Perturbed Earth rotation

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1 Introduction

Numerous astrometric studies are based on the dynamic theory of the Earth's rotation with respect to the center of mass [1–5]. It is known from astronomical observations (since the second half of the 19th century and possibly much earlier) that the Earth's rotation axis varies through time in both Earth-fixed and inertial reference frames. This means that the poles and latitudes evolve noticeably within a year.

Measurements indicate that this rather complex oscillatory process contains components with very diverse frequency and amplitude characteristics. For example, the small oscillations of the angular velocity vector in an Earth-fixed reference frame contain the main component with an amplitude of $0.20''$–$0.25''$ and a period of 430–440 sidereal days [1–4]. The considerable difference between the Chandler period and the period predicted by the classical theory of rigid bodies (the Euler precession period of 305 days for the rigid figure of the Earth) required an explanation. An explanation was partially given on the basis of a model of deformable Earth in the studies of W. Thomson, S. Newcomb, H. Poincaré, H. Geoffreys, A. Love, F. A. Sludskii, M. S. Molodenskii, et al. [4]. Traditionally, this motion is referred to as the free nutation of the deformable Earth or the Chandler wobble of the pole (to be definite, of the North pole).

There is also a noticeable component with an amplitude of $0.07''$–$0.08''$ and a pronounced period of one year (about 365.25 sidereal days). The observed pole oscillations have a wobbling character. The trajectory of the pole on the Earth's surface is a winding and unwinding spiral with a period of about six years.

Analyzing the pole trajectory and predicting its motion are of substantial interest in both scientific and applied aspects. The construction of a high-precision theoretical model of the deformable Earth's rotation, the identification of model parameters on the basis of IERS data, and a reliable prediction of the pole motion are very important in inertial navigation problems [6] on time intervals sufficiently large for practical purposes and in a number of astrometric and geophysical problems [1–5].

Let us give some remarks concerning the statement of the problem. To describe the rotation of the deformable Earth and the polar motion, we use the mechanical model of a viscoelastic solid [7]. The Earth is represented as a two-layer planet consisting of an absolutely firm rigid core (a ball) and a viscoelastic mantle. Considering a more complicated model of the figure of the Earth is unjustified, since in this case one would not be able to determine the necessary physical-mechanical characteristics of the planet on the basis of measurements with the accuracy needed. The authors stick to the obvious postulate that the complexity of a model at all stages of its construction should be strictly adequate to the problem to be solved and the accuracy of observation data.

From the viewpoint of theoretical mechanics, the analysis of oscillations of the Earth's pole is similar to the study of motions of the gyro axis under perturbing torques with viscoelastic strains taken into account in the quasistatic approximation.

At the initial stage of the analysis of the polar motion under perturbing torques, we consider the three-dimensional version of the two-body problem [7, 8]. We assume that the center of mass of a deformable planet (Earth) and a point satellite (Moon) perform a known rotatory-progressive motion around a common center of mass (barycenter), which moves on an elliptic orbit around the Sun.

In what follows, on the basis of an asymptotic analysis of the equations of motion in the osculating action-angle variables, we determine stable characteristics of the rotatory-progressive motion of the deformable Earth with respect to center of mass in the quasistatic approximation. First, we find the refined periods (frequencies) of the axial rotation and the Chandler wobble and make a comparison with the spectral analysis data [1–3]. Estimates of the free oscillation amplitudes of the angular velocity vector in the Earth-fixed reference frame are given, and we compare them with the observed values.

Using the kinematic and dynamic Euler equations, we construct a mathematical model of the first approximation to the Chandler wobble. In conclusion, we numerically determine the parameters of motion by the least squares method on the basis of daily IERS measurement data [2], construct the trajectory, and predict the polar motion, comparing the results with the IERS experimental data.

2 The first-approximation mathematical model of polar oscillations

To construct a model of the rotation with respect to the center of mass, we represent the equations in the form of the classical Euler dynamic equations with variable inertia tensor J [1, 3–5, 7–9]:

$$\begin{gathered} J\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times J\boldsymbol{\omega} = \mathbf{M}, \quad \boldsymbol{\omega} = (p, q, r)^T, \quad J = J^* + \delta J, \quad J^* = \text{const} \\ J^* = \operatorname{diag}(A^*, B^*, C^*), \qquad \delta J = \delta J(t), \qquad \|\delta J\| \ll \|J^*\| \end{gathered} \tag{2.1}$$

Here $\boldsymbol{\omega}$ is the angular velocity vector in an Earth-fixed reference frame [4] approximately coinciding with the principal central axes of inertia J^* of the "frozen" Earth with regard to the "equatorial bulge" [1–5, 7]. The additional perturbing terms obtained by the differentiation of the deformable Earth's moment of momentum (see [8, 9]) are included in the perturbing torque $\mathbf{M}$ of rather involved structure. It is considered that small variations in the inertia tensor δJ can contain various harmonic components caused by gravitational diurnal tides from the Sun and the Moon as well as possibly other components (annual, biannual, monthly, and semidiurnal). The main ingredients in the external perturbing torques $\mathbf{M}$ causing nutation oscillations are assumed to be gravitational. The possible presence of terms like $\dot{J}\boldsymbol{\omega}$ does not result in a refinement of the first-approximation model. Attempts to estimate these terms rigorously by taking into account geophysical factors are difficult and have not so far produced satisfactory results. The analysis of their effect in the existing literature is often completely speculative and scholastic, for it is not related to the actual determination of the torques (their amplitudes, direction, and frequency and phase characteristics).

The Euler kinematic equations determining the orientation of the Earth-fixed axes with respect to the orbital reference frame have the form [10]

$$\dot\theta = p\cos\varphi - q\sin\varphi - \omega_0(\nu)\sin\psi, \quad \dot\nu = \omega_0(\nu) = \omega_*(1+e\cos\nu)^2$$
$$\dot\psi = \frac{p\sin\varphi + q\cos\varphi}{\sin\theta} - \omega_0(\nu)\operatorname{ctg}\theta\cos\psi, \qquad e = 0.0167 \tag{2.2}$$
$$\dot\varphi = r - (p\sin\varphi + q\cos\varphi)\operatorname{ctg}\theta + \omega_0(\nu)\frac{\cos\psi}{\sin\theta}$$

Here $\nu(t)$ is the true anomaly, e is the eccentricity of the orbit, and ω_* is a constant determined by the gravitational and focal parameters. The analysis of system (2.1), (2.2) reveals that in the situation corresponding to the polar motion the terms proportional to ω_0 in Eqs. (2.2) prove to be much larger than p and q (approximately by a factor of 300) and are determining for $\dot\theta$ and $\dot\varphi$. This important property is not mentioned in scientific literature, and the above-mention terms were neglected without due justification (the orbital and rotatory motions were separated) [1–5].

Let us calculate the annual component of forced oscillations of the Earth's pole.

The expressions for the components of the gravitational torque exerted by the Sun have the following structure [9]:

$$M_q = 3\omega^2\left[(A^* + \delta A - (C^* + \delta C))\gamma_r\gamma_p + \delta J_{pq}\gamma_r\gamma_q + \right.$$
$$\left. + \delta J_{pr}(\gamma_r^2 - \gamma_p^2) - \delta J_{rq}\gamma_p\gamma_q\right], \qquad \omega = \omega_*(1+e\cos\nu)^{3/2} \tag{2.3}$$
$$\gamma_p = \sin\theta\sin\varphi, \qquad \gamma_q = \sin\theta\cos\varphi, \qquad \gamma_r = \cos\theta$$

To compute $M_{p,r}$, one makes a cyclic permutation of the subscripts p, q, and r in (2.3). It follows from the analysis of (2.3) that the annual component of polar oscillations can be caused by the term containing the products $\gamma_p\gamma_r$ and $\gamma_q\gamma_r$ of direction cosines. To compute them, we integrate Eqs. (2.3) in the first approximation:

$$r = r^0,\ \varphi \approx rt + \varphi^0,\ \nu \approx \omega_* t + \nu^0, \cos\theta(\nu) = a(\theta^0, \psi^0)\cos\nu$$
$$\theta(0) = \theta^0 = 66°33', \qquad 0.4 \le a \le 1, \qquad 0 \le \psi^0 \le 2\pi \tag{2.4}$$
$$\cos\theta\sin\theta = b(\theta^0,\psi^0)\cos\nu + d\cos 3\nu + \ldots,\ 0.4 \le b \le \frac{4}{3\pi},\ |d| \ll 1$$

The second and higher harmonics with respect to ν result in quantities smaller than the leading terms by a factor of 10^2–10^3 and hence are not taken into account.

Next, $B^* - A^*$ is also substantially smaller than $C^* - A^*$ (approximately by a factor of 160). By estimating the terms for p and q in Eqs. (2.1), by taking into account the expressions (2.4), and by averaging with respect to the fast phase φ, we obtain a simplified analytical model of the form

$$\begin{aligned} \dot{p} + N_p q &= \varkappa_q r^2 + 3b\omega_*^2 \chi_p \cos\nu, \qquad N_{p,q} \approx N = 2\pi/T_1 \approx (0.84 - 0.85)\omega_* \\ \dot{q} - N_q p &= -\varkappa_p r^2 - 3b\omega_*^2 \chi_q \cos\nu, \qquad p(0) = p^0, \qquad q(0) = q^0 \end{aligned} \tag{2.5}$$

Here κ_p and κ_q are the mean values of $\delta J_{pr}/B^*$ and $\delta J_{qr}/A^*$, which can be slow functions. The coefficients χ_p and χ_q are obtained by averaging the coefficients of $\cos\nu$ in the Sun's gravitational torque components with respect to φ, and $\chi_p = \chi_q$. As was already indicated, they are due to diurnal tides. The effect of the Moon's gravitational torques on nutation oscillations is relatively small owing to a significant difference in frequencies, and these torques are not taken into account. The right-hand sides of Eqs. (2.5) explicitly contain a harmonic term with a period of one year, which explains the mechanism of the nutation oscillations observed in the IERS records. Although the sensitivity of the coefficients $\kappa_{p,q}$ is by five orders of magnitude higher than that of $\chi_{p,q}$, the explicit regular mechanism of annual force-torque action by internal (atmospheric, oceanic, and seasonal) geophysical factors with estimated amplitude $M_h \sim 10^2\,\mathrm{kg{\cdot}m^2 s^{-2}}$ seems to be unfounded from the viewpoint of mechanics. The frequency analysis of the annual oscillations component also suggests that the geophysical interpretation is unsound [1].

The problem of constructing a gravitational-tidal oscillation mechanism with period sufficiently close to the Chandler period requires a separate investigation.

3 Numerical simulation results for the interpolation and prediction problems

It is of interest to study the efficiency of the interpolation and prediction of the polar motion by the remarkably simple mathematical model

$$x(\tau) = (\xi, f(\tau)), \quad y(\tau) = (\eta, f(\tau))$$

$$\xi = (\xi_1, \dots, \xi_6)^T, \quad \eta = (\eta_1, \dots, \eta_6)^T \tag{3.1}$$

$$f(\tau) = (1, \tau, \cos 2\pi\Omega\tau, \sin 2\pi\Omega\tau, \cos 2\pi\tau, \sin 2\pi\tau)^T, \quad \Omega = 0.845$$

on the basis of known IERS daily data [2].

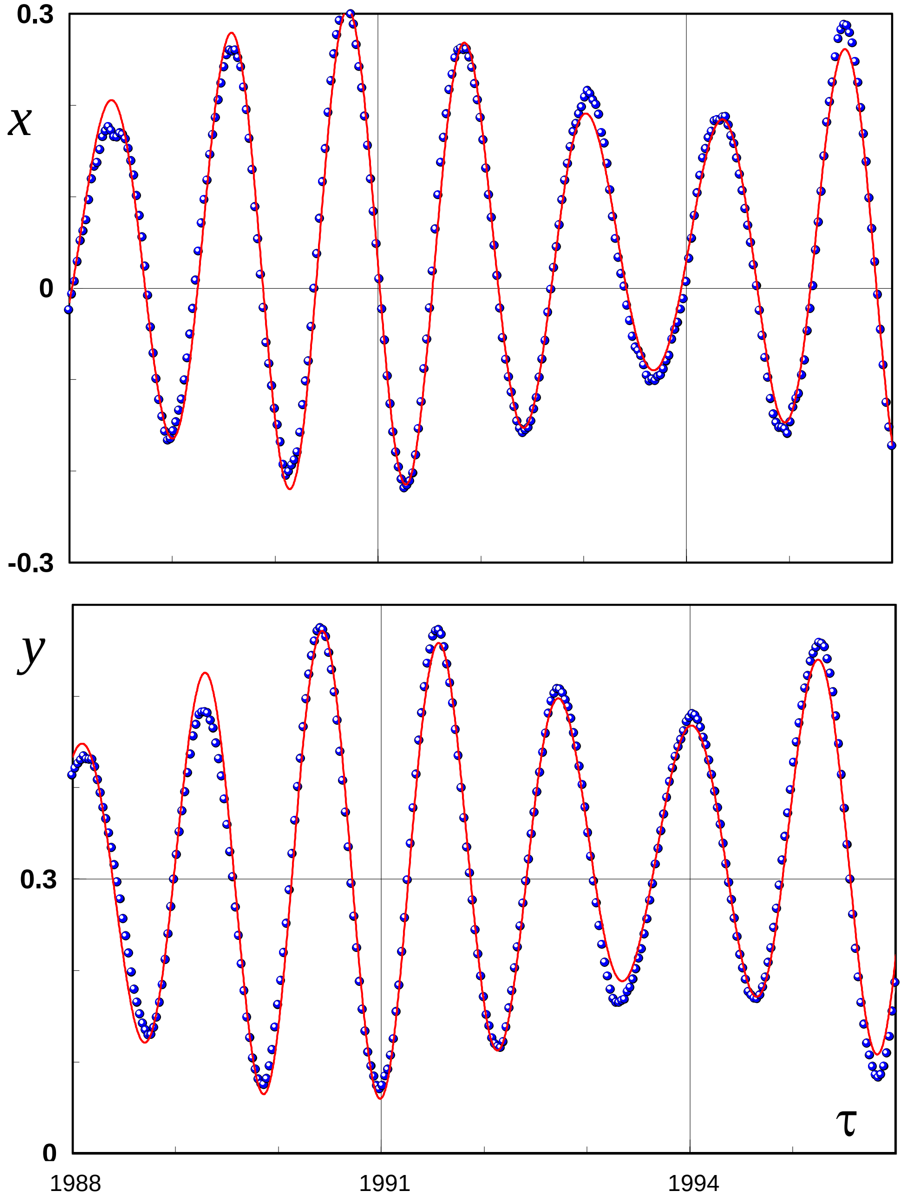

Fig. 01.

Figure 1 shows the theoretical curves $x^*(\tau)$ and $y^*(\tau)$ interpolating the daily measurements on the eight-year interval $0 \le \tau \le 8$ from 1988 to the end of 1995; open symbols indicate the measurement data. The standard deviations are

$\sigma_x = 0.014$ and $\sigma_y = 0.017$, which shows that the model (3.1) corresponding to the optimal values ξ^* and η^* has satisfactory accuracy, where

$$\begin{aligned} \xi^* &= (-0.041, -0.0004, -0.034, 0.194, -0.023, -0.065)^T \\ \eta^* &= (0.300, 0.005, 0.193, 0.033, -0.060, 0.020)^T \end{aligned} \tag{3.2}$$

The comparison of the coefficients ξ_3^* and η_4^* as well as ξ_4^* and η_3^*, determining the Chandler components of the oscillations, and also ξ_5^* and η_6^* as well as ξ_6^* and η_5^* (with regard to the factor $\Omega = 0.845$), corresponding to the annual component in (3.2), confirms the above-mentioned structural property of the model (3.1).

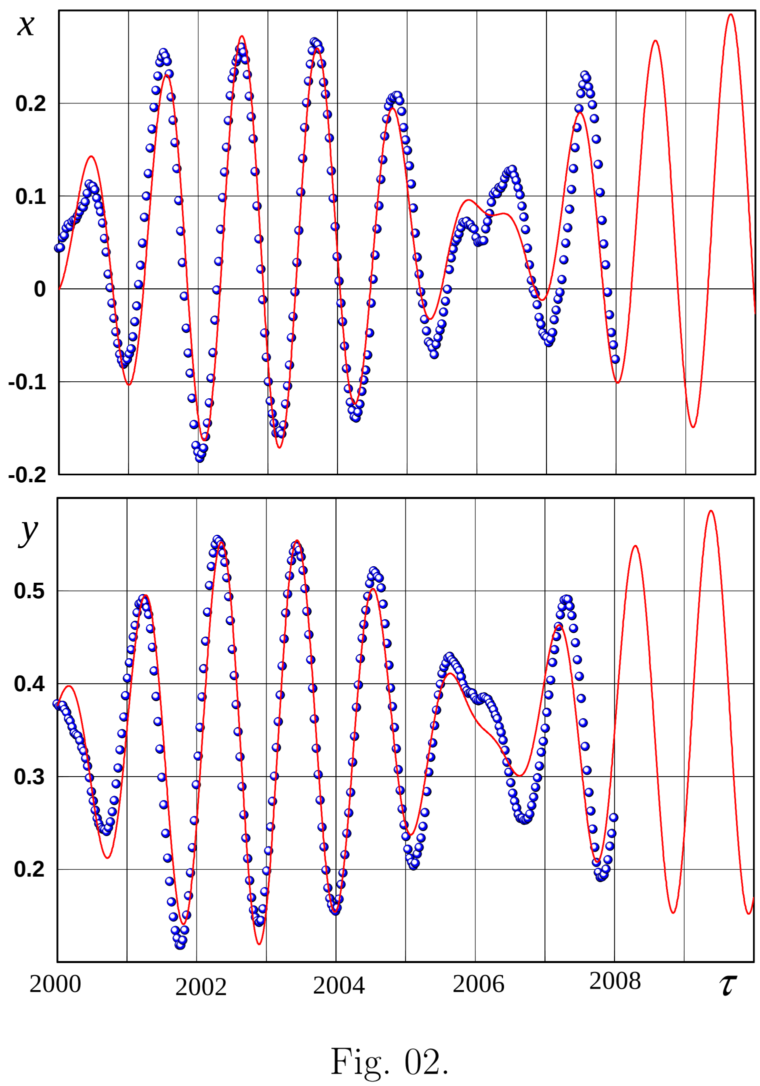

Fig. 02.

The IERS tries to provide forecasts of the polar motion for 100–150 days ahead in the annual reports [2] and on the basis of current data (IERS, EOR Product Center, http://hpiers.obspm.fr/eoppc/eop/eopc04/eopc04-xy.gif). They use their own model and technique, which result in a considerably imprecise and unstable forecast requiring weekly correction. The difference between forecasts can be comparable with the maximum deviation of the pole.

Figure 2 shows the authors' interpolation of observed polar motion on an eight-year interval of IERS daily observations (2000–2007). The corresponding optimal values and standard deviations are

$$\xi^* = (0.0407, 0.0032, 0.2306, 0.1205, -0.6452, -0.7881)^T, \quad \sigma_x = 0.1029$$

$$\eta^* = (0.3282, 0.0040, 0.1222, -0.0288, -0.0746, 0.0593)^T, \quad \sigma_y = 0.09597$$

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