Abel Prize 2009 as a necessitation to discussion on geometry, or physicist versus geometers.

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Abstract

Contemporary geometers do not acknowledge nonaxomatizable geometries. This fact means that our knowledge of geometry is poor. A perfect knowledge of geometry is important for "consumers of geometry" (physicists dealing with geometry of microcosm), because poor knowledge of geometry forces physicists to refuse from the orthodox geometrical paradigm and to use the quantum paradigm, which is a simple fitting, compensating our imperfect knowledge of geometry.

Situation in geometry, when we know only a negligible part of geometries serviceable for description of the space-time, should be qualified as a *poor knowledge of geometry.* Any geometry is a continual set of propositions on properties of geometrical objects. A geometry is axiomatizable, if the whole continual set of proposition can be deduced from a finite set of basic propositions (axioms) by means of rules of the formal logic. We know only axiomatizable geometries, because we do not enable to construct nonaxiomatizable geometries. However, our inaptitude of nonaxiomatizable geometries construction does not mean, that nonaxiomatizable geometries do not exist.

Of late one suggested a method of construction of physical geometries, that is geometries, which are described in terms and only in terms of the world function [1, 2, 3]. (The world function $\sigma(P,Q) = \frac{1}{2}\rho^2(P,Q)$, where $\rho(P,Q)$ is a distance between the points P and Q). Such an approach to geometry, when a geometry is described completely by a distance, is correlated better with our apprehension that a geometry is a science on disposition of geometrical objects, than with the conventional apprehension, when geometry is simply a logical construction.

The physical geometries are nonaxiomatizable as a rule, because the equivalence relation is intransitive in the physical geometries, in general. The equivalence relation is always transitive in axiomatizable geometries, and geometries with intransitive equivalence relation cannot be axiomatizable.

Almost all contemporary geometers do not acknowledge physical geometries. As far as I understand, the reason of this fact is very simple. A construction of a physical geometry does not refer to means of the geometry construction, which are characteristic for axiomatizable geometry (manifold, linear vector space, coordinate system, dimension, continuity). A physical geometry is formulated in terms of points and of distance (world function) between them. The physical geometry is formulated uniformly on arbitrary set of points (continuous and discrete, with unlimited divisibility and with restricted divisibility). A use of the physical geometry is reasonable and natural, if the geometry is considered as a science on disposition of geometrical objects. However, contemporary geometers consider means of the geometry description as necessary attributes of a geometry. They cannot imagine a geometry without such an attribute as the linear vector space.

They consider only geometries given on a manifold and they cannot imagine a geometry, given on a set of points without a mention of the set continuity.

In the nineteenth century all mathematicians did not acknowledge the non-Euclidean geometry, suggested by Lobachevsky and Bolyai. What was a reason of such abruption? I did not find any investigation of this question in literature. I think as follows. In that time the geometers dealt only with the Euclidean geometry. The Cartesian coordinate system can be introduced in the Euclidean geometry, but it cannot be introduced in the non-Euclidean geometry. The geometers considered the Cartesian coordinate system as an attribute of any geometry, because it was an attribute of the Euclidean geometry. The geometers did not acknowledge the non-Euclidean geometry as a real geometry, because one cannot introduce Cartesian coordinate system in it. It was the first crisis in geometry.

Now we have the second crisis. The contemporary geometers do not acknowledge the physical (nonaxiomatizable) geometries. The reason of such a relation is, in particular, the fact, that one cannot introduce a linear vector space in a physical geometry. It is supposed conventionally, that the linear vector space is a necessary attribute of any geometry. Accordingly to this approach the contemporary geometers suppose that any geometry is axiomatizable and try to construct nonaxiomatizable geometries, deducing them from some axiomatics. As far as it is impossible, the geometrical constructions appear to be inconsistent or overdetermined. For instance, the Riemannian geometry appears to be inconsistent (overdetermined) [4, 5].

Physicists, who use a geometry for description of the space-time, meet serious problems, connected with poor knowledge of geometry. Indeed, we know only negligible part of possible geometries. The real space-time geometry belongs to that part of geometries, which we do not know, and we are forced to invent different hypotheses, compensating a use of wrong space-time geometry. It happened as follows.

Considering physical phenomena in the microcosm, we may neglect influence of the matter distribution on the space-time geometry. Then the space-time geometry is to be uniform and isotropic. In the twentieth century the Riemannian geometry was considered as the most general geometry serviceable for description of the space-time. The geometry of Minkowski is the only Riemannian geometry, which is uniform and isotropic. Investigations show, that classical principles of dynamics together with the space-time geometry of Minkowski cannot explain experimental data. As far as our knowledge of geometry were poor, and we did not know uniform isotropic geometries other, than the geometry of Minkowski, we cannot modify space-time geometry, to agree predictions of the theory with experimental data. We were forced to modify principles of dynamics. As a result the quantum paradigm (variation of dynamic principles + fixed space-time geometry) has been existing for the twentieth century. The geometric paradigm (classical principles of dynamics + variated space-time geometry) could not appear until appearance of other (nonaxiomatizable) uniform isotropic space-time geometries.

The geometric paradigm is more reasonable and natural, than the quantum paradigm. The physical geometries are formalized in the sense, that all possible geometries are labelled by the world function. Giving the world function, one determines the geometry. It is necessary only to choose a proper world function. Principles of dynamics are not formalized (they are not labelled), and it is very difficult to guess true principles of dynamics. The quantum principles of dynamics work well for the nonrelativistic physical phenomena. Investigating the relativistic physical phenomena, we come to geometrical problems (strings, branes, compactification) again, and our poor knowledge of geometry cannot help us [6]. I have described the situation from the viewpoint of a physicist, who is a "consumer of a geometry"

The situation looks differently from the viewpoint of a geometer, who is a creator of a geometry. A geometer ignores the consume side of a geometry. He perceives the Riemannian geometry and the physical geometry as different matters, because they are constructed by different methods, although both geometries are sciences on the disposition of geometric objects. At the same time the geometer perceives the Euclidean geometry and the symplectic geometry as similar matters, because both geometries are logical constructions. He perceives the symplectic geometry as a geometry, although it has no relation to the geometry as a science on the disposition of geometric objects. He united Euclidean geometry and symplectic geometry by the same term 'geometry'. The fact, that the symplectic geometry is logical structure, as the Euclidean geometry, is more important for a geometer, than the fact, that the symplectic geometry has no relation to a geometry as a science on disposition of geometrical objects.

Situation may be illustrated by the legend on Alexander the Great, who hew the Gordian Knot by his sword instead of to untangle it. The Gordian cart was attached to the chancel by means of the Gordian Knot. From practical viewpoint (separation of the cart from the chancel) the action of Alexander is quite right. However, Alexander the Great did not manifest the art of untangling of knots, and he could not to awarded as a "champion of untangling of knots". If the separation of the cart from the chancel is restricted by the condition, that the Gordian Knot must be untangled, the separation becomes quite another problem, than that one, which had been solved by Alexander the Great.

In a like way geometers consider the problem of a geometry construction only under constraint, that this geometry is axiomatizable and it can be considered to be a logical construction. It does not mean, that geometers do not understand, that a geometry may be a science on disposition of geometrical objects. Geometers understand that a geometry may be a science on disposition of geometric objects, which is described in terms of only distance. There exists so called "distance geometry" [7, 8]. However, Blumental failed to construct pure metric geometry. He did not know the deformation principle. He was forced to introduce additional concept of a curve, which cannot be expressed via the concept of distance. As a result now geometers consider any geometry as a logical construction. In other words, any geometry is constructed under additional constraint, that the nonaxiomatizable geometries are impossible.

The Abel Prize 2009 (on geometry) has been awarded under this unfounded constraint, that the geometry may be only axiomatizable. Such approach would be justified twenty years ago, when nonaxiomatizable geometries were not known. Now, when such geometries appear to be possible [1, 2, 3], the Abel Prize to a geometer, dealing only with axiomatizable geometries looks as a stimulation of the art of deduction and of the art of complicated mathematical calculations. Of course, the art of deduction should be stimulated, awarding prizes to mathematicians. However, the awarder should declare, that the prize is awarded for the art of mathematical calculations and for the art of deduction, but not for a progress in the investigations of a geometry. The reason of awarding is to be clear for all people. The most people perceive the geometry as "consumers of a geometry", but not as its creators. From the viewpoint of the geometry consumer (physicist) the real progress in the geometry investigation is possible only, if the restriction on the geometry axiomatizability is removed. Attempt of the axiomatizable geometries construction, when even the "axiomatizable" Riemannian geometry appears to be inconsistent, is not a progress in geometry.

References

[1] Yu.A.Rylov, Geometry without topology as a new conception of geometry. *Int. Jour. Mat. & Mat. Sci. 30, iss. 12, 733-760, (2002),*

[2] Yu. A.Rylov, Non-Euclidean method of the generalized geometry construction and its application to space-time geometry in *Pure and Applied Differential geometry pp.238-246.* eds. Franki Dillen and Ignace Van de Woestyne. Shaker Verlag, Aachen, 2007.

[3] Yu. A.Rylov, Geometries with intransitive equivalence relation. *e-print 0807.2034*

[4] Yu. A.Rylov, New crisis in geometry? *e-print /math.GM/0503261* .

[5] Yu. A.Rylov, Crisis in the geometry development and its social consequences. *e-print math.GM/0609765.*

[6] Yu. A.Rylov, Why does the Standard Model fail to explain the elementary particles structure? *e-print 0810.0982*

[7] K. Menger, Untersuchen über allgemeine Metrik, *Mathematische Annalen,* **100**, 75-113, (1928).

[8] L. M. Blumenthal, *Theory and Applications of Distance Geometry*, Oxford, Clarendon Press, 1953.