geometry (T-geometry) is a generalization of the proper Euclidean geometry, founded on the property of σ-immanence. The proper Euclidean geometry can be described completely in terms of the world function $\sigma = \rho^2/2$, where ρ is the distance. This property is called the σ-immanence. Supposing that any physical geometry is σ-immanent, one obtains the T-geometry $\mathcal{G}$, replacing the Euclidean world function $\sigma_{\rm E}$ by means of σ in the σ-immanent presentation of the Euclidean geometry. One obtains the T-geometry $\mathcal{G}$, described by the world function σ. This method of the geometry construction is very simple and effective. T-geometry has a new geometric property: nondegeneracy of geometry. The class of homogeneous isotropic T-geometries is described by a form of a function of one parameter. Using T-geometry as the space-time geometry one can construct the deterministic space-time geometries with primordially stochastic motion of free particles and geometrized particle mass. Such a space-time geometry defined properly (with quantum constant as an attribute of geometry) allows one to explain quantum effects as a result of the statistical description of the stochastic particle motion (without a use of quantum principles). Geometrization of the particle mass appears to be connected with the restricted divisibility of the straight line segments. The statement, that the problem of the elementary particle mass spectrum is rather a problem of geometry, than that of dynamics, is a corollary of the particle mass geometrization.

1 Introduction

The proper Euclidean geometry has been constructed by Euclid many years ago. The Euclidean geometry was created as a logical construction, describing the mutual disposition of geometrical objects in the space. The main object of the Euclidean geometry is the point, i.e. the geometrical object, which has no size. The straight is the geometrical object which is a continuous set of points. The straight has a length, but it has no thickness. The plane is a continuous set of parallel straights. The complicated geometrical objects are combinations of more simple geometrical objects. The simplest geometrical objects (points, straights, planes, etc.) are called elementary geometrical objects (EGO). The Euclidean geometry is homogeneous and isotropic, and all identical EGOs have identical properties in all places of the space. Properties of elementary geometrical objects (EGO) are postulated, and properties of more complicated geometrical objects are obtained by logical reasonings from the axioms, describing properties of EGOs. The distance between two points P and Q in the Euclidean geometry is introduced as a number, describing the relation between the straight intercept PQ and some universal scale intercept AB. In the Euclidean presentation of the Euclidean geometry the distance is a derivative numerical quantity, which is not used at the Euclidean geometry construction.

On the one hand, the Euclidean geometry is a logical construction, on the other hand, it is a science on the mutual disposition of geometrical objects. Thus, there are two aspects of the Euclidean geometry. Any other geometry is obtained as the Euclidean geometry generalization. One can generalize the Euclidean geometry, considering it as a logical structure. In this case we change properties of EGOs, i.e. change axioms of the Euclidean geometry. As a result we obtain another homogeneous geometries (affine geometry, projective geometry, symplectic geometry, etc.). In general, any logical construction, which contains concepts of point and straight, can be considered to be a geometry. We shall refer to such a geometry as the mathematical geometry, because such geometries are interesting mainly for mathematicians, which train their mathematical and logical capacities, creating and investigating such geometries.

The geometry as a science on mutual disposition of geometrical objects is interesting mainly for physicists, which use it, describing physical phenomena in the space and in the space-time. Such a geometry will be referred to as a physical geometry. The main characteristic of the physical geometry is the distance between two arbitrary points of the space. A generalization of the Euclidean geometry is obtained as a result of a deformation of the Euclidean space. At such a deformation the distance between points is changed. The identical EGOs in different places becomes to be various and the obtained generalized geometry becomes inhomogeneous. In such a geometry one cannot use axioms, because the identical EGOs become various after inhomogeneous deformation. The generalization of the Euclidean geometry cannot be produced in the same way, as it is produced in the mathematical geometry. Well known mathematician Felix Klein [1] believed that only the homogeneous geometry deserves to be called a geometry. It is his opinion that the Riemannian geometry (in general, inhomogeneous geometry) should be qualified as a Riemannian geography, or a Riemannian topography. In other words, Felix Klein considered a geometry mainly as a logical construction. We seem that the qualification of the Riemannian geometry as a physical geometry is more appropriate, than the Riemannian topography, although this point is not essential. It is much more important that the physical geometry and the mathematical geometry are quite different buildings, because their construction is founded on different principles.

In this paper we shall consider only physical geometry, where the mutual position of geometrical objects is the principal object of investigation. The mutual position of geometrical objects can be described by distance ρ, which is given for all pairs of points $P, Q \in \Omega$, where Ω is the set of all points of the space. Usually instead of the distance ρ one considers the quantity $\sigma(P,Q) = \frac{1}{2}\rho^2(P,Q)$, known as the world function [2]. The world function is real even in the space-time geometry, where ρ may be imaginary. It is very important that the world function of the Euclidean geometry is the unique quantity, which is necessary for description of the proper Euclidean geometry. In other words, *the proper Euclidean geometry can be described completely in terms and only in terms of the world function* σ_{E} of the Euclidean space. This statement is a very important theorem of the Euclidean geometry, which can be proved in the framework of the Euclidean geometry. This theorem is a foundation for construction of all physical geometries.

The property of a physical geometry of being described completely by means of the world function will be referred to as the σ-immanence property of this geometry. We formulate this important theorem on the σ-immanence of the Euclidean geometry below, as soon as the necessary technique will be developed. Here we do note that the proper Euclidean geometry can be constructed as a mathematical geometry without a reference to the concept of the distance, or of the world function. The world function σ_{E} of the Euclidean space may be introduced, when the proper Euclidean geometry has been already constructed. Thus we do not need the world function for the proper Euclidean geometry construction.

The σ-immanence property of the Euclidean geometry was discovered rather recently [3, 4]. It has been proved that the Euclidean geometry can be presented in terms and only in terms of the function σ_{E}, provided the function σ_{E} satisfies a series of constraints, written in terms of σ_{E}. By definition, *any geometry is a totality of all geometric objects* $\mathcal{O}$ *and of all relations* $\mathcal{R}$ *between them.* The σ-immanence of the proper Euclidean geometry means that any geometric object $\mathcal{O}_{\mathrm{E}}$ and any relation $\mathcal{R}_{\mathrm{E}}$ of the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ can be presented in terms of the Euclidean world function σ_{E} in the form $\mathcal{O}_{\mathrm{E}}(\sigma_{\mathrm{E}})$ and $\mathcal{R}_{\mathrm{E}}(\sigma_{\mathrm{E}})$.

Let us suppose that any physical geometry $\mathcal{G}$ has the property of the σ-immanence. Then the geometry $\mathcal{G}$ may be constructed as a result of a deformation of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. Indeed, the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ is the totality of geometrical objects $\mathcal{O}_{\mathrm{E}}(\sigma_{\mathrm{E}})$ and relations $\mathcal{R}_{\mathrm{E}}(\sigma_{\mathrm{E}})$. We produce the change

$$\sigma_{\mathrm{E}} \rightarrow \sigma, \qquad \mathcal{O}_{\mathrm{E}}(\sigma_{\mathrm{E}}) \rightarrow \mathcal{O}_{\mathrm{E}}(\sigma), \qquad \mathcal{R}_{\mathrm{E}}(\sigma_{\mathrm{E}}) \rightarrow \mathcal{R}_{\mathrm{E}}(\sigma) \tag{1.1}$$

Then totality of geometrical objects $\mathcal{O}_{\mathrm{E}}(\sigma)$, relations $\mathcal{R}_{\mathrm{E}}(\sigma)$ and the world function σ form the physical geometry $\mathcal{G}$.

For instance, let the geometrical object $\mathcal{O}_{\mathrm{E}}(\sigma_{\mathrm{E}})$ be a sphere $\mathcal{S}_{\mathrm{E}P_0Q}$, passing through the point Q and with the center at the point P_0. We have in the proper

Euclidean geometry

$$\mathcal{O}_{\mathrm{E}}\left(\sigma_{\mathrm{E}}\right): \qquad \mathcal{S}_{\mathrm{E}P_0Q}=\left\{R|\sigma_{\mathrm{E}}\left(P_0,R\right)=\sigma_{\mathrm{E}}\left(P_0,Q\right)\right\} \tag{1.2}$$

The geometrical object $\mathcal{O}_{\mathrm{E}}\left(\sigma\right)$

$$\mathcal{O}_{\mathrm{E}}\left(\sigma\right): \qquad \mathcal{S}_{P_0Q}=\left\{R|\sigma\left(P_0,R\right)=\sigma\left(P_0,Q\right)\right\} \tag{1.3}$$

is the sphere $\mathcal{S}_{P_0Q}$ in the physical geometry $\mathcal{G}$.

Let $\mathcal{R}_{\mathrm{E}}\left(\sigma_{\mathrm{E}}\right)$ be the scalar product $\left(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2\right)_{\mathrm{E}}$ of two vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$ in $\mathcal{G}_{\mathrm{E}}$. It can be written in the σ-immanent form (i.e. in terms of the world function σ_{E})

$$\mathcal{R}_{\mathrm{E}}\left(\sigma_{\mathrm{E}}\right): \qquad \left(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2\right)_{\mathrm{E}}=\sigma_{\mathrm{E}}\left(P_0,P_1\right)+\sigma_{\mathrm{E}}\left(P_0,P_2\right)-\sigma_{\mathrm{E}}\left(P_1,P_2\right) \tag{1.4}$$

where index 'E' shows that the quantity relates to the Euclidean geometry. It is easy to see that (1.4) is a corollary of the Euclidean relations

$$\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{E}}^2=2\sigma_{\mathrm{E}}\left(P_0,P_1\right) \tag{1.5}$$

$$\left|\mathbf{P}_1\mathbf{P}_2\right|_{\mathrm{E}}^2=\left|\mathbf{P}_0\mathbf{P}_2-\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{E}}^2=\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{E}}^2+\left|\mathbf{P}_0\mathbf{P}_2\right|_{\mathrm{E}}^2-2\left(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2\right)_{\mathrm{E}} \tag{1.6}$$

According to (1.1) in the physical geometry $\mathcal{G}$ we obtain instead of (1.4)

$$\mathcal{R}_{\mathrm{E}}\left(\sigma\right): \qquad \left(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2\right)=\sigma\left(P_0,P_1\right)+\sigma\left(P_0,P_2\right)-\sigma\left(P_1,P_2\right) \tag{1.7}$$

Such a way of the physical geometry construction is very simple. It does not use any logical reasonings. It is founded on the supposition that *any physical geometry has the σ-immanence property*. It uses essentially the fact that the proper Euclidean geometry has been already constructed, and all necessary logical reasonings has been already produced in the proper Euclidean geometry.

The application of the replacement (1.1) to the construction of a physical geometry will be referred to as the deformation principle. Any change of distance ρ, or the world function σ between the points of the space Ω means a deformation of this space. We construe the concept of deformation in a broad sense. The deformation may transform a point into a surface and a surface into a point. The deformation may remove some points of the Euclidean space, violating its continuity, or decreasing its dimension. The deformation may add supplemental points to the Euclidean space, increasing its dimension. We may interpret any σ-immanent generalization of the Euclidean geometry as its deformation. In other words, the deformation principle is a very general method of the generalized geometry construction.

The physical geometry constructed on the basis of the deformation principle will be referred to as the tubular geometry (T-geometry) [3, 4, 5]. Such a name is used, because in the T-geometry the straight lines have, in general, a shape of hallow tubes, which in some T-geometries may degenerate into one-dimensional curves. At this point the T-geometry distinguishes from the Riemannian geometry, where the straight line (geodesic) is one-dimensional by its construction.

Construction of a nonhomogeneous geometry on the axiomatic basis is impossible practically, because there is a lot of different nonhomogeneous geometries. It is very difficult to invent axiomatics for a nonhomogeneous geometry, where identical objects have different properties in various places. Besides, one cannot invent axiomatics for each of these geometries. Thus, in reality there is no alternative to application of the deformation principle at the construction of the physical geometry. The real problem consists in the sequential application of the deformation principle. As far as the *deformation principle alone is sufficient for the construction of the physical geometry*, one may not use additional means of the geometry construction. At the physical geometry construction we do not use coordinate system and other means of descriptions.

Usually a construction of the Riemannian geometry is carried out in some coordinate system. The Riemannian geometry is obtained from the Euclidean one by means of the deformation principle, i.e. by the change infinitesimal Euclidean distance $dS_{\mathrm{E}} = \sqrt{g_{\mathrm{E}ik}dx^i dx^k}$ by means of the Riemannian one $dS_{\mathrm{R}} = \sqrt{g_{\mathrm{R}ik}dx^i dx^k}$. Properties of the Riemannian geometry are determined by the form of the metric tensor $g_{\mathrm{R}ik}$. But the form of the metric tensor depends on the choice of the coordinate system. If we use different coordinate systems, we obtain formally different description of the same Riemannian geometry. To separate the essential part of description, which relates to the geometry in itself, we are forced to consider description in all possible coordinate systems. The common part of descriptions in all coordinate systems (invariants of the group of the coordinate transformations) forms the description of the geometry in itself. Unfortunately, we cannot use all possible coordinate systems. Practically we use only continuous coordinates. The number of coordinates is fixed, and coincides with the dimension of the Riemannian geometry. We cannot solve definitely, whether the continuity is a property of the considered geometry, or maybe, it is a property of the coordinate description. As a result, most geometers believe that the continuity is the inherent property of the geometry. They admit that the discrete geometry may be constructed, but they do not know, how to do this, because one cannot use discrete coordinates for description of discrete geometries.

Mathematicians provide physicists with their geometrical construction, and physicists believe that the space-time is continuous. Continuity of the space-time cannot be tested experimentally, and the only reason of the space-time continuity is the fact that mathematicians are able to construct only continuous geometries, whereas they fail to construct discrete geometries.

We have the same situation with the space dimension. Geometers consider the dimension to be an inherent property of any geometry. They can imagine the n-dimensional Riemannian geometry, but they cannot imagine a geometry without a dimension, or a geometry of an indefinite dimension. The reason of these belief is the fact that the dimension of the manifold and its continuity are the starting points of the Riemannian geometry construction, and at this point one cannot separate the properties of the geometry from the properties of the manifold.

In the T-geometry we deal only with the geometry in itself, because it does not use any means of the description. As a result the T-geometry is insensitive to continuity or discreteness of the space, as well as to its dimension. Application of additional means of description can lead to inconsistency and to a restriction of the list of possible physical geometries.

Any generalization of the proper Euclidean geometry is founded on some property of the Euclidean geometry (or its objects). This property is conserved in all generalized geometries, whereas other properties of the Euclidean geometry are varied. Character and properties of the obtained generalized geometry depend essentially on the choice of the conserved property of the basic Euclidean geometry. For instance, the Riemannian geometry is such a generalization of the Euclidean one, where the one-dimensionality and continuity of the Euclidean straight line are conserved, whereas its curvature and torsion are varied. The straight line is considered to be the principal geometric object of the Euclidean geometry, and one supposes that such properties of the Euclidean straight line as continuity and one-dimensionality (absence of thickness) are to be conserved at the generalization. It means that the continuity and one-dimensionality of the straight line are to be the principal concepts of the generalized geometry (the Riemannian geometry). In accordance with such a choice of the conserved geometrical object one introduces the concept of the curve $\mathcal{L}$ as a continuous mapping of a segment of the real axis onto the space Ω

$$\mathcal{L}: \qquad [0,1] \to \Omega \tag{1.8}$$

To introduce the concept of the continuity, which is a basic concept of the generalization, one introduces the topological space, the dimension of the space Ω and other basic concepts of the Riemannian geometry, which are necessary for construction of the Riemannian generalization of the Euclidean geometry.

The σ-immanence of the Euclidean geometry is a property of the whole Euclidean geometry. Using the σ-immanence for generalization, we do not impose any constraints on the single geometric objects of the Euclidean geometry. As a result the σ-immanent generalization appears to be a very powerful generalization. Besides, from the common point of view the application of the whole geometry property for the generalization seems to be more reasonable, than a use of the properties of a single geometric object. Thus, using the property of the whole Euclidean geometry, the σ-immanent generalization seems to be more reasonable, than the Riemannian generalization, using the properties of the Euclidean straight line.

Now we list the most attractive features of the σ-immanent generalization of the Euclidean geometry:

1. It uses for the generalization the σ-immanence, which is a property of the Euclidean geometry as a whole (but not a property of a single geometric object as it takes place at the Riemannian generalization).

2. The σ-immanent generalization does not use any logical construction, and the σ-immanent generalization is automatically as consistent, as the Euclidean geometry, whose axiomatics is used implicitly. In particular, the T-geometry does

not contain any theorems. As a result the main problem of the T-geometry is a correct σ-immanent description of geometrical objects and relations of the Euclidean geometry. There are some subtleties in such a σ-immanent description, which are discussed below.

3. The σ-immanent generalization is a very powerful generalization. It varies practically all properties of the Euclidean geometry, including such ones as the continuity and the parallelism transitivity, which are conserved at the Riemannian generalization.

4. The σ-immanent generalization allows one to use the coordinateless description and ignore the problems, connected with the coordinate transformations as well as with the transformation of other means of description.

5. The T-geometry may be used as the space-time geometry. In this case the tubular character of straights explains freely the stochastic world lines of quantum microparticles. Considering the quantum constant $\hbar$ as an attribute of the space-time geometry, one can obtain the quantum description as the statistical description of the stochastic world lines [6]. Such a space-time geometry cannot be obtained in the framework of the Riemannian generalization of the Euclidean geometry.

In the paper we discuss the interplay between the Riemannian generalization of the Euclidean geometry and the σ-immanent one. The fact is that the world function σ has been introduced at first in the Riemannian geometry [2]. The world function σ plays an important, but not crucial role in the description of the Riemannian geometry, whereas in the T-geometry the world function σ is the only quantity, which is necessary for its construction and description.

Let us consider two generalization of the Euclidean geometry $\mathcal{G}_{\rm E}$ with the same world function σ. Let one of the generalization be the Riemannian geometry $\mathcal{G}_{\rm R}$ and the other one be a σ-immanent generalization $\mathcal{G}_{\sigma}$. Do the generalizations $\mathcal{G}_{\rm R}$ and $\mathcal{G}_{\sigma}$ coincide? Many important details of both generalizations coincide, but other details of the geometrical description are different. In general, the geometries $\mathcal{G}_{\rm R}$ and $\mathcal{G}_{\sigma}$ are different, although both geometries $\mathcal{G}_{\rm R}$ and $\mathcal{G}_{\sigma}$ are described by the same world function σ. The geometry $\mathcal{G}_{\sigma}$ is determined by the world function σ uniquely without any logical constructions. It means that the σ-immanent generalization $\mathcal{G}_{\sigma}$ cannot be inconsistent, because the basic geometry $\mathcal{G}_{\rm E}$ is consistent [7]. As to the Riemannian generalization $\mathcal{G}_{\rm R}$, it may be inconsistent, because it uses additional means of description (manifold, topology), which are not necessary for construction of the σ-immanent generalization of the Euclidean geometry. One should investigate to what extent the additional means of description are compatible between themselves and with the given world function σ, which is alone sufficient for the construction of the consistent σ-immanent generalization $\mathcal{G}_{\sigma}$.

Application of additional means of description leads to an overdetermination of the problem of the Euclidean geometry generalization. Some inconsistencies of the Riemannian generalization are corollaries of this overdetermination.

2 Euclidean geometry in the σ-immanent form

Definition 1 *The σ-space $V=\{\sigma,\Omega\}$ is the set Ω of points P with the given world function σ*

$$\sigma: \qquad \Omega\times\Omega\rightarrow\mathbb{R}, \qquad \sigma(P,P)=0, \qquad \forall P\in\Omega \tag{2.1}$$

Let the proper Euclidean geometry be given on the set Ω, and the quantity

$$\rho(P_0,P_1)=\sqrt{2\sigma(P_0,P_1)}, \qquad P_0,P_1\in\Omega \tag{2.2}$$

be the Euclidean distance between the points P_0, P_1.

Let the vector $\mathbf{P}_0\mathbf{P}_1=\{P_0,P_1\}$ be the ordered set of two points P_0, P_1. The point P_0 is the origin of the vector $\mathbf{P}_0\mathbf{P}_1$, and the point P_1 is its end. The length $|\mathbf{P}_0\mathbf{P}_1|$ of the vector $\mathbf{P}_0\mathbf{P}_1$ is defined by the relation

$$|\mathbf{P}_0\mathbf{P}_1|^2=2\sigma(P_0,P_1) \tag{2.3}$$

In the Euclidean geometry the scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2)$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_0\mathbf{P}_2$, having the common origin P_0, is expressed by the relation (1.7)

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2)=\sigma(P_0,P_1)+\sigma(P_1,P_0)-\sigma(P_1,P_2) \tag{2.4}$$

It follows from the expression (2.4), written for scalar products $(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{Q}_1)$ and $(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{Q}_0)$, and from the properties of the scalar product in the Euclidean space, that the scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ can be written in the σ-immanent form

$$\begin{aligned}(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) &= (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{Q}_1)-(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{Q}_0)= \\ &\sigma(P_0,Q_1)+\sigma(P_1,Q_0)-\sigma(P_0,Q_0)-\sigma(P_1,Q_1)\end{aligned} \tag{2.5}$$

Let $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$,...$\mathbf{P}_0\mathbf{P}_n$ be n vectors in the Euclidean space. The necessary and sufficient condition of their linear dependence is

$$F_n(\mathcal{P}^n)\equiv\det||(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)||=0, \qquad i,k=1,2,..n, \qquad \mathcal{P}^n=\{P_0,P_1,...P_n\} \tag{2.6}$$

where $F_n(\mathcal{P}^n)\equiv\det||(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)||$ is the Gram's determinant, constructed of the scalar products of vectors.

Let us formulate the theorem on the σ-immanence of the Euclidean geometry.

Theorem 1 *The σ-space $V=\{\sigma,\Omega\}$ is the n-dimensional proper Euclidean space, if and only if the world function σ satisfies the following conditions, written in terms of the world function σ.*

I. Condition of symmetry:

$$\sigma(P,Q)=\sigma(Q,P), \qquad \forall P,Q\in\Omega \tag{2.7}$$

II. Definition of the dimension:

$$\exists\mathcal{P}^n \equiv \{P_0, P_1, ...P_n\} \subset \Omega, \qquad F_n(\mathcal{P}^n) \neq 0, \qquad F_k\left(\Omega^{k+1}\right) = 0, \qquad k > n \quad (2.8)$$

where $F_n(\mathcal{P}^n)$ is the Gram's determinant (2.6). Vectors $\mathbf{P}_0\mathbf{P}_i$, $i = 1, 2, ...n$ are basic vectors of the rectilinear coordinate system K_n with the origin at the point P_0, and the metric tensors $g_{ik}(\mathcal{P}^n)$, $g^{ik}(\mathcal{P}^n)$, $i, k = 1, 2, ...n$ in K_n are defined by the relations

$$\sum_{k=1}^{k=n} g^{ik}(\mathcal{P}^n)\, g_{lk}(\mathcal{P}^n) = \delta_l^i, \qquad g_{il}(\mathcal{P}^n) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_l), \qquad i, l = 1, 2, ...n \quad (2.9)$$

$$F_n(\mathcal{P}^n) = \det ||g_{ik}(\mathcal{P}^n)|| \neq 0, \qquad i, k = 1, 2, ...n \quad (2.10)$$

III. Linear structure of the Euclidean space:

$$\sigma(P, Q) = \frac{1}{2}\sum_{i,k=1}^{i,k=n} g^{ik}(\mathcal{P}^n)\,(x_i(P) - x_i(Q))\,(x_k(P) - x_k(Q)), \qquad \forall P, Q \in \Omega \quad (2.11)$$

where coordinates $x_i(P)$, $i = 1, 2, ...n$ of the point P are covariant coordinates of the vector $\mathbf{P}_0\mathbf{P}$, defined by the relation

$$x_i(P) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}), \qquad i = 1, 2, ...n \quad (2.12)$$

IV: The metric tensor matrix $g_{lk}(\mathcal{P}^n)$ has only positive eigenvalues

$$g_k > 0, \qquad k = 1, 2, ..., n \quad (2.13)$$

V. The continuity condition: the system of equations

$$(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}) = y_i \in \mathbb{R}, \qquad i = 1, 2, ...n \quad (2.14)$$

considered to be equations for determination of the point P as a function of coordinates $y = \{y_i\}$, $i = 1, 2, ...n$ has always one and only one solution. Conditions II – V contain a reference to the dimension n of the Euclidean space.

This theorem states that the proper Euclidean space has the property of the σ-immanence, and hence any statement $\mathcal{S}$ of the proper Euclidean geometry can be expressed in terms and only in terms of the world function $\sigma_{\rm E}$ of the Euclidean geometry in the form $\mathcal{S}(\sigma_{\rm E})$. Producing the change $\sigma_{\rm E} \to \sigma$ in the statement $\mathcal{S}$, we obtain corresponding statement $\mathcal{S}(\sigma)$ of another T-geometry $\mathcal{G}$, described by the world function σ.

3 Construction of geometric objects in the σ-immanent form

In the T-geometry the geometric object $\mathcal{O}$ is described by means of the skeleton-envelope method [4]. It means that any geometric object $\mathcal{O}$ is considered to be a set of intersections and joins of elementary geometric objects (EGO).

The finite set $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\} \subset \Omega$ of parameters of the envelope function $f_{\mathcal{P}^n}$ is the skeleton of elementary geometric object (EGO) $\mathcal{E} \subset \Omega$. The set $\mathcal{E} \subset \Omega$ of points forming EGO is called the envelope of its skeleton $\mathcal{P}^n$. In the continuous generalized geometry the envelope $\mathcal{E}$ is usually a continual set of points. The envelope function $f_{\mathcal{P}^n}$

$$f_{\mathcal{P}^n}: \qquad \Omega \rightarrow \mathbb{R}, \tag{3.1}$$

determining EGO is a function of the running point $R \in \Omega$ and of parameters $\mathcal{P}^n \subset \Omega$. The envelope function $f_{\mathcal{P}^n}$ is supposed to be an algebraic function of s arguments $w = \{w_1, w_2, ...w_s\}$, $s = (n+2)(n+1)/2$. Each of arguments $w_k = \sigma(Q_k, L_k)$ is a σ-function of two arguments $Q_k, L_k \in \{R, \mathcal{P}^n\}$, either belonging to skeleton $\mathcal{P}^n$, or coinciding with the running point R. Thus, any elementary geometric object $\mathcal{E}$ is determined by its skeleton and its envelope function as the set of zeros of the envelope function

$$\mathcal{E} = \{R | f_{\mathcal{P}^n}(R) = 0\} \tag{3.2}$$

For instance, the cylinder $\mathcal{C}(P_0, P_1, Q)$ with the points P_0, P_1 on the cylinder axis and the point Q on its surface is determined by the relation

$$\begin{aligned} \mathcal{C}(P_0, P_1, Q) &= \{R | f_{P_0P_1Q}(R) = 0\}, \\ f_{P_0P_1Q}(R) &= F_2(P_0, P_1, Q) - F_2(P_0, P_1, R) \end{aligned} \tag{3.3}$$

$$F_2(P_0, P_1, Q) = \left| \begin{array}{cc} (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_1) & (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{Q}) \\ (\mathbf{P}_0\mathbf{Q}.\mathbf{P}_0\mathbf{P}_1) & (\mathbf{P}_0\mathbf{Q}.\mathbf{P}_0\mathbf{Q}) \end{array} \right| \tag{3.4}$$

Here $\sqrt{F_2(P_0, P_1, Q)}$ is the area of the parallelogram, constructed on the vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_0\mathbf{Q}$ and $\frac{1}{2}\sqrt{F_2(P_0, P_1, Q)}$ is the area of triangle with vertices at the points P_0, P_1, Q. The equality $F_2(P_0, P_1, Q) = F_2(P_0, P_1, R)$ means that the distance between the point Q and the axis, determined by the vector $\mathbf{P}_0\mathbf{P}_1$ is equal to the distance between R and the axis.

The elementary geometrical object $\mathcal{E}$ is determined in all physical geometries at once. In particular, it is determined in the proper Euclidean geometry, where we can obtain its meaning. We interpret the elementary geometrical object $\mathcal{E}$, using our knowledge of the proper Euclidean geometry. Thus, the proper Euclidean geometry is used as a sample geometry for interpretation of any physical geometry. In particular, the cylinder (3.3) is determined uniquely in any T-geometry with any world function σ.

In the Euclidean geometry the points P_0 and P_1 determine the cylinder axis. The shape of a cylinder depends on its axis and radius, but not on the disposition of points P_0, P_1 on the cylinder axis. As a result in the Euclidean geometry the cylinders $\mathcal{C}(P_0, P_1, Q)$ and $\mathcal{C}(P_0, P_2, Q)$ coincide, provided vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_0\mathbf{P}_2$ are collinear. In the general case of T-geometry the cylinders $\mathcal{C}(P_0, P_1, Q)$ and $\mathcal{C}(P_0, P_2, Q)$ do not coincide, in general, even if vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_0\mathbf{P}_2$ are collinear. Thus, in general, a deformation of the Euclidean geometry splits Euclidean geometrical objects.

We do not try to repeat subscriptions of Euclid at construction of the geometry. We take the geometrical objects and relations between them, prepared in the framework of the Euclidean geometry and describe them in terms of the world function. Thereafter we deform them, replacing the Euclidean world function σ_{E} by the world function σ of the geometry in question. In practice the construction of the elementary geometric object is reduced to the representation of the corresponding Euclidean geometrical object in the σ-immanent form, i.e. in terms of the Euclidean world function. The last problem is the problem of the proper Euclidean geometry. The problem of representation of the geometrical object (or relation between objects) in the σ-immanent form is a real problem of the T-geometry construction.

Application of the deformation principle is restricted by two constraints.

1. The deformation principle is to be applied separately from other methods of the geometry construction. In particular, one may not use topological structures in construction of a physical geometry, because for effective application of the deformation principle the obtained physical geometry must be determined only by the world function (metric).

2. Describing Euclidean geometric objects $\mathcal{O}(\sigma_{\mathrm{E}})$ and Euclidean relation $\mathcal{R}(\sigma_{\mathrm{E}})$ in terms of σ_{E}, we are not to use special properties of Euclidean world function σ_{E}. In particular, definitions of $\mathcal{O}(\sigma_{\mathrm{E}})$ and $\mathcal{R}(\sigma_{\mathrm{E}})$ are to have similar form in Euclidean geometries of different dimensions. They must not depend on the dimension of the Euclidean space.

The T-geometry construction is not to use coordinates and other methods of description, because the application of the means of description imposes constraints on the constructed geometry. Any means of description is a structure St given on the basic Euclidean geometry with the world function σ_{E}. Replacement $\sigma_{\mathrm{E}} \to \sigma$ is sufficient for construction of unique generalized geometry $\mathcal{G}_\sigma$. If we use an additional structure St for the T-geometry construction, we obtain, in general, other geometry $\mathcal{G}_{St}$, which coincides with $\mathcal{G}_\sigma$ not for all σ, but only for some of world functions σ. Thus, a use of additional means of description restricts the list of possible generalized geometries. For instance, if we use the coordinate description at construction of the generalized geometry, the obtained geometry appears to be continuous, because description by means of the coordinates is effective only for continuous geometries, where the number of coordinates coincides with the geometry dimension.

As far as the σ-immanent description of the proper Euclidean geometry is possible, it is possible for any T-geometry, because any geometrical object $\mathcal{O}$ and any relation $\mathcal{R}$ in the physical geometry $\mathcal{G}$ is obtained from the corresponding geometrical object $\mathcal{O}_{\mathrm{E}}$ and from the corresponding relation $\mathcal{R}_{\mathrm{E}}$ in the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ by means of the replacement $\sigma_{\mathrm{E}} \to \sigma$ in description of $\mathcal{O}_{\mathrm{E}}$ and $\mathcal{R}_{\mathrm{E}}$. For such a replacement be possible, the description of $\mathcal{O}_{\mathrm{E}}$ and $\mathcal{R}_{\mathrm{E}}$ is not to refer to special properties of σ_{E}, described by conditions II – V. A formal indicator of the conditions II – V application is a reference to the dimension n, because any of conditions II – V contains a reference to the dimension n of the proper Euclidean space.

Let us suppose that some geometrical object $\mathcal{O}_{\mathrm{E}_n}(\sigma_{\mathrm{E}_n}, n)$ is defined in the n-

dimensional Euclidean space, and this definition refers explicitly to the dimension of the Euclidean space n. Let us deform the n-dimensional Euclidean space E_n in the m-dimensional Euclidean space E_m. Then we must make the change

$$\mathcal{O}_{\mathrm{E}_n}\left(\sigma_{\mathrm{E}_n}, n\right) \rightarrow \mathcal{O}_{\mathrm{E}_n}\left(\sigma_{\mathrm{E}_m}, n\right) \tag{3.5}$$

On the other hand, we may define the same geometrical object directly in the m-dimensional Euclidean space E_m in the form $\mathcal{O}_{\mathrm{E}_m}\left(\sigma_{\mathrm{E}_m}, m\right)$. Equating this expression to (3.5), we obtain

$$\mathcal{O}_{\mathrm{E}_n}\left(\sigma_{\mathrm{E}_m}, n\right) = \mathcal{O}_{\mathrm{E}_m}\left(\sigma_{\mathrm{E}_m}, m\right), \qquad \forall m, n \in \mathbb{N} \tag{3.6}$$

It means that the definition of the geometrical object $\mathcal{O}$ is to be independent on the dimension of the Euclidean space.

If nevertheless we use one of special properties II – V of the Euclidean space in the σ-immanent description of a geometrical object $\mathcal{O}$, or relation $\mathcal{R}$, we refer to the dimension n and, ultimately, to the coordinate system, which is only a means of description.

Let us show this in the example of the determination of the straight in the Euclidean space. The straight $\mathcal{T}_{P_0Q}$ in the proper Euclidean space is defined by two its points P_0 and Q $(P_0 \neq Q)$ as the set of points R

$$\mathcal{T}_{P_0Q} = \{R \mid \mathbf{P}_0\mathbf{Q} || \mathbf{P}_0\mathbf{R}\} \tag{3.7}$$

where condition $\mathbf{P}_0\mathbf{Q}||\mathbf{P}_0\mathbf{R}$ means that vectors $\mathbf{P}_0\mathbf{Q}$ and $\mathbf{P}_0\mathbf{R}$ are collinear, i.e. the scalar product $(\mathbf{P}_0\mathbf{Q}.\mathbf{P}_0\mathbf{R})$ of these two vectors satisfies the relation

$$\mathbf{P}_0\mathbf{Q}||\mathbf{P}_0\mathbf{R}: \qquad (\mathbf{P}_0\mathbf{Q}.\mathbf{P}_0\mathbf{R})^2 = (\mathbf{P}_0\mathbf{Q}.\mathbf{P}_0\mathbf{Q})\,(\mathbf{P}_0\mathbf{R}.\mathbf{P}_0\mathbf{R}) \tag{3.8}$$

where the scalar product is defined by the relation (2.5). Thus, the straight line $\mathcal{T}_{P_0Q}$ is defined σ-immanently, i.e. in terms of the world function σ. We shall use two different names (straight and tube) for the geometric object $\mathcal{T}_{P_0Q}$. We shall use the term "straight", when we want to stress that $\mathcal{T}_{P_0Q}$ is a result of deformation of the Euclidean straight. We shall use the term "tube", when we want to stress that $\mathcal{T}_{P_0Q}$ may be a many-dimensional surface.

In the Euclidean geometry one can use another definition of collinearity. Vectors $\mathbf{P}_0\mathbf{Q}$ and $\mathbf{P}_0\mathbf{R}$ are collinear, if components of vectors $\mathbf{P}_0\mathbf{Q}$ and $\mathbf{P}_0\mathbf{R}$ are proportional in some rectilinear coordinate system. For instance, in the n-dimensional Euclidean space one can introduce rectilinear coordinate system, choosing $n+1$ points $\mathcal{P}^n = \{P_0, P_1, ...P_n\}$ and forming n basic vectors $\mathbf{P}_0\mathbf{P}_i$, $i = 1, 2, ...n$. Then the collinearity condition can be written in the form of n equations

$$\mathbf{P}_0\mathbf{Q}||\mathbf{P}_0\mathbf{R}: \qquad (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{Q}) = a\,(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{R}), \qquad i = 1, 2, ...n, \qquad a \in \mathbb{R}\setminus\{0\} \tag{3.9}$$

where $a \neq 0$ is some real constant. Relations (3.9) are relations for covariant components of vectors $\mathbf{P}_0\mathbf{Q}$ and $\mathbf{P}_0\mathbf{R}$ in the considered coordinate system with basic vectors $\mathbf{P}_0\mathbf{P}_i$, $i = 1, 2, ...n$. The definition of collinearity (3.9) depends on the dimension n of the Euclidean space. Let points $\mathcal{P}^n$ be chosen in such a way, that $(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{Q}) \neq 0$. Then eliminating the parameter a from relations (3.9), we obtain $n-1$ independent relations, and the geometrical object

$$\mathcal{T}_{Q\mathcal{P}^n} = \{R \mid \mathbf{P}_0\mathbf{Q} || \mathbf{P}_0\mathbf{R}\} = \bigcap_{i=2}^{i=n} \mathcal{S}_i, \tag{3.10}$$

$$\mathcal{S}_i = \left\{ R \left| \frac{(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{Q})}{(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{Q})} = \frac{(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{R})}{(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{R})} \right. \right\}, \qquad i = 2, 3, ...n \tag{3.11}$$

defined according to (3.9), depends on $n+2$ points $Q, \mathcal{P}^n$. This geometrical object $\mathcal{T}_{Q\mathcal{P}^n}$ is defined σ-immanently. It is a complex, consisting of the straight line and of the coordinate system, represented by $n+1$ points $\mathcal{P}^n = \{P_0, P_1, ...P_n\}$. In the Euclidean space the dependence on the choice of the coordinate system and on n points $\{P_1, ...P_n\}$, determining this system, is fictitious. The geometrical object $\mathcal{T}_{Q\mathcal{P}^n}$ depends essentially only on two points P_0, Q and coincides with the straight line $\mathcal{T}_{P_0Q}$ in the Euclidean space. But at deformations of the Euclidean space the geometrical objects $\mathcal{T}_{Q\mathcal{P}^n}$ and $\mathcal{T}_{P_0Q}$ are deformed differently. The points $P_1, P_2, ...P_n$ cease to be fictitious in definition of $\mathcal{T}_{Q\mathcal{P}^n}$, and geometrical objects $\mathcal{T}_{Q\mathcal{P}^n}$ and $\mathcal{T}_{P_0Q}$ become to be different geometric objects, in general. But being different, in general, they may coincide in some special cases.

What of the two geometrical objects in the deformed geometry $\mathcal{G}$ should be interpreted as a straight line, passing through the points P_0 and Q in the geometry $\mathcal{G}$? Of course, it is $\mathcal{T}_{P_0Q}$, because its definition does not contain a reference to a coordinate system, whereas definition of $\mathcal{T}_{Q\mathcal{P}^n}$ depends on the choice of the coordinate system, represented by points $\mathcal{P}^n$. In general, definitions of geometric objects and relations between them are not to refer to the means of description. Otherwise, the points determining the coordinate system are to be included in definition of the geometrical object.

But in the given case the geometrical object $\mathcal{T}_{P_0Q}$ is a $(n-1)$-dimensional surface, in general, whereas $\mathcal{T}_{Q\mathcal{P}^n}$ is an intersection of $(n-1)$ $(n-1)$-dimensional surfaces, i.e. $\mathcal{T}_{Q\mathcal{P}^n}$ is a one-dimensional curve, in general. The one-dimensional curve $\mathcal{T}_{Q\mathcal{P}^n}$ corresponds better to our ideas on the straight line, than the $(n-1)$-dimensional surface $\mathcal{T}_{P_0Q}$. Nevertheless, in physical geometry $\mathcal{G}$ it is $\mathcal{T}_{P_0Q}$, that is an analog of the Euclidean straight line.

It is very difficult to overcome our conventional idea that the Euclidean straight line cannot be deformed into many-dimensional surface, and *this idea has been prevent for years from construction of T-geometries.* Practically one uses such physical geometries, where deformation of the Euclidean space transforms the Euclidean straight lines into one-dimensional lines. It means that one chooses such geometries, where geometrical objects $\mathcal{T}_{P_0Q}$ and $\mathcal{T}_{Q\mathcal{P}^n}$ coincide.

$$\mathcal{T}_{P_0Q} = \mathcal{T}_{Q\mathcal{P}^n} \tag{3.12}$$

Condition (3.12) of coincidence of the objects $\mathcal{T}_{P_0Q}$ and $\mathcal{T}_{Q\mathcal{P}^n}$, imposed on the T-geometry, restricts the list of possible T-geometries.

In general, the condition (3.12) cannot be fulfilled, because lhs does not depend on points $\{P_1, P_2, ...P_n\}$, whereas rhs of (3.12) depends, in general. The tube $\mathcal{T}_{Q\mathcal{P}^n}$ does not depend on the points $\{P_1, P_2, ...P_n\}$, provided the distance $\sqrt{2\sigma(P_i, P_k)}$ between any two points $P_i, P_k \in \mathcal{P}^n$ is infinitesimal. In the Riemannian geometry the constraint (3.12) is fulfilled at the additional restriction.

$$\sqrt{2\sigma(P_i, P_k)} = \text{infinitesimal}, \qquad i, k = 1, 2, ...n \tag{3.13}$$

4 Interplay between metric geometry and T-geometry

Let us consider the metric geometry, given on the set Ω of points. The metric space $M = \{\rho, \Omega\}$ is given by the metric (distance) ρ.

$$\begin{aligned} \rho \quad &: \quad \Omega \times \Omega \to [0, \infty) \subset \mathbb{R} && (4.1)\\ \rho(P,P) &= 0, \qquad \rho(P,Q) = \rho(Q,P), \qquad \forall P, Q \in \Omega && (4.2)\\ \rho(P,Q) &\geq 0, \qquad \rho(P,Q) = 0, \quad \text{iff } P = Q, \qquad \forall P, Q \in \Omega && (4.3)\\ 0 &\leq \rho(P,R) + \rho(R,Q) - \rho(P,Q), \qquad \forall P, Q, R \in \Omega && (4.4) \end{aligned}$$

At first sight the metric space is a special case of the σ-space (2.1), and the metric geometry is a special case of the T-geometry with additional constraints (4.3), (4.4) imposed on the world function $\sigma = \frac{1}{2}\rho^2$. However it is not so, because the metric geometry is not equipped by the deformation principle. The fact, that the σ-immanence of the Euclidean geometry, as well as the complex of conditions (2.7) - (2.14), was not known until 1990, although any of relations (2.7) - (2.14) was well known. Additional (with respect to the σ-space) constraints (4.3), (4.4) are imposed to provide one-dimensionality of the straight lines. In the metric geometry the shortest (straight) line can be constructed only in the case, when it is one-dimensional.

Let us consider the set $\mathcal{EL}(P, Q, a)$ of points R

$$\mathcal{EL}(P, Q, 2a) = \{R | f_{P,Q,2a}(R) = 0\}, \qquad f_{P,Q,2a}(R) = \rho(P,R) + \rho(R,Q) - 2a \tag{4.5}$$

If the metric space coincides with the proper Euclidean space, this set of points is an ellipsoid with focuses at the points P, Q and the large semiaxis a. The relations $f_{P,Q,2a}(R) > 0$, $f_{P,Q,2a}(R) = 0$, $f_{P,Q,2a}(R) < 0$ determine respectively external points, boundary points and internal points of the ellipsoid. If $\rho(P, Q) = 2a$, we obtain the degenerate ellipsoid, which coincides with the segment $\mathcal{T}_{[PQ]}$ of the straight line, passing through the points P, Q. In the proper Euclidean geometry, the degenerate ellipsoid is one-dimensional segment of the straight line, but it is not evident that it is one-dimensional in the case of arbitrary metric geometry. For such a degenerate ellipsoid be one-dimensional in the arbitrary metric space, it is necessary that any degenerate ellipsoid $\mathcal{EL}(P, Q, \rho(P, Q))$ have no internal points. This constraint is written in the form

$$f_{P,Q,\rho(P,Q)}(R) = \rho(P,R) + \rho(R,Q) - \rho(P,Q) \geq 0 \tag{4.6}$$

Comparing relation (4.6) with (4.4), we see that the constraint (4.4) is introduced to provide the straight (shortest) line one-dimensionality (absence of internal points in the geometrical object determined by two points).

As far as the metric geometry does not use the deformation principle, it is a poor geometry, because in the framework of this geometry one cannot construct the scalar product of two vectors, define linear independence of vectors and construct such geometrical objects as planes. All these objects as well as other are constructed on the basis of the deformation of the proper Euclidean geometry.

Generalizing the metric geometry, Menger [8] and Blumenthal [9] removed the triangle axiom (4.4). They tried to construct the distance geometry, which would be a more general geometry, than the metric one. As far as they did not use the deformation principle, they could not determine the shortest (straight) line without a reference to the topological concept of the curve $\mathcal{L}$, defined as a continuous mapping (1.8), which cannot be expressed only via the distance. As a result the distance geometry appeared to be not a pure metric geometry (i.e. the geometry determined only by the distance).

Note that the Riemannian geometry uses the deformation principle in the coordinate form. The distance geometry cannot use it in such a form, because the metric and distance geometries are formulated in the coordinateless form. It is to use the deformation principle in the coordinateless form. But application of the deformation principle in the coordinateless form needs a use of the Euclidean geometry σ-immanence. K. Menger went to the concept of the σ-immanence, but he stopped in one step before the σ-immanence. Look at the K. Menger's theorem [8], written in our designations

Theorem 2 *The σ-space $V = \{\sigma, \Omega\}$ is isometrically embeddable in n-dimensional proper Euclidean space E_n, if and only if any set of $n+3$ points of Ω is isometrically embeddable in E_n.*

The theorem on the σ-immanence of the Euclidean geometry is obtained from the Menger's theorem, if instead of the condition "any set of $n+3$ points of Ω is isometrically embeddable in E_n" one writes the condition (2.11), which also contains $n+3$ points: $P, Q, \mathcal{P}^n$ and describes the fact that $\{P, Q, \mathcal{P}^n\} \subset E_n$. In this case the theorem condition contains only a reference to the properties of the world function of the Euclidean space, but not to the Euclideaness of the space. (continuity of the σ-space V is neglected in such a formulation.)

5 Conditions of the deformation principle application

Riemannian geometries satisfy the condition (3.12). The Riemannian geometry is a kind of inhomogeneous physical geometry, and, hence, it uses the deformation principle. Constructing the Riemannian geometry, the infinitesimal Euclidean distance is deformed into the Riemannian distance. The deformation is chosen in such a way that any Euclidean straight line $\mathcal{T}_{\mathrm{E}P_0Q}$, passing through the point P_0, collinear to the vector $\mathbf{P}_0\mathbf{Q}$, is transformed into the geodesic $\mathcal{T}_{P_0Q}$, passing through the point P_0, collinear to the vector $\mathbf{P}_0\mathbf{Q}$ in the Riemannian space.

Note that in T-geometries, satisfying the condition (3.12) for all points $Q,\mathcal{P}^n$, the straight line

$$\mathcal{T}_{Q_0;P_0Q}=\{R\mid \mathbf{P}_0\mathbf{Q}||\mathbf{Q}_0\mathbf{R}\} \tag{5.1}$$

passing through the point Q_0 collinear to the vector $\mathbf{P}_0\mathbf{Q}$, is not a one-dimensional line, in general. If the Riemannian geometries be T-geometries, they would contain non-one-dimensional geodesics (straight lines). But the Riemannian geometries are not T-geometries, because at their construction one uses not only the deformation principle, but some other methods, containing a reference to the means of description. In particular, in the Riemannian geometries the absolute parallelism is absent, and one cannot define a straight line (5.1), because the collinearity relation $\mathbf{P}_0\mathbf{Q}||\mathbf{Q}_0\mathbf{R}$ is not defined, if points P_0 and Q_0 do not coincide. On one hand, a lack of absolute parallelism allows one to go around the problem of non-one-dimensional straight lines. On the other hand, it makes the Riemannian geometries to be inconsistent, because they cease to be T-geometries, which are consistent by the construction (see for details [10]).

The fact is that the application of *only deformation principle* is sufficient for construction of a physical geometry. Besides, such a construction is consistent, because the original Euclidean geometry is consistent and, deforming it, we do not use any logical reasonings. If we introduce additional structure (for instance, a topological structure) we obtain a fortified physical geometry, i.e. a physical geometry with additional structure on it. The physical geometry, equipped with additional structure, is a more pithy construction, than the physical geometry simply. But it is valid only in the case, when we consider the additional structure as an addition to the physical geometry. If we use an additional structure in construction of the geometry, we identify the additional structure with one of structures of the physical geometry. If we demand that the additional structure be a structure of physical geometry, we restrict an application of the deformation principle and reduce the list of possible physical geometries, because coincidence of the additional structure with some structure of a physical geometry is possible not for all physical geometries, but only for some of them.

Let, for instance, we use concept of a curve $\mathcal{L}$ (1.8) for construction of a physical geometry. The concept of curve $\mathcal{L}$, considered as a continuous mapping, is a topological structure, which cannot be expressed only via the distance or via the world function. A use of the mapping (1.8) needs an introduction of topological space and, in particular, the concept of continuity. If we identify the topological curve (1.8) with the "metrical" curve, defined as a broken line

$$\mathcal{T}_{\mathrm{br}}=\bigcup_i\mathcal{T}_{[P_iP_{i+1}]},\qquad \mathcal{T}_{[P_iP_{i+1}]}=\left\{R|\sqrt{2\sigma(P_i,P_{i+1})}-\sqrt{2\sigma(P_i,R)}-\sqrt{2\sigma(R,P_{i+1})}\right\} \tag{5.2}$$

consisting of the straight line segments $\mathcal{T}_{[P_iP_{i+1}]}$ between the points P_i, P_{i+1}, we truncate the list of possible geometries, because such an identification is possible only in some physical geometries. Identifying (1.8) and (5.2), we eliminate all discrete physical geometries and those continuous physical geometries, where the segment $\mathcal{T}_{[P_iP_{i+1}]}$ of straight line is a surface, but not a one-dimensional set of points. Thus, additional structures may lead to (i) a fortified physical geometry, (ii) a restricted physical geometry and (iii) a restricted fortified physical geometry. The result depends on the method of the additional structure application.

Note that some constraints (continuity, convexity, lack of absolute parallelism), imposed on physical geometries are a result of a disagreement of the means of description, which are used at the geometry construction. In the T-geometry, which uses only the deformation principle, there is no such restrictions. Besides, the T-geometry has some new property of a physical geometry, which is not accepted by conventional versions of physical geometry. This property, called the geometry non-degeneracy, follows directly from the application of arbitrary deformations to the proper Euclidean geometry.

Definition 2 *The geometry is degenerate at the point P_0 in the direction of the vector $\mathbf{Q}_0\mathbf{Q}$, $|\mathbf{Q}_0\mathbf{Q}|\neq 0$, if the relations*

$$\mathbf{Q}_0\mathbf{Q}\uparrow\uparrow\mathbf{P}_0\mathbf{R}: \qquad (\mathbf{Q}_0\mathbf{Q}.\mathbf{P}_0\mathbf{R})=\sqrt{|\mathbf{Q}_0\mathbf{Q}|\cdot|\mathbf{P}_0\mathbf{R}|}, \qquad |\mathbf{P}_0\mathbf{R}|=a\neq 0 \tag{5.3}$$

considered as equations for determination of the point R, have not more, than one solution for any $a\neq 0$. Otherwise, the geometry is nondegenerate at the point P_0 in the direction of the vector $\mathbf{Q}_0\mathbf{Q}$.

Note that the first equation (5.3) is the condition of the parallelism of vectors $\mathbf{Q}_0\mathbf{Q}$ and $\mathbf{P}_0\mathbf{R}$.

The proper Euclidean geometry is degenerate, i.e. it is degenerate at all points in directions of all vectors. Considering the Minkowski geometry, one should distinguish between the Minkowski T-geometry and Minkowski geometry. The two geometries are described by the same world function and differ in the definition of the parallelism. In the Minkowski T-geometry the parallelism of two vectors $\mathbf{Q_0Q}$ and $\mathbf{P_0R}$ is defined by the first equation (5.3). This definition is based on the deformation principle. In the n-dimensional Minkowski geometry (n-dimensional pseudo-Euclidean geometry of index 1) the parallelism is defined by the relation of the type of (3.9)

$$\mathbf{Q}_0\mathbf{Q}\uparrow\uparrow\mathbf{P}_0\mathbf{R}: \qquad (\mathbf{P}_0\mathbf{P}_i.\mathbf{Q}_0\mathbf{Q})=a\,(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{R}), \qquad i=1,2,...n, \qquad a>0 \tag{5.4}$$

where points $\mathcal{P}^n=\{P_0,P_1,...P_n\}$ determine a rectilinear coordinate system with basic vectors $\mathbf{P}_0\mathbf{P}_i$, $i=1,2,..n$ in the n-dimensional Minkowski geometry . Dependence of the definition (5.4) on the points $(P_0,P_1,...P_n)$ is fictitious, but dependence on the number $n+1$ of points $\mathcal{P}^n$ is essential. Thus, definition (5.4) depends on the method of the geometry description.

The Minkowski T-geometry is degenerate at all points in direction of all timelike vectors, and it is nondegenerate at all points in direction of all spacelike vectors. The Minkowski geometry is degenerate at all points in direction of all vectors. Conventionally one uses the Minkowski geometry, ignoring the nondegeneracy in spacelike directions.

Considering the proper Riemannian geometry, one should distinguish between the Riemannian T-geometry and the Riemannian geometry. The two geometries are described by the same world function. They differ in the definition of the parallelism. In the Riemannian T-geometry the parallelism of two vectors $\mathbf{Q}_0\mathbf{Q}$ and $\mathbf{P}_0\mathbf{R}$ is defined by (5.3). In the Riemannian geometry the parallelism of two vectors $\mathbf{Q}_0\mathbf{Q}$ and $\mathbf{P}_0\mathbf{R}$ is defined only in the case, when the points P_0 and Q_0 coincide. Parallelism of remote vectors $\mathbf{Q}_0\mathbf{Q}$ and $\mathbf{P}_0\mathbf{R}$ is not defined, in general. This fact is known as absence of absolute parallelism.

The proper Riemannian T-geometry is locally degenerate, i.e. it is degenerate at all points P_0 in direction of all vectors $\mathbf{P}_0\mathbf{Q}$ with the origin at the point P_0. In the general case, when $P_0 \neq Q_0$, the proper Riemannian T-geometry is nondegenerate, in general. But it is degenerate locally as well as the proper Riemannian geometry. The proper Riemannian geometry is degenerate, because it is degenerate locally, whereas the nonlocal degeneracy is not defined in the Riemannian geometry, because of the lack of absolute parallelism. Conventionally one uses the Riemannian geometry (not Riemannian T-geometry) and ignores the property of the nondegeneracy completely.

From the viewpoint of the conventional approach to the physical geometry the nondegeneracy is an undesirable property of a physical geometry, although from the logical viewpoint and from viewpoint of the deformation principle the nondegeneracy is *an inherent property of a physical geometry*. The nonlocal nondegeneracy is ejected from the proper Riemannian geometry by denial of existence of the remote vector parallelism. Nondegeneracy in the spacelike directions is ejected from the Minkowski geometry by means of the redefinition of the two vectors parallelism. But the nondegeneracy is an important property of the real space-time geometry. To appreciate this, let us consider an example.

6 Simple example of nondegenerate space-time geometry

Let the space-time geometry $\mathcal{G}_{\mathrm{d}}$ be described by the T-geometry, given on 4-dimensional manifold $\mathcal{M}_{1+3}$. The world function σ_{d} is described by the relation

$$\sigma_{\mathrm{d}} = \sigma_{\mathrm{M}} + D\left(\sigma_{\mathrm{M}}\right) = \begin{cases} \sigma_{\mathrm{M}} + d & \text{if } \sigma_0 < \sigma_{\mathrm{M}} \\ \left(1 + \frac{d}{\sigma_0}\right)\sigma_{\mathrm{M}} & \text{if } 0 \leq \sigma_{\mathrm{M}} \leq \sigma_0 \\ \sigma_{\mathrm{M}} & \text{if } \sigma_{\mathrm{M}} < 0 \end{cases} \tag{6.1}$$

where $d \geq 0$ and $\sigma_0 > 0$ are some constants. The quantity σ_{M} is the world function in the Minkowski space-time geometry $\mathcal{G}_{\mathrm{M}}$. In the orthogonal rectilinear (inertial) coordinate system $x=\{t,\mathbf{x}\}$ the world function σ_{M} has the form

$$\sigma_{\mathrm{M}}\left(x,x'\right)=\frac{1}{2}\left(c^{2}\left(t-t'\right)^{2}-\left(\mathbf{x}-\mathbf{x}'\right)^{2}\right) \tag{6.2}$$

where c is the speed of the light.

Let us compare the broken line (5.2) in Minkowski space-time geometry $\mathcal{G}_{\mathrm{M}}$ and in the distorted geometry $\mathcal{G}_{\mathrm{d}}$. We suppose that $\mathcal{T}_{\mathrm{br}}$ is timelike broken line, and all links $\mathcal{T}_{[P_iP_{i+1}]}$ of $\mathcal{T}_{\mathrm{br}}$ are timelike and have the same length

$$\left|\mathbf{P}_i\mathbf{P}_{i+1}\right|_{\mathrm{d}}=\sqrt{2\sigma_{\mathrm{d}}\left(P_i,P_{i+1}\right)}=\mu_{\mathrm{d}}>0,\qquad i=0,\pm1,\pm2,... \tag{6.3}$$

$$\left|\mathbf{P}_i\mathbf{P}_{i+1}\right|_{\mathrm{M}}=\sqrt{2\sigma_{\mathrm{M}}\left(P_i,P_{i+1}\right)}=\mu_{\mathrm{M}}>0,\qquad i=0,\pm1,\pm2,... \tag{6.4}$$

where indices "d" and "M" mean that the quantity is calculated by means of σ_{d} and σ_{M} respectively. Vector $\mathbf{P}_i\mathbf{P}_{i+1}$ is regarded as the momentum of the particle at the segment $\mathcal{T}_{[P_iP_{i+1}]}$, and the quantity $\left|\mathbf{P}_i\mathbf{P}_{i+1}\right|=\mu$ is interpreted as its (geometric) mass. It follows from definition (2.5) and relation (6.1), that for timelike vectors $\mathbf{P}_i\mathbf{P}_{i+1}$ with $\mu>\sqrt{2\sigma_0}$

$$\left|\mathbf{P}_i\mathbf{P}_{i+1}\right|_{\mathrm{d}}^{2}=\mu_{\mathrm{d}}^{2}=\mu_{\mathrm{M}}^{2}+2d,\qquad \mu_{\mathrm{M}}^{2}>2\sigma_0 \tag{6.5}$$

$$\left(\mathbf{P}_{i-1}\mathbf{P}_i.\mathbf{P}_i\mathbf{P}_{i+1}\right)_{\mathrm{d}}=\left(\mathbf{P}_{i-1}\mathbf{P}_i.\mathbf{P}_i\mathbf{P}_{i+1}\right)_{\mathrm{M}}+d \tag{6.6}$$

Calculation of the shape of the segment $\mathcal{T}_{[P_0P_1]}\left(\sigma_{\mathrm{d}}\right)$ in $\mathcal{G}_{\mathrm{d}}$ gives the relation

$$r^{2}(\tau)=\left\{\begin{array}{ll}\tau^{2}\mu_{\mathrm{d}}^{2}\frac{\left(1-\frac{\tau d}{2(\sigma_0+d)}\right)^{2}}{\left(1-\frac{2d}{\mu_{\mathrm{d}}^{2}}\right)}-\frac{\tau^{2}\mu_{\mathrm{d}}^{2}\sigma_0}{(\sigma_0+d)}, & 0<\tau<\frac{\sqrt{2(\sigma_0+d)}}{\mu_{\mathrm{d}}}\\ \frac{3d}{2}+2d\left(\tau-1/2\right)^{2}\left(1-\frac{2d}{\mu_{\mathrm{d}}^{2}}\right)^{-1}, & \frac{\sqrt{2(\sigma_0+d)}}{\mu_{\mathrm{d}}}<\tau<1-\frac{\sqrt{2(\sigma_0+d)}}{\mu_{\mathrm{d}}}\\ \left(1-\tau\right)^{2}\mu_{\mathrm{d}}^{2}\left[\frac{\left(1-\frac{(1-\tau)d}{2(\sigma_0+d)}\right)^{2}}{\left(1-\frac{2d}{\mu_{\mathrm{d}}^{2}}\right)}-\frac{\sigma_0}{(\sigma_0+d)}\right], & 1-\frac{\sqrt{2(\sigma_0+d)}}{\mu_{\mathrm{d}}}<\tau<1\end{array}\right., \tag{6.7}$$

where $r\left(\tau\right)$ is the spatial radius of the segment $\mathcal{T}_{[P_0P_1]}\left(\sigma_{\mathrm{d}}\right)$ in the coordinate system, where points P_0 and P_1 have coordinates $P_0=\{0,0,0,0\}$, $P_1=\{\mu_{\mathrm{d}},0,0,0\}$ and τ is a parameter along the segment $\mathcal{T}_{[P_0P_1]}\left(\sigma_{\mathrm{d}}\right)$, $\left(\tau\left(P_0\right)=0,\ \tau\left(P_1\right)=1\right)$. One can see from (6.7) that the characteristic value of the segment radius is equal to $\sqrt{d}$.

Let the broken tube $\mathcal{T}_{\mathrm{br}}$ describe the "world tube" of a free particle. It means by definition that any link $\mathbf{P}_{i-1}\mathbf{P}_i$ is parallel to the adjacent link $\mathbf{P}_i\mathbf{P}_{i+1}$

$$\mathbf{P}_{i-1}\mathbf{P}_i\uparrow\uparrow\mathbf{P}_i\mathbf{P}_{i+1}:\qquad\left(\mathbf{P}_{i-1}\mathbf{P}_i.\mathbf{P}_i\mathbf{P}_{i+1}\right)-\left|\mathbf{P}_{i-1}\mathbf{P}_i\right|\cdot\left|\mathbf{P}_i\mathbf{P}_{i+1}\right|=0 \tag{6.8}$$

Definition of parallelism is different in geometries $\mathcal{G}_{\mathrm{M}}$ and $\mathcal{G}_{\mathrm{d}}$. As a result links, which are parallel in the geometry $\mathcal{G}_{\mathrm{M}}$, are not parallel in $\mathcal{G}_{\mathrm{d}}$ and vice versa.

Let $\mathcal{T}_{\mathrm{br}}\left(\sigma_{\mathrm{M}}\right)$ describe the world line of a free particle in the geometry $\mathcal{G}_{\mathrm{M}}$. The angle ϑ_{M} between the adjacent links in $\mathcal{G}_{\mathrm{M}}$ is defined by the relation

$$\cosh\vartheta_{\mathrm{M}}=\frac{\left(\mathbf{P}_{-1}\mathbf{P}_0.\mathbf{P}_0\mathbf{P}_1\right)_{\mathrm{M}}}{\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{M}}\cdot\left|\mathbf{P}_{-1}\mathbf{P}_0\right|_{\mathrm{M}}}=1 \tag{6.9}$$

The angle $\vartheta_{\rm M}=0$, and the geometrical object $\mathcal{T}_{\rm br}\left(\sigma_{\rm M}\right)$ is a timelike straight line on the manifold $\mathcal{M}_{1+3}$.

Let now $\mathcal{T}_{\rm br}\left(\sigma_{\rm d}\right)$ describe the world tube of a free particle in the geometry $\mathcal{G}_{\rm d}$. The angle $\vartheta_{\rm d}$ between the adjacent links in $\mathcal{G}_{\rm d}$ is defined by the relation

$$\cosh\vartheta_{\rm d}=\frac{\left(\mathbf{P}_{i-1}\mathbf{P}_{i}.\mathbf{P}_{i}\mathbf{P}_{i+1}\right)_{\rm d}}{\left|\mathbf{P}_{i}\mathbf{P}_{i+1}\right|_{\rm d}\cdot\left|\mathbf{P}_{i-1}\mathbf{P}_{i}\right|_{\rm d}}=1 \tag{6.10}$$

The angle $\vartheta_{\rm d}=0$ also. If we draw the broken tube $\mathcal{T}_{\rm br}\left(\sigma_{\rm d}\right)$ on the manifold $\mathcal{M}_{1+3}$, using coordinates of basic points P_i and measure the angle $\vartheta_{\rm dM}$ between the adjacent links in the Minkowski geometry $\mathcal{G}_{\rm M}$, we obtain for the angle $\vartheta_{\rm dM}$ the following relation

$$\cosh\vartheta_{\rm dM}=\frac{\left(\mathbf{P}_{i-1}\mathbf{P}_{i}.\mathbf{P}_{i}\mathbf{P}_{i+1}\right)_{\rm M}}{\left|\mathbf{P}_{i}\mathbf{P}_{i+1}\right|_{\rm M}\cdot\left|\mathbf{P}_{i-1}\mathbf{P}_{i}\right|_{\rm M}}=\frac{\left(\mathbf{P}_{i-1}\mathbf{P}_{i}.\mathbf{P}_{i}\mathbf{P}_{i+1}\right)_{\rm d}-d}{\left|\mathbf{P}_{i}\mathbf{P}_{i+1}\right|_{\rm d}^{2}-2d} \tag{6.11}$$

Substituting the value of $\left(\mathbf{P}_{i-1}\mathbf{P}_{i}.\mathbf{P}_{i}\mathbf{P}_{i+1}\right)_{\rm d}$, taken from (6.10), we obtain

$$\cosh\vartheta_{\rm dM}=\frac{\mu_{\rm d}^{d}-d}{\mu_{\rm d}^{2}-2d}\approx 1+\frac{d}{\mu_{\rm d}^{2}},\qquad d\ll\mu_{\rm d}^{2} \tag{6.12}$$

Hence, $\vartheta_{\rm dM}\approx\sqrt{2d}/\mu_{\rm d}$. It means, that the adjacent link is located on the cone of angle $\sqrt{2d}/\mu_{\rm d}$, and the whole line $\mathcal{T}_{\rm br}\left(\sigma_{\rm d}\right)$ has a random shape, because any link wobbles with the characteristic angle $\sqrt{2d}/\mu_{\rm d}$. The wobble angle depends on the space-time distortion d and on the particle mass $\mu_{\rm d}$. The wobble angle is small for the large mass of a particle. The random displacement of the segment end is of the order $\mu_{\rm d}\vartheta_{\rm dM}=\sqrt{2d}$, i.e. of the same order as the segment width. It is reasonable, because these two phenomena have the common source: the space-time distortion D.

One should note that the space-time geometry influences the stochasticity of particle motion nonlocally in the sense, that the form of the world function (6.1) for values of $\sigma_{\rm M}<\frac{1}{2}\mu_{\rm d}^{2}$ is unessential for the motion stochasticity of the particle of the mass $\mu_{\rm d}$.

Such a situation, when the world line of a free particle is stochastic in the deterministic geometry, and this stochasticity depends on the particle mass, seems to be rather exotic and incredible. But experiments show that the motion of real particles of small mass is stochastic indeed, and this stochasticity increases, when the particle mass decreases. From physical viewpoint a theoretical foundation of the stochasticity is desirable, and some researchers invent stochastic geometries, non-commutative geometries and other exotic geometrical constructions, to obtain the quantum stochasticity. But in the Riemannian space-time geometry the particle motion does not depend on the particle mass, and in the framework of the Riemannian space-time geometry it is difficult to explain the quantum stochasticity by the space-time geometry properties. The distorted geometry $\mathcal{G}_{\rm d}$ explains freely the stochasticity and its dependence on the particle mass. Besides, at proper choice of the distortion d the statistical description of stochastic $\mathcal{T}_{\rm br}$ leads to the quantum description (in terms of the Schrödinger equation) [6]. To do this, it is sufficient to set

$$d = \frac{\hbar}{2bc} \tag{6.13}$$

where $\hbar$ is the quantum constant, c is the speed of the light, and b is some universal constant, connecting the geometrical mass μ with the usual particle mass m by means of the relation $m = b\mu$. In other words, the distorted space-time geometry (6.1) is closer to the real space-time geometry, than the Minkowski geometry $\mathcal{G}_{\rm M}$.

Further development of the statistical description of geometrical stochasticity leads to a creation of the model conception of quantum phenomena (MCQP), which relates to the conventional quantum theory approximately in the same way as the statistical physics relates to the axiomatic thermodynamics. MCQP is the well defined relativistic conception with effective methods of investigation [11], whereas the conventional quantum theory is not well defined, because it uses incorrect space-time geometry, whose incorrectness is compensated by additional hypotheses (quantum principles). Besides, it has problems with application of the nonrelativistic quantum mechanical technique to the description of relativistic phenomena.

The geometry $\mathcal{G}_{\rm d}$, as well as the Minkowski geometry are homogeneous geometries, because the world function $\sigma_{\rm d}$ is invariant with respect to all coordinate transformations, with respect to which the world function $\sigma_{\rm M}$ is invariant. In this connection the question arises, whether one could invent some axiomatics for $\mathcal{G}_{\rm d}$ and derive the geometry $\mathcal{G}_{\rm d}$ from this axiomatics by means of proper reasonings. Note that such an axiomatics is to depend on the parameter d, because the world function $\sigma_{\rm d}$ depends on this parameter. If $d = 0$, this axiomatics is to coincide with the axiomatics of the Minkowski geometry $\mathcal{G}_{\rm M}$. If $d \neq 0$, this axiomatics cannot coincide with the axiomatics of $\mathcal{G}_{\rm M}$, because some axioms of $\mathcal{G}_{\rm M}$ are not satisfied in this case. In general, the invention of axiomatics, depending on the parameter d and in the general case on the distortion function D, seems to be a very difficult problem. Besides, why invent the axiomatics? We had derived the axiomatics for the proper Euclidean geometry, when we constructed it before. There is no necessity to repeat this process any time, when we construct a new geometry. It is sufficient to apply the deformation principle to the constructed Euclidean geometry written σ-immanently. Application of the deformation principle to the Euclidean geometry is a very simple and general procedure, which is not restricted by continuity, convexity and other artificial constraints, generated by our preconceived approach to the physical geometry. (Bias of the approach is displayed in the antecedent supposition on the one-dimensionality of any straight in any physical geometry).

Thus, we have seen that the nondegeneracy, as well as non-one-dimensionality of the straight are properties of the real physical geometries. The proper Euclidean geometry is a ground for all physical geometries, and it is a degenerate geometry. Nevertheless, it is beyond reason to deny an existence of nondegenerate physical geometries.

7 Corollaries of the nondegenerate space-time geometry

Possibility of the nondegenerate space-time geometry changes strongly the existing conception of the microcosm space-time. A small correction to the world function of the Minkowski space-time admits one to explain the enigmatic quantum nature of the microcosm. The quantum principles as an addition to the classical picture of the world appear to be not needed. The quantum principles become to be corollary of the space-time model. The microcosm space-time geometry changes radically. The universal transversal length $\sqrt{d}$ (6.13) appears as an attribute of the space-time. The particle motion becomes primordially stochastic. One does not need to search for the reason of stochasticity in the sense that the stochastic motion is a natural motion of any particle, whereas the deterministic motion is a motion of the particle of the extremely large mass.

In the classical mechanics (Newtonian or relativistic) the natural particle motion is deterministic. If the particle moves stochastically, one should search for the reason of the stochasticity. In the classical mechanics we reduce the stochastic particle motion to the natural deterministic motion. The stochastic motion is interpreted via the deterministic one. In the nondegenerate space-time we must be able to perceive the stochastic motion directly, without reducing it to the deterministic motion. It is a very difficult problem, because one needs to create a new conception of mechanics. Some ideas of such a conception of mechanics one can find in [16, 17].

8 Association problems instead of logical ones

Logical problems are absent in T-geometry, because they are supposed to be solved in the proper Euclidean geometry. Considering the space-time geometry as a T-geometry, we meet the problem of separation between the geometry and the dynamics.

The physical geometry is a science on mutual disposition of geometrical objects. But any geometrical object is an abstraction. As a set of points, any geometrical object does not exist in itself. In reality we may have some substance, having a shape of the geometrical object. The set of points (the geometrical object) is an abstraction of this fact. It is supposed, that we can always to separate the shape of the substance from its contents.

Classical physics and classical dynamics accept, that the substance may have any shape. Thus, it is accepted that any set of space points forms a geometrical object, because any set of points can be filled by the substance. This suggestion supposes the unlimited divisibility of the substance and, hence, the unlimited divisibility of geometrical objects. All this is valid for the usual space.

In the space-time we have another picture. In the classical physics the particle is the simplest element of substance. Existence of a particle in the space-time is described by the continuous tube, known as the particle world tube. If the particle is point-like, the world tube degenerates into the one-dimensional world line. The arbitrary geometrical object, i.e. an arbitrary set of events (space-time points), cannot exist. If we suppose that any substance consists of point-like particles, any geometrical object is an arbitrary set infinite one-dimensional world lines. Only such a set of events can be realized in the space-time as a geometrical object, because such a set may be filled by the substance. The geometrical object, which can be realized, i.e. filled with the substance, will be referred to as the physical object. The world line of a point-like particle is supposed to be indefinitely divisible, although this fact cannot be tested experimentally, because the world line is always infinite or closed.

The one-dimensional curve (1.8) and its partial case – the one-dimensional straight form the basis of the Riemannian geometry as well as of the classical dynamics. The property of the divisibility of the continuous one-dimensional straight is formulated as follows. Let the straight segment $\mathcal{T}_{[P_0P_1]}$ between the points P_0 and P_1

$$\mathcal{T}_{[P_0P_1]} = \{R|f_{\mathrm{s}P_0P_1}(R) = 0\} \tag{8.1}$$

$$f_{\mathrm{s}P_0P_1}(R) = \sqrt{2\sigma(P_0,R)} + \sqrt{2\sigma(R,P_1)} - \sqrt{2\sigma(P_0,P_1)} \tag{8.2}$$

be divided into two parts $\mathcal{T}_{[P_0Q]}$ and $\mathcal{T}_{[QP_1]}$ by the point Q. If the straight is continuous and one-dimensional, we obtain

$$\begin{aligned} \mathcal{T}_{[P_0P_1]} &= \mathcal{T}_{[P_0Q]} \cup \mathcal{T}_{[QP_1]}, \qquad \forall Q \in \mathcal{T}_{[P_0P_1]} \\ \mathcal{T}_{[P_0Q]} &= \{R|f_{\mathrm{s}P_0Q}(R) = 0\}, \qquad \mathcal{T}_{[QP_1]} = \{R|f_{\mathrm{s}QP_1}(R) = 0\} \end{aligned} \tag{8.3}$$

The relations (8.3) are satisfied for any points P_0, P_2, provided the geometry is Riemannian and all segments $\mathcal{T}_{[P_0P_1]}$ are one-dimensional. The property (8.3) of the one-dimensional straight (geodesic) will be referred to as the divisibility property.

Repeating division of the segment $\mathcal{T}_{[P_0P_1]}$ many times, we obtain

$$\mathcal{T}_{[P_0P_1]} = \bigcup_{i=0}^{i=N} \mathcal{T}_{[Q_iQ_{i+1}]}, \qquad Q_0 = P_0, \qquad Q_{N+1} = P_1 \tag{8.4}$$

$$Q_k \in \mathcal{T}_{[Q_{k-1}Q_{k+1}]}, \qquad \forall Q_k \in \mathcal{T}_{[P_0P_1]}, \qquad k = 1, 2, ...N \tag{8.5}$$

The divisibility admits one to divide the segment $\mathcal{T}_{[P_0P_1]}$ into infinitesimal segments and reestablish the primary segment $\mathcal{T}_{[P_0P_1]}$ by its infinitesimal parts. It is important that any infinitesimal segment $\mathcal{T}_{[Q_iQ_{i+1}]}$ is an element of the world line. It turns into several elements of the world line after further division.

The unlimited divisibility of a one-dimensional continuous curve is a ground for the infinitesimal analysis, created by Newton and Leibniz. As far as any geometrical objects of the Riemannian geometry and objects of classical mechanics (trajectories in the phase space and world lines in the space-time) may be considered as consisting of infinitesimal straight line segments, the infinitesimal analysis appears to be the mathematical tool of geometry and physics. The possibility of the world line restoration by its infinitesimal elements is a necessary property of the unlimited divisibility. Simultaneously it is a necessary condition of the infinitesimal analysis applicability.

In the nondegenerate geometry, where the straight is not one-dimensional, in general, we have,

$$\mathcal{T}_{[P_0Q]} \nsubseteq \mathcal{T}_{[P_0P_1]}, \qquad \mathcal{T}_{[QP_1]} \nsubseteq \mathcal{T}_{[P_0P_1]}, \qquad Q \in \mathcal{T}_{[P_0P_1]} \tag{8.6}$$

It means that we can divide the segment $\mathcal{T}_{[P_0P_1]}$ into parts, but we cannot reestablish it by its parts, in general, because the relation (8.3) does not take place.

Of course, we can divide the segment $\mathcal{T}_{[P_0Q]}$ into two parts, cutting it at the point $Q \neq P_0, P_1$. For instance,

$$\begin{aligned}
\mathcal{T}_{\mathrm{c}[P_0Q]P_1} &= \left\{R | f_{\mathrm{s}P_0P_1}(R) = 0 \wedge \sqrt{2\sigma(P_0, R)} \leq \sqrt{2\sigma(P_0, Q)}\right\}, \qquad Q \in \mathcal{T}_{[P_0P_1]} \\
\mathcal{T}_{\mathrm{c}[P_1Q]P_0} &= \left\{R | f_{\mathrm{s}P_0P_1}(R) = 0 \wedge \sqrt{2\sigma(P_1, R)} \leq \sqrt{2\sigma(P_1, Q)}\right\}, \qquad Q \in \mathcal{T}_{[P_0P_1]}
\end{aligned}$$

$$\mathcal{T}_{[P_0P_1]} = \mathcal{T}_{\mathrm{c}[P_0Q]P_1} \cup \mathcal{T}_{\mathrm{c}[P_1Q]P_0} \tag{8.7}$$

But the sets $\mathcal{T}_{\mathrm{c}[P_0Q]P_1}$ and $\mathcal{T}_{\mathrm{c}[P_1Q]P_0}$ do not coincide with the straight line segments $\mathcal{T}_{[P_0Q]}$ $\mathcal{T}_{[P_1Q]}$. In particular, the set $\mathcal{T}_{\mathrm{c}[P_0Q]P_1}$ depends on the external point $P_1 \notin \mathcal{T}_{\mathrm{c}[P_0Q]P_1}$. It means that we may not consider the sets $\mathcal{T}_{\mathrm{c}[P_0Q]P_1}$ and $\mathcal{T}_{\mathrm{c}[P_1Q]P_0}$ as a physical objects, i.e. as the set of points which can be filled by the substance. Besides, the fact that $\mathcal{T}_{\mathrm{c}[P_0Q]P_1}$ depends on the external point $P_1 \notin \mathcal{T}_{\mathrm{c}[P_0Q]P_1}$ means that the set $\mathcal{T}_{\mathrm{c}[P_0Q]P_1}$ is a part of $\mathcal{T}_{[P_0P_1]}$, whereas $\mathcal{T}_{[P_0Q]}$ may be considered to be a self-contained object, but not a part of other geometrical object. We shall formulate the difference between $\mathcal{T}_{\mathrm{c}[P_0Q]P_1}$ and $\mathcal{T}_{[P_0Q]}$ as follows. The set of points $\mathcal{T}_{[P_0Q]}$ forms a physical object, whereas the set of points $\mathcal{T}_{\mathrm{c}[P_0Q]P_1}$ is not a physical object. The set of points $\mathcal{T}_{\mathrm{c}[P_0Q]P_1}$ is a part of a physical object $\mathcal{T}_{[P_0P_1]}$. Here we meet a strange situation, when a part of a physical object may be not a physical object. In other words, the physical object has a finite size and cannot be divided into physical objects of infinitesimal size.

Lack of unlimited divisibility (finite divisibility) of the simplest geometrical object (the straight segment) leads to the finite divisibility of more complicated geometrical objects. The finite divisibility is associated in a way with the "quantum nature" of microcosm. The "quantum nature" of the microcosm is rather cloudy concept. It means a description of physical phenomena by means of the quantum mechanics formalism. The "quantum nature" means something connected rather with the dynamics, than with the geometry. The finite divisibility is a property of the nondegenerate space-time geometry. It is mainly a geometrical property, although there is some reference to the properties of the matter. But it is important, that the finite divisibility is such a property, which can be investigated by the geometrical methods, as soon as the world function of the space-time is known. In any case the concept of the finite divisibility is less cloudy, than the concept of "quantum nature".

9 The boundary between physics and geometry

Any physical phenomenon takes place in the space-time. Existence of the space-time is a common circumstance for all physical phenomena. Description of each physical phenomenon has two sides: (1) the side, common for all physical phenomenon, known as space-time geometry, (2) the side, specific for each physical phenomenon, known as dynamics. It means that some characteristics of the physical phenomenon are considered as geometrical characteristics, whereas other ones are considered as dynamical characteristics. The boundary between geometry and dynamics is not constant. This boundary varies, as the physics developed.

In the time of Isaac Newton the geometry was the most developed branch of mathematics. Geometrical methods of investigation were dominating. The famous Newtonian book [12] was written mainly in terms of geometry. Creation of the infinitesimal analysis by Newton and Leibniz, and its application in mechanics changed the situation. Analytical methods of description and investigation became dominating. Analytical methods penetrated into the geometry, and the analytical geometry arose. Further development of the geometry and, in particular, generalization of the Euclidean geometry was produced by the analytical methods in the framework of the differential geometry. The methods of the infinitesimal analysis were effective at description of only such physical processes, which could be divided into a set of local processes.

On the other hand, the role of the space-time geometry in the description of the dynamic systems increased. Some important physical concepts, considered at first as purely dynamical, appeared to have a geometrical origin. The energy-momentum conservation laws, considered at first as properties of dynamical systems, appeared to be generated by the space-time geometry. The quantum phenomena, considered in the XXth century as specific dynamic phenomena, are looking now as generated by the space-time properties. In particular, the quantum constant $\hbar$ appears to be a parameter of the space-time geometry. The particle mass appears to be a geometrical characteristic. Now one may not speak on the special quantum nature of physical phenomena in microcosm. All they are explained freely by means of the properties of the space-time geometry. As our knowledge on the microcosm physical phenomena develops, the role of the space-time geometry increases, and the geometry – dynamics boundary is shifted towards the dynamics.

In general, it seems rather reasonable, that all general physical laws are to have the geometrical origin, because the space-time is the only common circumstance, which takes place in all physical phenomena. For instance, the conservation law of the electric charge and multiplicity of any electrical charge to the elementary one are to have the geometrical origin, because these phenomena are universal. For the present these phenomena are not always connected with the space-time geometry.

In the nondegenerate space-time geometry there is no ground for the infinitesimal analysis, which supposes unlimited divisibility of the physical objects and a possibility of the primary physical object restoration by its parts. Note that the world tube divisibility cannot be tested experimentally. It is such a supposition which is tested by its corollaries.

In such a situation we may not imagine that we can divide the world tube of a real particle into parts. We are forced to suppose that the division of the world tube into elementary parts exists objectively, and such a division is not a result of our method of the world tube description. The different divisions of the world tube correspond to the world tubes of different particles. The world tube of a real particle is a broken tube, i.e. a chain of the connected finite segments $\mathcal{T}_{[P_k P_{k+1}]}$, whose length is proportional to the particle mass, whereas the vector $\mathbf{P}_k\mathbf{P}_{k+1}$ describes the particle momentum. In the nondegenerate space-time geometry one cannot abstract from the size of the link, which is connected with the particle mass. As a result the particle mass become to be a geometrical characteristic of a particle. In other words, the particle mass is geometrized. At the conventional consideration the particle mass is its dynamical characteristic. Thus, in the nondegenerate space-time geometry the boundary between geometry and dynamics is shifted towards the dynamics.

Any link has a finite size, and the conventional mathematical technique of the infinitesimal analysis does not work in the nondegenerate space-time geometry. The problem of adequate mathematical description of geometrical objects in the nondegenerate space-time geometry appears.

10 Interplay of geometric and analytical methods

Construction of T-geometry by means of the local analytical methods appeared to be ineffective. Application of analytical methods to the geometry needs an introduction of continuous coordinate systems, which restricts strongly the list of possible geometries. Application of nonlocal finite methods of description (the deformation principle) appears to be more effective at the geometry generalization. In the nondegenerate space-time with tubes instead of straights an application of the differential geometry appears to be ineffective at the construction of local characteristics of the space-time geometry.

The fact is that the methods of differential geometry are connected closely with concept of the one-dimensional curve as a primary geometrical object of the geometry. In the geometry, where the one-dimensional curve is not a physical object, the local analytical methods do not work effectively. One does not develop the nonlocal geometrical methods, which are based on the description of the whole geometrical object, but not on its infinitesimal elements. As a result we were forced to use conventional analytical methods in the quantum mechanics substantiation, founded on the geometry and statistics, In this case the distortion of the space-time geometry was considered as a correction to the Minkowski space-time geometry.

Let us consider a simple example, which shows that not only description in terms of infinitesimal analysis is possible. We consider solution of the Klein-Gordon equation in the Riemannian space-time. In the Minkowski space-time this equation has the form

$$\left(g^{ik}\partial_i\partial_k+\frac{m^2}{\hbar^2c^2}\right)\psi=0,\qquad g^{ik}=\text{diag}\left\{c^{-2},-1,-1,-1\right\}\tag{10.1}$$

The causal Green function for the equation (10.1) has the form [13]

$$\begin{aligned}
D^c\left(x-x'\right) &= \frac{1}{4\pi}\delta\left(2\sigma\right)-\frac{mc}{8\pi\hbar\sqrt{2\sigma}}\theta\left(\sigma\right)\left[J_1\left(\frac{mc\sqrt{2\sigma}}{\hbar}\right)-iN_1\left(\frac{mc\sqrt{2\sigma}}{\hbar}\right)\right]\\
&\quad +i\frac{mc}{4\pi^2\hbar\sqrt{-2\sigma}}\theta\left(-\sigma\right)K_1\left(-\frac{mc\sqrt{2\sigma}}{\hbar}\right), \qquad (10.2)\\
\theta\left(x\right) &= \begin{cases}1, & \text{if } x>0\\ 0, & \text{if } x<0\end{cases}
\end{aligned}$$

where σ is the world function

$$\sigma=\sigma\left(x,x'\right)=\frac{1}{2}\left(c^2\left(x^0-x'^0\right)^2-\left(\mathbf{x}-\mathbf{x}'\right)^2\right)\tag{10.3}$$

and J_1, N_1 and K_1 are the first order cylindric functions (respectively of Bessel, Neumann and Hankel).

Obtaining of the causal Green function (10.2) is equivalent to a solution of the Klein-Gordon equation (10.1).

In the Riemannian space-time the equation (10.1) turns into the equation

$$\left(g^{ik}\nabla_i\nabla_k+\frac{m^2}{\hbar^2c^2}\right)\psi=0,\tag{10.4}$$

where now g^{ik} is the metric tensor of the Riemannian space-time, and ∇_k is the covariant derivative in this space-time.

If we consider the process, which is described by the Klein-Gordon equation in the curved space-time, we may use two different methods:

1. To solve the equation (10.4).

2. To take the expression (10.2) for the causal Green function, where σ is the world function of the Riemannian space-time. It may be determined as the symmetric solution of the Jacobi-Hamilton equation

$$g^{ik}\left(\partial_i\sigma\right)\partial_k\sigma=2\sigma,\qquad \sigma\left(x,x'\right)=\sigma\left(x',x\right)\tag{10.5}$$

The second method is simpler in the sense that the equation (10.5) is the first order differential equation.

We are not sure that both methods lead to the same result. We do not discuss here, which of two methods is valid. We should like to pay attention to the following circumstance. The second (integral) method may be applied, *at least formally*, in the curved space-time and in the distorted space-time (6.1). In the last case the world function in (10.2) is taken in the form (6.1). The first (differential) method cannot be applied in the distorted space-time (6.1) even formally, because it is not clear how to take into account the space-time distortion.

11 Problems of the T-geometry perception

Although the T-geometry construction is very simple, it appears to be rather difficult for perception by mathematicians [14]. Its perception is especially difficult for professional geometers, which know and apply the mathematical formalism of the differential geometry. The fact is that the T-geometry cannot be constructed by means of the formalism of the differential geometry and that of the infinitesimal analysis, which is a foundation of the differential geometry.

The Riemannian geometry is constructed as a generalization of the proper Euclidean geometry, presented in the form of differential geometry. The generalization is produced on the basis of such a geometrical object as the Euclidean straight. Such properties of the Euclidean straight as continuity and one-dimensionality are conserved at the generalization, whereas such properties as vanishing curvature and vanishing torsion of the Euclidean straight may be violated. In the Euclidean geometry, as well as in the Riemannian geometry the concept of one-dimensional continuous curve is the primary object of the geometry. The concept of the curve is primary in the sense that any geometrical object may be considered as consisting of a set of curves. In turn any curve may be considered as consisting of infinite set of infinitesimal segments of a curve. Infinitesimal segments of a curve can be considered as infinitesimal segments of the straight. Thus, any geometrical object can be constructed of infinitesimal segments of a curve (straight), which are the primary objects of the Riemannian geometry.

On the other hand, the infinitesimal analysis has been invented by Newton and Leibniz for description of functions

$$y = f(x) \tag{11.1}$$

Such a function describes a curve on the plane (x, y). Any geometrical object can be constructed of infinitesimal segments of the curve (11.1). It means that the infinitesimal analysis is applicable for description of the differential geometry and, in particular, for description of the Riemannian geometry. Applicability of the infinitesimal analysis to problems of mechanics and physics is connected with the fact that problems of these sciences can be described in terms of curves, or geometrical objects, consisting of curves.

The T-geometry is the generalization of the Euclidean geometry, founded on the σ-immanence property of the Euclidean geometry, which can be presented in terms of the world function σ_{E}. The σ-immanence is the property of the whole Euclidean geometry, whereas the straight is only a geometrical object of the Euclidean geometry. The straight is very important geometrical object, but it is only one geometrical object among many others. Generalization made on the basis of the property of the whole Euclidean geometry is more preferable, than the generalization made on the basis of properties of one object. Thus, from the common viewpoint the T-geometry is more preferable, than the Riemannian geometry. The one-dimensional continuous curve is not a primary geometrical object of the T-geometry. It can be introduced as a secondary geometrical object, i.e. as a geometrical object, obtained by intersection of several primary geometrical objects, for instance, (3.10), (3.11). In some special cases the first order tube may degenerate into the one-dimensional straight. In this case we may use the means of the infinitesimal analysis. In the general case the one-dimensional curve is not a primary geometrical object and an application of the infinitesimal analysis appears to be ineffective.

Remark. In the Riemannian geometry the one-dimensional curve is the primary geometrical object of the geometry. The surface may be constructed of curves. This fact is reasonable and customary. In T-geometry the one-dimensional curve is a secondary (derivative) geometrical object, which can be constructed of some primary objects (surfaces) by means of their intersection. It seems to be incredible, that in T-geometry the curve is constructed of surfaces. I remember very well, that I could not perceive this fact for a long time.

The geometry can be described not only in terms of infinitesimal quantities by means of the infinitesimal analysis. It can be described also in terms of finite geometrical quantities. Unfortunately, the particle dynamics can be described only in terms of infinitesimal quantities by methods of the infinitesimal analysis. In contemporary physics the geometrical methods are not used in the particle dynamics. The particle dynamics is described in terms of one-dimensional curves (trajectories in the phase space, world lines in the space-time). The infinitesimal analysis describes these geometrical objects very well. The problem of the geometrical methods application did not arise in the contemporary particle dynamics.

In the nondegenerate space-time T-geometry (distorted space-time (6.1)) the particle world tube is not a one-dimensional curve. It is a chain consisting of finite non-one-dimensional links. In the simplest case any link is the straight segment $\mathcal{T}_{[P_k P_{k+1}]}$, whose length is proportional to the particle mass m. Quantum effects are connected with the thickness $\sqrt{d}$ and the length $\left|\mathcal{T}_{[P_k P_{k+1}]}\right|$ of the link, and we may not to tend the length $\left|\mathcal{T}_{[P_k P_{k+1}]}\right|$ to zero. Hence, we cannot reduce the problem of the particle dynamics directly to the description in terms of the one-dimensional curve and solve it by methods of the infinitesimal analysis. Unfortunately, in the particle dynamics we have no methods of description except for infinitesimal analysis. We are forced to describe the chain as a one-dimensional line. The link shape (thickness and length) was taken into account as some stochastic agent. This agent makes the one-dimensional world line to be stochastic. Statistical description of this stochastic motion leads to the quantum description in terms of the Schrödinger equation. But such a result we obtain in the simplest case of structure-less particle.

Analysis of the free Dirac particle, (i.e. the dynamic system $\mathcal{S}_{\mathrm{D}}$, described by the Dirac equation) shows [18], that the Dirac particle has a complex structure. This structure may be interpreted either dynamically, or geometrically. According to the dynamic interpretation the Dirac particle is a rotator, i.e. two constituents rotating around their common center of inertia. According to the geometrical interpretation the world tube of the Dirac particle is the chain of links $\mathcal{T}_{[P_i Q_i P_{i+1}]}$, $i = 0, \pm 1, ...$ Any link $\mathcal{T}_{[P_i Q_i P_{i+1}]}$ is a triangle with vertices at the points P_i, Q_i, P_{i+1}. In the case of the dynamical interpretation we have a confinement problem, i.e. we are to explain, what forces coupled the constituents of the Dirac particle. In the case of the geometrical interpretation we are not to explain anything. If in the simplest case the link may be a segment $\mathcal{T}_{[P_iP_{i+1}]}$ of the first order tube, then why cannot it be a segment $\mathcal{T}_{[P_iQ_iP_{i+1}]}$ of the second order tube, or even a segment $\mathcal{T}_{[P_iQ_iR_iP_{i+1}]}$ of the third order tube? The geometrical interpretation seems to be preferable, because it looks more natural, and besides, it does not contain the confinement problem. The geometrical approach looks especially attractive in the case of the segment $\mathcal{T}_{[P_iQ_iR_iP_{i+1}]}$, where three vertices P_i, Q_i, R_i are associated with three quarks inside the composite particle. The confinement problem is absent at the geometrical approach.

Although the model of the Dirac particle is not quite perfect, because internal degrees of freedom are described non-relativistically [19], but this model is the best model of the relativistic particle, and one should not ignore results of its investigation. Besides, the nonrelativistic character of this model can be corrected [19].

Calculation of the elementary particles mass spectrum is considered now as the main problem of the elementary particles theory. The problem of the elementary particles mass spectrum is considered conventionally as a dynamical problem, which reminds the problem of the atom electromagnetic emanation spectrum. One tries to solve this problem, searching for an appropriate dynamic system.

In the nondegenerate space-time geometry the particle mass is geometrized, and one should expect that the problem of the elementary particles mass spectrum is a geometrical problem, but not a dynamical one. In other words, we should search for appropriate structure of links, constituting the particle world tubes, but not for dynamic systems imitating this structure. Investigation of the dynamic system is based mainly on properties of groups, connected with the structure of the link. Such indirect investigation is more complicated, than a direct investigation of the geometrical structure of the link. Besides, considering the dynamical system, we are to take into account that the dynamical interaction propagates with the speed less, than the speed of the light. If we consider a structure with the purely geometrical couplings, we are free of this constraint, because the geometry is more fundamental part of the description, than the dynamics. The principles of relativity impose constraints only on dynamic systems, but not on the geometrical objects. We may solve the confinement problem without introduction of gluons.

The distorted space-time with particles, described by non-one-dimensional world tubes, sets the problem of adequate mathematical formalism. This formalism is to deal with the finite (but not infinitesimal) quantities. Besides, it must be oriented to different geometrical objects (whereas the infinitesimal analysis is oriented to an infinitesimal segment of one-dimensional curve). Construction of such a mathematical technique is a very complicated problem. Until such a mathematical technique is not created, we are forced to reduce the geometrical problems to the dynamical ones and to use the infinitesimal analysis.

12 Concluding remarks

The T-geometry is a very simple generalization of the Euclidean geometry. Construction of T-geometry expands the class of physical geometries appropriate for description of the space-time. Among homogeneous isotropic Riemannian geometries there is only one geometry appropriate for the space-time description. This is the Minkowski geometry. In the class of T-geometries there is a set of homogeneous isotropic geometries. Any homogeneous isotropic T-geometry is labelled by the distortion function $D(\sigma_{\mathrm{M}})$, describing the shape and thickness of the timelike straight segments. One may treat this circumstance as an introduction of the transverse universal length. The free particle motion is primordially deterministic only in the Minkowski space-time geometry with $D(\sigma_{\mathrm{M}}) = 0$. In all other homogeneous, isotropic T-geometries the free particle motion is primordially stochastic.

When in the beginning of XXth century it has been discovered, that the free motion of the small mass particle is primordially stochastic, one should to choose one of T-geometries with $D(\sigma_{\mathrm{M}}) \neq 0$ as a space-time geometry. Unfortunately, in that time neither the non-degenerate T-geometries, nor the nondegeneracy property were not known. Researchers were forced to use the Minkowski space-time geometry everywhere, including microcosm. To obtain the corollaries of the stochastic particle motion and to explain physical phenomena of microcosm, they are forced to introduce *additional hypotheses*, known as quantum principles. Because of lack of any alternative the quantum principles are accorded wide recognition, and now most researchers speak on the quantum origin of microcosm.

When the T-geometry had been constructed, it became clear, that the physical phenomena of microcosm can be freely explained by a true choice of the space-time geometry. There is no necessity to introduce the quantum principles and to speak about the quantum nature of the microcosm physical phenomena. The quantum nature of the microcosm prescribes to quantize all physical fields, including metrical fields, which describe the space-time properties. In the nondegenerate space-time geometry we have the distortion and the finite divisibility instead of the quantum nature, and there is no necessity to quantize the metrical fields. The electromagnetic and gravitational fields are metrical fields, because in the 5-dimensional Klein-Kaluza geometry they describe the properties of the space-time. One failed to quantize the gravitational field, and there is no problem with it. But the electromagnetic field has been quantized successfully, and at this point we have disagreement between the two approaches. There are no experimental evidence in favour of the quantization necessity. Experiments show that the electromagnetic field is emitted and absorbed in the form of quanta. This fact may be explained by the properties of the emitting or absorbing atom. But does the electromagnetic field *exist* in the form of quanta? We do not know experiments, which could answer positively this question. Besides, the dynamic equations for the electromagnetic and gravitational fields do not contain the quantum constant. Thus, there are doubts in the necessity of the electromagnetic field quantization.

Consideration of the nondegenerate T-geometry as a space-time geometry is the third essential revision of our space-time conception.

The first essential revision of the space-time conception was the transition from the Newtonian model of the space-time with two invariants to the Minkowski space-time geometry with one invariant. The second essential revision of the space-time conception was the transition from homogeneous space-time geometry to the non-homogeneous space-time geometry, where nonhomogeneity is generated by the substance distribution. The third revision of the space-time model is founded on existence of a new class of physical geometries, having such unknown unusual properties as the nondegeneracy and lack of the unlimited divisibility. These new properties of the space-time geometry concerns mainly microcosm. As any revision, the new conception of the space-time is difficult for perception. It is valid especially for those researchers, who believe that the geometry is a totality of coordinates, metric tensor, covariant derivatives etc.

Application of the T-geometry to the microcosm physics generates problems, connected with the further geometrization of physics. Interpretation of new properties of the T-geometry and a construction of an adequate mathematical formalism are the main problems of the new conception of the space-time.

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