Dynamic equations for tachyon gas

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Abstract

Dynamics of the tachyon gas is considered. It is interesting in the relation, that dark matter phenomenon is explained freely by existence of the tachyon gas. Tachyons have two unexpected properties: (1) a single tachyon cannot be detected and (2) the tachyon gas can be detected by its gravitational field. Although molecules (tachyons) of the tachyon gas moves with superluninal velocities, the mean motion of these molecules appears to be less, than the speed of the light. The tachyon gas properties differs from those of usual gas. The pressure of the tachyon gas is very high. It is not isotropic and depends on the gravitational potential. As a result the tachyon gas may form huge halos around galaxies. These halos have very large and almost constant density. This circumstance can explain the law of star velocities at the periphery of a galaxy. Properties of the tachyon gas admit one to consider it as a dark matter.

Key words: multivariant geometry, tachyon, tachyon gas, tachyon dynamics, dark matter

1 Introduction

The particles moving with the velocity, which is greater than the speed of the light, are called tachyons [1]-[5]. We shall use this name for particle whose world line is spacelike. Both definitions mean the same, if the world line is smooth, and one can define a derivative along the world line. This derivative is known as a velocity. We shall show that the world line of tachyons is not smooth. This property differ tachyons from tardions which are particles moving with velocity less, than the speed of the light.

The property of the space-time geometry called multivariance is essential for tachyons. It means that a vector **AB** at the point A has many equivalent vectors **CD, CD′, CD″,...**at the point C, but these vectors are not equivalent between themselves. Contemporary theorists do not accept the property of multivariance in geometry and try to remove it, if it appears by accident in geometry. For instance, when it appears in the Riemannian geometry, one removes this property, connecting any of numerous vectors **CD, CD′, CD″**,...at the point C with the path of their parallel transport from the point A and asserting absence of absolute parallelism in the Riemannian geometry.

Such a relation to multivariance is connected with the fact that beginning from Euclid one studied only proper Euclidean geometry, assuming that the space-time geometry cannot have any additional properties which are absent in the Euclidean geometry. The multivariance is denied in the Riemannian geometry, because one considers absence of absolute parallelism as a less defect of the geometry, than multivariance of the vector equivalence. Absence of smoothness (wobbling) of the tachyon world line is a corollary of multivariance of the spacelike vectors equality in the geometry of Minkowski. Let us present the tachyon world line in the form of a world chain, i.e. in the form of broken line, consisting of short straight line segment of the same length. Each link of the chain can be represented by a short spacelike vector. The adjacent vectors are equal for a free tachyon (or are in parallel, that is the same for vectors of the same length). At the presence of multivariance it leads to a wobbling of the world chain.

Multivariance is undesirable also in dynamics, but not only in geometry. It is to the point to remember about papers of Boltzman, who explained the gas dynamics via stochastic (multivariant) motion of its molecules. In this case the multivariant motion of a single molecule was explained by its collisions with other molecules. It was very difficult to credit that deterministic motion of the gas volume, containing many molecules, can be explained as a statistical description of multivariant motion of single molecules. However, some time later the dynamical multivariance has been accepted, when the Brownian motion has been detected experimentally. The Brownian motion can be interpreted as a motion of a gas, consisting of large molecules which could be observed in a usual microscope.

However the preconception against the geometric multivariance remains. It is very difficult to accept the geometric multivariance, if one takes into account that the geometric multivariance is connected with intransitivity of the equivalence relation and with nonaxiomatizability of the geometry, where equivalence of vectors is multivariant. The preconception against the geometric multivariance reminds the preconception against the concept of inertia, which existed in the time, when the Aristotelian mechanics dominated. Concept of inertia was absent in the Aristotelian mechanics. We are accomplished on the basis of the Newtonian mechanics, and now it is very difficult to imagine what was a reason of the preconception of adherents of the Aristotelian mechanics against the concept of inertia. But such a preconception existed certainly.

Unlikely it was accidental that the first law of mechanics (the law of inertia) has been formulated by Newton in the form of a separate law, although it is a special case of the second law (when the force is equal to zero). It is unlikely that Newton did not understand this. Apparently the law of inertia has been formulated as a separate law, because the concept of inertia need a separate formulation to stress the importance of the concept of inertia, which was not common in that time.

Rationale of the multivariance concept as a fundamental property of geometry can be found in [6, 7]. However, the contemporary theorists prefer an experimental test instead of rationale, and we consider such a test.

Let us consider the space-time with the geometry of Minkowski. Equality of two vectors in such a geometry may be defined doubly:

(1) Conventional definition: Two vectors $\mathbf{AB}$ and $\mathbf{CD}$ are equal, if their components $(\mathbf{AB})_k$, and $(\mathbf{CD})_k$ are equal

$$\mathbf{AB}\mathrm{eqv}\mathbf{CD}: \quad (\mathbf{AB})_k = (\mathbf{CD})_k\,, \quad k = 0, 1, 2, 3 \tag{1.1}$$

(2) Coordinateless definition: Two vectors $\mathbf{AB}$ and $\mathbf{CD}$ are equal, if they are in parallel ($\mathbf{AB} \upuparrows \mathbf{CD}$) and their lengths $|\mathbf{AB}|$ and $|\mathbf{CD}|$ are equal

$$(\mathbf{AB} \upuparrows \mathbf{CD}): \quad (\mathbf{AB}.\mathbf{CD}) = |\mathbf{AB}| \cdot |\mathbf{CD}| \tag{1.2}$$

$$|\mathbf{AB}| = |\mathbf{CD}|: \quad \sigma(A, B) = \sigma(C, D) \tag{1.3}$$

Here $\sigma(A, B)$ is the world function of the space-time geometry between the points A and B. The space-time geometry is described completely by the world function σ

$$\sigma: \quad \Omega \times \Omega \to \mathbb{R}, \quad \sigma(P, Q) = \sigma(Q, P)\,, \quad \sigma(P, P) = 0, \quad \forall P, Q \in \Omega \tag{1.4}$$

where Ω is the set points, where geometry is given. The world function is defined by the relation $\sigma(P, Q) = \frac{1}{2}d^2(P, Q)$, where $d(P, Q)$ is the space-time distance between the points P and Q. The world function has been introduced by J.L Synge for description of Riemannian geometry of the space-time [8]. In the Euclidean space the scalar product $(\mathbf{AB}.\mathbf{CD})$ of two vectors is expressed by the formula

$$(\mathbf{AB}.\mathbf{CD}) = \sigma(A, D) + \sigma(B, C) - \sigma(A, C) - \sigma(B, D) \tag{1.5}$$

The same formula (1.5) take place in other space-time geometries, in particular in the geometry of Minkowski.

In the proper Euclidean geometry of any dimension both definitions (1.1) and (1.2), (1.3) coincide because of special properties of the world function of the proper Euclidean geometry. In the geometry of Minkowski the conventional definition contains four equations and vector $\mathbf{CD}$ is determined uniquely, if the vector $\mathbf{AB}$ is given. Multivariance is absent in the conventional definition of the two vectors equivalence.

The coordinateless definition (1.2), (1.3) contains only two equations, whereas any vector is described by four coordinates. In general, equations (1.2), (1.3) admit a multivariant solution. Nevertheless for timelike vectors four relations (1.1) are equivalent to two equations (1.2), (1.3). It is conditioned by special properties of the world function in the geometry of Minkowski. But for spacelike vectors the equivalence of the two definitions is absent, and equality of spacelike vectors **AB** and **CD** is multivariant.

What of two definitions are true? The coordinateless definition (1.2), (1.3) is true, because it does not use mathematical technique of linear vector space. The coordinateless definition (1.2), (1.3) is a fundamental pure geometric definition, whereas the conventional definition (1.1) uses properties of the linear vector space, which cannot be introduced in arbitrary space-time geometry. In particular, the linear vector space cannot be introduced in a multivariant geometry.

In the present paper the dynamic equations for the tachyon gas are deduced. It appears that the tachyon gas is a substance, which can be considered as a dark matter, which is discovered in astrophysical observations [9]. As a result these observations may be interpreted in favour of the coordinateless definition (1.2), (1.3). It follows from this circumstance that the existence of the tachyon gas may be considered as acknowledged experimentally.

2 Dynamic equations for a single tachyon

Dynamic equations for tachyons are deduced in the framework of metric approach to geometry, when the geometry is described completely by means of only world function. We shall consider space-time geometry as a geometry of Minkowski with slight gravitational field in the space-time. In this case the world function has the form

$$\sigma\left(x,x'\right)=\frac{1}{2}\left(\left(c^{2}-2V\left(\mathbf{y}\right)\right)\left(x_{0}-x_{0}'\right)^{2}-\left(\mathbf{x}-\mathbf{x}'\right)^{2}\right),\quad \mathbf{y}=\frac{\mathbf{x}+\mathbf{x}'}{2} \tag{2.1}$$

where $\{x^0,\mathbf{x}\}=\{x^0,x^1,x^2,x^1\}$ are coordinates in some inertial coordinate system, $V=V\left(x\right)$ is the gravitational potential ($V\ll c^2$). The geometry, described completely by a world function, is called the physical geometry.

In the physical geometry the particle dynamics is described by the skeleton conception [7], where instead of the continuous world line one uses the world chain $\mathcal{C}$ (broken line), whose links are vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ of the same length μ

$$\mathcal{C}=\bigcup_s \mathbf{P}_s\mathbf{P}_{s+1},\quad \left|\mathbf{P}_s\mathbf{P}_{s+1}\right|=\mu=\text{const},\quad s=...0,1,2,... \tag{2.2}$$

For free particle the adjacent vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$ are equivalent ($\mathbf{P}_s\mathbf{P}_{s+1}\text{eqv}\mathbf{P}_{s+1}\mathbf{P}_{s+2}$). Then the equivalence conditions (1.2) - (1.5) can be written in the form

$$\sigma\left(P_s,P_{s+2}\right)=4\sigma\left(P_s,P_{s+1}\right),\quad \sigma\left(P_s,P_{s+1}\right)=\sigma\left(P_{s+1},P_{s+2}\right),\quad s=0,\pm1,\pm2,... \tag{2.3}$$

If there exist the limit $\mu\rightarrow0$, the world chain (2.2) turns into a smooth world line. Keeping in mind that world line $\sigma\left(P_s,P_{s+1}\right)=\frac{1}{2}d^2\left(P_s,P_{s+1}\right)$, where d is the distance between the points P_s and P_{s+1}, one can see, that in the proper Euclidian geometry the relation (2.3) describes the rule of straight line construction by means of only compasses.

In the case of tachyon $\sigma(P_s, P_{s+1}) < 0$ and μ is imaginary $\mu^2 = -|\mu^2|$. We consider three adjacent points P_0, P_1, P_2 of the world chain

$$P_0 = \{x_0, \mathbf{x}\}, \quad P_1 = \{x_0 + p_0, \mathbf{x} + \mathbf{p}\}, \quad P_2 = \{x_0 + 2p_0 + \alpha_0, \mathbf{x} + 2\mathbf{p} + \boldsymbol{\alpha}\} \tag{2.4}$$

The 4-vector $\alpha = \{\alpha_0, \boldsymbol{\alpha}\}$ is a discrete analog of the acceleration vector. We write equations (2.3) for the points (2.4). The quantities $x = \{x_0, \mathbf{x}\}$ and $\{x_0 + p_0, \mathbf{x} + \mathbf{p}\}$ are supposed to be given, and the four components of the 4-vector $\alpha = \{\alpha_0, \boldsymbol{\alpha}\}$ are to be determined from two equations (2.3)

One obtains for tachyon

$$\left(c^2 - 2V\right)(p_0 + \alpha_0)^2 - (\mathbf{p} + \boldsymbol{\alpha})^2 = \left(c^2 - 2V\right)p_0^2 - \mathbf{p}^2 = \mu^2, \tag{2.5}$$

$$\left(c^2 - 2V\right)(2p_0 + \alpha_0)^2 - (2\mathbf{p} + \boldsymbol{\alpha})^2 = 4\left(\left(c^2 - 2V\right)p_0^2 - \mathbf{p}^2\right), \tag{2.6}$$

It follows from (2.5) that

$$p_0 = \sqrt{\frac{\mathbf{p}^2 - |\mu|^2}{c^2 - 2V}} = \frac{p}{c}\sqrt{\frac{1 - \frac{|\mu|^2}{p^2}}{1 - 2\frac{V}{c^2}}} \tag{2.7}$$

We consider separately two different cases: (1) $p_0 \neq 0$ and (2) $p_0 = 0$

2.1 The case $p_0 \neq 0$

After transformation of equations (2.5), (2.6) one obtains two relations

$$\alpha_0 = \frac{\boldsymbol{\alpha}\mathbf{p}}{p_0\left(c^2 - 2V\right)}, \quad \mathbf{v} = \frac{\mathbf{p}}{p_0} \tag{2.8}$$

$$\left(\frac{c^2 - 2V - v^2}{c^2 - 2V}\right)\alpha_{\parallel}^2 + \boldsymbol{\alpha}_{\perp}^2 = 0, \quad v = \frac{p}{p_0} \tag{2.9}$$

where

$$\boldsymbol{\alpha}_{\parallel} = \mathbf{p}\frac{(\boldsymbol{\alpha}\mathbf{p})}{\mathbf{p}^2}, \quad \boldsymbol{\alpha}_{\perp} = \boldsymbol{\alpha} - \boldsymbol{\alpha}_{\parallel}, \quad \alpha_{\parallel}^2 = \frac{(\boldsymbol{\alpha}\mathbf{p})^2}{\mathbf{p}^2}, \quad \alpha_{\parallel} = \frac{\boldsymbol{\alpha}\mathbf{p}}{p}, \quad p = |\mathbf{p}| \tag{2.10}$$

Here $\boldsymbol{\alpha}_{\parallel}$ is the component of 3-vector $\boldsymbol{\alpha}$ which is in parallel with the 3-vector $\mathbf{p}$, whereas $\boldsymbol{\alpha}_{\perp}$ is the component of 3-vector $\boldsymbol{\alpha}$, which is perpendicular to the 3-vector $\mathbf{p}$.

Solution of equation (2.9) is nonunique

$$\alpha_{\parallel} = \frac{r\sqrt{c^2 - 2V}}{\sqrt{(v^2 - c^2 + 2V)}}, \quad v = \frac{p}{p_0} \tag{2.11}$$

$$\alpha_{\perp 1} = r\cos\phi, \quad \alpha_{\perp 2} = r\sin\phi \quad v = \frac{p}{p_0} = \frac{p\sqrt{(c^2-2V)}}{\sqrt{\mathbf{p}^2 - |\mu|^2}} \tag{2.12}$$

$$\alpha_0 = \frac{\boldsymbol{\alpha}\mathbf{p}}{p_0\left(c^2-2V\right)} = \frac{p}{p_0}\left(\frac{r}{\sqrt{\left(v^2-c^2+2V\right)\left(c^2-2V\right)}}\right) \tag{2.13}$$

Here r, ϕ are arbitrary real numbers $r \geq 0$. The length $|\boldsymbol{\alpha}|$ of multivariant 3-vector $\boldsymbol{\alpha}$ is of the order r

$$|\boldsymbol{\alpha}|^2 = r^2 \frac{v^2}{\left(v^2-c^2+2V\right)} \tag{2.14}$$

Components of the multivariant particle velocity $\mathbf{u}$ are defined by relations

$$\mathbf{u} = \frac{\mathbf{p}+\boldsymbol{\alpha}}{p_0+\alpha_0}, \quad u^0 = 1 \tag{2.15}$$

In the orthogonal coordinate system these components have the form

$$\begin{aligned} u_{\parallel} &= \frac{p+\alpha_{\parallel}}{p_0+\alpha_0} = \left(p + \frac{r\sqrt{c^2-2V}}{\sqrt{\left(v^2-c^2+2V\right)}}\right)\left(p_0 + \frac{pr}{p_0\sqrt{\left(v^2-c^2+2V\right)\left(c^2-2V\right)}}\right)^{-1} \\ &= \frac{p_0\left(c^2-2V\right)}{p} + \mathcal{O}\left(r^{-1}\right) \end{aligned} \tag{2.16}$$

$$u_{\perp 1} = \frac{\alpha_{\perp 1}}{p_0+\alpha_0} = \frac{p_0\sqrt{\left(v^2-c^2+2V\right)\left(c^2-2V\right)}\cos\phi}{p} + \mathcal{O}\left(r^{-1}\right) \tag{2.17}$$

$$u_{\perp 2} = \frac{\alpha_{\perp 2}}{p_0+\alpha_0} = \frac{p_0\sqrt{\left(v^2-c^2+2V\right)\left(c^2-2V\right)}\sin\phi}{p} + \mathcal{O}\left(r^{-1}\right) \tag{2.18}$$

The components of the 3-vector $\mathbf{u}$ do not depend on parameters r at $r \to \infty$. As far as the components of $\mathbf{u}$ do not depend practically on r, we may average components of $\mathbf{u}$ with any weight function. We choose the weight function in such a way, as if the integration is produced over infinite volume in the spherical coordinate system (r, θ, ϕ). Components of 3-vector $\mathbf{u}$ do not depend on r, θ. The mean values $\langle\mathbf{u}\rangle$ and $\langle\mathbf{u}^2\rangle$ of variables $\mathbf{u}$ and $\mathbf{u}^2$ are obtained as a result of averaging

$$\langle u\rangle = \lim_{R\to\infty}\frac{1}{N}\int_0^R r^2 dr \int_0^{\pi}\sin\theta d\theta \int_0^{2\pi} u\left(r,\phi\right)d\phi, \quad N = \frac{4\pi R^3}{3} \tag{2.19}$$

The variables $\mathbf{u}$ are represented by formulas (2.16) - (2.18) in the form

$$\mathbf{u}\left(r,\phi\right) = \mathbf{u}\left(\phi\right) + \mathcal{O}\left(r^{-1}\right) \tag{2.20}$$

Representation (2.20) does not mean that the term $\mathcal{O}\left(r^{-1}\right)$ has a singularity at $r = 0$. According to (2.16) - (2.18), (2.13) all components of $\mathbf{u}$ are regular at $r = 0$. Integration of the term $\mathcal{O}\left(r^{-1}\right)$ gives term of the order R^2/N, which vanishes at

$R\to\infty$, because of normalizing factor $1/N$, which is proportional R^{-3}. It is valid for all positive powers of $\mathbf{u}$. At such an averaging one obtains

$$\left\langle u_{\parallel}\right\rangle =\frac{\sqrt{c^2-2V}}{p}\sqrt{\mathbf{p}^2-|\mu^2|}=c\sqrt{1-2\frac{V}{c^2}}\sqrt{1-\frac{|\mu^2|}{\mathbf{p}^2}} \tag{2.21}$$

$$\left\langle \mathbf{u}_{\perp}\right\rangle =0 \tag{2.22}$$

$$\left\langle u_{\parallel}^2\right\rangle =\left\langle u_{\parallel}\right\rangle^2=\left\langle \mathbf{u}\right\rangle^2=c^2\left(1-2\frac{V}{c^2}\right)\left(1-\frac{|\mu^2|}{\mathbf{p}^2}\right)<c^2 \tag{2.23}$$

Using expressions (2.12), (2.13), (2.7) one obtains

$$\begin{aligned}\left\langle \mathbf{u}_{\perp}^2\right\rangle &= \frac{p_0^2\left(v^2-c^2+2V\right)\left(c^2-2V\right)}{p^2}=\left(c^2-2V\right)\left(1-\frac{p_0^2\left(c^2-2V\right)}{p^2}\right)\\ &= \frac{|\mu|^2}{p^2}\left(c^2-2V\right)=c^2-2V-\left\langle u_{\parallel}^2\right\rangle \end{aligned} \tag{2.24}$$

$$\left\langle \mathbf{u}^2\right\rangle =\left\langle u_{\parallel}^2\right\rangle +\left\langle \mathbf{u}_{\perp}^2\right\rangle =c^2-2V \tag{2.25}$$

2.2 The case $p_0=0$

This case corresponds to the special case $\mathbf{p}^2=|\mu^2|$. In this case according to (2.7) $p_0=0$, and equations (2.5), (2.6) take the form

$$\left(c^2-2V\right)\alpha_0^2-\left(\mathbf{p}+\boldsymbol{\alpha}\right)^2=-\mathbf{p}^2=-\left|\mu\right|^2, \tag{2.26}$$

$$\left(c^2-2V\right)\alpha_0^2-\left(2\mathbf{p}+\boldsymbol{\alpha}\right)^2=-4\mathbf{p}^2, \tag{2.27}$$

$$p^2=\left|\mu^2\right|$$

These equations are reduced to the form

$$\boldsymbol{\alpha}\mathbf{p}=\alpha_{\parallel}p=0,\quad \left(c^2-2V\right)\alpha_0^2-\boldsymbol{\alpha}_{\perp}^2-\alpha_{\parallel}^2=0 \tag{2.28}$$

Solution of equations (2.28) has the form

$$\alpha_0=\frac{r}{\sqrt{c^2-2V}},\qquad \alpha_{\parallel}=0,\qquad \alpha_{\perp 1}=r\cos\phi,\qquad \alpha_{\perp 2}=r\sin\phi, \tag{2.29}$$

Averaging by means of the formula (2.19), one obtains

$$\left\langle u_{\parallel}\right\rangle =\left\langle \frac{p\sqrt{c^2-2V}}{r}\right\rangle =0,\quad \left\langle \mathbf{u}_{\perp}\right\rangle =0 \tag{2.30}$$

$$\left\langle \mathbf{u}_{\perp}^2\right\rangle =\left\langle \left|\frac{\boldsymbol{\alpha}_{\perp}}{\alpha_0}\right|^2\right\rangle =\left\langle \frac{r^2}{r^2}\left(c^2-2V\right)\right\rangle =c^2-2V \tag{2.31}$$

$$\left\langle u_{\parallel}^2\right\rangle =\left\langle u_{\parallel}\right\rangle^2=0,\quad \left\langle \mathbf{u}^2\right\rangle =\left\langle u_{\parallel}^2\right\rangle +\left\langle \mathbf{u}_{\perp}^2\right\rangle =c^2-2V \tag{2.32}$$

One can see from (2.30) - (2.32) that results for $\langle u_\parallel \rangle$, $\langle \mathbf{u}_\perp \rangle$, $\left\langle u_\parallel^2 \right\rangle$, $\langle \mathbf{u}_\perp^2 \rangle$, taken for $p^2 = |\mu^2|$, that corresponds to the case $p_0 = 0$, coincide with results (2.21) - (2.24) . It means, that the case $p_0 = 0$ is a special case of the case $p_0 \neq 0$. The case $p_0 = 0$ can be obtained from the general case $p_0 \neq 0$, setting $p^2 = |\mu^2|$. We shall consider farther only the general case $p_0 \neq 0$.

3 Dynamic equations for tachyon gas

Motion of a single tachyon is multivariant (stochastic). This multivariance is a geometric one. Let us consider a statistical ensemble of many tachyons. Such a statistical ensemble form a tachyon gas, where the pressure tensor $P^{\alpha\beta}$ has the form

$$P^{\alpha\beta} = \rho\frac{1}{2}\left(l_{(1)}^\alpha l_{(1)}^\beta + l_{(2)}^\alpha l_{(2)}^\beta\right)\left(\langle \mathbf{u}^2 \rangle - \langle \mathbf{u} \rangle^2\right) = \rho\frac{1}{2}\left(l_{(1)}^\alpha l_{(1)}^\beta + l_{(2)}^\alpha l_{(2)}^\beta\right)\langle \mathbf{u}_\perp^2 \rangle \tag{3.1}$$

where ρ is the mass density of the gas and 3-vectors $l_{(1)}^\alpha$, $l_{(2)}^\alpha$ are unit 3-vectors, which are orthogonal to 3-vector $\mathbf{p}$ and between themselves. Anisotropy of the pressure is connected with the fact that the vector

$$\mathbf{u}_\perp = \frac{\boldsymbol{\alpha}_\perp}{p_0 + \alpha_0}$$

which is responsible for pressure, is orthogonal to the vector $\mathbf{p}$. Using relations (2.23) - (2.25), one obtains from (3.1)

$$P^{\alpha\beta} = \frac{1}{2}\rho\left(l_{(1)}^\alpha l_{(1)}^\beta + l_{(2)}^\alpha l_{(2)}^\beta\right)\left(c^2 - 2V - \langle \mathbf{u} \rangle^2\right) \tag{3.2}$$

Anisotropy of the pressure is explained by the fact that the state of the tachyon gas is described by its density ρ, velocity $\mathbf{u} = \langle \mathbf{u} \rangle$ and polarization 3-vector $\mathbf{p}/|\mathbf{p}|$. According to (2.21), (2.22) the tachyon gas velocity $\langle \mathbf{u} \rangle$ is in parallel with polarization 3-vector $\mathbf{p}/|\mathbf{p}|$.

$$\langle \mathbf{u} \rangle = \frac{\mathbf{p}}{p}c\sqrt{1 - 2\frac{V}{c^2}}\sqrt{1 - \frac{|\mu^2|}{\mathbf{p}^2}}, \quad \langle \mathbf{u}_\perp \rangle = 0 \tag{3.3}$$

In the case, when $\mathbf{p}^2 = |\mu^2|$ and $\langle \mathbf{u} \rangle = 0$, the unit polarization 3-vector $\mathbf{p}/|\mathbf{p}|$ does not vanish. General-covariant description of the polarization is realized by means of bivector $l_{(1)}^i l_{(2)}^k - l_{(1)}^k l_{(2)}^i$, where 4-vectors $l_{(1)}^i$, $l_{(2)}^k$, $k = 0, 1, 2, 3$ are unit 4-vectors which are orthogonal to 4-vectors $(p_0, \mathbf{0})$ and $(0, \mathbf{p})$ and between themselves

$$l_{(a)}^0 p_0 = 0, \quad l_{(a)}^\alpha p_\alpha = 0, \quad a = 1, 2; \quad g_{ik} l_{(1)}^i l_{(2)}^k = 0 \tag{3.4}$$

In the case, when $\langle \mathbf{u} \rangle \neq 0$

$$\left(l_{(1)}^\alpha l_{(1)}^\beta + l_{(2)}^\alpha l_{(2)}^\beta\right) = \delta^{\alpha\beta} - \frac{\langle u^\alpha \rangle \langle u^\beta \rangle}{\langle \mathbf{u} \rangle^2} \tag{3.5}$$

and the pressure tensor can be written in the form

$$P^{\alpha\beta} = \frac{1}{2}\rho\left(\delta^{\alpha\beta} - \frac{\langle u^\alpha\rangle\langle u^\beta\rangle}{\langle \mathbf{u}\rangle^2}\right)\left(c^2 - 2V - \langle \mathbf{u}\rangle^2\right) \tag{3.6}$$

Remark. Tachyon gas may be a mixture of tachyon gases with different polarization vector. Components of this mixture have different mean velocities. These components do not interact between themselves and move freely one through another.

Dynamic equations for the tachyon gas have the form

$$\nabla_k T^{ik} = 0 \tag{3.7}$$

where T^{ik} is the energy-momentum tensor of the tachyon gas and ∇ is the covariant derivative in the space-time with the metric tensor

$$g_{ik} = \operatorname{diag}\left(c^2 - 2V, -1, -1, -1\right) \tag{3.8}$$

The Christoffel symbols γ^i_{kl} are defined by the relations

$$\gamma^i_{kl} = \frac{1}{2}g^{is}\left(g_{is,k} + g_{sk,i} - g_{ik,s}\right)$$

Here and further comma means differentiation.

$$g_{is,k} \equiv \partial_k g_{is} \equiv \frac{\partial g_{is}}{\partial x^k}$$

Only following Christoffel symbols do not vanish in the space-time geometry (3.8)

$$\begin{aligned}
\gamma^0_{00} &= \frac{1}{2}g^{00}g_{00,0} = \frac{1}{2}\frac{\partial}{\partial t}\log\left(c^2 - 2V\right) \approx -c^{-2}\partial_0 V, \\
\gamma^\alpha_{00} &= -\frac{1}{2}g^{\alpha\alpha}g_{00,\alpha} = \frac{1}{2}\frac{\partial}{\partial x^\alpha}\left(c^2 - 2V\right) = -\partial_\alpha V \\
\gamma^0_{\alpha 0} &= \frac{1}{2}g^{00}g_{00,\alpha} = \frac{1}{2}\frac{\partial}{\partial x^\alpha}\log\left(c^2 - 2V\right) \approx -c^{-2}\partial_\alpha V
\end{aligned} \tag{3.9}$$

Then the dynamic equations (3.7) have the form

$$\nabla_k T^{0k} = \partial_k T^{0k} + \partial_0 \log\left(c^2 - 2V\right) T^{00} + \frac{3}{2}\partial_\alpha \log\left(c^2 - 2V\right) T^{0\alpha} = 0 \tag{3.10}$$

$$\nabla_k T^{\beta k} = \partial_k T^{\beta k} + \frac{1}{2}\partial_\beta\left(c^2 - 2V\right) T^{00} + \frac{1}{2}\partial_0 \log\left(c^2 - 2V\right) T^{\beta 0} + \frac{1}{2}\partial_\beta \log\left(c^2 - 2V\right) T^{0\beta} = 0 \tag{3.11}$$

In the nonrelativistic approximation they have the form

$$\partial_k T^{0k} = 0, \quad \partial_k T^{\beta k} = T^{00}\partial_\beta V \tag{3.12}$$

The energy-momentum tensor T^{ik} is the energy-momentum tensor of ideal gas. It has the form

$$T^{ik} = \rho \left\langle u^i u^k \right\rangle \tag{3.13}$$

where 4-velocity u^k is defined by the relation (2.15). Averaging is made by means of the formula (2.19). One obtains

$$T^{00} = \rho, \quad T^{\alpha 0} = T^{0\alpha} = \rho \left\langle u^\alpha \right\rangle \tag{3.14}$$

$$T^{\alpha\beta} = \rho \left\langle u^\alpha \right\rangle \left\langle u^\beta \right\rangle + P^{\alpha\beta}, \quad \alpha, \beta = 1, 2, 3 \tag{3.15}$$

where the pressure tensor $P^{\alpha\beta}$ is determined by relations (3.1), (3.2). Omitting for brevity the symbol of averaging and replacing $\left\langle u^\alpha \right\rangle$ by u^α, one obtains from (3.12)

$$\partial_0 \rho + \partial_\alpha \left(\rho u^\alpha \right) = 0 \tag{3.16}$$

$$\partial_0 \left(\rho u^\beta \right) + \partial_\alpha \left(\rho u^\beta u^\alpha \right) + \partial_\alpha P^{\alpha\beta} = \rho \partial_\beta V \tag{3.17}$$

Using equation (3.16), equation (3.17) is reduced to the form

$$\partial_0 u^\beta + u^\alpha \partial_\alpha u^\beta + \frac{1}{\rho} \partial_\alpha P^{\alpha\beta} = \partial_\beta V \tag{3.18}$$

If $\mathbf{u} \neq \mathbf{0}$ one may use the expression (3.6) for the energy-momentum tensor $P^{\alpha\beta}$. One obtains

$$\partial_0 u^\beta + u^\alpha \partial_\alpha u^\beta + \frac{1}{\rho} \partial_\beta \left(\frac{1}{2} \rho \left(c^2 - 2V - \mathbf{u}^2 \right) \right) - \frac{1}{\rho} \partial_\alpha \left(\frac{1}{2} \rho \frac{u^\alpha u^\beta}{\mathbf{u}^2} \left(c^2 - 2V - \mathbf{u}^2 \right) \right) = \partial_\beta V \tag{3.19}$$

or

$$\begin{gathered} \partial_0 u^\beta + u^\alpha \partial_\alpha u^\beta + \frac{1}{2} \frac{\partial_\beta \rho}{\rho} \left(c^2 - 2V - \mathbf{u}^2 \right) - \partial_\beta \left(2V - \frac{\mathbf{u}^2}{2} \right) \\ - \frac{1}{\rho} \partial_\alpha \left(\frac{1}{2} \rho \frac{u^\alpha u^\beta}{\mathbf{u}^2} \left(c^2 - 2V - \mathbf{u}^2 \right) \right) = 0 \end{gathered} \tag{3.20}$$

If $\mathbf{u} = \mathbf{0}$, the last term in (3.20) becomes indefinite. To make it definite, one is to use the tachyon gas parametrization and to replace $u^\alpha u^\beta / \mathbf{u}^2$ by $p^\alpha p^\beta / \mathbf{p}^2$.

4 Balanced state of the tachyon gas

Let us consider the balanced state of the tachyon gas in the gravitational field of a galaxy. Equations (3.20) are written in spherical coordinate system (r, θ, ϕ). For simplicity we shall consider a spherically symmetric field $V = V(r)$. We suppose that the tachyon gas is practically at rest. The velocity components have the form $u_r = 0$, $u_\theta = 0$ The azimuth component u_ϕ is very small, and one may set $\mathbf{u} = \mathbf{0}$ everywhere in (3.20) except for the multiplier $u^\alpha u^\beta / \mathbf{u}^2$. We are interested in the density $\rho = \rho(r)$ of the tachyon gas in the stationary gravitational field. More exactly we are interested, whether the tachyon gas density can be enough great to explain the dark matter.

Two equations of (3.20) corresponding to $\beta = \theta$ and to $\beta = \phi$ are identities of the form $0 = 0$. The equation for $\beta = r$ takes the form

$$\frac{1}{2\rho}\frac{\partial\rho}{\partial r}\left(c^2 - 2V\right) - 2\frac{\partial}{\partial r}V - \frac{1}{\rho}\frac{\partial}{r\partial\phi}\left(\frac{1}{2}\rho\frac{u^\phi u^\phi}{\mathbf{u}^2}\left(c^2 - 2V\right)\right) = 0 \tag{4.1}$$

or, as far as $\partial/\partial\phi = 0$, one obtains

$$\frac{1}{\rho}\frac{\partial\rho}{\partial r}\left(c^2 - 2V\left(r\right)\right) = 4\frac{\partial}{\partial r}V\left(r\right) \tag{4.2}$$

Integration of equation (4.2) gives

$$\rho = \frac{\rho_0 c^4}{\left(c^2 - 2V\left(r\right)\right)^2} \tag{4.3}$$

where ρ_0 is the integration constant. In the case, when $V\left(r\right) \ll c^2$, one obtains

$$\rho = \rho_0\left(1 + 4\frac{V\left(r\right)}{c^2}\right) \tag{4.4}$$

The density of the tachyon gas changes rather slowly, and the gravitational field of the tachyon gas is sufficient to imitate the gravitational field of the dark matter. Such a capacity of the tachyon gas is connected with a very high pressure of the tachyon gas.

Remark. Someone argue that the mass of the halo

$$m_{\mathrm{h}} = 4\pi\int_0^R \rho\left(r\right)r^2 dr \approx \frac{4\pi}{3}\rho_0 R^3 \tag{4.5}$$

tends to ∞ at $R \to \infty$ and concludes that the halo of the tachyon gas is impossible. In reality, such a divergence is a problem of all stationary stellar atmospheres. For instance, density of isothermal stationary atmosphere is defined by the relation

$$\rho = \rho_0 \exp\left(-\frac{GMm_{\mathrm{m}}}{kTr}\right) \tag{4.6}$$

where ρ_0 is the atmosphere density on the stellar surface, M is the stellar mass and m_{m} is the mass of a gas molecule. One obtains the following estimation for the atmosphere mass

$$m_{\mathrm{a}} = 4\pi\int_{r_0}^R \rho_0\exp\left(-\frac{GM}{kTr}\right)r^2 dr = 4\pi\int_{1/R}^{1/r_0}\rho_0\exp\left(-\frac{GM}{kT}\xi\right)\frac{d\xi}{\xi^4} \approx 4\pi\rho_0\frac{R^5}{5} \tag{4.7}$$

which diverges at $R \to \infty$. In reality the stellar atmospheres exist, but they are not stationary [11]. The same may be valid for tachyon gas. Besides, such a situation is possible that the whole universe is uniformly filled by tachyons. The density of tachyons is greater inside regions in vicinity of galaxies. These regions form halos filled with a dark matter. In general, calculation of such regions is an important problem, but it is a complicated gas dynamic problem which is not considered in this paper.

5 Discussion

Thus, it follows from the condition of vector equality (1.1) that tachyons cannot exist. It follows from the coordinateless conditions (1.2), (1.3), that a single tachyon cannot be discovered, whereas the tachyon gas is the best candidate for role of the dark matter, because it have the maximal possible pressure. It means that the metric approach to the space-time geometry and coordinateless conditions (1.2), (1.3) are true, whereas the conditions of vector equality (1.1) are wrong for spacelike vectors. (For timelike vectors conditions (1.1) give the same result as coordinateless conditions (1.2), (1.3).

From general consideration the coordinateless equations are better, than conditions which uses the means of description (coordinate system). But why are the conditions (1.1) wrong for spacelike vectors? To answer this question we consider the proper Euclidean geometry, where conditions (1.1) and conditions (1.2), (1.3) are equivalent and where the spacelike vectors are absent.

The geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$ is a result of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ generalization. However, a generalization of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ depends on the way of the proper Euclidean geometry representation [10]. Conventional representation of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ is based on a use of the linear vector space formalism. Any generalized geometry is also based on the linear vector space formalism, and all properties of the linear vector space are conserved at the generalization, because all axioms of the linear vector space take place in the generalized geometry. The conventional representation of the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ will be referred to as axiomatic approach to the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$.

However, there exists a metric approach to the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$, when it is described in terms of the world function and only in terms of the world function. At a generalization of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ the Euclidean world function σ_{E} is replaced by the world function σ of the geometry in question in all definitions of $\mathcal{G}_{\mathrm{E}}$. Besides, the world function σ_{E} has some specific properties of $\mathcal{G}_{\mathrm{E}}$. These special properties can be described in terms of the world function σ_{E}. These properties are described by the relations [7]

If σ_{E} is the world function of n-dimensional Euclidean space, it satisfies the following relations.

I. Definition of the dimension and introduction of the rectilinear coordinate system:

$$\exists\mathcal{P}^n \equiv \{P_0, P_1, ...P_n\} \subset \Omega, \qquad F_n\left(\mathcal{P}^n\right) \neq 0, \qquad F_k\left(\Omega^{k+1}\right) = 0, \qquad k > n \tag{5.1}$$

where $F_n\left(\mathcal{P}^n\right)$ is the Gram's determinant

$$F_n\left(\mathcal{P}^n\right) \equiv \det ||(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)|| \quad i, k = 1, 2, ...n \tag{5.2}$$

Here $(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)$ is the scalar product of two vectors $\mathbf{P}_0\mathbf{P}_i$ and $\mathbf{P}_0\mathbf{P}_k$ defined by the relation

$$(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k) = \sigma_{\mathrm{E}}\left(P_0, P_i\right) + \sigma_{\mathrm{E}}\left(P_0, P_k\right) - \sigma_{\mathrm{E}}\left(P_i, P_k\right) \tag{5.3}$$

Vectors $\mathbf{P}_0\mathbf{P}_i,\ \ i=1,2,...n$ are basic vectors of the rectilinear coordinate system K_n with the origin at the point P_0. The covariant metric tensor $g_{ik}\left(\mathcal{P}^n\right),\ \ i,k=1,2,...n$ and the contravariant one $g^{ik}\left(\mathcal{P}^n\right),\ \ i,k=1,2,...n$ in K_n are defined by the relations

$$\sum_{k=1}^{k=n} g^{ik}\left(\mathcal{P}^n\right) g_{lk}\left(\mathcal{P}^n\right)=\delta_l^i, \qquad g_{il}\left(\mathcal{P}^n\right)=\left(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_l\right), \qquad i,l=1,2,...n \tag{5.4}$$

$$F_n\left(\mathcal{P}^n\right)=\det\left|\left|g_{ik}\left(\mathcal{P}^n\right)\right|\right|\neq 0, \qquad i,k=1,2,...n \tag{5.5}$$

II. Linear structure of the Euclidean space:

$$\sigma_{\mathrm{E}}\left(P,Q\right)=\frac{1}{2}\sum_{i,k=1}^{i,k=n} g^{ik}\left(\mathcal{P}^n\right)\left(x_i\left(P\right)-x_i\left(Q\right)\right)\left(x_k\left(P\right)-x_k\left(Q\right)\right), \qquad \forall P,Q\in\Omega \tag{5.6}$$

where coordinates $x_i\left(P\right),\ i=1,2,...n$ of the point P are covariant coordinates of the vector $\mathbf{P}_0\mathbf{P}$, defined by the relation

$$x_i\left(P\right)=\left(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}\right), \qquad i=1,2,...n \tag{5.7}$$

III: The metric tensor matrix $g_{lk}\left(\mathcal{P}^n\right)$ has only positive (or only negative) eigenvalues

$$g_k>0, \qquad k=1,2,...,n \tag{5.8}$$

IV. The continuity condition: the system of equations

$$\left(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}\right)=y_i\in\mathbb{R}, \qquad i=1,2,...n \tag{5.9}$$

considered to be equations for determination of the point P as a function of coordinates $y=\{y_i\},\ \ i=1,2,...n$ has always one and only one solution.

Conditions I -IV describe possibility of a use of the linear vector space formalism. These conditions are fulfilled in the case of the proper Euclidean geometry. This formalism involves a use of a coordinate system. The geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$ is rather close to the proper Euclidean geometry. Nevertheless the condition (5.8) is not fulfilled in $\mathcal{G}_{\mathrm{M}}$, and one has a surprise with tachyons. To introduce a metric dimension of a geometry and to use the coordinate system, the conditions (5.1), (5.2) are to be fulfilled.

Let the space-time geometry be discrete. For instance, let the world function have the form

$$\sigma=\sigma_{\mathrm{M}}+\frac{\lambda_0^2}{2}\operatorname{sgn}\left(\sigma_{\mathrm{M}}\right)$$

where σ_{M} is the world function of the geometry of Minkowski and λ_0 is the elementary length. In this case conditions (5.1), (5.2) are not fulfilled, and one cannot use coordinate system and a definite metric dimension. One cannot use the linear vector space formalism. One is forced to use coordinateless description [7]. Formally this circumstance is conditioned by multivariance of the discrete geometry. Unfortunately, most scientists believe that coordinate system can be used in any space-time geometry and do not believe in existence of the multivariant space-time geometry. This paper is important in the relation, that it explains freely the dark matter phenomenon by means of multivariant properties of the space-time geometry even in the case of the geometry of Minkowski. Multivariant properties of the space-time geometry are important not only in applications to tachyons.

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