SPACETIME DISTORTION
AS A REASON FOR QUANTUM STOCHASTICITY

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ABSTRACT

One developes a theory which relates to the conventional quantum mechanics in the same way, as the statistical physics relates to the axiomatic thermodynamics. Quantum effects are shown to be a result of some spacetime deformation (distortion) which converts one-dimensional world lines of free pointlike particles into three-dimensional world tubes. The world tubes are random, and their thickness is determined by the Planck constant describing the distortion magnitude. A statistical description of the world tubes in the associated Minkowski spacetime is equivalent to the quantum description (Schrödinger equation). Application of the statistical description to the dynamic system described by the Pauli equation shows that the electron spin has a classical analog, which is a collective property of the statistical ensemble of electrons (not a property of a single electron).

A quantum system is a stochastic system in the sense that experiments with a single quantum system are irreproducible in general. As far as the quantum constant $\hbar$ is an universal constant, and quantum phenomena are universal, it should expect that the reason of quantum stochasticity is also universal and connected with the spacetime properties. We are trying to interpret quantum effects and quantum stochasticity as a result of some spacetime deformation (distortion). It is impossible in frames of the conventional Riemannian spacetime model, and one needs to look at the spacetime from a more general point of view which can be formulated as follows.

All properties of the spacetime are described exhaustively by the interval $S(P,Q)$ between any two points P and Q of the spacetime, or by the world function $\sigma = S^2/2$. Manifold is a derivative concept which can be described in terms of the world function. Mathematical possibility of such a construction was founded [1]. A crucial point of this construction is a new geometrical object: tube of nth order $(n = 0, 1, \ldots)$ defined by n spacetime points. The geometry constructed on the basis of world function and tubes will be referred to as T-geometry, (a geometry of tubes), or the metric approach to geometry (MAG).

Constructing T-geometry is rather simple. Relations of the Euclidean geometry written in terms of the world function are claimed as relations of T-geometry. For instance, vector $\mathbf{P}_0\mathbf{Q}$ is defined as an ordered set of two point P_0 and Q. P_0 is the origin, and Q is the end of the vector $\mathbf{P}_0\mathbf{Q}$. The length $\mid \mathbf{P}_0\mathbf{Q} \mid$ of the vector $\mathbf{P}_0\mathbf{Q}$ is defined by the relation

$$\mid \mathbf{P}_0\mathbf{Q} \mid = S(P_0, Q) = [2\sigma(P_0, Q)]^{1/2}. \tag{1}$$

In T-geometry the scalar product $(\mathbf{P}_0\mathbf{P}.\mathbf{P}_0\mathbf{Q})$ of two vectors $\mathbf{P}_0\mathbf{P}$ and $\mathbf{P}_0\mathbf{Q}$, having a common origin P_0, and the angle ϑ between them are defined through the world function as follows

$$2 \mid \mathbf{P}_0\mathbf{P} \mid\mid \mathbf{P}_0\mathbf{Q} \mid \cos\vartheta = 2(\mathbf{P}_0\mathbf{P}.\mathbf{P}_0\mathbf{Q}) = \mid \mathbf{P}_0\mathbf{P} \mid^2 + \mid \mathbf{P}_0\mathbf{Q} \mid^2 - \mid \mathbf{PQ} \mid^2 \tag{2}$$

where the module $\mid \mathbf{PQ} \mid$ of the vector $\mathbf{PQ}$ is defined by Eq.(1). In the Euclidean space Eq.(2) is the cosine theorem.

By definition two vectors $\mathbf{P}_0\mathbf{P}$ and $\mathbf{P}_0\mathbf{Q}$ are parallel ($\uparrow\uparrow$), antiparallel ($\uparrow\downarrow$), or collinear ($\|$), if the relations $\cos\vartheta = 1, -1, \pm 1$ respectively take place. The tube $\mathcal{T}_{P_0P}$ (of the first order) determined by two points P_0, P is defined as a set of such points Q, that $\mathbf{P}_0\mathbf{Q} \parallel \mathbf{P}_0\mathbf{P}$. According to Eq.(2) it means

$$\mathcal{T}_{P_0P}: \qquad (\mathbf{P}_0\mathbf{P}.\mathbf{P}_0\mathbf{Q})^2 - \mid \mathbf{P}_0\mathbf{P} \mid^2 \mid \mathbf{P}_0\mathbf{Q} \mid^2 = 0. \tag{3}$$

One equation (3) in the n-dimensional space describes in general a tube as a $(n-1)$-dimensional surface. However in the n-dimensional Euclidean space the tube degenerates into one-dimensional straight line, because there only one unit vector $\mathbf{P}_0\mathbf{Q}$ parallel to $\mathbf{P}_0\mathbf{P}$ exists. Nondegeneracy of timelike tubes in the spacetime will be referred to as a spacetime distortion.

Application of MAG to the Minkowski spacetime M permits to answer such a very simple question why world lines of free real particles are timlike. The conventional geometry and physics cannot answer this simple question. The answer of MAG is as follows. Timelike tubes and spacelike ones of the Minkowski spacetime M have different shape. The timelike geodesics of M are tubes of M, whereas spacelike geodesics of M are not tubes of M. Spacelike tubes of M are two three-dimensional planes tangent to a set of light cones which have their vertices on the spacelike geodesic.

The spacetime distortion can be considered as a result of some deformation of the Minkowski spacetime M. Indeed at MAG any deformation of the spacetime leads to a variation of its world function. Vice versa, any variation of the world function means some deformation of the spacetime. In particular, there are such spacetime deformations which convert the Minkowski spacetime M into non-Riemannian space and cannot be expressed in terms of a manifold, i.e. in terms of conventional Riemannian geometry. Distortion is one of such spacetime deformations.

A uniform distortion is defined as follows [2]:

$$\sigma_M \rightarrow \sigma_D = \sigma_M + D(\sigma_M) \tag{4}$$

where σ_M is the world function of the Minkowski spacetime, σ_D is the world function of the distorted spacetime (DST). D is a distortion function. The real spacetime is DST. A distortion transforms timelike straight lines of the Minkowski spacetime into timelike three-dimensional tubes. The uniform distortion does not violate the spacetime isotropy and uniformity.

The world tube of a particle of the mass m is defined as a broken timelike tube.

$$\mathcal{T}_{br} = \bigcup_i \mathcal{T}_{[P_i,P_{i+1}]}, \qquad S_D(P_i, P_{i+1}) = \mu = m/b, \qquad i = 0, \pm 1, \pm 2, \ldots \tag{5}$$

where $\mathcal{T}_{[P_i,P_{i+1}]}$ is a segment of the tube $\mathcal{T}_{P_i,P_{i+1}}$ between the points P_i, P_{i+1}, b is an universal constant of dimensionality $[b] = g/\text{cm}$. According to the first Newton law for a free particle the momenta $p = \mathbf{P}_i\mathbf{P}_{i+1}$ of different links are parallel, i.e. $\mathbf{P}_i\mathbf{P}_{i+1} \uparrow\downarrow \mathbf{P}_i\mathbf{P}_{i-1}, \quad i = 0, \pm 1, \pm 2 \ldots$ Together with Eq.(5) it means

$$(\mathbf{P}_i\mathbf{P}_{i+1}.\mathbf{P}_i\mathbf{P}_{i-1}) = - \mid \mathbf{P}_i\mathbf{P}_{i+1} \mid\mid \mathbf{P}_i\mathbf{P}_{i-1} \mid = -\mu^2, \qquad i = 0, \pm 1, \pm 2, \ldots \tag{6}$$

In the Minkowski spacetime M the relations (5),(6) considered as equations for P_{i+1} with P_{i-1}, P_i fixed have a sole solution. It means that any link $\mathbf{P}_{i-1}\mathbf{P}_i$ of the world line (5) determines single-valuedly the next link $\mathbf{P}_i\mathbf{P}_{i+1}$. In DST the same equations (5), (6) have many solutions, in general. At fixed link $\mathbf{P}_{i-1}\mathbf{P}_i$ a position of the next link $\mathbf{P}_i\mathbf{P}_{i+1}$ is not determined single-valuedly, in general, because in DST there are many vectors $\mathbf{P}_i\mathbf{P}_{i+1}$ of the length μ antiparallel to $\mathbf{P}_i\mathbf{P}_{i-1}$. It is a reason of the stochasticity (uncertainty) of world tubes of free particles in the deterministic DST.

Thus, if the spacetime is distorted, world tubes of free pointlike particles are random [3]. Statistical description of stochastic world tubes of free non-relativistic particles is equivalent to the Schrödinger equation, provided at large σ_M the distortion function has the asymptotical form $D(\sigma_M) = \hbar/(2bc)$, where b is the universal constant, $\hbar$ is the Planck constant, and c is the speed of the light. Thus, at this approach the Planck constant describes the spacetime distortion, i.e. properties of the spacetime. Stochasticity of a world tube of a free particle in the **deterministic** DST is connected with an existence of many unit vectors $\mathbf{P}_0\mathbf{Q}$ parallel to a given vector $\mathbf{P}_0\mathbf{P}$.

Now let us define what does the term "statistical description" mean. Let $\mathcal{S}$ be a stochastic physical system. It means that experiments with a single system $\mathcal{S}$ are irreproducible. There are no dynamic equations for a single stochastic system. In order to investigate the stochastic system $\mathcal{S}$, let us consider a set $\mathcal{E}$ (statistical ensemble) of many like independent stochastic systems $\mathcal{S}$. Experiments with the statistical ensemble $\mathcal{E}$ are reproducible, and $\mathcal{E}$ can be considered as a deterministic dynamic system, associated with the stochastic system $\mathcal{S}$. There exist dynamic equations for $\mathcal{E}$. By definition the statistical ensemble $\mathcal{E}$ of stochastic systems $\mathcal{S}$ carries out a statistical description of the stochastic systems $\mathcal{S}$. In other words, the statistical description is a way of reduction of a stochastic system $\mathcal{S}$ to a deterministic system $\mathcal{E}$.

The main concept of the statistical description is a deterministic dynamic system called the statistical ensemble (not the probability density, or probability amplitude). Such a statistical description will be referred to as the statistical ensemble technique (SET). It is the most general way of the statistical description. The classical statistical physics and the quantum mechanics are special cases of SET, when the statistical ensemble is described respectively in terms of a probability density, or a probability amplitude.

Finally, any statistical description substitutes "a dynamics of stochastic systems" by a dynamics of the statistical ensemble. SET carries out a connection between the stochastic system and the statistical ensemble directly and independently of the statistical emsemble interpretation, i.e. it is of no importance what rules are used for calculation of average values.

The statistical physics and quantum mechanics associate a stochastic system with a statistical ensemble by means of intermediate concepts (respectively probability density and probability amplitude) which are connected with an interpretation of the statistical ensemble, i.e. with the rules of calculation of average values. These rules are different for classical and quantum stochastic systems and have nothing to do with the dynamics. A usage of these intermediate concepts leads to a conceptual distinction between the dynamics of classical sysems and the dynamics of quantum systems. As a result many people consider that all that is not quantum is classical and vice versa. In reality, an existence of such a conceptual boundary is not necessary, and not all that is not quantum is necessarily classical.

In application to stochastic world tubes the SET means a construction of a statistical ensemble $\mathcal{E}$ consisitng of stochastic world tubes. In the Minkowski spacetime M the statistical ensemble $\mathcal{E}$ of free classical deterministic particles of the mass m is described by the action

$$\mathcal{A}[\mathbf{x}]=-\int mc\sqrt{1-c^{-2}(d\mathbf{x}/dt)^2}dtd\boldsymbol{\xi}, \tag{7}$$

$$\mathbf{x}=\mathbf{x}(t,\boldsymbol{\xi}),\qquad \mathbf{x}=\{x^1,x^2,x^3\},\qquad \boldsymbol{\xi}=\{\xi^1,\xi^2,\xi^3\} \tag{8}$$

where $t,\mathbf{x}$ are coordinates of a particle in M, and $\boldsymbol{\xi}$ are Lagrangian coordinates labelling the ensemble particles. Using Eulerian coordinates $t,\mathbf{x}$ as independent variables instead of $t,\boldsymbol{\xi}$, the action (7) reduces to the form

$$\mathcal{A}[\rho,\mathbf{j},\xi_0,\boldsymbol{\xi}] = \int\{-\sqrt{m^2c^2j^ij_i} - j^i\partial_i\xi_0 - j^ig^\alpha(\boldsymbol{\xi})\partial_i\xi_\alpha\}dtd\mathbf{x} \qquad (9)$$

where $j^i = \{\rho,\mathbf{j}\}$ is 4-flux of particles, $g^\alpha(\boldsymbol{\xi})$, $\alpha = 1,2,3$ are arbitrary functions of their arguments, ξ_0 is the action variable (temporal Lagrangian coordinate), and $j_i = g_{ik}j^k$, g_{ik}=diag$\{c^2,-1,-1,-1\}$. Indeed, the Jacobians

$$j^i = \frac{\partial(x^i,\xi_1,\xi_2,\xi_3)}{\partial(x^0,x^1,x^2,x^3)}, \qquad i = 0,1,2,3 \qquad (10)$$

satisfy identically dynamic equations $\delta\mathcal{A}/\delta\xi_i = 0$, $i = 0,1,2,3$. Adding relations (10) to the action (9) as additional constraints, one does not change the variational problem. Then eliminating j^i from Eq.(9) by means of Eq.(10) and reversing relations $\boldsymbol{\xi} = \boldsymbol{\xi}(t,\mathbf{x})$, one obtains Eq.(7) with $\mathbf{x} = \mathbf{x}(t,\boldsymbol{\xi})$. Let us note that the variational problems (7) and (9) are equivalent provided $m =$const. If m depends on the ensemble state, the equivalency of Eqs. (7) and (9) violates.

Let us imagine that the mass m in Eq.(9) depends on the ensemble state derivatives $\partial_i\rho, \partial_i\mathbf{j}$. Then the action (9) will describe an ensemble of stochastic particles, and the action (9) cannot reduce to Eq.(9). In particular, replacing m^2 in Eq.(9) by

$$m_q^2 = m^2 + (\frac{\hbar}{2c}\nabla\log\rho)^2, \qquad m = \text{const} \qquad (11)$$

setting $g^\alpha(\boldsymbol{\xi}) = 0$, $\alpha = 1,2,3$ and using non-relativistic approximation $\mid\mathbf{j}/c\mid\ll\rho$, the action (9) reduces to the form

$$\mathcal{A}_q[\rho,\mathbf{j},\varphi] = \int\{\frac{\mathbf{j}^2}{2m\rho} - \rho\partial_t\varphi - \mathbf{j}\nabla\varphi - \frac{\hbar^2}{8m}\rho(\nabla\log\rho)^2\}dtd\mathbf{x}, \qquad (12)$$

$$\varphi = \xi_0 - mc^2t.$$

Substitution

$$\rho = \psi^*\psi, \qquad \mathbf{j} = -\frac{i\hbar}{2m}(\psi^*\nabla\psi - \nabla\psi^*\psi), \qquad \psi = \sqrt{\rho}\exp(i\varphi/\hbar) \qquad (13)$$

transforms this action into the action for the Schrödinger equation

$$\mathbf{A}_S[\psi,\psi] = \int[\frac{i\hbar}{2}(\psi^*\frac{\partial\psi}{\partial t} - \frac{\partial\psi^*}{\partial t}\psi) - \frac{\hbar^2}{2m}\nabla\psi^*\nabla\psi]d^4x \qquad (14)$$

Thus the dependence (11) of the mass on the ensemble state ρ leads in the non-relativistic approximation to the Schrödinger equation.

Let us define the mass in the action (9) by means of the relation

$$m = b < \mu_{oM}(P) >, \qquad (15)$$

where the mean oriented mass $< \mu_{oM} >$ is the projection of the momentum $p = \mathbf{P}_i\mathbf{P}_{i+1}$ onto the direction of the mean velocity $< u(P) >=< \mathbf{P}_i\mathbf{P} \mid \mathbf{P}_i\mathbf{P} \mid_M^{-1} > \mid< \mathbf{P}_i\mathbf{P} \mid \mathbf{P}_i\mathbf{P} \mid_M^{-1} >\mid_M^{-1}$, b is the universal constant. Index "M" means that the corresponding scalar product and the module are calculated by means of the world function σ_M, i.e in the Minkowski space associated with DST by Eq.(4), whereas segments $\mathcal{T}_{[P_i P_{I+1}]}$ of the broken tube are calculated by means of the world function σ_D of DST. Angular brackets mean averaging over the ensemble. Vectors $\mathbf{P}_i\mathbf{P}_{i+1}$ and $\mathbf{P}_i\mathbf{P} \mid \mathbf{P}_i\mathbf{P} \mid_M^{-1}$, $P \in \mathcal{T}_{[P_i P_{I+1}]}$ are parallel in DST, but they are not parallel in the associated Minkowski space M. If the distortion vanishes $D(\sigma_M) = 0$, the mass μ_{oM} does not depend on the point P, and m=const. If $D(\sigma_M) \neq 0$, the momentum $\mathbf{P}_i\mathbf{P}_{i+1}$ and the 4-velocity $\mathbf{P}_i\mathbf{P} \mid \mathbf{P}_i\mathbf{P} \mid_M^{-1}$ are not parallel in M. Then the mass $< \mu_{oM} >= p_k < u^k(P) >$ depends on the ensemble state.

In the coordinate system, where spatial components of the 4-momentum vanish

$$p_i = \{\mu_M, \mathbf{0}\}, \quad < u^i >= \{(1-\boldsymbol{\beta}^2)^{-1/2}, \boldsymbol{\beta}(1-\boldsymbol{\beta}^2)^{-1/2}\}, \quad \mu_M =\mid \mathbf{P}_i\mathbf{P}_{i+1} \mid_M \qquad (16)$$

one has

$$< \mu_{oM} >= p_i < u^i >= \mu_M(1-\boldsymbol{\beta}^2)^{-1/2} \simeq \mu_M(1+\boldsymbol{\beta}^2/2), \qquad \beta \ll 1. \qquad (17)$$

Here $\boldsymbol{\beta}c$ is the mean 3-velocity in the coordinate system, where $\mathbf{p} = 0$. ϑ=arctanhβ is the angle between vectors p_i and $< u^i >$ in M.

If the mean 3-velocity $\boldsymbol{\beta}c$ is determined by the relation

$$\boldsymbol{\beta}c = -\frac{\hbar}{2b\mu_M}\nabla \log \rho \qquad (18)$$

then the mass m defined by relations (15),(17),(18) coincides with m_q and in the non-relativistic approximation the action (9) for the ensemble of stochastic world tubes takes the form (12). In this case the statistical description of stochastic world tubes coincides with the quantum mechanic description. What form must have the distortion function $D(\sigma_M)$, in order that the relation (18) takes place? The answer is such [3]

$$\sigma_D = \sigma_M + \sigma_0/2, \qquad D(\sigma_M) = \sigma_0/2, \qquad \text{for} \qquad \sigma_M > \mu_e^2 = m_e^2/b^2, \qquad (19)$$

where $\sigma_0 = \hbar/bc$ =const and m_e is the mass of the lightest massive particle, i.e. m_e is the electron mass.

This result is obtained as follows. Let us use notations

$$\mu_D=[2\sigma_D(P_0,P_1)]^{1/2},\qquad \mu_M=[2\sigma_M(P_0,P_1)]^{1/2} \tag{20}$$
$$\mu_M=g(\mu_D),\qquad \mu_D=g^{-1}(\mu_M)=[\mu_M^2+2D(\mu_M^2/2)]^{1/2},$$

where D is the distortion function defined by Eq.(4). The radius $R_M(\mu_M)$ of the tube segment $\mathcal{T}_{[P_0P_1]}$ measured at the point P in M can be presented in the form

$$R_M(\mu_D)=[g^2(\mu_D)/4-g^2(\mu_D/2)]^{1/2}. \tag{21}$$

The point P is located at the middle of the intercept P_0P_1. The mean 3-velocity at the point P is determined by the contributions of adjacent world tubes. Any tube at the point P' contributes the 3-velocity $\beta c=R_M/\mu_M$ in the direction $\mathbf{P'P}$ with the weight $\rho(P')$. Averaging over all directions, one obtains the following result

$$\boldsymbol{\beta}c=c\boldsymbol{\vartheta}=-\frac{2}{3}\frac{R_M^2}{\mu_M}\frac{\nabla\rho}{\rho}. \tag{22}$$

Comparison of Eqs. (18) and (22) leads to a functional equation $R_M^2(\mu_D)=3\sigma_0/4$ for the function g which has the solution $g^2(\mu_D)=\mu_D^2-\sigma_0$. It corresponds to the result (19).

The quantity $(\sigma_0)^{1/2}$ is of the order of the tube radius and can be treated as an elementary length. Its value can be estimated on the base of the supposition that $\mu_M^2>0$ for any $\mu_D^2\geq\mu_e^2=m_e^2/b^2$. Setting $\mu_D=\mu_e$ and using that $\mu_D^2<\sigma_0=\hbar/bc$, one obtains $b<m_e^2c/\hbar\simeq 10^{-16.5}g/cm$, $(\sigma_0)^{1/2}>\hbar/m_ec\simeq 10^{-10.5}cm$. If it appears that the distortion can explain quantum effects in other cases, but not only for free particles, one does not need the quantum principles, and the spacetime distortion can be treated as a reason of quantum effects.

Let us apply SET to the dynamic system $\mathcal{S}_P$ described by the Pauli equation for the non-relativistic electron. The action has the form

$$\mathbf{A}_P[\psi,\psi]=\int[\frac{i\hbar}{2}(\psi^*\frac{\partial\psi}{\partial t}-\frac{\partial\psi^*}{\partial t}\psi)-\frac{\hbar^2}{2m}\nabla\psi^*\nabla\psi]d^4x \tag{23}$$
$$\psi=\binom{\psi_1}{\psi_2},\qquad \psi^*=(\psi_1^*,\psi_2^*)$$

with $j^0=\rho$ and $\mathbf{j}$ defined by the equation

$$j^0=\rho=\psi^*\psi,\qquad \mathbf{j}=-\frac{i\hbar}{2m}(\psi^*\nabla\psi-\nabla\psi^*\psi).$$

By means of the transformation

$$\psi=\binom{\psi_1}{\psi_2}=\rho^{1/2}\exp(i\varphi)\boldsymbol{\sigma\eta}\binom{\chi_1}{\chi_2},\qquad \chi=\binom{\chi_1}{\chi_2}=\text{ const},$$
$$\chi^*\chi=1,\qquad \mathbf{b}=\chi^*\boldsymbol{\sigma}\chi,\qquad \boldsymbol{\eta}=\frac{\boldsymbol{\xi}}{\mid\boldsymbol{\xi}\mid} \tag{25}$$

the action for $\mathcal{S}_P$ reduces to the form

$$\mathcal{A}_P[\rho,\mathbf{j},\varphi,\boldsymbol{\xi}] = \mathcal{A}_{Pc}[\rho,\mathbf{j},\varphi,\boldsymbol{\xi}] + \mathcal{A}_{Pq}[\rho,\mathbf{j},\varphi,\boldsymbol{\xi}] \tag{26}$$

$$\mathcal{A}_{Pc}[\rho,\mathbf{j},\varphi,\boldsymbol{\xi}] = \int (\frac{m}{2\rho}\mathbf{j}^2 - \hbar j^i\partial_i\varphi - \hbar j^i\varepsilon_{\alpha\beta\gamma}\eta_\alpha\partial_i\eta_\beta b_\gamma)d^4x, \qquad j^0 \equiv \rho, \tag{27}$$

$$\mathcal{A}_{Pq}[\rho,\mathbf{j},\varphi,\boldsymbol{\xi}] = \int \frac{\hbar^2}{8m}\rho[4(\varepsilon_{\alpha\beta\gamma}\eta_\alpha\nabla\eta_\beta b_\gamma)^2 - 4(\partial_\alpha\eta_\beta)(\partial_\alpha\eta_\beta) - \frac{(\nabla\rho)^2}{\rho^2}]d^4x \tag{28}$$

where $\varepsilon_{\alpha\beta\gamma}$ is the Levi-Chivita pseudo-tensor, $\mathbf{b}$ is a constant unit vector, $\boldsymbol{\sigma}$ are the Pauli matrices

$$\mathbf{s} = 2\boldsymbol{\eta}(\mathbf{b}\boldsymbol{\eta}) - \mathbf{b}, \qquad \boldsymbol{\eta} = \boldsymbol{\xi}/\mid\boldsymbol{\xi}\mid = (\mathbf{b}+\mathbf{s})[2(1+\mathbf{bs})]^{-1/2} \tag{29}$$
$$\mathbf{s} = \psi^*\boldsymbol{\sigma}\psi/(\psi^*\psi),$$

Eq.(27) describes a classical part of the action (26) which can reduce to the form

$$\mathcal{A}_{cl}[\mathbf{x}] = \int \frac{m}{2}(\frac{d\mathbf{x}}{dt})^2 dt d\xi, \tag{30}$$

The action (30) describes a statistical ensemble $\mathcal{E}_{Pc}$ of independent classical pointlike particles. It means that the action (26) describes the statistical ensemble of quantum particles. Indeed, supposing that the wave function ψ and the action $\mathcal{A}_P$ describe a single quantum electron, one cannot explain, why in the quasi-uniform limit

$$\mid \frac{\hbar\rho}{m\mid\mathbf{j}\mid}\nabla\log u \mid \ll 1, \qquad u = \rho,\eta_1,\eta_2,\eta_3, \tag{31}$$

when one can neglect $\mathcal{A}_{Pq}$ as compared with $\mathcal{A}_{Pc}$, the action $\mathcal{A}_P = \mathcal{A}_{Pc}$ describes a classical statistical ensemble, but not a single classical particle. The statement that the dynamic system $\mathcal{S}_P$ is a statistical ensemble is the crucial point, where statistics penetrates into quantum dynamics.

The action for the ensemble of spinless particles described by the Schrödinger equation has the form (12). The classical part $\mathcal{A}_{Sc}$ of the spinless particle action (12) distinguishes from Eq.(27) by the term depending on the spin variables $\boldsymbol{\eta}$ and responsible for vorticity of the vector field $\mathbf{v} = \mathbf{j}/\rho$.

$$\nabla\times\mathbf{v} = \nabla\times\frac{\mathbf{j}}{\rho} = \frac{\hbar}{m}\varepsilon_{\alpha\beta\gamma}\nabla\eta_\alpha\times\nabla\eta_\beta b_\gamma \tag{32}$$

Both $\mathcal{A}_{Pc}$ and $\mathcal{A}_{Sc}$ can reduce to the form (30). It means that both classical ensembles $\mathcal{A}_{Pc}$ and $\mathcal{A}_{Sc}$ consist of the same pointlike particles. At the same time the actions $\mathcal{A}_{Pc}$ and $\mathcal{A}_{Sc}$ distinguish by the term containing the spin variable $\boldsymbol{\eta}$.

Then one concludes that the electron spin has a classical analog which can be interpreted as a vorticity of the electron ensemble flow. The electron spin is a collective (statistical) property of the electron ensemble (not of a single electron).

Important remark. Electron spin as a statistical property cannot be understood from the standpoint of quantum axiomatics, because the conventional quantum dynamics does not contain any mention about statistical description. (For the same reason the temperature as a statistical property cannot be understood from the standpoint of axiomatic thermodynamics. It can be understood only from the standpoint of statistical physics.)

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