Dynamical methods of investigation in application to the Dirac particle

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Abstract

The Dirac particle $\mathcal{S}_{\rm D}$ is investigated by means of dynamic methods, i.e. without a use of the principles of quantum mechanics. It is shown that the Pauli particle $\mathcal{S}_{\rm P}$ and the nonrelativistic approximation $\mathcal{S}_{\rm nD}$ of the Dirac particle $\mathcal{S}_{\rm D}$ are different dynamic systems. $\mathcal{S}_{\rm nD}$ contains the high frequency degrees of freedom, which are absent in the dynamic system $\mathcal{S}_{\rm P}$. It means that the nonrelativistic Dirac particle $\mathcal{S}_{\rm nD}$ is composite (i.e. it has internal degrees of freedom), whereas the Pauli particle $\mathcal{S}_{\rm P}$ is a pointlike particle with the spin. In the absence of the electromagnetic field the world line of the classical Pauli particle $\mathcal{S}_{\rm Pcl}$ is a timelike straight, whereas that of the classical nonrelativistic Dirac particle $\mathcal{S}_{\rm nDcl}$ is a helix. The characteristic frequency $\Omega = 2mc^2/\hbar$ of this helix is the threshold frequency of the pair production. Using dynamic methods, one shows freely that the Copenhagen interpretation, when the wave function is a specific quantum object describing the state of individual particle, is incompatible with the quantum mechanics formalism. Besides, it is shown that the momentum distribution in quantum mechanics is in reality the mean momentum distribution. Effectiveness of different investigation strategies is discussed and compared.

1 Introduction

It is common practice to think, that the Pauli particle, i.e. the dynamic system $\mathcal{S}_{\rm P}$, described by the Pauli equation, is a nonrelativistic approximation of the Dirac particle, i.e. the dynamic system $\mathcal{S}_{\rm D}$, described by the Dirac equation [1]. In reality, it appears that the nonrelativistic approximation $\mathcal{S}_{\rm nD}$ of the Dirac particle $\mathcal{S}_{\rm D}$ is a composite particle, which is more complicated than the Pauli particle $\mathcal{S}_{\rm P}$, because it has additional degrees of freedom, which absent at the Pauli particle. It can be explained as follows.

The Dirac equation is the system of four first order complex equations for four complex dependent variables, whereas the Pauli equation is the system of two complex first order equations for two complex dependent variables (the order of differential equation is determined by the highest order of time derivative). If the Pauli equation is a nonrelativistic approximation of the Dirac equation, then why its order is lower. The reduction of the order of differential equations is explained usually by the fact that the coefficients before the highest temporal derivatives are small in the nonrelativistic approximation. These terms are neglected and the order of the differential equation is reduced. On the other hand, it is well known that neglecting the highest derivatives, we loss high frequency solutions. Indeed, if the temporal frequency is large enough, the term with the highest derivative may become very large, even if the coefficient before the derivative is small. It means that we may not neglect the highest derivatives without producing a proper investigation.

In the present paper we produce such an investigation, which shows that one cannot neglect the terms, which are connected with the internal structure of the Dirac particle. Recently [2] it has been shown that the classical Dirac particle $\mathcal{S}_{\mathrm{Dcl}}$ has internal structure (additional degrees of freedom). In the present paper it is shown that in the nonrelativistic approximation the internal structure of the Dirac particle takes place also.

From formal viewpoint the neglect of the high frequency solution is a mathematical mistake in the investigation of the Dirac dynamic system $\mathcal{S}_{\mathrm{D}}$, and a very interesting question arises. Why was the mathematical mistake in the transition to the nonrelativistic approximation remaining to be unnoticed for eighty years after invention of the Dirac equation? The answer is very simple. Nobody looked for this mistake. The quantum theory developed by means of the experimental-fitting methods, when the logical structure of a theory was a secondary circumstance. It was necessary to explain the enigmatic microcosm by any means. As a result the quantum theory is founded on the enigmatic quantum principles, which are nonrelativistic. There is nothing bad in application of the experimental-fitting method for explanation of concrete experiments and experimental data, because this method admits one to introduce new concepts, which are characteristic for the considered physical phenomena. However, it leads to undesirable consequences, when the experimental-fitting method is applied to a construction of a physical theory. In this case the method turns into a theoretical-fitting method. Application of the theoretical-fitting method to a construction of a theory is ineffective, because it admits one to introduce new hypotheses and new concepts, but it does not admit one to establish logical connections between different concepts and explain some concepts via other more fundamental concepts. The main goal of a physical theory is a determination of logical connections between the physical concepts and deduction of all concepts via some fundamental concepts. Any progress in reduction of the number of the fundamental concepts and explanation of other concepts in terms of the fundamental concepts is a real progress of a physical theory. Introducing new hypotheses, the experimental-fitting method enables to introduce new concepts, but it does not admit one to establish logical connection between the physical concepts and reduce the number of fundamental basic concepts. One should not use the theoretical-fitting method to determine the logical connections between the physical concepts and to construct a satisfactory theory . Instead of the fitting method one should use the Newtonian deductive method with its slogan "Hypotheses non fingo".

If we have serious problems with new physical phenomena, what are we to do? According to the Newtonian slogan we should look for mistakes in the existing physical theory, find them and correct. The finding of mistakes in the foundation of the existing physical theory is a very difficult problem, because these mistakes appear to be located in other branches of science (geometry, theory of dynamic systems and theory of stochastic systems). In the beginning of the 20th century, when the quantum theory arose, these mistakes were not discovered. The researchers of the 20th century were forced to invent additional hypotheses (quantum principles), which could compensate unknown mistakes in the foundation of the physical theory of microcosm phenomena. In the same way Ptolemeus constructed his doctrine of the celestial mechanics, where the mistake concerning the Earth motion was compensated by additional suppositions. The Ptolemaic doctrine described correctly the motion of planets, but it cannot be used for discovery of the Newtons gravitation law and for calculation of trajectories of rockets in their travel to other planets. In the same way the contemporary quantum theory describes correctly the atomic spectra and other nonrelativistic quantum phenomena, but it fails in the description of the specific relativistic quantum phenomena. The Ptolemaic doctrine (as well as the contemporary quantum theory) was a list of the prescriptions, which lead to true experimentally tested predictions. But not all these prescription were connected logically between themselves. Some of them were compatible only in some region of parameters of the theory, and the doctrine cannot be applied outside this region. List of these prescriptions did not form a logical structure, and the reason was an incorrect fundamental supposition, concerning the Earth motion. The list of the Ptolemaic prescriptions did not form a logical structure. It could be applied only to the planet motion, but it could not be extended to motion of other celestial bodies (comets, rockets). The Copernicus doctrine was a logical structure, because it did not contain mistake concerning the Earth motion. It could be extended to the motion of any celestial bodies.

Analogous situation takes place in the contemporary quantum theory, which also forms a list of prescriptions, but not a logical structure. These prescriptions work very well in the nonrelativistic phenomena of microcosm, but one fails to extend them to relativistic phenomena of microcosm. The reason of this failure is conditioned by the incorrect statements (mistakes) in the foundation of the contemporary quantum theory.

We list the incorrect statements (mistakes), which must be corrected for a construction of a satisfactory theory of microcosm phenomena.

1. *The straight is a one-dimensional line in any space-time geometry.* This state-

ment forbids space-time geometries, where the motion of free particles is primordially stochastic.

2. *Any statistical description is produced in terms of the probability theory.* This statement forbids the dynamical conception of the statistical description, which does not use the concept of the probability density as a main concept of the statistical description.

3. *The free particle Hamiltonian function H and its energy E, taken with the opposite sign, always coincide* $(E = -H)$.

The fourth problem, which should be overcome, is not a mistake. It was a purely mathematical problem. Without solving this problem, one cannot obtain the correct interpretation of the wave function as a method of an ideal fluid description. We consider this problem in the second section.

Two first points concern the quantum theory as a whole, whereas the third point concerns only relativistic quantum theory. It concerns the problem of the pair production and depreciates many papers on the relativistic quantum field theory. We show this in the example of the second quantization of the nonlinear Klein-Gordon equation

$$\partial_0^2\psi - \boldsymbol{\nabla}^2\psi + m^2\psi =: \lambda\psi^+\psi\psi : \tag{1.1}$$

where the speed of the light $c = 1$, the quantum constant $\hbar = 1$, and λ is the constant of self-action.

At the secondary quantization the nonlinear term in rhs of (1.1) provides the pair production, if one imposes some additional constraint [3, 4, 5, 6]

$$\psi P_k - P_k\psi = -i\hbar\frac{\partial\psi}{\partial x^k}, \qquad P^k = \int T^{0k}d\mathbf{x}, \qquad k = 0, 1, 2, 3 \tag{1.2}$$

where T^{ik} is the energy-momentum tensor. Conventionally the condition (1.2) is considered to be the condition, which is necessary for the secondary quantization. Nobody does not consider the conditions (1.2) as some additional constraints, which are not necessary for the secondary quantization, and nobody tests compatibility of constraints (1.2) with the dynamic equation (1.1). However, the secondary quantization of the equation (1.1) is possible without imposition of constraints (1.2) [7]. It means that the conditions (1.2) are additional constraints and compatibility of constraints (1.2) with the dynamic equation (1.1) is to be tested. The test has been made in [7]. It has been shown, that the relations (1.2) and (1.1) are compatible only in the case, when the self-action constant $\lambda = 0$, and the dynamic equation (1.1) is linear.

Thus, although the statement of the pair production problem in the form of two relations (1.1) and (1.2) leads to the pair production effect [3, 4, 5, 6], but this result is not reliable, because the statement of the problem is inconsistent. Besides, the mathematical formalism is imperfect, because it uses perturbation theory and renormalization. Combination of the inconsistent statement of the problem with the imperfect mathematical technique admits one to obtain any desirable result (in the given case the effect of pair production).

On the other hand, the secondary quantization of the equation (1.1) without imposition of (1.2) provides the consistent statement of the problem with the perfect nonperturbative mathematical technique (without renormalization). However, the pair production effect is absent at such a consistent statement of the problem [7].

What is the physical ground of the constraint (1.2), which leads to the pair production effect? If $k = 0$ the relation (1.2) describes the well known fact that for the free particle $H = -E$, where H is the Hamilton function, defined as the quantity canonically conjugate to the time t, and E is the particle energy, defined as an integral of the component T^{00} of the energy-momentum tensor. But this relation is valid only in the case, when the pair production is absent [8]. In the general case, when there is the pair production, the imposition of constraint (1.2) means that the description is produced in terms of particles and antiparticles, which are considered as different dynamic systems [8]. The number of objects is indefinite, and one is forced to use the perturbative methods. On the contrary, absence of the constraint (1.2) means that the description is produced in terms of world lines, which are considered as the fundamental objects of dynamics. The number of these extended objects is fixed, and one may use nonperturbative methods of investigation (see details in [7]).

Dynamics, where the dynamic system (particle) exists only some time and disappears at some time moment after collision with the dynamic anti-system (antiparticle), is inconsistent. The technique of classical dynamic systems does not admit one to use such a dynamical description. However, the same tecnique admits one to describe this collision, if a particle and an antiparticle are different states of the same physical object (world line), and evolution of the dynamic system is determined by a parameter changing monotone along the world line. In this case the collision leads only to a transition from one state to another. Mathematical technique of quantum theory also cannot overcome the difficulty, connected with particle and antiparticle as different dynamic systems. The belief, that we can overcome this difficulty, introducing creation and annihilation operators, is delusive. It is a reason, why the relations (1.1) and (1.2) are incompatible, and it is the third point in the list of mistakes.

It means that a simple addition of the nonlinear term to the linear Klein-Gordon equation does not provide the pair production effects. The reasons, generating the pair production, have a more complicated structure, than a simple product of the creation and annihilation operators. Besides, these reasons have a classical analog in the form of specific force fields. (See for details [9]).

We see in the considered example, that the quantum principles do not work in application to the relativistic quantum systems, or at least, they are not effective in application to them. There is a hope that the relativistic quantum systems can be investigated more effectively by dynamical methods, which do not use the quantum principles.

The above-mentioned mistakes look very simple, and it is very difficult to believe that a correction of these incorrect statements could lead to a construction of a satisfactory theory of the microcosm phenomena. However, these mistakes underlie of the contemporary quantum theory of microcosm phenomena, and correction of them is very important for further development of the microcosm phenomena theory. In particular, correction of the first mistake leads to construction of the new conception of geometry [10] and to a revision of our space-time conception in the microcosm. The scale of this revision is comparable with the scale of revision connected with appearance of the relativity theory [11]. Correction of the second mistake lead to a construction of the dynamical conception of the statistical description. Dynamical methods of this conception are used in the present paper. But we apply the dynamical methods without a reference to their physical foundation. The fact, that the dynamical methods have appeared in accordance with the Newtonian investigation strategy as a result of correction of a mistake, is very important from the logical viewpoint.

Unfortunately, this fact is of no importance for contemporary pragmatic theorists, educated on the experimental-fitting method of investigation. They do not believe in any foundation and trust only in effectiveness of the applied investigation methods. In the given case I prefer to use their rules, in order the paper were transparent for most readers, educated on the experimental-fitting method of investigation.

As concerns the third mistake, it is not yet corrected properly in the sense that the effective theory of the pair production effect is not yet constructed. It is clear only, that application of the quantum principles in solution of this problem leads to the blind alley.

In this paper we show that the dynamical methods of investigation (without a use of quantum principles) are founded logically. Besides, they are more effective in application to the investigation of the Dirac particle, than the conventional methods, based on the application of quantum principles.

2 Dynamical methods of investigation

We use a more developed mathematical technique for a description of quantum systems. This technique supposes that all essential information on the quantum dynamical system is contained in the dynamic system itself. Such specific quantum concepts as the wave function and principles of quantum mechanics appear to be only the means of description. The wave function as the means of description may be applied to both quantum and classical dynamic systems. But the quantum principles may be applied only to quantum dynamic systems, because they contains some constraints, which are not satisfied for classical systems. The quantum system and classical system distinguish dynamically (in additional terms in the action), but not in the way of description. This fact becomes to be clear, when both systems are described in the same terms. For instance, the quantum system and the corresponding classical system may be described in terms of the wave function, or both systems may be described in terms of the particle position and momentum. The difference between the various methods of description lies only in the convenience of their application. A use of the wave function is effective in description of quantum systems, because in this case the dynamic equations are linear. On the contrary, the description in terms of the particle position is convenient for description of classical systems, where dynamic equations are ordinary differential equations.

Progress in the development of the mathematical technique has a *mathematical ground: integration of dynamic equations.* This pure mathematical achievement has physical consequences. It appears that the quantum mechanics may be considered to be a statistical description of randomly moving particles. We underline that we investigate well known quantum systems, and all new results are *corollaries of the more developed methods of investigation.* It is meaningless to argue against the new obtained results by a reference to experimental data, because such arguments are arguments against the considered dynamic systems, but not against the methods of investigation. Experimental data may not be arguments against the mathematical methods of investigation in principle. As to the investigated dynamic systems, we admit that they may be imperfect and need an improvement, but this problem lies outside the framework of the paper.

We show new mathematical methods of investigation in the simple example of the Schrödinger particle $\mathcal{S}_{\mathrm{S}}$, i.e. the dynamic system $\mathcal{S}_{\mathrm{S}}$, described by the Schrödinger equation.

The action for the free nonrelativistic quantum particle $\mathcal{S}_{\mathrm{S}}$ has the following form

$$\mathcal{S}_{\mathrm{S}}: \qquad \mathcal{A}_{\mathrm{S}}[\psi,\psi^*] = \int \left\{ \frac{i\hbar}{2}\left(\psi^*\partial_0\psi - \partial_0\psi^*\cdot\psi\right) - \frac{\hbar^2}{2m}\boldsymbol{\nabla}\psi^*\boldsymbol{\nabla}\psi \right\} dtd\mathbf{x} \qquad (2.1)$$

where $\psi = \psi(t,\mathbf{x})$ is a complex one-component wave function, $\psi^* = \psi^*(t,\mathbf{x})$ is the complex conjugate to ψ, and m is the particle mass. The action (2.1) generates dynamic equations

$$i\hbar\partial_0\psi = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi, \qquad -i\hbar\partial_0\psi^* = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi^* \qquad (2.2)$$

The 4-current j^k and the energy-momentum tensor T_l^k are the canonical quantities associated with the action $\mathcal{A}_{\mathrm{S}}[\psi,\psi^*]$. They are determined by the relations

$$j^k = \{\rho,\mathbf{j}\} = \frac{i}{\hbar}\left(\frac{\partial\mathcal{L}}{\partial(\partial_k\psi^*)}\psi^* - \frac{\partial\mathcal{L}}{\partial(\partial_k\psi)}\psi\right) = \left\{\psi^*\psi, -\frac{i\hbar}{2m}\left(\psi^*\boldsymbol{\nabla}\psi - \boldsymbol{\nabla}\psi^*\cdot\psi\right)\right\} \qquad (2.3)$$

$$T_l^k = \frac{\partial\mathcal{L}}{\partial(\partial_k\psi^*)}\partial_l\psi^* + \frac{\partial\mathcal{L}}{\partial(\partial_k\psi)}\partial_l\psi - \delta_l^k\mathcal{L}, \qquad k,l = 0,1,2,3 \qquad (2.4)$$

where $\mathcal{L}$ is the Lagrangian density for the action (2.1)

$$\mathcal{L} = \frac{i\hbar}{2}\left(\psi^*\partial_0\psi - \partial_0\psi^*\cdot\psi\right) - \frac{\hbar^2}{2m}\boldsymbol{\nabla}\psi^*\boldsymbol{\nabla}\psi \qquad (2.5)$$

The dynamic system $\mathcal{S}_{\rm S}$ is determined completely by dynamic equations (2.2) and expressions (2.3), (2.4) for the 4-current and the energy-momentum tensor. Only connection between the particle and the wave functions is not described by these relations. This connection is described by means of the relations

$$\langle F(\mathbf{x},\mathbf{p})\rangle = B\int \mathrm{Re}\left\{\psi^* F(\mathbf{x},\hat{\mathbf{p}})\,\psi\right\} d\mathbf{x}, \qquad \hat{\mathbf{p}} = -\mathbf{i}\hbar\boldsymbol{\nabla}, \qquad B = \left(\int \psi^*\psi d\mathbf{x}\right)^{-1} \tag{2.6}$$

which define the mean value $\langle F(\mathbf{x},\mathbf{p})\rangle$ of any function $F(\mathbf{x},\mathbf{p})$ of the particle coordinates $\mathbf{x}$ and momentum $\mathbf{p}$. Application of the rules (2.6) is restricted by some conditions. They demand that the dynamic equations be linear and the wave function be a vector in the Hilbert space of states. We shall refer to the relations (2.6) together with the restrictions imposed on its applications as the quantum principles, because von Neumann has shown [12], that the quantum mechanics can be deduced from relations of the type (2.6), provided they are valid for all observable quantities. Thus, the interpretation of the wave function is carried out on the basis of the quantum principles, which are something external with respect to the dynamic system $\mathcal{S}_{\rm S}$.

In reality, the quantum principles are not necessary for interpretation of the dynamic system $\mathcal{S}_{\rm S}$. It is sufficient to make a proper change of dynamic variables and to describe the dynamic system $\mathcal{S}_{\rm S}$ in terms of the particle coordinates $\mathbf{x}$. Such a description does not contain the enigmatic wave function, whose meaning is unclear, and one does not need the quantum principles (2.6) for its interpretation. The Schrödinger particle $\mathcal{S}_{\rm S}$ is a partial case of the generalized Schrödinger particle $\mathcal{S}_{\rm gS}$, which is the dynamic system $\mathcal{S}_{\rm gS}$, described by the action

$$\mathcal{A}_{\rm gS}[\psi,\psi^*] = \int\left\{\frac{i\hbar}{2}\left(\psi^*\partial_0\psi - \partial_0\psi^*\cdot\psi\right) - \frac{\hbar^2}{2m}\boldsymbol{\nabla}\psi^*\boldsymbol{\nabla}\psi + \frac{\hbar^2}{8m}\sum_{\alpha=1}^{\alpha=3}(\boldsymbol{\nabla}s_\alpha)^2\rho\right\}\mathrm{d}^4x \tag{2.7}$$

$$\rho\equiv\psi^*\psi, \qquad \mathbf{s}\equiv\frac{\psi^*\boldsymbol{\sigma}\psi}{\rho}, \qquad \boldsymbol{\sigma}=\{\sigma_\alpha\}, \qquad \alpha=1,2,3, \tag{2.8}$$

Here $\psi = \binom{\psi_1}{\psi_2}$, $\psi^* = (\psi_1^*,\psi_2^*)$ is the two-component wave function, and σ_α are the Pauli matrices. The 4-current is defined by the relation (2.3) with two-component wave function ψ. In the case, when components ψ_1 and ψ_2 are linear dependent (for instance, $\psi = \binom{\psi_1}{0}$), the mean spin vector $\mathbf{s}$ =const, and the last term in the action (2.7) vanishes. In this case the dynamic system $\mathcal{S}_{\rm gS}$ turns into the dynamic system (2.1).

One can show, that the dynamic system $\mathcal{S}_{\rm gS}$ is another representation of the dynamic system $\mathcal{E}[\mathcal{S}_{\rm st}]$, i.e. the action for $\mathcal{S}_{\rm gS}$ can be obtained from the action for the dynamic system $\mathcal{E}[\mathcal{S}_{\rm st}]$ by means of a proper change of variables [13].

The dynamic system $\mathcal{E}[\mathcal{S}_{\rm st}]$ is a statistical ensemble of stochastic particles $\mathcal{S}_{\rm st}$.

It is described by the action

$$\mathcal{E}\left[\mathcal{S}_{\rm st}\right]: \qquad \mathcal{A}_{\mathcal{E}[\mathcal{S}_{\rm st}]}\left[\mathbf{x},\mathbf{u}_{\rm st}\right]=\int\left\{\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^{2}+\frac{m}{2}\mathbf{u}_{\rm st}^{2}-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}_{\rm st}\right\}dtd\boldsymbol{\xi} \tag{2.9}$$

where $\mathbf{u}_{\rm st}=\mathbf{u}_{\rm st}\left(t,\mathbf{x}\right)$ is a vector function of arguments $t,\mathbf{x}$ (not of $t,\boldsymbol{\xi}$), and $\mathbf{x}=\mathbf{x}\left(t,\boldsymbol{\xi}\right)$ is a 3-vector function of independent variables $t,\boldsymbol{\xi}=\{\xi_{1,}\xi_{2},\xi_{3}\}$. Dynamic equations for the dynamic system $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$ are obtained as a result of variation of the action (2.9) with respect to dependent dynamic variables $\mathbf{x},\mathbf{u}_{\rm st}$. In the action (2.9) the variables $\boldsymbol{\xi}$ label stochastic systems $\mathcal{S}_{\rm st}$, constituting the statistical ensemble. The operator $\boldsymbol{\nabla}$ is defined in the space of coordinates $\mathbf{x}$ by the relation

$$\boldsymbol{\nabla}=\{\partial_{1},\partial_{2},\partial_{3}\}\equiv\left\{\frac{\partial}{\partial x^{1}},\frac{\partial}{\partial x^{2}},\frac{\partial}{\partial x^{3}}\right\} \tag{2.10}$$

The 3-vector $\mathbf{u}_{\rm st}$ describes the mean value of the stochastic component of the particle motion, which is considered to be a function of the variables $t,\mathbf{x}$. The first term $\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^{2}$ describes the energy of the regular component of the stochastic particle motion. The second term $m\mathbf{u}_{\rm st}^{2}/2$ describes the energy of the random component of velocity. The components $\frac{d\mathbf{x}}{dt}$ and $\mathbf{u}_{\rm st}$ of the total velocity are connected with different degrees of freedom, and their energies should be added in the expression for the Lagrange function density. The last term $-\hbar\boldsymbol{\nabla}\mathbf{u}_{\rm st}/2$ describes interplay between the velocity $\frac{d\mathbf{x}}{dt}$ of the regular component and the random one $\mathbf{u}_{\rm st}$.

The action (2.9) is a sum (integral) of actions for independent stochastic systems $\mathcal{S}_{\rm st}$, labelled by the parameters $\boldsymbol{\xi}=\{\xi_{1},\xi_{2},\xi_{3}\}$. Any stochastic system $\mathcal{S}_{\rm st}$ is a stochastic particle, whose state is described by its coordinate $\mathbf{x}\left(t\right)$. The action for the stochastic system $\mathcal{S}_{\rm st}$ is obtained from the action (2.9) for $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$. It has the form

$$\mathcal{S}_{\rm st}: \qquad \mathcal{A}_{\mathcal{S}_{\rm st}}\left[\mathbf{x},\mathbf{u}_{\rm st}\right]=\int\left\{\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^{2}+\frac{m}{2}\mathbf{u}_{\rm st}^{2}-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}_{\rm st}\right\}dt \tag{2.11}$$

where $\mathbf{x}=\mathbf{x}\left(t\right)$. In reality, the action (2.11) is not well defined mathematically, if $\hbar\neq0$. It is only symbolic, because the operator (2.10) is defined in the vicinity of the point $\mathbf{x}$, but not at the point $\mathbf{x}$ itself. As a result the dynamic equations for the stochastic system $\mathcal{S}_{\rm st}$ do not exist, if $\hbar\neq0$. This fact agrees with the stochasticity of $\mathcal{S}_{\rm st}$. By definition the system $\mathcal{S}_{\rm st}$ is stochastic, if there exist no dynamic euations for $\mathcal{S}_{\rm st}$. If we cut off interaction with the stochastic agent, setting $\hbar=0$ in the action (2.11) (or remove two last terms), we obtain the well defined action for the free nonrelativistic deterministic particle $\mathcal{S}_{\rm d}$

$$\mathcal{S}_{\rm d}: \qquad \mathcal{A}_{\mathcal{S}_{\rm d}}\left[\mathbf{x},\mathbf{u}_{\rm st}\right]=\int\left\{\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^{2}+\frac{m}{2}\mathbf{u}_{\rm st}^{2}\right\}dt, \qquad \mathbf{x}=\mathbf{x}\left(t\right) \tag{2.12}$$

The Schrödinger particle $\mathcal{S}_{\rm S}$ (2.1) is a partial case of the dynamic system $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$ (2.9), whereas the generalized Schrödinger particle $\mathcal{S}_{\rm gS}$ (2.7) coincide with the dynamic system $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$ (2.9). The action (2.7) may be obtained from the action (2.9) mathematically by means of a proper change of variables. (see Appendix A).

Interpretation of the dynamic system (2.9) is very simple, but dynamic equations for $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ are rather complicated. They have the form

$$\frac{\delta \mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}}{\delta \mathbf{x}} = -m\frac{d^2\mathbf{x}}{dt^2} + \boldsymbol{\nabla}\left(\frac{m}{2}\mathbf{u}_{\mathrm{st}}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}_{\mathrm{st}}\right) = 0 \tag{2.13}$$

$$\frac{\delta \mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}}{\delta \mathbf{u}_{\mathrm{st}}} = m\mathbf{u}_{\mathrm{st}}\rho + \frac{\hbar}{2}\boldsymbol{\nabla}\rho = 0, \tag{2.14}$$

where ρ is the function of derivatives of $\mathbf{x}$ with respect to $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, determined by the relation

$$\rho = \left[\frac{\partial\left(x^1, x^2, x^3\right)}{\partial\left(\xi_1, \xi_2, \xi_3\right)}\right]^{-1} = \frac{\partial\left(\xi_1, \xi_2, \xi_3\right)}{\partial\left(x^1, x^2, x^3\right)} \tag{2.15}$$

Resolving the relation (2.14) with respect to $\mathbf{u}_{\mathrm{st}}$ in the form

$$\mathbf{u}_{\mathrm{st}} = -\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho, \tag{2.16}$$

and eliminating $\mathbf{u}_{\mathrm{st}}$ from (2.13), we obtain

$$m\frac{d^2\mathbf{x}}{dt^2} = -\boldsymbol{\nabla}U\left(\rho, \boldsymbol{\nabla}\rho\right), \qquad U\left(\rho, \boldsymbol{\nabla}\rho\right) = \frac{\hbar^2}{8m}\left(\frac{\left(\boldsymbol{\nabla}\rho\right)^2}{\rho^2} - 2\frac{\boldsymbol{\nabla}^2\rho}{\rho}\right) \tag{2.17}$$

Thus, dynamic equations, generated by the action (2.9), describe the regular motion component of any particle $\mathcal{S}_{\mathrm{st}}$, as a motion in a very complicated potential field U, depending on the distribution of all particles of the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$. Of course, the trajectories $\mathbf{x} = \mathbf{x}\left(t, \boldsymbol{\xi}\right)$ do not describe the motion of individual stochastic particles. They describe only statistical average motion of stochastic particles. The situation reminds situation in the gas dynamics. The dynamic equations of the gas dynamics describe the motion of the "gas particles", which contain many molecules. Motion of the gas molecules is random and chaotic. It cannot be described by the gas dynamics equations, which describe only regular component of the molecule motion.

Note, that the term $\frac{m}{2}\mathbf{u}_{\mathrm{st}}^2$ in (2.11) looks as a kinetic energy, but according to (2.16) it does not depend on the temporary derivative $\dot{\mathbf{x}}$, and in dynamic equations it acts as a potential energy.

The statistical ensemble (2.9) may be considered to be some fluid. We may speak about the flow of the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$, keeping in mind, that dynamic equation (2.17) for the dynamic system $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ may be interpreted as hydrodynamic equation for some "quantum" fluid.

On the contrary, the dynamic equations, generated by the action (2.1), are linear and rather simple, whereas their interpretation is very complicated, because it uses the principles of quantum mechanics (2.6). Thus, the description by means of the action (2.9) admits a simple interpretation, but dynamic equations are very complicated for a solution.

If the actions (2.1) and (2.9) describe the same dynamic system (further for brevity we shall speak about equivalence of dynamic systems (2.1) and (2.9), although in reality the action (2.1) is only a partial case of the action (2.9)), it is reasonable to use the dynamic system $\mathcal{E}\left[\mathcal{S}_{\text{st}}\right]$ as starting point for the statement of the problem and for interpretation of the results obtained, whereas the dynamic system $\mathcal{S}_{\text{S}}$ will be used only for solution of dynamic equations, which have a simple form in terms of the wave function.

Why was this evident circumstance not used before? Why was the problem of the stochastic motion of microparticles stated in terms of enigmatic wave function? The answer is very simple. The connection between two different forms (2.1) and (2.9) of the action for the Schrödinger particle *has not been known for a long time.*

It is known, that the Schrödinger equation can be written in the hydrodynamical form [14]. D. Bohm [15] used this circumstance for the hydrodynamic interpretation of quantum mechanics. But it was only interpretation of the quantum principles in the hydrodynamical terms. He failed to eliminate the quantum principles and the wave function from the foundation of the quantum mechanics, and the wave function remained to be an enigmatic object – the vector in the Hilbert space. One failed to connect the wave function with the hydrodynamic variables: the density ρ and the velocity $\mathbf{v}$. In more exact terms the connection between the wave function and hydrodynamic variables ρ, $\mathbf{v}$ was established, but it was a one-way connection. In the case of the irrotational flow the hydrodynamical variables can be expressed via the wave function ψ, but one cannot do this in the case of the irrotational flow. Hence, one can transit from the description in terms of the wave function to the description in terms of ρ, $\mathbf{v}$, but one cannot transit from the hydrodynamic description in terms of ρ, $\mathbf{v}$ to a description in terms of ψ, because, in general, the fluid flow is rotational, and the dynamic system (2.9) cannot be described in terms of the one-component wave function.

Let us present the wave function in the form

$$\psi = \sqrt{\rho}e^{i\varphi}, \tag{2.18}$$

substitute it in the Schrödinger equation (2.2) and separate the real and imaginary parts of the equation. We obtain two real equations

$$\partial_0 \ln \rho = -\frac{\hbar}{m}\left(\nabla^2\varphi + \nabla \ln \rho \nabla\varphi\right) \tag{2.19}$$

$$\partial_0\varphi + \frac{\hbar}{2m}\left(\nabla\varphi\right)^2 = \frac{\hbar}{2m}\left(\frac{1}{2}\nabla^2 \ln \rho + \left(\frac{1}{2}\nabla \ln \rho\right)^2\right) \tag{2.20}$$

To obtain hydrodynamic equation, one needs to take gradient of the equation (2.20) and introduce the velocity $\mathbf{v} = \{v^1, v^2, v^3\}$ by means of the relation

$$\mathbf{v} = \frac{\hbar}{m}\boldsymbol{\nabla}\varphi \tag{2.21}$$

We obtain

$$\partial_0 \rho + \partial_\alpha (\rho v^\alpha) = 0, \qquad \partial_0 v^\alpha + v^\beta \partial_\beta v^\alpha = -\frac{1}{\rho} \partial_\beta P^{\alpha\beta}, \qquad \alpha = 1, 2, 3 \tag{2.22}$$

where $P^{\alpha\beta}$ is the tension tensor

$$P^{\alpha\beta} = \frac{\hbar^2}{4m^2} \left(\frac{(\partial_\alpha \rho) \partial_\beta \rho}{\rho} - \partial_\alpha \partial_\beta \rho \right) \tag{2.23}$$

The hydrodynamic equations (2.22) are obtained as a result of differentiation of the equation (2.2), written in terms of the wave function. It means that to transit from the hydrodynamic equations (2.22) to the equation, written in terms of the wave function, one needs to integrate the hydrodynamic equations (2.22). Besides, in the case of the irrotational flow the wave function is presented in terms of ρ and hydrodynamical potential φ. The same is valid in the general case, but the number of the hydrodynamical potentials is to be more than one, and it is necessary to introduce additional hydrodynamic (Clebsch) potentials.

The problem of integration of the hydrodynamical equations is rather complicated problem, which has been solved only in the end of eighties [16]. To solve this problem, it was necessary to develop a special Jacobian technique [13], which was used already by Clebsch [17, 18].

As soon as the hydrodynamic equations for the ideal fluid have been integrated, it becomes clear, that the wave function is simply a method of the ideal fluid description. The wave function ψ ceases to be an enigmatic vector of the Hilbert space, whose meaning was obtained only via quantum principles. Now one can determine the chain of the dynamic variable transformations which turn the action (2.9) into the action (2.7) (for details see Appendix A). As a result the action (2.9) may be used as a starting point for the description of the quantum Schrödinger particle $\mathcal{S}_\mathrm{S}$. At such a description the quantum principles (2.6) are not needed, because they are only a tool for interpretation of the wave function.

The statistical ensemble (2.9) as the starting point of the quantum description has a series of advantages over the action (2.1):

1. The statistical ensemble (2.9) is a very transparent construction founded on the simple physical idea, that the quantum particle is a stochastically moving particle.

2. It does not use quantum principles, which are nonrelativistic and cannot be extended properly to the relativistical case.

3. Statistical ensemble (2.9) is a more general construction, because the action (2.1) is a partial case of the action (2.9).

4. In the statistical description, founded on the action (2.9), there are three different aspects: dynamical factor, statistical factor and random factor. Each of these factors can be separated as a corresponding term in the action and investigated apart.

5. Description in terms of the dynamic system (2.9) is a statistical description. As any statistical description it contains two objects: the individual stochastic particle $\mathcal{S}_{\mathrm{st}}$ and the statistical average particle $\langle \mathcal{S}_{\mathrm{st}} \rangle$. Respectively there are two kinds of measurements: individual measurement (S-measurement) produced over the individual particle $\mathcal{S}_{\mathrm{st}}$ and the massive measurement (M-measurement) produced over the statistical average particle $\langle \mathcal{S}_{\mathrm{st}} \rangle$. These measurements have different properties, and their identification is inadmissible.

The complexity of dynamic equations (2.17) is the only defect of the statistical description (2.9).

We underline that the transition from the action (2.9) as a starting point to the action (2.1) is motivated mathematically. No additional physical arguments have been used for the substantiation of the statistical ensemble (2.9) as a starting point of the quantum description.

If we consider stationary states of the statistical ensemble, we are interested only in the value of the magnetic moment (which is supposed to be connected with the value of the total spin). In this case the spin origin is of no importance. But if we investigate the individual particle structure, it is important, whether the spin is generated by the individual particle, or it is generated by vorticity of the fluid flow. It is meaningless to classify the particles over their spin, if the spin has a collective origin, and the individual particle has not its own angular moment. If we use the conventional approach to the quantum mechanics, i.e. if we start from the action (2.1) we cannot separate dynamical and collective properties directly. Only starting from the action (2.9), we can try to solve this problem for concrete dynamic systems (for instance, the Dirac particle $\mathcal{S}_{\mathrm{D}}$ and the Pauli particle $\mathcal{S}_{\mathrm{P}}$). If a researcher stands on the viewpoint of the Copenhagen interpretation, where the wave function describes the state of individual particle, the statement of the problem seems to be incorrect for him.

The collective origin of the spin can be perceived, only using statistical approach presented by the action (2.9). The statistical description, founded on the action (2.9) leads to the statement that wave function describes a state of the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$, but not a state of a single quantum particle. Discussion of the question, what object is described by the wave function, has a long history. Some researchers [19] believe, that the wave function describes the state of a single quantum particle, whereas other ones [20, 21] believe that the wave function describes the state of the statistical ensemble. There is a long list of different opinions about this question, but we do not present them, because this problem is not a question of a belief. It can and must be solved on the basis of the mathematical formalism.

The problem is set as follows. What dynamic system is described by the action (2.1)? A single quantum particle, or a statistical ensemble of single particles? Let us go to the limit $\hbar \to 0$. Then the action (2.1) will describe the classical dynamic system $\mathcal{S}_{\mathrm{Scl}}$. If the dynamic system $\mathcal{S}_{\mathrm{Scl}}$ is a single classical particle, then the wave function describes the state of a single particle. If the dynamic system $\mathcal{S}_{\mathrm{cl}}$ is a statistical ensemble of classical particles, then the wave function describes the state of a statistical ensemble of single particles. One cannot go to the limit $\hbar \to 0$ in the action (2.1) directly, because the description of the dynamic system $\mathcal{S}_{\mathrm{S}}$ degenerates.

We make the change of variables

$$\psi \to \Psi_b = |\psi| \exp\left(\frac{\hbar}{b} \log \frac{\psi}{|\psi|}\right), \qquad \psi = |\Psi_b| \exp\left(\frac{b}{\hbar} \log \frac{\Psi_b}{|\Psi_b|}\right) \tag{2.24}$$

where $b \neq 0$ is some real constant. After this change of variables the action (2.1) turns into

$$\mathcal{S}_{\mathrm{S}}: \qquad \mathcal{A}_{\mathrm{S}}\left[\Psi_b, \Psi_b^*\right] = \int \left\{ \frac{ib}{2} \left(\Psi_b^* \partial_0 \Psi_b - \partial_0 \Psi_b^* \cdot \Psi_b\right) - \frac{b^2}{2m} \mathbf{\nabla} \Psi_b^* \mathbf{\nabla} \Psi_b - \frac{\hbar^2 - b^2}{2m} \left(\mathbf{\nabla} |\Psi_b|\right)^2 \right\} dt d\mathbf{x} \tag{2.25}$$

The dynamic equation takes the form

$$ib\partial_0 \Psi_b = -\frac{b^2}{2m} \mathbf{\nabla}^2 \Psi_b - \frac{\hbar^2 - b^2}{8m} \left(\frac{(\mathbf{\nabla}\rho)^2}{\rho^2} + 2\mathbf{\nabla} \frac{\mathbf{\nabla}\rho}{\rho} \right) \Psi_b, \qquad \rho \equiv \Psi_b^* \Psi_b \tag{2.26}$$

Instead of (2.3), we obtain

$$\rho = \Psi_b^* \Psi_b, \qquad \mathbf{j} = -\frac{ib}{2m} \left(\Psi_b^* \mathbf{\nabla} \Psi_b - \mathbf{\nabla} \Psi_b^* \cdot \Psi_b\right) \tag{2.27}$$

Setting $\hbar = 0$ in (2.25), (2.26), we obtain

$$\mathcal{S}_{\mathrm{Scl}}: \qquad \mathcal{A}_{\mathrm{Scl}}\left[\Psi_b, \Psi_b^*\right] = \int \left\{ \frac{ib}{2} \left(\Psi_b^* \partial_0 \Psi_b - \partial_0 \Psi_b^* \cdot \Psi_b\right) - \frac{b^2}{2m} \mathbf{\nabla} \Psi_b^* \mathbf{\nabla} \Psi_b + \frac{b^2}{2m} \left(\mathbf{\nabla} |\Psi_b|\right)^2 \right\} dt d\mathbf{x} \tag{2.28}$$

$$ib\partial_0 \Psi_b = -\frac{b^2}{2m} \mathbf{\nabla}^2 \Psi_b + \frac{b^2}{8m} \left(\frac{(\mathbf{\nabla}\rho)^2}{\rho^2} + 2\mathbf{\nabla} \frac{\mathbf{\nabla}\rho}{\rho} \right) \Psi_b, \qquad \rho \equiv \Psi_b^* \Psi_b \tag{2.29}$$

The action (2.28) describes the statistical ensemble of free classical particles and, hence, the wave function describes the statistical ensemble, but not a single particle. The action (2.28) may not describe a single classical particle, because the dynamic system (2.28) has infinite number of the freedom degrees. As far as the description (2.28) in terms of the wave function Ψ_b is a limit $\hbar \to 0$ of the description in terms of the wave function ψ, the wave function ψ in (2.1) may not describe a single quantum particle. Thus, the *supposition that the wave function describes a state of a single particle is incompatible with the quantum mechanics formalism.*

According to the Copenhagen interpretation of quantum mechanics the wave function ψ describes the state of a single quantum particle, whereas the state of a classical particle is described by its position $\mathbf{x}$ and its momentum $\mathbf{p}$. It is supposed that the wave function is a specific quantum quantity, which has no classical analog. In accordance with this approach one may not go to the limit $\hbar \to 0$ in the action (2.1), because the action vanishes, and the description degenerates.

The transformation (2.24) changes only the scale of the wave function phase $\ln(\psi/|\psi|)$, and this change may be very slight. The wave function Ψ_b is the valid wave function, which can be used, in particular, for calculation of average values by means of the relation (2.6). This calculation may be produced for any value of the constant b. The wave function Ψ_b describes the same state of $\mathcal{S}_{\mathrm{S}}$ at different values of the parameter b, because the state of the dynamic system does not determine the wave function uniquely, and the same state of $\mathcal{S}_{\mathrm{S}}$ may be described by different wave functions. From viewpoint of the statistical description (2.9) the wave function is not uniquely defined, because it is constructed of hydrodynamic potentials, i.e. it is a result of integration of uniquely defined velocity $\mathbf{v}$. The parameter b in the transformation (2.6) is a constant of integration.

We may set $b = \hbar$ in the relations (2.28), (2.29) and obtain a description of "classical particle " in the form containing the quantum constant $\hbar$.

$$\mathcal{S}_{\mathrm{Scl}}: \qquad \mathcal{A}_{\mathrm{Scl}}[\psi, \psi^*] = \int \left\{ \frac{i\hbar}{2}\left(\psi^* \partial_0 \psi - \partial_0 \psi^* \cdot \psi\right) - \frac{\hbar^2}{2m} \nabla \psi^* \nabla \psi \right.$$
$$\left. + \frac{\hbar^2}{2m} \left(\nabla |\psi|\right)^2 \right\} dt d\mathbf{x} \tag{2.30}$$

$$i\hbar \partial_0 \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi + \frac{\hbar^2}{8m} \left(\frac{(\nabla \rho)^2}{\rho^2} + 2\nabla \frac{\nabla \rho}{\rho} \right) \psi, \qquad \rho \equiv \psi^* \psi \tag{2.31}$$

The same result may be obtained from (2.28), (2.29) by means of the transformation inverse to the transformation (2.24). Formally the action (2.30) distinguishes from the action (2.1) in the last term, which describes a lack of quantum effects. The quantum constant in two first terms has no relation to quantum effects. The dependence on $\hbar$ is conditioned by a special choice of the arbitrary constant b.

The action (2.30) describes the dynamic system $\mathcal{S}_{\mathrm{Scl}} = \mathcal{E}[\mathcal{S}_{\mathrm{d}}]$ in the "quantum language", i.e. in terms of the wave function. The action

$$\mathcal{S}_{\mathrm{Scl}} = \mathcal{E}[\mathcal{S}_{\mathrm{d}}]: \qquad \mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{d}}]}[\mathbf{x}] = \int \frac{m}{2} \left(\frac{d\mathbf{x}}{dt} \right)^2 dt d\boldsymbol{\xi} \tag{2.32}$$

where $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$, describes the same dynamic system in the "classical language", i.e. in terms of classical variables $\mathbf{x}, \mathbf{p}$. In the same way the action (2.1) describes the dynamic system $\mathcal{S}_{\mathrm{S}} = \mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ in quantum language, whereas the action (2.9) describes the same dynamic system in the classical language. It is reasonable that the quantum system $\mathcal{S}_{\mathrm{S}}$ is described simpler in the quantum language, whereas the classical system $\mathcal{S}_{\mathrm{Scl}} = \mathcal{E}[\mathcal{S}_{\mathrm{d}}]$ is described simpler in the classical language. However, it is not a reason for the statement that the quantum system is to be described in the quantum language (in terms of the wave function).

Two different description of the classical system $\mathcal{S}_{\mathrm{cl}}$ can be used for interpretation of the rule (2.6) and for interpretation of the correspondence principle. The obtained results may be applied to the quantum system $\mathcal{S}_{\mathrm{S}}$, because the difference between the dynamic systems $\mathcal{S}_{\mathrm{S}} = \mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ and $\mathcal{S}_{\mathrm{Scl}} = \mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$, described respectively by actions (2.1) and (2.30), manifests itself only in the additional nonlinear term in the dynamic equation. The possibility of description $\mathcal{S}_{\mathrm{S}} = \mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ and $\mathcal{S}_{\mathrm{Scl}} = \mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$ in both languages (classical and quantum) shuts the door before the Copenhagen interpretation, where the wave function is supposed to describe a single particle. Thus, *there is neither reason nor excuse for application of the Copenhagen interpretation.*

The rules (2.6) are statistical relations, which can be applied to both classical and quantum statistical ensembles. Some results of their application appear to be rather curious. For instance, the momentum distribution

$$w\left(\mathbf{p}\right) = \psi_{\mathbf{p}}^{*}\psi_{\mathbf{p}}, \qquad \psi_{\mathbf{p}} = \frac{1}{\left(2\pi\right)^{3}}\int e^{\frac{i}{\hbar}\mathbf{p}\mathbf{x}}\psi\left(\mathbf{x}\right)d\mathbf{x} \tag{2.33}$$

at the state described by the wave function ψ appears to be rather the mean momentum distribution, than the momentum distribution [22]. Let us manifest the difference between the momentum distribution and the mean momentum distribution (distribution over mean momenta) in the example of the ideal gas.

Let us consider a gas, moving with the constant velocity $\mathbf{u}\left(\mathbf{x}\right) = \mathbf{u} =$const. As any fluid such a gas motion may be described by the wave function. It has the form

$$\psi\left(t,\mathbf{x}\right) = A\exp\left(-\frac{im\mathbf{u}\mathbf{x}}{\hbar} - \frac{im\mathbf{u}^{2}}{2\hbar}t\right), \qquad A = \mathrm{const} \tag{2.34}$$

where m is the mass of the gas molecule. The density ρ and the velocity $\mathbf{u}$, described by the formulas (2.3), (2.34)

$$\rho = \psi^{*}\psi, \qquad \mathbf{u} = \frac{\mathbf{j}}{\rho} = -\frac{i\hbar}{2m\psi^{*}\psi}\left(\psi^{*}\boldsymbol{\nabla}\psi - \boldsymbol{\nabla}\psi^{*}\cdot\psi\right) \tag{2.35}$$

are constant and satisfy the hydrodynamic equations with arbitrary form of the internal energy.

Calculation by means of the formula (2.33) gives

$$w\left(\mathbf{p}\right)d\mathbf{p} = \psi_{\mathbf{p}}^{*}\psi_{\mathbf{p}}d\mathbf{p} = B\delta\left(\mathbf{p} - m\mathbf{u}\right)d\mathbf{p} \tag{2.36}$$

where B is a constant and δ is the Dirac δ-function. Chaotic motion of molecules is described by the Maxwell distribution

$$F\left(\mathbf{x},\mathbf{p}\right)d\mathbf{p} = \frac{1}{\left(2\pi mkT\right)^{3/2}}\exp\left\{-\frac{\left(\mathbf{p} - m\mathbf{u}\left(\mathbf{x}\right)\right)^{2}}{2mkT}\right\}d\mathbf{p} \tag{2.37}$$

It depends on the gas temperature T and has nothing to do with the distribution (2.36). The gas motion is described by the gas dynamic equations, which do not take into account chaotic molecular motion and do not contain a reference to the Maxwell distribution. What is the distribution (2.36)?

Let us divide the volume V of the gas flow into similar cubic cells $V_1, V_2, ...V_N$, $N \gg 1$. Let the following conditions be satisfied

$$l_\mathrm{c} \ll L, \qquad |v_\mathrm{t}\tau_c| \ll L, \qquad |\mathbf{u}(\mathbf{x})| \ll v_\mathrm{t} = \sqrt{\frac{3kT}{m}} \tag{2.38}$$

where L is the linear size of the cell, l_c is the mean path between the molecule collisions, τ_c is the mean time between the collisions and v_t is the mean thermal velocity of molecules.

Let us calculate the mean momentum $\langle \mathbf{p}_i \rangle$ of the gas molecule in the cell V_i. We obtain $\langle \mathbf{p}_i \rangle = m\mathbf{u}(\mathbf{x})$, $\mathbf{x} \in V_i$, $i = 1, 2, ...N$. The set of all $\langle \mathbf{p}_i \rangle$, $i = 1, 2, ...N$ forms the mean momentum distribution. This distribution is determined completely by the gas flow, and it has nothing to do with the Maxwell momentum distribution (2.37), which describes both the regular and random components of the molecule momenta. Under conditions (2.38) the mean momentum distribution is much narrower, than the Maxwell distribution, because the Maxwell distribution takes into account the random component of the molecule velocity, and in the given case the random component is much larger, than the regular one. In the given case the relation (2.36) may be rewritten in the form

$$w(\langle \mathbf{p} \rangle)\, d\langle \mathbf{p} \rangle = \psi_\mathbf{p}^* \psi_\mathbf{p} d\langle \mathbf{p} \rangle = B\delta(\langle \mathbf{p} \rangle - m\mathbf{u})\, d\langle \mathbf{p} \rangle \tag{2.39}$$

where $\langle \mathbf{p} \rangle$ is the mean particle momentum.

Besides, any $\langle \mathbf{p}_i \rangle$ is labelled by the index i, or by the coordinate $\mathbf{x}_i$ of the volume V_i. It means, that variables $\mathbf{x}$ and $\langle \mathbf{p}_i \rangle$ are not independent, and mutual coordinate-momentum distribution does not exist. In the Copenhagen interpretation the lack of the mutual coordinate-momentum distribution is explained by the noncommutativity of operators $\mathbf{x}$ and $\mathbf{p} = -i\hbar\nabla$, and the distribution (2.33) is considered to be a distribution over the stochastic component of the momentum (some quantum analog of the Maxwell distribution). There are other unexpected characteristics of the rule (2.6).

The rule (2.6) is only a method to obtain the information contained in the investigated dynamic system. This information can be obtained from the dynamic system by other methods. An application of the rule (2.6) does not add any real information beyond that one, which is contained in the investigated dynamic system.

3 Dynamic disquantization

The quantum langauge, i.e. the description, containing the quantum constant $\hbar$, may be used for a description of a classical dynamic system, because the quantum constant $\hbar$ may be used instead of the arbitrary dynamical constant b. Replacement of dynamical constant b by the quantum constant is produced to make the dynamic equations to be linear. For instance, in the action (2.25) the quantum constant $\hbar$

is used naturally, i.e. in the sense that setting $\hbar = 0$, we suppress the quantum effects. In the action (2.1) for the same dynamic system the quantum constant $\hbar$ is used artificially in the sense that setting $\hbar = 0$, we do not suppress the quantum effects. Furthermore, setting $\hbar = 0$, we destroy any description. But the action (2.1) generates linear dynamic equation, and this circumstance is a reason of the artificial identification $b = \hbar$, when the dynamical constant b is identified with the quantum constant $\hbar$.

Such an artificial identification may be produced in other quantum systems (for instance, in $\mathcal{S}_{\mathrm{D}}$ and $\mathcal{S}_{\mathrm{P}}$), and we cannot be sure, that setting $\hbar = 0$, we suppress the quantum effects. Besides, we cannot be sure that, using the transformation of the type (2.24), we can separate the quantum terms from dynamical and statistical ones.

We need a more effective formal dynamical procedure, which could suppress the stochastic terms. Let us compare dynamic equations (2.17) for $\mathcal{S}_{\mathrm{S}} = \mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ written in the form

$$\frac{d\mathbf{p}}{dt} = -\boldsymbol{\nabla} U\left(\rho, \boldsymbol{\nabla}\rho\right), \qquad \frac{d\mathbf{x}}{dt} = \frac{\mathbf{p}}{m}, \qquad U\left(\rho, \boldsymbol{\nabla}\rho\right) = \frac{\hbar^2}{8m}\left(\frac{\left(\boldsymbol{\nabla}\rho\right)^2}{\rho^2} - 2\frac{\boldsymbol{\nabla}^2\rho}{\rho}\right) \tag{3.1}$$

with the dynamic equations for $\mathcal{S}_{\mathrm{Scl}} = \mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$, which have the form

$$\frac{d\mathbf{p}}{dt} = 0, \qquad \frac{d\mathbf{x}}{dt} = \frac{\mathbf{p}}{m} \tag{3.2}$$

where $\mathbf{x} = \mathbf{x}\left(t, \boldsymbol{\xi}\right)$, $\mathbf{p} = \mathbf{p}\left(t, \boldsymbol{\xi}\right)$. Dynamic equations (3.1), are the partial differential equations, because ρ is defined by the relation (2.15), containing derivatives with respect to ξ_α, $\alpha = 1, 2, 3$, whereas dynamic equations (3.2) are ordinary differential equations. Equations (3.2) contain derivatives only in one direction in the space of independent variables $\{t, \boldsymbol{\xi}\}$, whereas equation (3.1) contain derivatives in different directions of the space of independent variables $\{t, \boldsymbol{\xi}\}$. This property is conserved at any change of independent dynamical variables, and, in particular, at the change $\{t, \boldsymbol{\xi}\} \rightarrow \{t, \mathbf{x}\}$. If a system of partial differential equations contains derivative only in one direction of the space of independent variables, this system can be reduced to the system of ordinary differential equations by means of a proper change of variables.

If we want to suppress the quantum effects, we must to reduce the system of partial differential equations to the system of ordinary differential equations. To make this, we should project derivatives in the space of independent variables onto some direction. Then the system will contain derivatives only in one direction, and hence it may be reduced to the system of ordinary differential equations. Onto what direction should derivatives in the system (3.1) be projected, to obtain the system (3.2)?

Such a direction is described by the 4-current $j^k = \{\rho, \mathbf{j}\} = \left\{j^k\right\}$, $k = 0, 1, 2, 3$ in the space-time. The projection should be made in the space of independent variables $\{t, \mathbf{x}\}$, i.e. in the space-time. It is convenient to choose dependent variables in such a way, that the 4-current j^k were one of dependent variables. We take the action (A.22) for the dynamic system $\mathcal{S}_{\mathrm{S}} = \mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}\left[\varphi, \boldsymbol{\xi}, j\right] = \int \left\{ \frac{m}{2}\frac{j^\alpha j^\alpha}{\rho} - bj^k\left(\partial_k\varphi + g^\alpha\left(\boldsymbol{\xi}\right)\partial_k\xi_\alpha\right) - \frac{\hbar^2}{8m}\frac{\left(\boldsymbol{\nabla}\rho\right)^2}{\rho} \right\} d^4x, \quad (3.3)$$

where according to (A.15) and (A.19)

$$j^k = \{\rho, \mathbf{j}\} = \left\{\rho, \frac{b\rho}{m}\left(\boldsymbol{\nabla}\varphi + g^\alpha\left(\boldsymbol{\xi}\right)\boldsymbol{\nabla}\xi_\alpha\right)\right\} \qquad (3.4)$$

and $g^\alpha\left(\boldsymbol{\xi}\right)$, $\alpha = 1, 2, 3$ are arbitrary functions of argument $\boldsymbol{\xi}$.

The second term in the action (3.3) contains derivatives only in the direction of the 4-vector j^k. In the last term of (3.3) the derivatives are to be projected onto the vector j^k. We are to make the change

$$\partial_l \to \partial_{||l} = \frac{j_l j^k}{j_s j^s}\partial_k, \qquad l = 0, 1, 2, 3 \qquad (3.5)$$

in the action (3.3). We obtain

$$\frac{\left(\boldsymbol{\nabla}\rho\right)^2}{\rho} = \frac{\left(\partial_\alpha\rho\right)\left(\partial_\alpha\rho\right)}{\rho} \to \frac{j_\alpha j_\alpha\left(j^i\partial_i\rho\right)^2}{\rho\left(j^s j_s\right)^2} \qquad (3.6)$$

$$j_\alpha j_\alpha = \mathbf{j}^2 = \rho^2\mathbf{v}^2, \qquad j^s j_s = c^2\rho^2 - \rho^2\mathbf{v}^2$$

In the nonrelativistic approximation, when the velocity $|\mathbf{v}| \ll c$, we obtain the following estimation

$$\frac{\left(\boldsymbol{\nabla}\rho\right)^2}{\rho} \approx \frac{\mathbf{v}^2\left(j^i\partial_i\rho\right)^2}{c^4\rho^3} \qquad (3.7)$$

In the nonrelativistic approximation $c \to \infty$ the last term in the action (3.3) is to be neglected after the change (3.5). Thus, in the case of the Schrödinger particle $\mathcal{S}_{\mathrm{S}}$ the change (3.5) leads to a suppression of quantum effects.

We shall refer to the procedure (3.5) as the dynamical disquantization, because it transforms the Schrödinger particle $\mathcal{S}_{\mathrm{S}} = \mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ into the classical system $\mathcal{S}_{\mathrm{Scl}} = \mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$. The dynamical disquantization is the relativistic dynamical procedure. It does not refer to the quantum constant and suppresses any stochasticity regardless of its origin. From here on we shall use the dynamical disquantization for the suppression of stochasticity in quantum systems.

Strictly, the dynamical disquantization is to be applied to the dynamic equations. But in many cases the application of the dynamical disquantization to the action leads to the same result, as its application to the dynamic equations.

4 Nonrelativistic Dirac particle

The Dirac particle is the dynamic system $\mathcal{S}_{\mathrm{D}}$, described by the Dirac equation. The action $\mathcal{A}_{\mathrm{D}}$ for the dynamic system $\mathcal{S}_{\mathrm{D}}$ has the form

$$\mathcal{S}_{\mathrm{D}}: \qquad \mathcal{A}_{\mathrm{D}}[\bar{\psi},\psi] = c^2 \int (-mc\bar{\psi}\psi + \frac{i}{2}\hbar\bar{\psi}\gamma^l\partial_l\psi - \frac{i}{2}\hbar\partial_l\bar{\psi}\gamma^l\psi - \frac{e}{c}A_l\bar{\psi}\gamma^l\psi)d^4x \quad (4.1)$$

where m and e are respectively the mass and the charge of the Dirac particle, and c is the speed of the light. Here ψ is four-component complex wave function, ψ^* is the Hermitian conjugate wave function, and $\bar{\psi} = \psi^*\gamma^0$ is the conjugate one. γ^i, $i = 0,1,2,3$ are 4×4 complex constant matrices, satisfying the relation

$$\gamma^l\gamma^k + \gamma^k\gamma^l = 2g^{kl}I, \qquad k,l = 0,1,2,3. \quad (4.2)$$

where I is the 4×4 identity matrix, and g^{kl} =diag$(c^{-2},-1,-1,-1)$ is the metric tensor. The quantity A_k, $k = 0,1,2,3$ is the electromagnetic potential. The action (4.1) generates dynamic equation for the dynamic system $\mathcal{S}_{\mathrm{D}}$, known as the Dirac equation

$$\gamma^l\left(-i\hbar\partial_l + \frac{e}{c}A_l\right)\psi + mc\psi = 0 \quad (4.3)$$

and expressions for physical quantities: the 4-flux j^k of particles and the energy-momentum tensor T_l^k

$$j^k = c^2\bar{\psi}\gamma^k\psi, \qquad T_l^k = \frac{ic^2}{2}\left(\bar{\psi}\gamma^k\partial_l\psi - \partial_l\bar{\psi}\cdot\gamma^k\psi\right) \quad (4.4)$$

Here we obtain nonrelativistic approximation of the Dirac particle. Our investigation differs from the conventional derivation of this approximation (see for instance [1]) by consideration of the high frequency solutions. To obtain the nonrelativistic approximation $\mathcal{S}_{\mathrm{nD}}$ of the Dirac particle $\mathcal{S}_{\mathrm{D}}$, we use the following representation of the 4×4 Dirac γ-matrices

$$\gamma^0 = \frac{1}{c}\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma^\mu = \begin{pmatrix} 0 & -\sigma_\mu \\ \sigma_\mu & 0 \end{pmatrix}, \qquad \gamma^0\gamma^\mu = \frac{1}{c}\begin{pmatrix} \sigma_\mu & 0 \\ 0 & -\sigma_\mu \end{pmatrix} \quad (4.5)$$

where σ_μ, $\mu = 1,2,3$ are 2×2 Pauli matrices, and I is the 2×2 identity matrix. We use designations

$$\pi_l \equiv i\hbar\partial_l - \frac{e}{c}A_l, \qquad \pi_l^* \equiv -i\hbar\partial_l - \frac{e}{c}A_l, \qquad l = 0,1,2,3 \quad (4.6)$$

and representation of 4-component wave functions ψ in the form

$$\psi = \frac{\exp\left(-\frac{i}{2}\Omega t\right)}{\sqrt{2}}\begin{pmatrix} \psi_1 + \psi_2 \\ \psi_1 - \psi_2 \end{pmatrix}, \qquad \psi^* = \frac{\exp\left(\frac{i}{2}\Omega t\right)}{\sqrt{2}}\left(\psi_1^* + \psi_2^*, \psi_1^* - \psi_2^*\right), \quad (4.7)$$

$$\Omega = \frac{2mc^2}{\hbar} \quad (4.8)$$

where ψ_1 and ψ_2 are two-component wave functions, and asterisk (*) means the Hermitian conjugation. The action (4.1) for the Dirac particle $\mathcal{S}_{\rm D}$ can be written in the form

$$\mathcal{A}_{\rm D}\left[\psi_1,\psi_1^*,\psi_2,\psi_2^*\right]=\frac{1}{2}\int\left\{\psi_1^*\pi_0\psi_1+\psi_2^*\pi_0\psi_2+c\psi_1^*\sigma_\mu\pi_\mu\psi_2\right.$$

$$\left.+c\psi_2^*\sigma_\mu\pi_\mu\psi_1+2mc^2\psi_2^*\psi_2\right\}d^4x+\text{c.c.} \qquad (4.9)$$

where (c.c.) means the complex conjugate expression with respect to the previous term.

Dynamic equations have the form

$$\delta\psi_1^*\ :\qquad \pi_0\psi_1=-c\sigma_\mu\pi_\mu\psi_2 \qquad (4.10)$$

$$\delta\psi_2^*\ :\qquad \left(\pi_0+2mc^2\right)\psi_2=-c\sigma_\mu\pi_\mu\psi_1 \qquad (4.11)$$

Expressions (4.4) for the 4-current j^k and the energy-momentum tensor T_k^0 turn into

$$j^0=\left(\psi_1^*\psi_1+\psi_2^*\psi_2\right),\qquad j^\mu=c\left(\psi_1^*\sigma_\mu\psi_2+\psi_2^*\sigma_\mu\psi_1\right),\qquad \mu=1,2,3 \qquad (4.12)$$

$$T_0^0=2mc^2\left(\psi_1^*\psi_1+\psi_2^*\psi_2\right)+\frac{i\hbar}{2}\left(\psi_1^*\partial_0\psi_1+\psi_2^*\partial_0\psi_2\right)+\text{c.c.} \qquad (4.13)$$

$$T_\mu^0=\frac{i\hbar}{2}\left(\psi_1^*\partial_\mu\psi_1+\psi_2^*\partial_\mu\psi_2\right)+\text{c.c.},\qquad \mu=1,2,3 \qquad (4.14)$$

$$T_0^\nu=2mc^2\left(\psi_1^*\sigma_\nu\psi_2+\psi_2^*\sigma_\nu\psi_1\right)+\frac{i\hbar}{2}\left(\psi_1^*\sigma_\nu\partial_0\psi_2+\psi_2^*\sigma_\nu\partial_0\psi_1\right)+\text{c.c.},\qquad \nu=1,2,3$$

In the nonrelativistic case, when the speed of the light $c\to\infty$, we have $\left|\pi_\mu\psi_1\right|\ll mc\left|\psi_1\right|$, and according to (4.11) ψ_2 is a small quantity, provided the temporal frequency of the quantity ψ_2 is not too large. For simplicity we shall consider the case, when $A_0=0$ and $\pi_0=i\hbar\partial_0$. In this case, resolving equations (4.10) and (4.11) with respect to ψ_1, we obtain

$$\left(i\hbar\partial_0+2mc^2\right)i\hbar\partial_0\psi_1=c^2\pi_\nu\sigma_\nu\sigma_\mu\pi_\mu\psi_1 \qquad (4.15)$$

Using identity

$$\sigma_\mu\sigma_\nu\equiv\delta_{\mu\nu}I+i\varepsilon_{\mu\nu\alpha}\sigma_\alpha,\qquad \mu,\nu=1,2,3 \qquad (4.16)$$

where I is the 2×2 identity matrix and $\varepsilon_{\mu\nu\alpha}$ is the Levi-Chivita pseudotensor, we can transform the dynamic equation (4.15) to the form

$$\left(\frac{i}{\Omega}\partial_0+1\right)i\hbar\partial_0\psi_1=\hat{H}_{\rm P}\left(m\right)\psi_1,\qquad \Omega=\frac{2mc^2}{\hbar} \qquad (4.17)$$

Here Ω is the threshold frequency (4.8) of the pair production, and $\hat{H}_{\rm P}\left(m\right)$ is the Hamiltonian for the Pauli equation. It has the form

$$\hat{H}_{\rm P}\left(m\right)=\frac{\pi_\mu\pi_\mu}{2m}+\frac{ie\hbar}{2mc}\varepsilon_{\nu\mu\alpha}\partial_\nu A_\mu\sigma_\alpha=\frac{\boldsymbol{\pi}^2}{2m}+\frac{ie\hbar}{2mc}\mathbf{H}\boldsymbol{\sigma} \qquad (4.18)$$

where $\boldsymbol{\pi} = \{\pi_1, \pi_2, \pi_3\}$, $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ and $\mathbf{H} = \{H_1, H_2, H_3\}$ is the magnetic field

$$H_\alpha = \varepsilon_{\nu\mu\alpha}\partial_\nu A_\mu, \qquad \alpha = 1, 2, 3 \tag{4.19}$$

Let $u_\pm$ be the low frequency ($|\partial_0 u| \ll \Omega\,|u|$) solutions of the Pauli equations

$$i\hbar\partial_0 u_+ = \hat{H}_\mathrm{P}\,(m)\,u_+, \qquad i\hbar\partial_0 u_- = \hat{H}_\mathrm{P}\,(-m)\,u_- \tag{4.20}$$

In the nonrelativistic case, when $|\boldsymbol{\pi} u_\pm| \ll |mcu_\pm|$, the solution $u_\pm$ of equations (4.20) is a low frequency solution, i.e. $|\partial_0 u_\pm| \ll \Omega\,|u_\pm|$, as it follows from (4.20). The low frequency solution $u_+\,(t, \mathbf{x})$ is a solution of the equation (4.17), because the first term in lhs of (4.17) is small as compared with the second one.

Let us consider the case, when $\psi_1 = u_+\,(t, \mathbf{x})$ is the low frequency solution of the first equation (4.20). In this case the first equation (4.20) coincides with (4.15). It follows from (4.11), that

$$\psi_2 = -\frac{\sigma_\mu \pi_\mu}{2mc}\psi_1 \tag{4.21}$$

and $|\psi_2| \ll |\psi_1|$, because in the nonrelativistic approximation $|\pi_\mu\psi_1| \ll mc\,|\psi_1|$. It follows from (4.12)

$$j^0 = \psi_1^*\psi_1 \tag{4.22}$$

$$j^\mu = -\frac{1}{2m}\left(\psi_1^*\pi_\mu\psi_1 + \left(\pi_\mu^*\psi_1^*\right)\psi_1 + i\varepsilon_{\mu\nu\alpha}\psi_1^*\sigma_\alpha\pi_\nu\psi_1 - i\varepsilon_{\mu\nu\alpha}\left(\pi_\nu^*\psi_1^*\right)\sigma_\alpha\psi_1\right)$$

or

$$\mathbf{j} = -\frac{i\hbar}{2m}\left(\psi_1^*\boldsymbol{\nabla}\psi_1 - \boldsymbol{\nabla}\psi_1^*\cdot\psi_1\right) + \frac{e}{c}\mathbf{A}\psi_1^*\psi_1 + \frac{\hbar}{2m}\boldsymbol{\nabla}\times\left(\psi_1^*\boldsymbol{\sigma}\psi_1\right) \tag{4.23}$$

The high frequency expression $\exp(i\Omega t)\,u_-\,(t, \mathbf{x})$ is also a solution of (4.17). Indeed, substituting it in (4.17), we obtain after transformation of lhs

$$e^{i\Omega t}\left(\frac{i}{\Omega}\partial_0 - 1\right)i\hbar\partial_0 u_-\,(t, \mathbf{x}) = -e^{i\Omega t}\hat{H}_\mathrm{P}\,(-m)\,u_-\,(t, \mathbf{x}) \tag{4.24}$$

As far as $u_-\,(t, \mathbf{x})$ is a low frequency quantity ($|\partial_0 u_-| \ll \Omega\,|u_-|$), the first term in lhs of (4.24) is small as compared with the second one, and the function $u_-\,(t, \mathbf{x})$ is a solution of the second equation (4.20). Thus, the general solution of (4.17) has the form

$$\psi_1 = u_+\,(t, \mathbf{x}) + e^{i\Omega t}u_-\,(t, \mathbf{x}) \tag{4.25}$$

where $u_+\,(t, \mathbf{x})$ and $u_-\,(t, \mathbf{x})$ are two independent low frequency solutions of (4.20). Equations for $u_+\,(t, \mathbf{x})$ and $u_-\,(t, \mathbf{x})$ are the Pauli equations with different sign of the mass m.

The quantity ψ_2 is determined by the equation (4.11), whose general solution has the form

$$\psi_2 = \exp(i\Omega t)\left(w_0 - \int \frac{c\pi_\mu\sigma_\mu}{i\hbar}\psi_1\,(t, \mathbf{x})\exp(-i\Omega t)\,dt\right) \tag{4.26}$$

where Ω is determined by the relation (4.8), and w_0 is an indefinite constant, which can be included in the indefinite integral in (4.26). (The quantity w_0 is a constant, but not a function of $\mathbf{x}$, because the quantity ψ_2 is to satisfy equation (4.10), and it is possible only if w_0 =const). In the general case, when ψ_1 has the form (4.25), we obtain for ψ_2

$$\psi_2 = w_0 e^{i\Omega t} - e^{i\Omega t} \frac{c\pi_\mu \sigma_\mu}{i\hbar} \int \left(u_+ (t, \mathbf{x}) e^{-i\Omega t} + u_- (t, \mathbf{x})\right) dt \qquad (4.27)$$

In the limit $\Omega \to \infty$, the first term in the integral of (4.27) is small as compared with the second term (if $u_- (t, \mathbf{x}) \neq 0$). Integrating the first term in integral of (4.27), we consider $u_+ (t, \mathbf{x})$ as independent of t. We obtain

$$\psi_2 (t, \mathbf{x}) = -\frac{c\sigma_\mu \pi_\mu}{i\hbar} u (t, \mathbf{x}) e^{i\Omega t} - \frac{c\pi_\mu \sigma_\mu}{\hbar \Omega} u_+ (t, \mathbf{x}) \qquad (4.28)$$

where

$$u (t, \mathbf{x}) = \int u_- (t, \mathbf{x}) dt \qquad (4.29)$$

and the arbitrary constant w_0 is included in the indefinite integral.

Substituting expressions (4.25), (4.28) in (4.12), we express the 4-current via solutions u_+ and u_- of the Pauli equation (4.20). The 4-current j^k has regular component j^k_{reg} and oscillating component j^k_{os}, which oscillates with the high frequency Ω. We obtain for j^k

$$j^k = j^k_{\text{reg}} + j^k_{\text{os}}, \qquad k = 0, 1, 2, 3$$

$$j^0_{\text{reg}} = u_+^* u_+ + u_-^* u_- + \frac{c^2}{\hbar^2} \left(\pi_\mu^* u^*\right) \sigma_\mu \sigma_\nu \pi_\nu u + \frac{1}{4m^2 c^2} \left(\pi_\mu^* u_+^*\right) \sigma_\mu \sigma_\nu \pi_\nu u_+ \qquad (4.30)$$

$$j^\mu_{\text{reg}} = -\frac{1}{2m} \left(u_+^* \sigma_\mu \sigma_\nu \pi_\nu u_+ - 2i \frac{mc^2}{\hbar} u_-^* \sigma_\mu \sigma_\nu \pi_\nu u\right) + \text{c.c.}, \qquad \mu = 1, 2, 3 \qquad (4.31)$$

$$\begin{aligned} j^0_{\text{os}} &= \left(u_+^* u_- + u_-^* u_+\right) \cos (\Omega t) + i \left(u_-^* u_+ - u_+^* u_-\right) \sin (\Omega t) \\ &+ \frac{1}{2m\hbar} \left(i\pi_\mu^* u^* \sigma_\mu \sigma_\nu \pi_\nu u_+ - i\pi_\mu^* u_+^* \sigma_\mu \sigma_\nu \pi_\nu u\right) \cos (\Omega t) \\ &+ \frac{1}{2m\hbar} \left(\pi_\mu^* u^* \sigma_\mu \sigma_\nu \pi_\nu u_+ + \pi_\mu^* u_+^* \sigma_\mu \sigma_\nu \pi_\nu u\right) \sin (\Omega t) \end{aligned} \qquad (4.32)$$

$$\begin{aligned} j^\mu_{\text{os}} = &-\frac{1}{2m} \left(u_-^* \sigma_\mu \sigma_\nu \pi_\nu u_+ - 2i \frac{mc^2}{\hbar} u_+^* \sigma_\mu \sigma_\nu \pi_\nu u\right) \cos (\Omega t) + \text{c.c} \\ &-\frac{1}{2m} \left(-iu_-^* \sigma_\mu \sigma_\nu \pi_\nu u_+ + 2\frac{mc^2}{\hbar} u_+^* \sigma_\mu \sigma_\nu \pi_\nu u\right) \sin (\Omega t) + \text{c.c.} \end{aligned} \qquad (4.33)$$

where (c.c.) means the expression, which is complex conjugate to the previous term.

Note that the Dirac particle is charged. It means that the states of the Dirac particle, containing oscillating charge density ej^0, or oscillating current density $e\mathbf{j}$, are unstable with respect to electromagnetic radiation. As a result of the electromagnetic emanation, the Dirac particle transits to such a state, where j^k does not depend on time. Such a situation takes place for the states of the electron in the atom. Only stationary states, where j^k does not depend on time, are stable. The frequency $\Omega = 2mc^2/\hbar$ is very high, and the instability is very strong in the sense, that the time of transition to the stable state is very short.

It follows from the expressions (4.32) and (4.33), that if the state of the Dirac particle contains only low frequencies ($u_+ \neq 0, \quad u = 0$), or only high frequencies ($u_+ = 0, \quad u \neq 0$), the oscillating 4-current vanishes ($j_{\text{os}}^k = 0,\ k = 0,1,2,3$). Is it possible such a situation, that the oscillating 4-current j_{os}^k vanishes at the state, where there are low frequency components and the high frequency ones together?

To investigate this problem, we use the relation (4.29) and rewrite the expression (4.32) for j_{os}^0 in the form

$$j_{\text{os}}^0 = \left(u_+^* \partial_0 u - \frac{i}{2m\hbar}\left(\pi_\mu^* u_+^*\right)\sigma_\mu \sigma_\nu \pi_\nu u\right)\cos(\Omega t) + \text{c.c.}$$

$$+\left(i\left(\partial_0 u^*\right) u_+ + \frac{1}{2m\hbar}\left(\pi_\nu^* u^*\right)\sigma_\nu \sigma_\mu \pi_\mu u_+\right)\sin(\Omega t) + \text{c.c.} \tag{4.34}$$

The condition of vanishing j_{os}^0 has the form

$$u_+^* \partial_0 u - \frac{i}{2m\hbar}\left(\pi_\mu^* u_+^*\right)\sigma_\mu \sigma_\nu \pi_\nu u = 0 \tag{4.35}$$

We consider the case, when the electromagnetic field does not depend on t, and hence operator ∂_0 commutes with operators π_μ, defined by (4.6). Then according to (4.29) and (4.20) the function u satisfies the equation

$$i\hbar\partial_0 u = \hat{H}_{\text{P}}(-m)\, u + w_0 = -\frac{1}{2m}\sigma_\mu \sigma_\nu \pi_\mu \pi_\nu u + w_0 \tag{4.36}$$

where w_0 is an arbitrary complex two-component constant. We obtain

$$-\frac{1}{2m}u_+^* \sigma_\mu \sigma_\nu \pi_\mu \pi_\nu u + \frac{1}{2m}\left(\pi_\mu^* u_+^*\right)\sigma_\mu \sigma_\nu \pi_\nu u + u_+^* w_0 = 0 \tag{4.37}$$

Condition of vanishing $\mathbf{j}_{\text{os}}$ has the form

$$\pi_\nu^* u_+^* \sigma_\mu \sigma_\nu u_- - 2i\frac{mc^2}{\hbar}u_+^* \sigma_\mu \sigma_\nu \pi_\nu u = 0, \qquad \mu = 1,2,3 \tag{4.38}$$

5 Plane waves of nonrelativistic Dirac particle

We describe plane waves of the Dirac particle in terms of the four-component wave function Ψ, defined by the relation

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \frac{\exp\left(-\frac{i}{2}\Omega t\right)}{\sqrt{2}}\begin{pmatrix} I & I \\ I & -I \end{pmatrix}\psi \tag{5.1}$$

where ψ is the dynamical variable of the action (4.1), ψ_1, ψ_2 are dynamical variables of the action (4.9), and I is the 2×2 identity matrix. We have two kinds of wave function describing the plane waves: the low frequency wave function $\Psi_{\rm lf}$ and the high frequency one $\Psi_{\rm hf}$

$$\Psi_{\rm lf}=\left(\begin{array}{c}\psi_{\rm lf1}\\ \psi_{\rm lf2}\end{array}\right)=\exp\left(-i\frac{\mathbf{k}^2}{2m\hbar}t+\frac{i\mathbf{kx}}{\hbar}\right)\left(\begin{array}{c}\chi\\ \frac{\sigma_\mu k_\mu}{2mc}\chi\end{array}\right) \tag{5.2}$$

$$\Psi_{\rm hf}=\left(\begin{array}{c}\psi_{\rm hf1}\\ \psi_{\rm hf2}\end{array}\right)=\exp\left(2i\frac{mc^2t}{\hbar}+i\frac{\mathbf{k}^2}{2m\hbar}t+\frac{i\mathbf{kx}}{\hbar}\right)\left(\begin{array}{c}-\frac{\sigma_\mu k_\mu}{2mc}\eta\\ \eta\end{array}\right) \tag{5.3}$$

where χ and η are two-component constant quantities, and $\mathbf{k}=\{k_1,k_2,k_3\}$ =const ($\mathbf{k}^2\ll m^2c^2$) is the canonical momentum of the plane wave. The plane waves (5.2) and (5.3) are associated with the nonrelativistic particle and antiparticle. All wave functions $\Psi_{\rm lf}$, $\Psi_{\rm hf}$ satisfy the nonrelativistic approximation of the Dirac equation (4.10), (4.11).

The quantities (4.12), (4.13), (4.14) have the form

$$j_{\rm lf}^0 = \left(1+\frac{\mathbf{k}^2}{4m^2c^2}\right)\chi^*\chi, \qquad j_{\rm hf}^0=\left(1+\frac{\mathbf{k}^2}{4m^2c^2}\right)\eta^*\eta \tag{5.4}$$

$$j_{\rm lf}^\mu = \frac{k_\mu}{m}j_{\rm lf}^0 \qquad j_{\rm hf}^\mu=-\frac{k_\mu}{m}j_{\rm hf}^0, \tag{5.5}$$

$$T_{\rm lf0}^0=\left(mc^2+\frac{\mathbf{k}^2}{2m}\right)j_{\rm lf}^0, \qquad T_{\rm hf0}^0=-\left(mc^2+\frac{\mathbf{k}^2}{2m}\right)j_{\rm hf}^0, \tag{5.6}$$

$$T_{\rm lf0}^\mu=k_\mu j_{\rm lf}^0, \qquad T_{\rm hf0}^\mu=-k_\mu j_{\rm hf}^0 \tag{5.7}$$

Expressions (5.4) – (5.7) can be obtained also by conventional method, i.e. as a nonrelativistic approximation of exact solutions of the Dirac equation in the form of plane waves. The quantities with index 'lf' are obtained from solution for the positive value of the temporal component $k_0=\left|\sqrt{m^2c^2+\mathbf{k}^2}\right|$ of the canonical momentum, whereas the quantities with index 'hf' are obtained from solution for the negative value of the temporal component $k_0=-\left|\sqrt{m^2c^2+\mathbf{k}^2}\right|$ of the canonical momentum.

Let us return to investigation of stability conditions (4.35), (4.38). We consider only the case, when $A_k=0,\quad \pi_\mu=i\hbar\partial_\mu$. The low frequency plane wave is associated with the nonvanishing quantity u_+, and the high frequency plane wave is associated with the nonvanishing quantity u_- or with the quantity (4.29). For the plane waves the quantities u_+, u have the form

$$u_+=\exp\left(i\frac{p_\alpha x^\alpha}{\hbar}-i\frac{\mathbf{p}^2}{2m\hbar}t\right)\chi_+, \qquad u=-i\frac{2m\hbar}{\mathbf{k}^2}\exp\left(i\frac{k_\alpha x^\alpha}{\hbar}+i\frac{\mathbf{k}^2}{2m\hbar}t\right)\chi_- \tag{5.8}$$

$$\mathbf{k}^2,\mathbf{p}^2\ll m^2c^2, \qquad \chi_+,\chi_-=\text{const} \tag{5.9}$$

where χ_+, χ_- are two-component constant quantities. Substituting (5.8) in the constraints (4.37), (4.38), we obtain

$$\chi_+^*\chi_- -\frac{1}{\mathbf{k}^2}p_\mu k_\nu\chi_+^*\sigma_\mu\sigma_\nu\chi_- = 0 \tag{5.10a}$$

$$\chi_+^*\sigma_\mu\sigma_\nu\chi_-\left(-p_\nu+\frac{4m^2c^2}{\mathbf{k}^2}k_\nu\right) = 0, \qquad \mu=1,2,3 \tag{5.10b}$$

Eliminating $\chi_+^*\sigma_\mu\sigma_\nu\chi_-$ from equations (5.10a) and (5.10b), we obtain

$$\left(-p_\mu p_\mu + 4m^2c^2\right)\chi_+^*\chi_- = 0 \tag{5.11}$$

As far as $|\mathbf{p}| \ll 2mc$, the bracket in (5.11) cannot vanish, and we obtain

$$\chi_+^*\chi_- = 0 \tag{5.12}$$

Applying the identity (4.16) to the relation (5.10b) and taking into account (5.12), we obtain

$$-i\varepsilon_{\mu\nu\alpha}\chi_+^*\sigma_\alpha\chi_-\left(-p_\nu + \frac{4m^2c^2}{\mathbf{k}^2}k_\nu\right) = 0, \qquad \mu = 1,2,3 \tag{5.13}$$

As far as the bracket in (5.13) cannot vanish, we obtain from (5.13)

$$\chi_+^*\sigma_\mu\chi_- = q_\mu, \qquad q_\mu = A\left(-p_\mu + \frac{4m^2c^2}{\mathbf{k}^2}k_\mu\right), \qquad \mu = 1,2,3 \tag{5.14}$$

where A is some complex number.

Let us represent the quantities χ_+, χ_- in the form

$$\chi_+ = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \qquad \chi_- = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \tag{5.15}$$

Then relations (5.12) and (5.13) take the form

$$a_1^*b_1 + a_2^*b_2 = 0, \qquad a_1^*b_2 + a_2^*b_1 = q_1,$$

$$-ia_1^*b_2 + ia_2^*b_1 = q_2, \qquad a_1^*b_1 - a_2^*b_2 = q_3$$

These equations are transformed to the form

$$a_1^*b_1 = \frac{q_3}{2}, \qquad a_2^*b_1 = \frac{q_1 - iq_2}{2}, \qquad a_1^*b_2 = \frac{q_1 + iq_2}{2}, \qquad a_2^*b_2 = -\frac{q_3}{2} \tag{5.16}$$

It follows from the first two relations (5.16) and from the last two relations (5.16)

$$\frac{a_1^*}{a_2^*} = \frac{q_3}{(q_1 - iq_2)}, \qquad \frac{a_1^*}{a_2^*} = -\frac{q_1 + iq_2}{q_3} \tag{5.17}$$

Two relations (5.17) are compatible, only if

$$q_3^2 + q_1^2 + q_2^2 = A^2\left(-p_\mu + \frac{4m^2c^2}{\mathbf{k}^2}k_\mu\right)\left(-p_\mu + \frac{4m^2c^2}{\mathbf{k}^2}k_\mu\right) = 0 \tag{5.18}$$

As far as the brackets in (5.18) cannot vanish, the relation (5.18) can be satisfied, only if $A = 0$ and $q_\mu = 0$, $\mu = 1,2,3$. Then we obtain from (5.16)

$$a_1^*b_1 = 0, \qquad a_2^*b_2 = 0, \qquad a_1^*b_2 = 0, \qquad a_2^*b_1 = 0 \tag{5.19}$$

If $a_1 \neq 0 \vee a_2 \neq 0$, the equations (5.19) can be satisfied only if $b_1 = 0 \wedge b_2 = 0$. If $b_1 \neq 0 \vee b_2 \neq 0$, the equations (5.19) can be satisfied only if $a_1 = 0 \wedge a_2 = 0$. It means that any linear combination of the low frequency solution and of the high frequency solution is unstable with respect to electromagnetic radiation.

Stable superposition of Ψ_{lf} and of Ψ_{hf} is impossible. In the stable states Ψ_{lf} and Ψ_{hf} can be considered as states of different dynamic systems. In other words, in the stable states of the Dirac particle the superselection rule takes place.

6 Classical approximation of the nonrelativistic Dirac particle

To make the dynamic disquantization, we need to introduce hydrodynamic variables, where the current j^k were the dependent variable instead of ψ. Transforming the action (4.1), we use the mathematical technique [23, 24], where the wave function ψ is considered to be a function of hypercomplex numbers γ and coordinates x. In this case the physical quantities are obtained by means of a convolution of expressions $\psi^* O\psi$ with the zero divisor. This technique allows one to work without fixing the γ-matrices representation.

Using designations

$$\gamma_5 = c\gamma^{0123} \equiv c\gamma^0\gamma^1\gamma^2\gamma^3, \tag{6.1}$$

$$\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3,\} = \{-i\gamma^2\gamma^3, -i\gamma^3\gamma^1, -i\gamma^1\gamma^2\} \tag{6.2}$$

we make the change of variables

$$\psi = Ae^{i\varphi+\frac{1}{2}\gamma_5\kappa} \exp\left(-\frac{i}{2}\gamma_5\boldsymbol{\sigma\eta}\right) \exp\left(\frac{i\pi}{2}\boldsymbol{\sigma}\mathbf{n}\right) \Pi \tag{6.3}$$

$$\psi^* = A\Pi \exp\left(-\frac{i\pi}{2}\boldsymbol{\sigma}\mathbf{n}\right) \exp\left(-\frac{i}{2}\gamma_5\boldsymbol{\sigma\eta}\right) e^{-i\varphi-\frac{1}{2}\gamma_5\kappa} \tag{6.4}$$

where (*) means the Hermitian conjugation, and the quantity

$$\Pi = \frac{1}{4}(1 + c\gamma^0)(1 + \mathbf{z}\boldsymbol{\sigma}), \qquad \mathbf{z} = \{z^\alpha\} = \text{const}, \qquad \alpha = 1, 2, 3; \qquad \mathbf{z}^2 = 1 \tag{6.5}$$

is the zero divisor (projector). The quantities A, κ, φ, $\boldsymbol{\eta} = \{\eta^\alpha\}$, $\mathbf{n} = \{n^\alpha\}$, $\alpha = 1, 2, 3$, $\mathbf{n}^2 = 1$ are eight real parameters, determining the wave function ψ. These parameters may be considered as new dependent variables, describing the state of dynamic system $\mathcal{S}_\text{D}$. The quantity φ is a scalar, and κ is a pseudoscalar. Six remaining variables A, $\boldsymbol{\eta} = \{\eta^\alpha\}$, $\mathbf{n} = \{n^\alpha\}$, $\alpha = 1, 2, 3$, $\mathbf{n}^2 = 1$ can be expressed through the flux 4-vector $j^l = \bar{\psi}\gamma^l\psi$ and spin 4-pseudovector

$$S^l = i\bar{\psi}\gamma_5\gamma^l\psi, \qquad l = 0, 1, 2, 3 \tag{6.6}$$

Because of two identities

$$S^l S_l \equiv -j^l j_l, \qquad j^l S_l \equiv 0. \tag{6.7}$$

there are only six independent components among eight components of quantities j^l, and S^l.

Mathematical details of the dependent variables transformation can be found in [2], where the action is calculated for the case $c = 1$ and vanishing electromagnetic field $A_l = 0$. As a result we have the following form of the action, written in the hydrodynamical form

$$\mathcal{S}_\text{D}: \qquad \mathcal{A}_D[j, \varphi, \kappa, \boldsymbol{\xi}] = \int \mathcal{L}d^4x, \qquad \mathcal{L} = \mathcal{L}_\text{cl} + \mathcal{L}_\text{q1} + \mathcal{L}_\text{q2} \tag{6.8}$$

$$\mathcal{L}_{\rm cl}=-mc\rho-\hbar j^l\partial_l\varphi-\frac{e}{c}A_lj^l-\frac{\hbar j^l}{2\left(1+\boldsymbol{\xi}\mathbf{z}\right)}\varepsilon_{\alpha\beta\gamma}\xi^\alpha\partial_l\xi^\beta z^\gamma,\qquad\rho\equiv\sqrt{j^lj_l}\tag{6.9}$$

$$\mathcal{L}_{\rm q1}=2mc\rho\sin^2(\frac{\kappa}{2})-\frac{\hbar}{2}S^l\partial_l\kappa,\tag{6.10}$$

$$\mathcal{L}_{\rm q2}=\frac{\hbar(\rho+cj^0)}{2}\varepsilon_{\alpha\beta\gamma}\partial^\alpha\frac{j^\beta}{(\rho+cj^0)}\xi^\gamma-\frac{\hbar}{2(\rho+cj^0)}\varepsilon_{\alpha\beta\gamma}\left(\partial^0j^\beta\right)j^\alpha\xi^\gamma\tag{6.11}$$

where $\varepsilon_{\alpha\beta\gamma}$ is the Levi-Chivita 3-pseudotensor. The Lagrangian density $\mathcal{L}$ is a function of 4-vector j^l, scalar φ, pseudoscalar κ, and the unit 3-pseudovector $\boldsymbol{\xi}$, which is connected with the spin 4-pseudovector S^l by means of the relations

$$\xi^\alpha=\rho^{-1}\left[S^\alpha-\frac{j^\alpha S^0}{(\rho+cj^0)}\right],\qquad\alpha=1,2,3;\qquad\rho\equiv\sqrt{j^lj_l}\tag{6.12}$$

$$S^0=\mathbf{j}\boldsymbol{\xi},\qquad S^\alpha=\rho\xi^\alpha+\frac{(\mathbf{j}\boldsymbol{\xi})j^\alpha}{\rho+cj^0},\qquad\alpha=1,2,3\tag{6.13}$$

Producing the dynamical disquantization (3.5) in (6.8) - (6.11), we obtain

$$\mathcal{A}_{\rm Dqu}[j,\varphi,\boldsymbol{\xi}]=\int\left\{-\kappa_0m\rho-\frac{e}{c}A_lj^l-\hbar j^i\left(\partial_i\varphi+\frac{\varepsilon_{\alpha\beta\gamma}\xi_\alpha\partial_i\xi_\beta z_\gamma}{2\left(1+\boldsymbol{\xi}\mathbf{z}\right)}\right)\right.$$
$$\left.+\frac{\hbar j^k}{2(\rho+j_0)\rho}\varepsilon_{\alpha\beta\gamma}\left(\partial_kj^\beta\right)j^\alpha\xi_\gamma\right\}d^4x\tag{6.14}$$

where $\kappa_0=\pm1$ is the solution of the dynamic equation $\delta\mathcal{A}_{\rm Dqu}/\delta\kappa=0$, which does not contain derivatives, because the last term of (6.10) vanishes after dynamical disquantization (3.5) in virtue of the second identity (6.7).

We introduce the Lagrangian coordinates $\tau=\{\tau_0,\boldsymbol{\tau}\}=\{\tau_i(x)\}$, $i=0,1,2,3$ as functions of coordinates x in such a way that only coordinate τ_0 changes along the direction j^l, i.e.

$$j^k\partial_k\tau_\mu=0,\qquad\mu=1,2,3\tag{6.15}$$

Considering the variables $\tau=\{\tau_0,\boldsymbol{\tau}\}$ as independent variables in (6.14), we obtain after calculations (See mathematical details in [2])

$$\mathcal{A}_{\rm Dqu}[x,\boldsymbol{\xi}]=\int\left\{-\kappa_0mc\sqrt{\dot{x}^l\dot{x}_l}-\frac{e}{c}A_l\dot{x}^l+\hbar\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{2(1+\boldsymbol{\xi}\mathbf{z})}+\hbar\frac{(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}}{2\sqrt{\dot{x}^s\dot{x}_s}(\sqrt{\dot{x}^s\dot{x}_s}+c\dot{x}^0)}\right\}d^4\tau\tag{6.16}$$

where period means the total derivative $\dot{x}^s\equiv dx^s/d\tau_0$. The quantities $x=\{x^0,\mathbf{x}\}=\{x^i\}$, $i=0,1,2,3$, and $\boldsymbol{\xi}=\{\xi_\alpha\}$, $\alpha=1,2,3$ are considered to be functions of the Lagrangian coordinates τ_0, $\boldsymbol{\tau}=\{\tau_1,\tau_2,\tau_3\}$. Here and in what follows the symbol $\times$ means the vector product of two 3-vectors. The quantity $\mathbf{z}$ is the constant unit 3-vector (6.5). In fact, variables x depend on $\boldsymbol{\tau}$ as on parameters, because the action (6.16) does not contain derivatives with respect to τ_α, $\alpha=1,2,3$. Lagrangian density of the action (6.16) does not contain independent variables τ explicitly. Hence, it may be written in the form

$$\mathcal{A}_{\mathrm{Dqu}}[x,\boldsymbol{\xi}]=\int\mathcal{A}_{\mathrm{Dcl}}[x,\boldsymbol{\xi}]d\boldsymbol{\tau},\qquad d\boldsymbol{\tau}=d\tau_1 d\tau_2 d\tau_3 \tag{6.17}$$

where

$$\mathcal{A}_{\mathrm{Dcl}}[x,\boldsymbol{\xi}]=\int\left\{-\kappa_0 mc\sqrt{\dot{x}^i\dot{x}_i}-\frac{e}{c}A_l\dot{x}^l+\hbar\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{2(1+\boldsymbol{\xi}\mathbf{z})}+\hbar\frac{(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}}{2\sqrt{\dot{x}^s\dot{x}_s}(\sqrt{\dot{x}^s\dot{x}_s}+c\dot{x}^0)}\right\}d\tau_0 \tag{6.18}$$

It is easy to see that the action (6.18) is invariant with respect to transformation $\tau_0\rightarrow\tilde{\tau}_0=F(\tau_0)$, where F is an arbitrary monotone function. This invariance admits one to choose the variable $t=x^0$ as a parameter τ_0. In this case we obtain instead of (6.18)

$$\begin{aligned}\mathcal{A}_{\mathrm{Dcl}}[\mathbf{x},\boldsymbol{\xi}] &= \int\left\{-\kappa_0 mc^2\sqrt{1-\frac{\dot{\mathbf{x}}^2}{c^2}}-\frac{e}{c}A_0-\frac{e}{c}\mathbf{A}\dot{\mathbf{x}}+\hbar\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{2(1+\boldsymbol{\xi}\mathbf{z})}\right.\\ &\left.+\frac{\hbar}{2c^2}(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}\left(1-\frac{\dot{\mathbf{x}}^2}{c^2}+\sqrt{1-\frac{\dot{\mathbf{x}}^2}{c^2}}\right)^{-1}\right\}dt\end{aligned} \tag{6.19}$$

In the nonrelativistic approximation, when $c\rightarrow\infty$, the coefficient $\frac{\hbar}{2c^2}$ before the last term tends to zero. Nevertheless, we may not omit the last term in the action (6.19), because the last term contains the highest derivative. In the dynamic equations this term generates the term with the small parameter before the highest derivative. Such a term may not be omitted, because it is of the same order as the other terms. This term may generate oscillations with the frequency of the order $\Omega=2mc^2/\hbar$, and $\Omega\hbar/mc^2\approx 1$.

In the nonrelativistic approximation the action (6.19) turns into

$$\mathcal{A}_{\mathrm{Dcl}}[\mathbf{x},\boldsymbol{\xi}]=\int\left\{\frac{1}{2}\kappa_0 m\dot{\mathbf{x}}^2+\frac{\hbar}{4c^2}(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}-\frac{e}{c}\left(A_0+\mathbf{A}\dot{\mathbf{x}}\right)+\hbar\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{2(1+\boldsymbol{\xi}\mathbf{z})}\right\}dt \tag{6.20}$$

where the first term $-\kappa_0 mc^2$ is omitted, because it gives no contribution in the dynamic equations. Two first terms in (6.20) describe dynamics and structure of the classical Dirac particle. The third term describes interaction with the electromagnetic field.

7 Solution of dynamic equation for the classical nonrelativistic Dirac particle

Dynamic equations for the classical nonrelativistic Dirac particle $\mathcal{S}_{\mathrm{nDcl}}$ generated by the action (6.20) have the form

$$-\kappa_0 m\ddot{\mathbf{x}}-e\mathbf{E}-\frac{e}{c}\left(\dot{\mathbf{x}}\times\mathbf{H}\right)+\hbar\frac{d}{dt}\frac{(\boldsymbol{\xi}\times\ddot{\mathbf{x}})}{2c^2}-\hbar\frac{d}{dt}\frac{(\dot{\mathbf{x}}\times\dot{\boldsymbol{\xi}})}{4c^2}=0 \tag{7.1}$$

where

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} + \frac{1}{c}\boldsymbol{\nabla} A_0, \qquad \mathbf{H} = \boldsymbol{\nabla} \times \mathbf{A}, \qquad A_k = \{A_0, \mathbf{A}\} \tag{7.2}$$

$$\boldsymbol{\xi} \times \left(-\hbar\frac{(\dot{\boldsymbol{\xi}} \times \mathbf{z})}{2(1+\boldsymbol{\xi}\mathbf{z})} + \hbar\frac{d}{dt}\frac{(\mathbf{z} \times \boldsymbol{\xi})}{2(1+\boldsymbol{\xi}_\mu \mathbf{z})} - \hbar\frac{\left((\dot{\boldsymbol{\xi}} \times \boldsymbol{\xi})\mathbf{z}\right)}{2(1+\boldsymbol{\xi}\mathbf{z})^2}\mathbf{z} + \hbar\frac{(\dot{\mathbf{x}} \times \ddot{\mathbf{x}})}{4c^2} \right) = 0 \tag{7.3}$$

Vector product in (7.3) is a corollary of the constraint $\boldsymbol{\xi}^2 = 1$.

After simplification the dynamic equation (7.3) is reduced to the form (See Appendix B of [2])

$$\dot{\boldsymbol{\xi}} = \frac{\boldsymbol{\xi} \times (\dot{\mathbf{x}} \times \ddot{\mathbf{x}})}{2c^2} \tag{7.4}$$

This equation describes rotation of the unit vector $\boldsymbol{\xi}$ with the angular frequency $\omega = c^{-2}\left(\dot{\mathbf{x}} \times \ddot{\mathbf{x}}\right)/2$. In general, we may not neglect the rhs of (7.4) in the nonrelativistic approximation, if $\dot{\mathbf{x}}$ oscillates with the frequency of the order of $mc^2/\hbar$. Solving dynamic equations (7.1), (7.3) we shall see that such frequencies are possible.

When the electromagnetic field is absent ($\mathbf{E} = 0$ and $\mathbf{H} = 0$), the equation (7.1) is reduced to the form

$$-\kappa_0 m\ddot{\mathbf{x}} + \hbar\frac{d}{dt}\frac{(\boldsymbol{\xi} \times \ddot{\mathbf{x}})}{2c^2} - \hbar\frac{d}{dt}\frac{(\dot{\mathbf{x}} \times \dot{\boldsymbol{\xi}})}{4c^2} = 0 \tag{7.5}$$

$$-\kappa_0 m\dot{\mathbf{x}} + \hbar\frac{(\boldsymbol{\xi} \times \ddot{\mathbf{x}})}{2c^2} - \hbar\frac{(\dot{\mathbf{x}} \times \dot{\boldsymbol{\xi}})}{4c^2} = -\mathbf{p} = \text{const} \tag{7.6}$$

The equation (7.6) can be solved exactly, if $\dot{\mathbf{x}} \times \ddot{\mathbf{x}} = a\boldsymbol{\xi}$, where a is an arbitrary quantity. Then according to (7.4)

$$\boldsymbol{\xi} = \text{const} \tag{7.7}$$

The equation (7.6) turns into

$$-\kappa_0 m\dot{\mathbf{x}} + \hbar\frac{(\boldsymbol{\xi} \times \ddot{\mathbf{x}})}{2c^2} = -\mathbf{p} \tag{7.8}$$

The general solution of (7.8) has the form

$$\dot{\mathbf{x}} = \frac{\mathbf{p}}{\kappa_0 m} + \mathbf{V}\cos(\omega t + \phi) + \boldsymbol{\xi} \times \mathbf{V}\sin(\omega t + \phi), \qquad |\mathbf{V}| \ll c \tag{7.9}$$

where ϕ is an arbitrary constant. The quantities $\mathbf{V}$ and $\mathbf{p}$ are the constant vectors satisfying the constraints

$$\mathbf{V}\boldsymbol{\xi} = 0, \qquad \boldsymbol{\xi}^2 = 1 \tag{7.10}$$

and the frequency ω is determined by the relation

$$\omega = -\kappa_0\frac{2mc^2}{\hbar}, \qquad \kappa_0 = \pm 1 \tag{7.11}$$

After integration of (7.9) we obtain

$$\mathbf{x}=\mathbf{X}+\frac{\mathbf{p}t}{\kappa_0 m}-\mathbf{V}\frac{\kappa_0\hbar}{2mc^2}\sin\left(\omega t+\phi\right)+\boldsymbol{\xi}\times\mathbf{V}\frac{\kappa_0\hbar}{2mc^2}\cos\left(\omega t+\phi\right),\qquad \mathbf{X}=\mathrm{const} \tag{7.12}$$

Thus, the world line of the free nonrelativistic classical Dirac particle $\mathcal{S}_{\mathrm{nDcl}}$ is a helix. According to the condition $|\mathbf{V}|\ll c$ the radius r of the helix is much less, than the Compton wave length $\lambda_{\mathrm{C}}=\hbar/mc$. This result agrees with the result of investigation of the relativistic classical Dirac particle $\mathcal{S}_{\mathrm{Dcl}}$ [2], where the world line is also a helix, but without the constraint $|\mathbf{V}|\ll c$. In the limit $c\rightarrow\infty$ the oscillating terms vanish in expression (7.12) for $\mathbf{x}$. However, they are not vanish in the expression (7.9) for $\dot{\mathbf{x}}$. In the limit $c\rightarrow\infty$ the helix turns into straight line, but the velocity of circular motion does not vanish.

The angular momentum generated by the solution (7.12), (7.9) has the form

$$\mathbf{M}=m\left(\mathbf{x}\times\dot{\mathbf{x}}\right)=\frac{\boldsymbol{\xi}\mathbf{V}^2 m}{\omega}+\mathbf{M}_{\mathrm{os}} \tag{7.13}$$

where $\mathbf{M}_{\mathrm{os}}$ is the oscillating part of the angular momentum. Averaging over the time, the mean value $\langle\mathbf{M}_{\mathrm{os}}\rangle$ of $\mathbf{M}_{\mathrm{os}}$ vanishes. Then

$$\langle\mathbf{M}\rangle=m\left(\mathbf{x}\times\dot{\mathbf{x}}\right)=-\kappa_0\frac{\hbar}{2}\boldsymbol{\xi}\frac{\mathbf{V}^2}{c^2},\qquad \kappa_0=\pm1 \tag{7.14}$$

This result is applicable only in the nonrelativistic case, when $V^2\ll c^2$. The mean angular momentum $\langle\mathbf{M}\rangle$ is directed along the unit vector $\boldsymbol{\xi}$. Its module is equal to $\hbar/2$, provided $V=c$.

In the general relativistic case the velocity amplitude $|\mathbf{V}|$ and the frequency ω are connected between themselves, and the solution (7.12) turns [2] into the relation

$$\mathbf{x}=\mathbf{X}+\frac{\mathbf{V}}{\Omega_{\mathrm{Dcl}}}\sin\left(\Omega_{\mathrm{Dcl}}t+\phi\right)-\frac{\boldsymbol{\xi}\times\mathbf{V}}{\Omega_{\mathrm{Dcl}}}\cos\left(\Omega_{\mathrm{Dcl}}t+\phi\right),\qquad \mathbf{X}=\mathrm{const} \tag{7.15}$$

where

$$\mathbf{V}=c\frac{\sqrt{\gamma^2-1}}{\gamma}\mathbf{n},\qquad \Omega_{\mathrm{Dcl}}=-\kappa_0\frac{2mc^2}{\hbar\gamma^2},\qquad \mathbf{n}^2=1,\qquad \mathbf{n}\boldsymbol{\xi}=0 \tag{7.16}$$

and $\gamma\geq1$ is an arbitrary constant (Lorentz-factor of rotation). The regular momentum $\mathbf{p}=0$, because only in this case one succeeded to solve exactly the relativistic dynamic equations. At $\gamma\rightarrow1$ the relation (7.15) turns into (7.12) with $\mathbf{p}=0$.

In the relativistic case the mean magnetic moment has the form

$$\langle\mathbf{M}_{\mathrm{Dcl}}\rangle=m\left(\mathbf{x}\times\dot{\mathbf{x}}\right)=-\kappa_0 V^2\frac{\hbar}{2c^2}\gamma^2\boldsymbol{\xi}=-\kappa_0\left(\gamma^2-1\right)\frac{\hbar}{2}\boldsymbol{\xi} \tag{7.17}$$

In the Dirac dynamic system $\mathcal{S}_{\mathrm{D}}$ the internal degrees of freedom are described nonrelativistically [25]. This defect can be corrected [25]. After such a correction the classical Dirac particle $\mathcal{S}_{\rm Dcl}$ turns into the modified classical Dirac particle $\mathcal{S}_{\rm mDcl}$. In this case we have instead of (7.15) - (7.17)

$$\mathbf{x}=\mathbf{X}+\frac{\mathbf{V}}{\Omega_{\rm mDcl}}\sin\left(\Omega_{\rm mDcl}t+\phi\right)-\frac{\boldsymbol{\xi}\times\mathbf{V}}{\Omega_{\rm mDcl}}\cos\left(\Omega_{\rm mDcl}t+\phi\right),\qquad \mathbf{X}=\text{const} \tag{7.18}$$

$$\mathbf{V}=c\frac{\sqrt{\gamma^2-1}}{\gamma}\mathbf{n},\qquad \Omega_{\rm mDcl}=-\kappa_0\frac{2mc^2}{\hbar\left(2\gamma^2-1\right)^2},\qquad \mathbf{n}^2=1,\qquad \mathbf{n}\boldsymbol{\xi}=0 \tag{7.19}$$

$$\left\langle\mathbf{M}_{\rm mDcl}\right\rangle=m\left(\mathbf{x}\times\dot{\mathbf{x}}\right)=-\kappa_0\frac{\left(\gamma^2-1\right)}{\gamma^2}\left(2\gamma^2-1\right)^2\frac{\hbar}{2}\boldsymbol{\xi} \tag{7.20}$$

In the nonrelativistic case, when $\gamma-1\ll1$, the results (7.20), (7.17) coincide with (7.14) and between themselves.

Conventionally, the nonrelativistic approximation is obtained by other method (See, for instance, [1], sec. 70). One derives dynamic equations for nonrelativistic Dirac particle, and thereafter one transits to the classical approximation (dynamic disquantization). As a result one obtains the relation (7.12) without two last terms, i.e. $\mathbf{V}\equiv0$, and one obtains a straight line instead of a helix. The loss of the two last terms in (7.12) takes place at the stage of transition from the Dirac equation to the Pauli equation. Formally the loss of two last terms in (7.12) is justified by the fact that these terms are proportional to c^{-2} and small in the limit $c\to\infty$.

We are to remark here that the nonrelativistic approximation is described by the inequality $|\dot{\mathbf{x}}|\ll c$. It concerns only velocities and does not impose any constraints on the position $\mathbf{x}$. As to the velocity, all terms in the relation (7.9) for the velocity are of the same order, and we may not neglect the two last terms in (7.9). The additional terms give a very small contribution to the particle position, but they introduce additional degrees of freedom, which are rather rigid . These degrees of freedom cannot be excited at the low energies, characteristic for the atomic spectra. The characteristic energy, connected with these degrees of freedom is of the order $\hbar\omega=2mc^2$. In other words, it is a characteristic threshold energy of the pair production.

8 Relativistic corrections to nonrelativistic classical Dirac particle

Note that the action (6.20) for the nonrelativistic classical Dirac particle as well as the dynamic equation (7.1) does not contain the term, describing interaction of the magnetic moment with the magnetic field, what is characteristic for the nonrelativistic Pauli equation. This interaction may be obtained, if we take the high frequency solution (7.9) and average the action (6.20) over the frequency ω, determined by the relation (7.11). In reality, it is necessary to average only the term $-\frac{e}{c}\mathbf{A}\left(t,\mathbf{x}\right)\dot{\mathbf{x}}$. We obtain

$$\left\langle-\frac{e}{c}\mathbf{A}\left(t,\mathbf{x}\right)\dot{\mathbf{x}}\right\rangle=-\frac{e}{c}\left\langle\mathbf{A}\left(t,\mathbf{x}\right)\delta\dot{\mathbf{x}}+\delta x^\mu\partial_\mu\mathbf{A}\left(t,\mathbf{x}\right)\delta\dot{\mathbf{x}}\right\rangle \tag{8.1}$$

where $\delta\mathbf{x}$ and $\delta\dot{\mathbf{x}}$ are determined by the relations (7.12) and (7.9)

$$\delta\mathbf{x} = -\frac{\mathbf{V}}{\omega}\sin(\omega t) + \boldsymbol{\xi}\times\frac{\mathbf{V}}{\omega}\cos(\omega t), \qquad \omega = -\kappa_0\frac{2mc^2}{\hbar}, \qquad \kappa_0 = \pm 1 \tag{8.2}$$

$$\delta\dot{\mathbf{x}} = \frac{\mathbf{p}}{\kappa_0 m} + \mathbf{V}\cos(\omega t) + \boldsymbol{\xi}\times\mathbf{V}\sin(\omega t), \qquad \mathbf{V}\boldsymbol{\xi} = 0, \qquad \boldsymbol{\xi}^2 = 1 \tag{8.3}$$

and angular brackets mean the averaging over the argument ωt. Substituting relations (8.2) and (8.3) in the relation (8.1) and taking into account that

$$\left\langle\cos^2(\omega t)\right\rangle = \left\langle\sin^2(\omega t)\right\rangle = \frac{1}{2}, \qquad \langle\sin(\omega t)\cos(\omega t)\rangle = 0 \tag{8.4}$$

we obtain

$$\left\langle -\frac{e}{c}\mathbf{A}\dot{\mathbf{x}}\right\rangle + \frac{e}{c}\mathbf{A}\frac{\mathbf{p}}{\kappa_0 m} = +\frac{e}{2c\omega}V_\alpha A_{\alpha,\mu}\varepsilon_{\mu\nu\sigma}\xi_\nu V_\sigma - \frac{e}{2c\omega}A_{\alpha,\mu}\varepsilon_{\alpha\beta\gamma}V_\mu\xi_\beta V_\gamma \tag{8.5}$$

In vector form the expression (8.5) takes the form

$$\left\langle -\frac{e}{c}\mathbf{A}\dot{\mathbf{x}}\right\rangle + \frac{e}{c}\mathbf{A}\frac{\mathbf{p}}{\kappa_0 m} = -\frac{\kappa_0 e\hbar}{4mc^3}\mathbf{V}\left(\left(\left(\boldsymbol{\xi}\times\mathbf{V}\right)\cdot\boldsymbol{\nabla}\right)\mathbf{A} - \nabla\left(\mathbf{A}\cdot\left(\boldsymbol{\xi}\times\mathbf{V}\right)\right)\right) \tag{8.6}$$

Let us transform the last term in rhs of (8.6) by means of the vector formula

$$\boldsymbol{\nabla}(\mathbf{F}\cdot\mathbf{G}) = (\mathbf{F}\cdot\boldsymbol{\nabla})\mathbf{G} + (\mathbf{G}\cdot\boldsymbol{\nabla})\mathbf{F} + \mathbf{F}\times(\boldsymbol{\nabla}\times\mathbf{G}) + \mathbf{G}\times(\boldsymbol{\nabla}\times\mathbf{F})$$

Setting $\mathbf{F} = \mathbf{A}$, $\mathbf{G} = \boldsymbol{\xi}\times\mathbf{V}$ and taking into account, that $\boldsymbol{\xi}$ =const, $\mathbf{V}$ =const, $(\boldsymbol{\xi}\mathbf{V}) = 0$, we obtain instead of (8.6)

$$\begin{aligned}
\left\langle -\frac{e}{c}\mathbf{A}\dot{\mathbf{x}}\right\rangle &= -\frac{e}{c}\mathbf{A}\frac{\mathbf{p}}{\kappa_0 m} + \frac{\kappa_0 e\hbar}{4mc^3}\mathbf{V}\left(\left(\boldsymbol{\xi}\times\mathbf{V}\right)\times\left(\boldsymbol{\nabla}\times\mathbf{A}\right)\right) \\
&= -\frac{e}{c}\mathbf{A}\frac{\mathbf{p}}{\kappa_0 m} + \frac{\kappa_0 e\hbar}{4mc^3}\mathbf{V}\left(\left(\boldsymbol{\xi}\times\mathbf{V}\right)\times\mathbf{H}\right) \\
&= -\frac{e}{c}\mathbf{A}\frac{\mathbf{p}}{\kappa_0 m} - \frac{\kappa_0 e\hbar}{4mc}\left(\frac{\mathbf{V}}{c}\right)^2(\boldsymbol{\xi}\mathbf{H})
\end{aligned} \tag{8.7}$$

where

$$\mathbf{H} = \boldsymbol{\nabla}\times\mathbf{A}$$

is the magnetic field.

After averaging the high frequency relativistic term turns into

$$\left\langle \frac{\hbar}{4c^2}(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}\right\rangle \to \kappa_0\frac{m\mathbf{V}^2}{2} = \text{const} \tag{8.8}$$

This term is constant, and it does not contribute to the dynamic equations.

Let us substitute (8.7) in the action (6.20). Taking into account, that the average low frequency velocity $\dot{\mathbf{x}} = \frac{\mathbf{p}}{\kappa_0 m}$, we obtain instead of (6.20)

$$\mathcal{A}_{\mathrm{Dcl}}[\mathbf{x},\boldsymbol{\xi}] = \int \left\{ \frac{1}{2}\kappa_0 m\dot{\mathbf{x}}^2 - \frac{e}{c}A_0 - \frac{e}{c}\mathbf{A}\dot{\mathbf{x}} - \frac{\kappa_0 e\hbar}{4mc}\left(\frac{\mathbf{V}}{c}\right)^2 (\boldsymbol{\xi}\mathbf{H}) + \hbar\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{2(1+\boldsymbol{\xi}\mathbf{z})} \right\} dt \tag{8.9}$$

The action (8.9) differs from the action for the classical Pauli particle in the sense that it depends on the free parameter V. This parameter describes the intensity of the high frequency rotation. To obtain the action for the classical Pauli particle we should identify the variable $\boldsymbol{\xi}$ with the particle spin and set $V = c\sqrt{2}$. Although this value of the velocity is relativistic and unreal, the contribution of the high frequency term in the action (8.9) has the same form as in the action for the classical Pauli particle.

Note that taking for averaging the relativistical expressions (7.16), or (7.19) for V and ω, we obtain instead of $V = c\sqrt{2}$ another expressions, where $|V| < c$.

9 Concluding remarks

This paper is devoted to comparison of the Newtonian strategy and the experimental-fitting strategy in their application to the microcosm investigation. In this comparison we underline the role of mistakes in the foundation of physical theory. These mistakes are actual only in the theory of microcosm phenomena. It was the mistakes, that have lead to a replacement of the Newtonian strategy, dominating in 19th century, by the Ptolemaic experimental-fitting approach, dominating in microcosm investigations of 20th century. The Ptolemaic approach and experimental-fitting way of thinking are the main obstacles on the path of development of the satisfactory fundamental microcosm theory. The Ptolemaic approach works very well at investigation of concrete physical phenomena, because it is insensitive to mistakes in the foundation of the theory. However, it is not adequate for construction of a fundamental physical theory, because it create only list of predictions, but not a logical structure. Extension of a fundamental theory to the new region of relativistic microcosm phenomena is produced easier, if the theory is a logical structure, but not a list of prescriptions. Discovery and correction of mistakes is the only way for construction of a logical structure instead of the Ptolemaic list of predictions. In such a situation it is very important to distinguish between a mistake and a simple deficiency of our knowledge, as well as between the mistake and incorrect hypothesis. A mistake is an incorrect information, whereas a deficiency of knowledge is simply a lack of information. A hypothesis may be correct in some situation and invalid in other situation. A hypothesis may be justified or removed, because it lies outside the logical structure of the satisfactory theory. On the contrary, the mistake must be discovered and corrected. It is contained in the logical structure of the satisfactory theory and it may not be ignored. Unfortunately, the experimental-fitting approach does not distinguish between a mistake and an incorrect hypothesis, because at this approach a theory is a list of prescriptions having equal importance, but not a logical structure.

The most contemporary researchers of microcosm are educated in the Ptolemaic experimental-fitting approach. The Newtonian approach is unknown for them, although it is not a new approach. They believe that any theory, which explain experimental data, is a good theory. Of course, the experimental-fitting approach has an historical reason, but, when the main mistakes are discovered and corrected, the list of prescriptions can and must be replaced by a logical structure. Now there is no reason for a use of the Ptolemaic approach for construction of the theory of microcosm phenomena.

Rejecting the quantum principles and using dynamical methods of investigation, we obtain results, which cannot be obtained by means of the conventional technique, based on the axiomatic representation of the quantum mechanics. In particular, dynamical methods lead to such results: (1) formalization of the procedure of transition to classical approximation, (2) composite structure of the Dirac particle and (3) nonrelativistic description of the internal degrees of freedom of the Dirac particle. These results cannot be obtained by conventional methods of investigation.

Mistakes in the foundation of a theory are rather specific. This is not logical, or mathematical mistakes. These mistakes are associative delusions, where one uses incorrect associations between our ideas on the properties of the phenomena of the real world. It is rather difficult to discover the associative delusions. We illustrate this in the example of discovery of the nonrelativistic character of the Dirac equation.

The Dirac equation can be written in the relativistically covariant form. It is common practice to think, that it means that the Dirac equation describes relativistical processes and has the Lorentz symmetry, i.e. the set of all its solutions is transformed to the same set of solutions at any Lorentz transformation. This opinion has been existing for many years, and we try to understand the reason of this viewpoint.

The relativistic character of dynamic equations associates with the representation of these equations in the relativistically covariant form. However, this association is valid only at some additional conditions, which are fulfilled practically always, and as a result these conditions are not mentioned usually in the conventional formulation of the relativistic invariance (compatibility of dynamic equations with the principles of relativity). Unfortunately, in the case of the Dirac equation these additional conditions are not fulfilled, and the Dirac equation appears to be formally nonrelativistic. In reality, only internal degrees of freedom are nonrelativistic. If these internal degrees of freedom are ignored, the Dirac particle appears to be relativistic.

The additional constraint in the formulation of the relativistical invariance changes the formulation. The correct formulation looks as follows. *Symmetry of dynamic equations, written in the relativistically covariant form coincides with the symmetry of their absolute objects* [26]. The absolute objects are such quantities, which are the same for all solutions. Usually such an absolute object is the metric tensor, which has the form $g_{ik}=\mathrm{diag}\{c^2,-1,-1,-1\}$. The group of symmetry of g_{ik} is the Lorentz group, and the symmetry group of dynamic equations appears to be the

Lorentz group. The Maxwell equations, the Klein-Gordon equation and many other dynamic equations for real dynamic systems contain only the metric tensor as an absolute object, and the formulation of relativistical invariance is simplified. It looks as follows. *The symmetry group of dynamic equations, written in the relativistically covariant form is the Lorentz group.* In such a form it used by most researchers.

The Dirac equation does not contain the metric tensor. Instead it contains the γ-matrices γ^i, $i = 0, 1, 2, 3$. The γ-matrices form a matrix 4-vector, whose symmetry group is lower, than the Lorentz group. As a result the Dirac equation appears to have not a symmetry of the Lorentz group. In other words, the Dirac equation appears to be nonrelativistic equation.

What physical situation is behind this result? Why does the dynamic equation, written in the relativistically covariant form, become to be nonrelativistic, if it contains an absolute vector? To answer this question, we consider an example of a charged classical particle, moving in the given electromagnetic field F^{ik}.

Dynamic equation for the relativistic particle may written in the noncovariant form

$$\frac{d}{dt}\frac{m\dot{x}^\mu}{\sqrt{1-\frac{\dot{\mathbf{x}}^2}{c^2}}} = \frac{e}{c}F^{\mu 0} + \frac{e}{c}F^{\mu\nu}g_{\nu\beta}\dot{x}^\beta, \qquad \mu = 1, 2, 3, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \tag{9.1}$$

and in the relativistically covariant form

$$m\frac{d^2x^k}{d\tau^2} = \frac{e}{c}F^{kl}g_{ls}\frac{dx^s}{d\tau}, \qquad k = 0, 1, 2, 3 \tag{9.2}$$

where τ is the proper time, e, m are respectively the particle charge and the particle mass.

If the particle is nonrelativistic the dynamic equation is written in the noncovariant form

$$m\frac{d^2x^\mu}{dt^2} = \frac{e}{c}F^{\mu 0} + \frac{e}{c}F^{\mu\nu}g_{\nu\beta}\frac{dx^\beta}{dt}, \qquad \mu = 1, 2, 3 \tag{9.3}$$

Can the dynamic equations (9.3) for the nonrelativistic particle be written in the relativistically covariant form? The answer is yes, although most researchers believe that it is impossible. In the relativistically covariant form the dynamic equations (9.3) have the form

$$m\frac{d}{d\tau}\left[\left(l_k\dot{x}^k\right)^{-1}\dot{x}^i - \frac{1}{2}g^{ik}l_k\left(l_j\dot{x}^j\right)^{-2}\dot{x}^s g_{sl}\dot{x}^l\right] = \frac{e}{c}F^{il}g_{lk}\dot{x}^k; \qquad i = 0, 1, 2, 3 \tag{9.4}$$

where $\dot{x}^k \equiv dx^k/d\tau$. The quantity l_k, $k = 0, 1, 2, 3$ is a constant timelike unit 4-vector

$$g^{ik}l_il_k = 1; \tag{9.5}$$

Using the special choice of $l_k = \{c, 0, 0, 0\}$ and substituting it in (9.4), it is easy to verify, that we obtain the dynamic equations (9.3) for $i = 1, 2, 3$. For $i = 0$ we obtain dynamic equation, which is a corollary of (9.3).

As far as dynamic equations for both relativistic and nonrelativistic particles can be written in the noncovariant form and in the relativistically covariant one, it is clear that the difference between the relativistic and nonrelativistic descriptions is not connected with form of dynamic equations. There is anything else, which distinguishes the relativistic conception from the nonrelativistic one. It is well known that the difference lies in different space-time conceptions. In the Newtonian conception there is an absolute simultaneity and there are two invariant quantities: absolute time t and absolute space distance r, whereas in the relativistic space-time conception there exists only one absolute quantity: the space-time interval $s=\sqrt{c^2t^2-r^2}$. The Newtonian space-time $\mathcal{S}_{\rm N}$ has seven-parametric continuous group of motion, whereas the Minkowski space-time $\mathcal{S}_{\rm M}$ has ten-parametric continuous group of motion. Besides, the Newtonian space-time $\mathcal{S}_{\rm N}$ may be considered to be the Minkowski space-time $\mathcal{S}_{\rm M}$ with additional geometric structure $\mathcal{L}$, given in it. In other words, $\mathcal{S}_{\rm N}=\mathcal{S}_{\rm M}\wedge\mathcal{L}$. The additional structure $\mathcal{L}$ is a specific timelike direction in $\mathcal{S}_{\rm M}$, described by the constant timelike vector l_k. Introduction of $\mathcal{L}$ admits one to construct two invariants in $\mathcal{S}_{\rm M}\wedge\mathcal{L}$

$$t=l_kx^k,\qquad r=\sqrt{x^kx_k+(l_kx^k)^2} \tag{9.6}$$

for a vector x^k, whereas in $\mathcal{S}_{\rm M}$ we have only one invariant $s=\sqrt{x^kx_k}$. It does not refer to $\mathcal{L}$.

The Newtonian space-time $\mathcal{S}_{\rm N}$ considered as $\mathcal{S}_{\rm M}\wedge\mathcal{L}$ admits only such motions of $\mathcal{S}_{\rm M}$, which transform vector l_k into the same vector l_k and do not violate the structure $\mathcal{L}$. The condition of the structure $\mathcal{L}$ conservation at the space-time motion reduces the ten-parametric group of motion of $\mathcal{S}_{\rm M}$ to seven-parametric group of motion of $\mathcal{S}_{\rm M}\wedge\mathcal{L}$. In general, at the relativistically covariant description the absolute objects, introduced by Anderson [26] may be considered as the quantities, describing additional structures in $\mathcal{S}_{\rm M}$. It means, that any system of dynamic equations may be written in the relativistically covariant form, provided the proper absolute objects (additional structures) are introduced. Thus, to determine, whether the dynamic equations are compatible with the principles of relativity, we may write them in the relativistically covariant form and determine whether or not they contain absolute objects and what are properties of these absolute objects. If the dynamic equations contain the constant timelike vector l_k, we have nonrelativistic dynamic system, because l_k describes the additional space-time structure, characteristic for the Newtonian space $\mathcal{S}_{\rm N}$ represented as $\mathcal{S}_{\rm M}\wedge\mathcal{L}$.

Such an approach is convenient in the sense, that it does not contain a reference to the coordinate system, which is simply a method of description. Relativistic character of dynamic equation is connected directly with absence of additional space-time structures in $\mathcal{S}_{\rm M}$, but not with the relativistically covariant form of the dynamic equations, because any dynamic equations can be always written in the relativistically covariant form, provided the proper geometrical structure is introduced in $\mathcal{S}_{\rm M}$. The relativistically covariant dynamic equation is relativistic, provided it does not contain a reference to some additional structure. However, such a formulation is unreliable, because the reservation of a reference to additional structure may be omitted by mistake. In this case the relativistic character of dynamic equations appear to be connected with the relativistic covariance of these equations, but not with the additional structure $\mathcal{L}$ in $\mathcal{S}_{\mathrm{M}}$. It is this case that takes place in reality. As a result we have an associative mistake, when the relativistic invariance is associated with the relativistic covariance, although in reality the relativistic invariance is associated with an absence of additional geometrical structures in $\mathcal{S}_{\mathrm{M}}$.

The experimental-fitting style of investigation, applied everywhere, is the main defect of the contemporary investigation strategy. This style is applied not only in investigation of concrete physical phenomena of microcosm, where its application is admissible. It is applied also at construction of the fundamental physical theory, that is not admissible, because the fundamental physical theory is a systematization of our knowledge and establishment of logical connection between the fundamental concepts. The fundamental physical theory is a logical structure, but not a list of rules, which should be used for explanation experimental data. The list of the rules may contain the rules, which are contradictory between themselves, but the statements of a logical structure must not be contradictory.

Unfortunately, the Newtonian investigation strategy is not used practically, because it cannot be used, if our fundamental concepts contain mistakes. Some of them were listed in introduction. These mistakes were not actual in the 19th century, when the microcosm was not investigated, and the Newtonian investigation strategy was dominating. Existence of these mistakes during the 20th century was the reason, why at the microcosm investigations the Newtonian strategy was replaced by the experimental-fitting investigation strategy. The last has the advantage, that it is insensitive to mistakes in the foundation of a physical theory. Following the experimental-fitting strategy, the researcher of microcosm did not try to find mistakes of their predecessors. Further more, doubts in results, obtained by great predecessors were considered to be a bad form, generated by self-conceit. Instead of searching for mistakes, that was prescribed by the Newtonian strategy, researchers invented new hypotheses. *Three generations* of the microcosm researchers were educated in ideas of the experimental-fitting strategy. Having investigated the Dirac equation and obtaining the first result, that the Pauli particle is the nonrelativistic approximation of the Dirac particle, they did not try to carry out the investigation completely, because the experimental-fitting strategy does not demand this. Why is it necessary, if the obtained result explains the experimental data? The fact that the investigation is not complete, and the Dirac particle is a composite particle was not considered, although such incomplete investigation was incorrect mathematically, and the mistake could be found at the scrupulous mathematical investigation.

Preconceptions of the experimental-fitting style of thinking are very strong. Even the author of this paper is not free of them, although he is an adherent of the Newtonian strategy and tries to use this strategy in his investigations. For instance, he could not find the mistake in the nonrelativistic approximation of the Dirac equation. He has paid attention on the reduction of the order of the dynamic system and on the small parameter before the highest derivatives only after he had discovered, that the Dirac particle was composite. The last result was obtained from other consideration [2, 25].

Mathematical Appendix

A Transformation of the action for the statistical ensemble

Let us transform the action

$$\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]: \qquad \mathcal{A}_{\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]}\left[\mathbf{x},\mathbf{u}_{\mathrm{st}}\right] = \int \left\{ \frac{m\dot{\mathbf{x}}^2}{2} - \frac{e}{c}A_0 - \frac{e}{c}\mathbf{A}\frac{d\mathbf{x}}{dt} + \frac{m\mathbf{u}_{\mathrm{st}}^2}{2} - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}_{\mathrm{st}} \right\} dt d\boldsymbol{\xi} \tag{A.1}$$

for the statistical ensemble of stochastic particles, moving in the given electromagnetic field $A = \{A_0, \mathbf{A}\} = \{A_0, A_1, A_2, A_3\}$. Here $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$, $\mathbf{u}_{\mathrm{st}} = \mathbf{u}_{\mathrm{st}}(t, \mathbf{x})$ are dependent dynamic variables, and $\boldsymbol{\nabla} = \{\partial_1, \partial_2, \partial_3\} = \left\{\frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right\}$. The variable $\mathbf{x}$ describes the regular component of the stochastic particle motion. The dynamic variable $\mathbf{u}_{\mathrm{st}}$ is a function of $t, \mathbf{x}$ and depends on $\boldsymbol{\xi}$ via $\mathbf{x}$. The quantity $\mathbf{u}_{\mathrm{st}}$ may be regarded as the mean velocity of the stochastic component, whereas $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$ describes the regular component of the particle motion. The last term in (A.1) describes influence of the stochasticity on the regular evolution component.

To eliminate variable $\mathbf{u}_{\mathrm{st}}$, we should to solve dynamic equations $\delta\mathcal{A}/\delta\mathbf{u}_{\mathrm{st}} = 0$ with respect to $\mathbf{u}_{\mathrm{st}}$. As far as $\mathbf{u}_{\mathrm{st}}$ is a function of $t, \mathbf{x}$, we should go to independent variables $t, \mathbf{x}$ in the action (A.1). We obtain instead of (A.1)

$$\mathcal{A}_{\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]}\left[\boldsymbol{\xi},\mathbf{u}_{\mathrm{st}}\right] = \int \left\{ \frac{m\dot{\mathbf{x}}^2}{2} - \frac{e}{c}A_0 - \frac{e}{c}\mathbf{A}\frac{d\mathbf{x}}{dt} + \frac{m\mathbf{u}_{\mathrm{st}}^2}{2} - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}_{\mathrm{st}} \right\} \rho(t, \mathbf{x})\, dt d\mathbf{x} \tag{A.2}$$

where $\boldsymbol{\xi}$, $\mathbf{u}_{\mathrm{st}}$ are dependent variables, whereas $t, \mathbf{x}$ are independent variables. Here ρ and $\dot{\mathbf{x}} = \mathbf{u}$ are functions of $\boldsymbol{\xi}$, defined by the relations

$$\rho = \frac{\partial(\xi_1, \xi_2, \xi_3)}{\partial(x^1, x^2, x^3)}, \qquad \dot{\mathbf{x}} \equiv \mathbf{u} \equiv \frac{\partial(\mathbf{x}, \xi_1, \xi_2, \xi_3)}{\partial(t, \xi_1, \xi_2, \xi_3)} = \frac{1}{\rho}\frac{\partial(\mathbf{x}, \xi_1, \xi_2, \xi_3)}{\partial(t, x^1, x^2, x^3)}, \tag{A.3}$$

Variation of (A.2) with respect $\mathbf{u}_{\mathrm{st}}$ gives

$$\frac{\delta\mathcal{A}_{\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]}}{\delta\mathbf{u}_{\mathrm{st}}} = m\mathbf{u}_{\mathrm{st}}\rho + \frac{\hbar}{2}\boldsymbol{\nabla}\rho = 0 \tag{A.4}$$

Resolving the equation (A.4) with respect to $\mathbf{u}_{\mathrm{st}}$ in the form

$$\mathbf{u}_{\mathrm{st}} = -\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho, \tag{A.5}$$

we obtain instead of (A.2)

$$\mathcal{A}_{\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]}\left[\boldsymbol{\xi}\right] = \int \left\{ \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 - \frac{e}{c}A_0 - \frac{e}{c}\mathbf{A}\frac{d\mathbf{x}}{dt} - \frac{\hbar^2}{8m}\frac{(\boldsymbol{\nabla}\rho)^2}{\rho^2} \right\} \rho\, dt d\mathbf{x} \tag{A.6}$$

where ρ and $\frac{d\mathbf{x}}{dt}$ are functions of space-time derivatives of $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, determined by the relations (A.3). The action (A.6) describes some ideal charged fluid with the internal energy per unit mass

$$U(\rho, \boldsymbol{\nabla}\rho) = \frac{\hbar^2}{8m} \frac{(\boldsymbol{\nabla}\rho)^2}{\rho^2} \tag{A.7}$$

Let us introduce new dependent variables $j = \{\rho, \rho\mathbf{u}\} = \{j^k\}$, $k = 0, 1, 2, 3$ by means of relations (A.3). From formal viewpoint it is convenient to represent the hydrodynamic variables $j = \{\rho, \rho\mathbf{u}\} = \{j^k\}$, $k = 0, 1, 2, 3$ in the form

$$j^k = \frac{\partial\left(x^k, \xi_1, \xi_2, \xi_3\right)}{\partial\left(x^0, x^1, x^2, x^3\right)} = \frac{\partial J}{\partial \xi_{0,k}}, \qquad k = 0, 1, 2, 3 \tag{A.8}$$

where the Jacobian

$$J = \frac{\partial\left(\xi_0, \xi_1, \xi_2, \xi_3\right)}{\partial\left(x^0, x^1, x^2, x^3\right)} = \det||\xi_{i,k}||, \qquad \xi_{l,k} \equiv \partial_k \xi_l, \qquad l, k = 0, 1, 2, 3 \tag{A.9}$$

is considered to be a function of variables $\xi_{l,k} \equiv \partial_k \xi_l$, $l, k = 0, 1, 2, 3$. The variable ξ_0 is the new dependent variable (temporal Lagrangian coordinate), which appears to be fictitious.

We introduce new dynamic variables by the Lagrange multipliers $p = \{p_k\}$, $k = 0, 1, 2, 3$, and obtain instead of (A.6)

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}[\xi, j, p] = \int \left\{ \frac{m}{2\rho} j^\alpha j^\alpha - \frac{e}{c} A_0 \rho - \frac{e}{c} A_\alpha j^\alpha - p_k \left(j^k - \frac{\partial J}{\partial \xi_{0,k}} \right) - \frac{\hbar^2}{8m} \frac{(\boldsymbol{\nabla}\rho)^2}{\rho} \right\} d^4x \tag{A.10}$$

where $\xi = \{\xi_k\}$, $k = 0, 1, 2, 3$.

Variation of the action (A.10) with respect to ξ_l leads to the dynamic equations

$$\frac{\delta \mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}}{\delta \xi_l} = \partial_s \left(p_k \frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{l,s}} \right) = 0, \qquad l = 0, 1, 2, 3 \tag{A.11}$$

As far as the variable ξ_0 is fictitious, there are only three independent equations among four equations (A.11).

Using identities

$$\frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{l,s}} \equiv J^{-1} \left(\frac{\partial J}{\partial \xi_{0,k}} \frac{\partial J}{\partial \xi_{l,s}} - \frac{\partial J}{\partial \xi_{0,s}} \frac{\partial J}{\partial \xi_{l,k}} \right) \tag{A.12}$$

$$\frac{\partial J}{\partial \xi_{i,l}} \xi_{k,l} \equiv J \delta_k^i, \qquad \partial_l \frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{i,l}} \equiv 0 \tag{A.13}$$

and designations (A.8), we can eliminate the variables $\boldsymbol{\xi}$ from the equations (A.11). We obtain

$$j^k \partial_l p_k - j^k \partial_k p_l = 0, \qquad l = 0, 1, 2, 3 \tag{A.14}$$

Variation of (A.10) with respect to j^β and $j^0=\rho$ gives respectively

$$p_\beta = m\frac{j^\beta}{\rho} - \frac{e}{c}A_\beta, \qquad \beta = 1,2,3 \tag{A.15}$$

$$p_0 = -\frac{m}{2\rho^2}j^\alpha j^\alpha - \frac{e}{c}A_0 + \frac{\hbar^2}{8m}\left(2\frac{\boldsymbol{\nabla}^2\rho}{\rho} - \frac{(\boldsymbol{\nabla}\rho)^2}{\rho^2}\right) \tag{A.16}$$

Eliminating p_k from the equations (A.14) by means of relations (A.15), (A.16), we obtain hydrodynamic equations for the ideal charged fluid in the conventional form

$$\left(\partial_0 + v^\alpha\partial_\alpha\right)v^\mu = \frac{e}{mc}F_{\mu 0} + \frac{e}{mc}F_{\mu\alpha}v^\alpha - \frac{1}{m\rho}\partial_\mu p, \qquad \mu = 1,2,3 \tag{A.17}$$

where the pressure p and the electromagnetic field F_{ik} are defined by the relations

$$p = \frac{\hbar^2}{8m}\left(\frac{(\boldsymbol{\nabla}\rho)^2}{\rho^2} - 2\frac{\boldsymbol{\nabla}^2\rho}{\rho}\right), \qquad F_{ik} = \partial_k A_i - \partial_i A_k, \qquad i,k = 0,1,2,3 \tag{A.18}$$

The wave function is constructed of potentials. The equations (A.17) does not contain potentials $\boldsymbol{\xi}$ and A_k, and they cannot be used for description of the fluid in terms of the wave function. To construct a description in terms of the wave function, we should not to eliminate potentials $\boldsymbol{\xi}$ from the equations (A.11). Instead, we should integrate them. The dynamic equations (A.11) may be considered to be linear partial differential equations with respect to variables p_k. They can be solved in the form

$$p_k = b\left(\partial_k\varphi + g^\alpha(\boldsymbol{\xi})\,\partial_k\xi_\alpha\right), \qquad k = 0,1,2,3 \tag{A.19}$$

where $g^\alpha(\boldsymbol{\xi})$, $\alpha = 1,2,3$ are arbitrary functions of the argument $\boldsymbol{\xi} = \{\xi_1,\xi_2,\xi_3\}$, b is an arbitrary real constant, and φ is the variable ξ_0, which ceases to be fictitious.

One can test by the direct substitution that the relation (A.19) is the general solution of linear equations (A.11). Indeed, using (A.12) and the second identity (A.13), the equations (A.11) may be written in the form

$$\frac{\partial^2 J}{\partial\xi_{0,k}\partial\xi_{l,s}}\partial_s p_k = J^{-1}\left(\frac{\partial J}{\partial\xi_{0,k}}\frac{\partial J}{\partial\xi_{l,s}} - \frac{\partial J}{\partial\xi_{0,s}}\frac{\partial J}{\partial\xi_{l,k}}\right)\partial_s p_k = 0 \tag{A.20}$$

Substituting (A.19) in (A.20) and taking into account antisymmetry of the bracket in (A.20) with respect to indices k and s, we obtain

$$J^{-1}\left(\frac{\partial J}{\partial\xi_{0,k}}\frac{\partial J}{\partial\xi_{l,s}} - \frac{\partial J}{\partial\xi_{0,s}}\frac{\partial J}{\partial\xi_{l,k}}\right)\frac{\partial g^\alpha(\boldsymbol{\xi})}{\partial\xi_\mu}\xi_{\mu,s}\xi_{\alpha,k} = 0 \tag{A.21}$$

The relation (A.21) is the valid equality, as it follows from the first identity (A.13).

Let us substitute (A.19) in the action (A.10). Taking into account the first identity (A.13) and omitting the term

$$\frac{\partial J}{\partial\xi_{0,k}}\partial_k\varphi = \frac{\partial\left(\varphi,\xi_1,\xi_2,\xi_3\right)}{\partial\left(x^0,x^1,x^2,x^3\right)}$$

which does not contribute to the dynamic equation, we obtain

$$\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]: \qquad \mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}\left[\varphi,\boldsymbol{\xi},j\right]=\int\left\{\frac{m}{2}\frac{j^{\alpha}j^{\alpha}}{j^{0}}-\frac{e}{c}A_{k}j^{k}-j^{k}p_{k}-\frac{\hbar^{2}}{8m}\frac{\left(\boldsymbol{\nabla}\rho\right)^{2}}{\rho}\right\}d^{4}x, \tag{A.22}$$

Here quantities p_k are determined by the relations (A.19).

The action in the form (A.22) is remarkable in the sense, that it contains information on initial values of the fluid velocities $\mathbf{v}=\mathbf{j}/\rho$. Dynamic equations, generated by the action (A.22), are partial differential equations, and one needs to give initial values for variables $\varphi,\boldsymbol{\xi}$. But initial values for variables $\varphi,\boldsymbol{\xi}$ determine only labelling of the fluid particles, and they may be chosen universal. For instance, we may choose for all fluid flows

$$\varphi\left(0,\mathbf{x}\right)=\varphi_{\mathrm{in}}\left(\mathbf{x}\right)=0, \qquad \boldsymbol{\xi}\left(0,\mathbf{x}\right)=\boldsymbol{\xi}_{\mathrm{in}}\left(\mathbf{x}\right)=\mathbf{x} \tag{A.23}$$

Then the functions $\mathbf{g}\left(\boldsymbol{\xi}\right)$ are determined by the initial values of the velocity $\mathbf{v}\left(0,\mathbf{x}\right)=\mathbf{v}_{\mathrm{in}}\left(\mathbf{x}\right)$ in the form [13]

$$\mathbf{g}\left(\boldsymbol{\xi}\right)=\mathbf{v}_{\mathrm{in}}\left(\boldsymbol{\xi}\right) \tag{A.24}$$

The initial value $\rho\left(0,\mathbf{x}\right)=\rho_{\mathrm{in}}\left(\mathbf{x}\right)$ of the density ρ may be also included in the action (A.22). It is necessary only to redefine the connection between the quantities j^k and $\boldsymbol{\xi}$, substituting the relations (A.8) by the relations [13]

$$j^{k}=\rho_{0}\left(\boldsymbol{\xi}\right)\frac{\partial\left(x^{k},\xi_{1},\xi_{2},\xi_{3}\right)}{\partial\left(x^{0},x^{1},x^{2},x^{3}\right)}, \qquad k=0,1,2,3 \tag{A.25}$$

where $\rho_0\left(\boldsymbol{\xi}\right)$ is an arbitrary function of $\boldsymbol{\xi}$. At the initial conditions (A.24) this arbitrary function is to be chosen in the form

$$\rho_{0}\left(\mathbf{x}\right)=\rho_{\mathrm{in}}\left(\mathbf{x}\right)=\rho\left(0,\mathbf{x}\right)$$

Now we eliminate the variables $\mathbf{j}=\{j^1,j^2,j^3\}$ from the action (A.22), using relation (A.15). We obtain

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}\left[\rho,\varphi,\boldsymbol{\xi}\right]=\int\left\{-p_{0}-\frac{e}{c}A_{0}-\frac{\left(p_{\beta}+\frac{e}{c}A_{\beta}\right)\left(p_{\beta}+\frac{e}{c}A_{\beta}\right)}{2m}-\frac{\hbar^{2}}{8m}\frac{\left(\boldsymbol{\nabla}\rho\right)^{2}}{\rho^{2}}\right\}\rho d^{4}x, \tag{A.26}$$

where the quantities p_k, $k=0,1,2,3$ are determined by the relation (A.19).

Instead of dependent variables $\rho,\varphi,\boldsymbol{\xi}$ we introduce the n-component complex function $\psi=\{\psi_\alpha\}$, $\alpha=1,2,\ldots,n$, which is defined by the relations [13]

$$\psi_{\alpha}=\sqrt{\rho}e^{i\varphi}u_{\alpha}(\boldsymbol{\xi}), \qquad \psi_{\alpha}^{*}=\sqrt{\rho}e^{-i\varphi}u_{\alpha}^{*}(\boldsymbol{\xi}), \qquad \alpha=1,2,\ldots,n, \tag{A.27}$$

$$\psi^{*}\psi\equiv\sum_{\alpha=1}^{n}\psi_{\alpha}^{*}\psi_{\alpha}, \tag{A.28}$$

where (*) means the complex conjugate. The quantities $u_\alpha(\boldsymbol{\xi})$, $\alpha = 1, 2, \ldots, n$ are functions of only variables $\boldsymbol{\xi}$, and satisfy the relations

$$-\frac{i}{2}\sum_{\alpha=1}^{n}\left(u_\alpha^*\frac{\partial u_\alpha}{\partial \xi_\beta} - \frac{\partial u_\alpha^*}{\partial \xi_\beta}u_\alpha\right) = g^\beta(\boldsymbol{\xi}), \qquad \beta = 1, 2, 3, \qquad \sum_{\alpha=1}^{n} u_\alpha^* u_\alpha = 1. \tag{A.29}$$

The number n is such a natural number that the equations (A.29) admit a solution. In general, n depends on the form of the arbitrary integration functions $\mathbf{g} = \{g^\beta(\boldsymbol{\xi})\}$, $\beta = 1, 2, 3$. The functions $\mathbf{g}$ determine vorticity of the fluid flow. If $\mathbf{g} = 0$, equations (A.29) have the solution $u_1 = 1$, $u_\alpha = 0$, $\alpha = 2, 3, ...n$. In this case the function ψ may have one component, and the fluid flow is irrotational.

In the general case it is easy to verify that

$$\rho = \psi^*\psi, \qquad \rho p_0(\varphi, \boldsymbol{\xi}) = -\frac{ib}{2}(\psi^*\partial_0\psi - \partial_0\psi^* \cdot \psi) \tag{A.30}$$

$$\rho p_\alpha(\varphi, \boldsymbol{\xi}) = -\frac{ib}{2}(\psi^*\partial_\alpha\psi - \partial_\alpha\psi^* \cdot \psi), \qquad \alpha = 1, 2, 3, \tag{A.31}$$

The variational problem with the action (A.22) appears to be equivalent to the variational problem with the action functional

$$\begin{aligned} &\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}[\psi, \psi^*] \\ = &\int\left\{\frac{ib}{2}(\psi^*\partial_0\psi - \partial_0\psi^* \cdot \psi) - \frac{e}{c}A_0\rho \right. \\ &\left. -\frac{\rho}{2m}\left(-\frac{ib}{2\rho}(\psi^*\boldsymbol{\nabla}\psi - \boldsymbol{\nabla}\psi^* \cdot \psi) + \frac{e}{c}\mathbf{A}\right)^2 - \frac{\hbar^2}{8m}\frac{(\boldsymbol{\nabla}\rho)^2}{\rho}\right\}\mathrm{d}^4x \end{aligned} \tag{A.32}$$

or

$$\begin{aligned} &\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}[\psi, \psi^*] \\ = &\int\left\{\frac{ib}{2}(\psi^*\partial_0\psi - \partial_0\psi^* \cdot \psi) - \frac{e}{c}A_0 + \frac{b^2}{8m\rho}(\psi^*\boldsymbol{\nabla}\psi - \boldsymbol{\nabla}\psi^* \cdot \psi)^2 \right. \\ &\left. +\frac{ibe}{2mc}\mathbf{A}(\psi^*\boldsymbol{\nabla}\psi - \boldsymbol{\nabla}\psi^* \cdot \psi) - \frac{\hbar^2}{8m}\frac{(\boldsymbol{\nabla}\rho)^2}{\rho} - \frac{\rho}{2m}\left(\frac{e}{c}\mathbf{A}\right)^2\right\}\mathrm{d}^4x \end{aligned} \tag{A.33}$$

For the two-component function ψ $(n = 2)$ the following identity takes place

$$(\boldsymbol{\nabla}\rho)^2 - (\psi^*\boldsymbol{\nabla}\psi - \boldsymbol{\nabla}\psi^* \cdot \psi)^2 \equiv 4\rho\boldsymbol{\nabla}\psi^*\boldsymbol{\nabla}\psi - \rho^2\sum_{\alpha=1}^{\alpha=3}(\boldsymbol{\nabla}s_\alpha)^2, \tag{A.34}$$

$$\rho \equiv \psi^*\psi, \qquad \mathbf{s} \equiv \frac{\psi^*\boldsymbol{\sigma}\psi}{\rho}, \qquad \boldsymbol{\sigma} = \{\sigma_\alpha\}, \qquad \alpha = 1, 2, 3, \tag{A.35}$$

where σ_α are the Pauli matrices. In virtue of the identity (A.34) the action (A.32) reduces to the form

$$\begin{aligned} & \mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}[\psi, \psi^*] \\ = & \int \left\{ \frac{ib}{2} \left(\psi^* \partial_0 \psi - \partial_0 \psi^* \cdot \psi\right) - \frac{e}{c} A_0 - \frac{1}{2m} \left(-ib\boldsymbol{\nabla}\psi^* - \frac{e}{c}\mathbf{A}\psi^*\right) \left(ib\boldsymbol{\nabla}\psi - \frac{e}{c}\mathbf{A}\psi\right) \right. \\ & \left. + \frac{b^2 - \hbar^2}{8\rho m} (\boldsymbol{\nabla}\rho)^2 + \frac{b^2}{8m} \sum_{\alpha=1}^{\alpha=3} (\boldsymbol{\nabla} s_\alpha)^2 \rho \right\} \mathrm{d}^4 x, \end{aligned} \tag{A.36}$$

where $\mathbf{s}$ and ρ are defined by the relations (A.35). One should expect, that the two-component wave function describes the general case, because the number of real components of the two-component wave function coincides with the number of hydrodynamic variables $\{\rho, \mathbf{j}\}$. But this statement is not yet proved.

In the case of irrotational flow, when the two-component function ψ has linear dependent components, for instance $\psi = \{\psi_1, 0\}$, the 3-vector $\mathbf{s}$ =const, and the term containing 3-vector $\mathbf{s}$ vanishes. In the special case, when the electromagnetic potentials $A_k = 0$, the action (A.36) for $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ coincides with the action (2.25) for $\mathcal{S}_{\mathrm{S}}$.

Finally, if we choose the arbitrary constant b in the form $b = \hbar$ and set $A_k = 0$, we obtain the action (2.1) for the Schrödinger particle.

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