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# Multiaxial Fatigue Criteria and Durability of Titanium Compressor Disks in Low- and Giga-Cycle Fatigue Modes

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**Abstract** The Crossland, Findley and Sines fatigue fracture models are used to estimate the durability of the compressor disk for cases of low-cycle fatigue (LCF) and giga-cycle fatigue (GCF). The model parameters are determined by using the data of uniaxial fatigue tests for various stress ratios.

## 1 Introduction

The phenomenon of in-service gas turbine engine (GTE) compressor disks fatigue fracture is well-known. Usually compressor disks are manufactured from Ti-based alloy Ti-6Al-4V. According to fractured disk analysis in most cases the fatigue fracture is observed near the contact zone of disk and blade.

The finite element model is created and 3D strain-stress state is calculated for GTE compressor disk contact structure (disk-blades-pins-shroud ring) taking in to account centrifugal, aerodynamic and contact cyclic loading. Several multiaxial fatigue criteria used and results of simulated durability are compared with flight service data. The giga-cycle fatigue (GCF) due to observed high frequency axial vibrations of shroud ring is also studied. Because of absence of experimentally proved GCF multiaxial criteria the known low-cycle fatigue (LCF) criteria are generalized.

## 2 Fatigue durability estimation models based on the stress-strain state

Analysis of fatigue durability is based on results of uniaxial cyclic loading tests for different values of stress ratio $R = \sigma_{\min}/\sigma_{\max}$, where $\sigma_{\max}$ and $\sigma_{\min}$ are the maximal

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and minimal stresses during the cycle. The stress amplitude is $\sigma_a = (\sigma_{\max} - \sigma_{\min})/2$ and the stress range is $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$. The experimental data of uniaxial tests in LCF mode are described by Wohler curves that can be represented by the Basquin relation

$$\sigma = \sigma_u + \sigma_c N^{\beta} \tag{1}$$

where $\sigma_u$ is the fatigue limit, $\sigma_c$ is the fatigue strength factor, $\beta$ is the fatigue strength exponent, and $N$ is the number of cycles to fracture. See typical curve in Fig. 1. It has two branches according to low ($N < 10^7$) and giga ($N > 10^8$) cycle fatigue.

According to Sines [5], the uniaxial fatigue curve (1) can be generalized to the case of multiaxial stress state as

$$\Delta\tau/2 + \alpha_s \sigma_{\text{mean}} = S_0 + AN^{\beta} \tag{2}$$

where $\sigma_{\text{mean}}$ is the mean sum of principal stresses over a loading cycle, $\Delta\tau$ is the change in the octahedral tangent stress per cycle, $\Delta\tau/2$ is the octahedral tangent stress amplitude, and $\alpha_s$ , $S_0$, $A$ and $\beta$ are parameters to be determined from experimental data.

According to Crossland [3], the uniaxial fatigue curve can be generalized to the case of multiaxial stress state as

$$\Delta\tau/2 + \alpha_c(\overline{\sigma}_{\max} - \Delta\tau/2) = S_0 + AN^{\beta} \tag{3}$$

where $\overline{\sigma}_{\max}$ is the maximum sum of principal stresses in a loading cycle.

The form of the Findley [4] model for the case of multiaxial stress state is:

$$(\Delta\tau_s/2 + \alpha_F \sigma_n)_{\max} = S_0 + AN^{\beta} \tag{4}$$

where $\tau_s$, $\sigma_n$ are the modules of the tangent stress and normal stress for the plane with normal $n_i$, for this plane combination $\Delta\tau_s/2 + \alpha_F \sigma_n$ takes a maximum value. The criteria parameters $\alpha_s$ , $S_0$, $A$ and $\beta$ are determined in [1] from uniaxial fatigue curves and experimental values $\sigma_u$, $\sigma_{u0}$ , $\sigma_B$, where $\sigma_u$ and $\sigma_{u0}$ are the fatigue limits

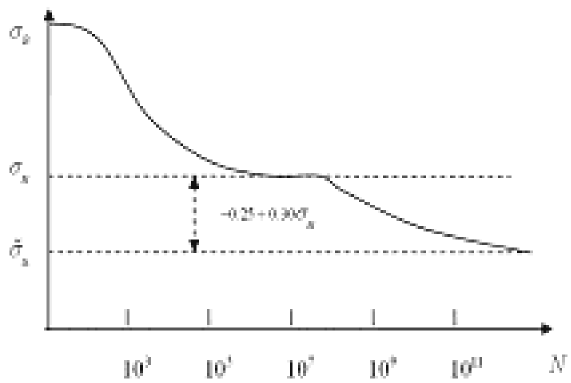

**Fig. 1** Wohler's curve for metals.

for $R = 1$ and $R = 0$ respectively, $\sigma_B$ is limit strength. Here are approximate values of the parameters for titanium alloy: the limit strength is $\sigma_B = 1100MPa$, the fatigue limits according to the curves $\sigma_a(N)$ for R = 1 and R = 0 are equal $\sigma_u = 450$ MPa and $\sigma_{u0} = 350MPa$, respectively, the exponent in the power-law dependence on the number of cycles is $\beta = -0.45$.

## 3 Example of durability estimation in LCF and GCF modes

The three-dimensional stress-strain state of the contact system of the compressor (disk-blades-pins-shroud ring) is analyzed numerically using finite-element method (for details see [2]). The centrifugal forces, the distributed aerodynamic pressures on the blades, and the forces of nonlinear contact interaction between structural elements are taken into account for LCF mode. In addition for GCF mode the small cyclic changes of stress-strain state due to shroud ring vibrations are calculated. Details are highlighted in [1] and [2].

For LCF mode (basic stress state) of flight cycles (takeoff-flight-landing) the input parameters are the following: the angular velocity of rotation $\omega = 314rad/s$ (3000 revolutions per minute), the flow velocity $200m/s$. The material properties are as follows: $E = 116Ga$, $n = 0.32$, and $\rho = 4370kg/m^3$ for the disk (titanium alloy), $E = 69GPa$, $n = 0.33$, and $\rho = 2700kg/m^3$ for the blades (aluminum alloy).

Known cryteria for LCF mode are used for GCF mode. The GCF parameters for these criteria are detected by using right branch of one-dimensional fatigue curves in the same way as left branch is used in the LCF case. The similarity between left and right branches of fatigue curve is used by substitution $\sigma_B \rightarrow \sigma_u$, $\sigma_u \rightarrow \tilde{\sigma}_u$, $\sigma_{u0} \rightarrow \tilde{\sigma}_{u0}$. Here $\tilde{\sigma}_u$ and $\tilde{\sigma}_{u0}$ are new fatigue limits for right branch of fatigue curve for asymmetry factors $R = -1$  $R = 0$. The following parameter values for titanium alloy are used $\tilde{\sigma}_u = 250MPa$, $\tilde{\sigma}_{u0} = 200MPa$. Axial displacements of shroud ring

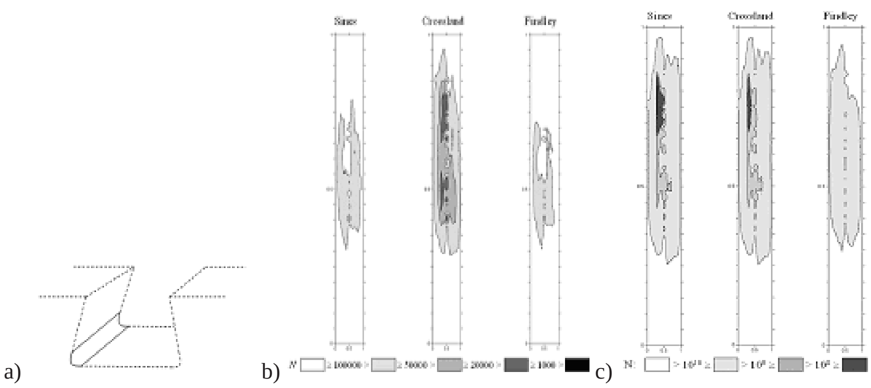


**Fig. 2** (a) - place of crack initiation in the slot of disk-blade connection and calculated durability distributions for LCF (b) and GCF (c) modes

are caused by its vibrations. For disc-blade sector calculation the right side of shroud ring displacement is equal to zero while the left side displacement is equal to maximal vibration amplitude $\pm 1mm$ (Fig. 2c) for frequency of $3000 rpm$. Vibration stress state is imposed on the basic stress state.

The computations [2] show that the most dangerous areas are situated near the "swallow tail" contact regions between the disk and the blades. Fig. 2a shows the zone of maximum tensile stresses concentration at the left (rounded) corner of the groove where the blade is inserted. In Fig. 2b, the computed numbers of flight cycles before fracture (for various criteria of multiaxial fatigue fracture) are displayed for most dangerous area of groove. In Fig, 2c the computed numbers of vibrations before fracture are presented. In Fig. 2 (b) and (c) the horizontal axis represents the dimensionless coordinate of the rounding of the groove's left corner, the vertical axis represents the dimensionless coordinate across the groove depth. For LCF mode the Sines and Findley criteria provide estimates of the service life of gas turbine engine disks of about 20000-50000 flight cycles. The Crossland criterion predicts the possibility of fatigue fracture after less than 20000 flight cycles and it corresponds to exploitation time of 50 000 hours. For GCF mode generalized criteria of Sines, Crossland and Findley provide estimates of the service life of gas turbine engine disks of about $10^9 \div 10^{10}$ vibration cycles and it again corresponds to exploitation time of 50 000 hours. Though the presented durability estimations are rather approximate they point onto possibility of fatigue development in considered structure elements for both cases of LCF (flight cycles) and GCF (high frequency low amplitude vibrations). The most serious danger may happen due to mutual action of mentioned mechanisms because they may develop almost simultaneously in one and the same place. On the whole, all these criteria give similar pictures of the fatigue fracture regions location.

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