Dynamical methods of investigations in application to the Schrödinger particle

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Abstract

Quantum systems are dynamic systems restricted by the principles of quantum mechanics (linearity of dynamic equations, linear transformation of the wave function etc.). One suggests to investigate the quantum systems simply as dynamic systems, ignoring the quantum principles and constraints imposed by them. Such dynamic methods of investigation appear to be more adequate and effective, than the conventional quantum methods of investigation. Using these methods, which ignore the quantum principles, one can overcome the principal problem of quantum field theory: join of nonrelativistic quantum principles with the relativity principles. Investigation of dynamic system $\mathcal{S}_{\mathrm{S}}$, described by the Schrödinger equation, admits one to prove that the Copenhagen interpretation is incompatible with quantum mechanics formalism. Besides, it is shown that sometimes the application of quantum principles leads to incorrect results.

1 Introduction

One uses new dynamical methods of investigation, which ignore the quantum principles. Elimination of the quantum principles admits one to eliminate the principal problem of QFT: *join* of the *relativity principles* with the *principles of quantum mechanics.* After this elimination the relativistic quantum theory is constructed on the basis of relativistic dynamics only, whereas the conventional approach uses in addition the linearity of dynamic equations. This linearity is based on a special gauge of the wave function. A use of such an artificial property of the wave function for construction of the relativistic quantum theory seems to be unjustified, although the linearity is very convenient, because the linear equations are simple for solution. We show in the simple example of Schrödinger particle $\mathcal{S}_{\mathrm{S}}$ (dynamic system described by the Schrödinger equation), that the difference between the quantum system and the classical one is purely dynamical, and the quantum system may be described without a reference to quantum principles.

A fundamental physical theory $\mathcal{T}_{\rm f}$ must be a logical structure. It means, that the theory contains a few fundamental propositions. All predictions of the theory $\mathcal{T}_{\rm f}$ as well as other propositions $\mathcal{P}_{\rm i}$ intermediate between the fundamental propositions $\mathcal{P}_{\rm f}$ and experimental data $\mathcal{D}_{\rm e}$ are deduced from the fundamental propositions $\mathcal{P}_{\rm f}$ by means of logical reasonings and mathematical calculations. Practical derivation of predictions and explanations from the fundamental propositions $\mathcal{P}_{\rm f}$ may be difficult and complicated, because of long reasonings and complicated calculations. In this case one uses a set of intermediate propositions $\mathcal{P}_{\rm i}$, which are valid in some region of physical phenomena. Practical application of the intermediate propositions $\mathcal{P}_{\rm i}$ as some physical theory $\mathcal{T}_{\rm c}$ may appear to be more effective and simpler, than a use of the fundamental theory $\mathcal{T}_{\rm f}$.

In general, the intermediate propositions $\mathcal{P}_{\rm i}$ are to be corollaries of the fundamental propositions $\mathcal{P}_{\rm f}$. However, if it is impossible to discover the true fundamental propositions $\mathcal{P}_{\rm f}$, we may guess intermediate propositions $\mathcal{P}_{\rm i}$ and consider them as a curtailed physical theory $\mathcal{T}_{\rm c}$. The set of prescriptions $\mathcal{P}_{\rm i}$ is chosen in such a way to explain some set of experimental data. The curtailed theory $\mathcal{T}_{\rm c}$ is not a logical structure, it is simply a list of prescriptions, which are not connected logically between themselves. The logical structure appears, only if we discover and add the fundamental propositions $\mathcal{P}_{\rm f}$, which generate this list of prescriptions $\mathcal{P}_{\rm i}$ under some conditions. In this case the intermediate propositions $\mathcal{P}_{\rm i}$ may be eliminated from the formulation of the fundamental theory $\mathcal{T}_{\rm f}$, because the propositions $\mathcal{P}_{\rm i}$ are corollaries of fundamental propositions $\mathcal{P}_{\rm f}$.

Thus, we may use the curtailed physical theory $\mathcal{T}_{\rm c}$ instead of the fundamental physical theory $\mathcal{T}_{\rm f}$. For instance, the axiomatic thermodynamics is a curtailed theory with respect to kinetic theory, which may be considered to be a fundamental theory. The conventional quantum mechanics is a curtailed theory, whereas the corresponding fundamental theory is not known yet. Such an application of $\mathcal{T}_{\rm c}$ is possible only under those conditions (for instance, in the nonrelativistic case), when the intermediate propositions $\mathcal{P}_{\rm i}$ are valid. However, extension of the curtailed theory $\mathcal{T}_{\rm c}$ to other conditions (for instance, to the relativistical case) may appear to be impossible, because the intermediate propositions $\mathcal{P}_{\rm i}$ are only corollaries of the fundamental propositions $\mathcal{P}_{\rm f}$ in the nonrelativistic case. Under another conditions (for instance, in the relativistical case) the fundamental propositions $\mathcal{P}_{\rm f}$ may generate another intermediate propositions $\mathcal{P}_{\rm i}^{*}$, which does not coincide with $\mathcal{P}_{\rm i}$ and cannot be obtained from $\mathcal{P}_{\rm i}$, because the set of $\mathcal{P}_{\rm i}$ is simply a list of prescriptions (a list of corollaries of $\mathcal{P}_{\rm f}$), but not a logical structure.

This fact is shown in the scheme of Figure.1. The direct way from the conventional nonrelativistic quantum theory to the relativistic quantum theory is very difficult, because the conventional nonrelativistic quantum theory is a kind of curtailed theory. The quantum principles form the essential part of this theory. In general, the quantum principles are to be corollaries of fundamental propositions and of nonrelativistic constraints. Unfortunately, we cannot be sure that the quantum principles are logical corollaries of fundamental principles, because the fundamental propositions were not known, when the quantum principles were formulated. The quantum principles have been nicked, and we cannot be sure that they have nicked correctly in all details. Even if the quantum principles has been nicked correctly in all details, we cannot decide what is a corollary of fundamental propositions and what is a corollary of nonrelativistic constraints. To construct the relativistic quanum theory, we are to clean out the nonrelativistic quantum theory from its nonrelativistic features. The only reliable method of such a refinement is a return to the fundamental propositions. Unfortunately, the fundamental propositions are not known.

In particular, the particle production effect is the essential part of relativistic quantum theory. However, this effect is present neither in classical relativistic physics, nor in the nonrelativistic quantum principles. The contemporary relativistic quantum theory takes into account the particle production only formally, introducing the creation and annihilation operators. At such a consideration of the particle production effect the creation of particles is possible not only by pairs, but by terns, by quaternaries, etc. It depends on the form of the Lagrangian. It means that the particle production effect is taken into account on the dynamical level, whereas it should be taken into account on the more fundamental level, because in reality the particles are produced only by pairs. It is conditioned by the fact that the particle production is connected with the turn of the particle world line in time.

In this paper we try to obtain the fundamental proposition (the starting point of the quantum theory) in the example of the Schrödinger particle

2 Artificiality of conventional method of description

The action for the Schrödinger particle $\mathcal{S}_{\mathrm{S}}$ has the form

$$\mathcal{A}_{\mathrm{S}}[\psi] = \int \left\{ \frac{i\hbar}{2} \left(\psi^* \partial_0 \psi - \partial_0 \psi^* \cdot \psi \right) - \frac{\hbar^2}{2m} \boldsymbol{\nabla} \psi^* \boldsymbol{\nabla} \psi \right\} d^4 x \tag{2.1}$$

where $\psi = \psi(t, \mathbf{x})$ is a complex wave function. The action carries out the complete description of the Schrödinger particle $\mathcal{S}_{\mathrm{S}}$, because it generates the dynamic equation

$$i\hbar \partial_0 \psi = -\frac{\hbar^2}{2m} \boldsymbol{\nabla}^2 \psi, \tag{2.2}$$

and corresponding canonical quantities: the 4-current j^k and the energy-momentum tensor T_l^k

$$j^k = \{\rho, \mathbf{j}\} = \left\{ \psi^* \psi, -\frac{i\hbar}{2m} \left(\psi^* \boldsymbol{\nabla} \psi - \boldsymbol{\nabla} \psi^* \cdot \psi \right) \right\} \tag{2.3}$$

Connection between the particle and the wave function is described by the relations.

$$\langle F(\mathbf{x},\mathbf{p})\rangle = B\int \mathrm{Re}\left\{\psi^* F(\mathbf{x},\hat{\mathbf{p}})\psi\right\}d\mathbf{x}, \qquad \hat{\mathbf{p}} = -i\hbar\boldsymbol{\nabla}, \qquad B = \left(\int \psi^*\psi d\mathbf{x}\right)^{-1} \tag{2.4}$$

which define the mean value $\langle F(\mathbf{x},\mathbf{p})\rangle$ of any function $F(\mathbf{x},\mathbf{p})$ of position $\mathbf{x}$ and momentum $\mathbf{p}$. We shall refer to these relations together with the restrictions imposed on its applications as the quantum principles, because von Neumann [1] has shown, that all proposition of quantum mechanics can be deduced from relations of this type.

Setting $\hbar = 0$ in the action (2.1), we hope to obtain a description of classical particle. Instead, the description disappears, and in this sense the description (2.1) is artificial.

3 Natural description of the Schrödinger particle

To obtain the natural description of the Schrödinger particle, we produce the change of variables (another gauge of the wave function phase)

$$\psi \to \Psi_b: \quad \psi = |\Psi_b|\exp\left(\frac{b}{\hbar}\log\frac{\Psi_b}{|\Psi_b|}\right) \qquad b = \mathrm{const} \neq 0 \tag{3.1}$$

in the action (2.1). We obtain

$$\mathcal{A}_{\mathrm{S}}[\Psi_b] = \int\left\{\frac{ib}{2}\left(\Psi_b^*\partial_0\Psi_b - \partial_0\Psi_b^*\cdot\Psi_b\right) - \frac{b^2}{2m}\boldsymbol{\nabla}\Psi_b^*\boldsymbol{\nabla}\Psi_b \right.$$

$$\left. + \frac{b^2}{2m}\left(\boldsymbol{\nabla}|\Psi_b|\right)^2 - \frac{\hbar^2}{2m}\left(\boldsymbol{\nabla}|\Psi_b|\right)^2\right\} dt d\mathbf{x} \tag{3.2}$$

This change of variables leads to the replacement $\hbar \to b$ and to appearance of two nonlinear terms which compensate each other, if $b = \hbar$.

The dynamic equation becomes to be nonlinear, if $b^2 \neq \hbar^2$

$$ib\partial_0\Psi_b = -\frac{b^2}{2m}\boldsymbol{\nabla}^2\Psi_b - \frac{\hbar^2 - b^2}{8m}\left(\frac{(\boldsymbol{\nabla}\rho)^2}{\rho^2} + 2\boldsymbol{\nabla}\frac{\boldsymbol{\nabla}\rho}{\rho}\right)\Psi_b, \tag{3.3}$$

$$\rho = \Psi_b^*\Psi_b, \qquad \mathbf{j} = -\frac{ib}{2m}\left(\Psi_b^*\boldsymbol{\nabla}\Psi_b - \boldsymbol{\nabla}\Psi_b^*\cdot\Psi_b\right) \tag{3.4}$$

However, the description becomes to be natural in the sense, that after setting $\hbar = 0$, the action $\mathcal{A}_{\mathrm{S}}[\Psi_b]$ turns into the action

$$\mathcal{A}_{\mathrm{Scl}}[\Psi_b] = \int\left\{\frac{ib}{2}\left(\Psi_b^*\partial_0\Psi_b - \partial_0\Psi_b^*\cdot\Psi_b\right) - \frac{b^2}{2m}\boldsymbol{\nabla}\Psi_b^*\boldsymbol{\nabla}\Psi + \frac{b^2}{2m}\left(\boldsymbol{\nabla}|\Psi_b|\right)^2\right\} dt d\mathbf{x} \tag{3.5}$$

which describes the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{d}}]$ of free classical particles $\mathcal{S}_{\mathrm{d}}$. The action $\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{d}}]}$ for this statistical ensemble can be represented in the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{d}}]}[\mathbf{x}] = \int \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 dt d\boldsymbol{\xi} \tag{3.6}$$

where $\mathbf{x} = \mathbf{x}(t,\boldsymbol{\xi})$ is a 3-vector function of independent variables $t, \boldsymbol{\xi} = \{\xi_1,\xi_2,\xi_3\}$. The variables (Lagrangian coordinates) $\boldsymbol{\xi}$ label particles $\mathcal{S}_{\mathrm{d}}$ of the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{d}}]$. The statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{d}}]$ is a dynamical system of the hydrodynamical type. One can show that the dynamic system, described by the action $\mathcal{A}_{\mathrm{Scl}}[\Psi_b]$ (3.5) is a partial case (irrotational flow) of the dynamic system $\mathcal{E}[\mathcal{S}_{\mathrm{d}}]$ [2].

Connection between the Schrödinger equation and hydrodynamical description is well known [3, 4]. But a connection between the description in terms of wave function and the hydrodynamic description was one-way. One can transit from the Schrödinger equation to the hydrodynamic equations, but one cannot transit from hydrodynamic equations to the description in terms of the wave function, because one *needs* to *integrate* hydrodynamic equations. Indeed, the Schrödinger equation consists of two real first order equations for the density ρ and the phase φ, whereas the system of the hydrodynamical equations consists of four first order equations for the density ρ and for the velocity $\mathbf{v}$. To obtain four hydrodynamic equations one needs to take gradient of the equation for the phase φ. On the contrary, if we transit from the hydrodynamic description to the description in terms of the wave function, we are to integrate hydrodynamic equations. In the general case this integration was not known for a long time.

Change of variables, leading from the action $\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{d}}]}[\mathbf{x}]$ to the action $\mathcal{A}_{\mathrm{Scl}}[\Psi_b]$ contains integration (see [2] or mathematical appendices to papers [6, 7]). The constant b in the action $\mathcal{A}_{\mathrm{Scl}}[\Psi_b]$ is an arbitrary constant of integration (gauge constant). Arbitrary integration functions are "hidden" inside the wave function Ψ_b. Thus, the limit of Schrödinger particle (3.2) at $\hbar \to 0$ is a statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{d}}]$, but not an individual particle $\mathcal{S}_{\mathrm{d}}$. It means, that the *wave function describes a statistical ensemble of particles, but not an individual particle*, and Copenhagen interpretation, where the wave function describes an individual particle, is *incompatible* with the quantum mechanics formalism.

A use of Copenhagen interpretation does not generate any problems, until we consider mathematical formalism of QM, because in the framework of this formalism we have the only object of investigation. It is of no importance what is the name of the investigated object. But at the consideration of the measurement we have two different kinds of measurement:

1. Individual measurement (S-measurement), which is produced over the individual stochastic particle $\mathcal{S}_{\mathrm{st}}$.

2. Massive measurement (M-measurement), which is produced over the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$, or over the statistical average particle $\langle\mathcal{S}\rangle$.

4 Statistical ensemble as a starting point of quantum theory

One can show, that the Schrödinger particle $\mathcal{S}_{\mathrm{S}}$ is a special case of the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ of stochastically moving free particles $\mathcal{S}_{\mathrm{st}}$, i.e. the action (3.2) for $\mathcal{S}_{\mathrm{S}}$ can be obtained from the action for the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ by means of a proper change of variables [2].

The statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ of *stochastic* particles $\mathcal{S}_{\mathrm{st}}$ is a dynamic system, described by the action

$$\mathcal{A}_{\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]}\left[\mathbf{x},\mathbf{u}_{\mathrm{df}}\right]=\int\left\{\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^{2}+\frac{m}{2}\mathbf{u}_{\mathrm{df}}^{2}-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}_{\mathrm{df}}\right\}dtd\boldsymbol{\xi} \tag{4.1}$$

where $\mathbf{u}_{\mathrm{df}}=\mathbf{u}_{\mathrm{df}}\left(t,\mathbf{x}\right)$ is the diffusion velocity, describing the mean value of the stochastic component of the velocity, whereas $\frac{d\mathbf{x}}{dt}\left(t,\boldsymbol{\xi}\right)$ describes the regular component of the particle velocity, and $\mathbf{x}=\mathbf{x}\left(t,\boldsymbol{\xi}\right)$ is a 3-vector function of independent variables $t,\boldsymbol{\xi}=\left\{\xi_{1},\xi_{2},\xi_{3}\right\}$. The variables $\boldsymbol{\xi}$ label stochastic particles $\mathcal{S}_{\mathrm{st}}$, constituting the statistical ensemble. The operator $\boldsymbol{\nabla}$ is defined in the space of coordinates $\mathbf{x}$. Dynamic equations have the form

$$\frac{\delta\mathcal{A}_{\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]}}{\delta\mathbf{x}}=-m\frac{d^{2}\mathbf{x}}{dt^{2}}+\boldsymbol{\nabla}\left(\frac{m}{2}\mathbf{u}_{\mathrm{df}}^{2}-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}_{\mathrm{df}}\right)=0 \tag{4.2}$$

$$\frac{\delta\mathcal{A}_{\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]}}{\delta\mathbf{u}_{\mathrm{df}}}=m\rho\mathbf{u}_{\mathrm{df}}+\frac{\hbar}{2}\boldsymbol{\nabla}\rho=0, \tag{4.3}$$

$$\rho=\left[\frac{\partial\left(x^{1},x^{2},x^{3}\right)}{\partial\left(\xi_{1},\xi_{2},\xi_{3}\right)}\right]^{-1}=\frac{\partial\left(\xi_{1},\xi_{2},\xi_{3}\right)}{\partial\left(x^{1},x^{2},x^{3}\right)} \tag{4.4}$$

Resolving (4.3) with respect to $\mathbf{u}_{\mathrm{df}}$ in the form

$$\mathbf{u}_{\mathrm{df}}=-\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho, \tag{4.5}$$

and eliminating $\mathbf{u}_{\mathrm{df}}$ from equation (4.2), we obtain the dynamic equations of the hydrodynamical type

$$m\frac{d^{2}\mathbf{x}}{dt^{2}}=-\boldsymbol{\nabla}U\left(\rho,\boldsymbol{\nabla}\rho\right),\qquad U\left(\rho,\boldsymbol{\nabla}\rho\right)=\frac{\hbar^{2}}{8m}\left(\frac{\left(\boldsymbol{\nabla}\rho\right)^{2}}{\rho^{2}}-2\frac{\boldsymbol{\nabla}^{2}\rho}{\rho}\right) \tag{4.6}$$

Hydrodynamic equations (4.6) may be written in terms of the wave function [2, 6, 7]. The proper change of the variables together with *integration* turns the action (4.1) for $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ into the action containing the quantum constant $\hbar$ and an arbitrary integration constant b.

The wave function is not a specific quantum quantity. The wave function is a method of description of *any ideal fluid*. Quantum and classical dynamic systems distinguish *dynamically* (by the form of their actions). The form of description (in terms of wave function, or in terms of position and momentum of the particle) is of no importance.

Considering the statistical ensemble of stochastic particles as a starting point of the quantum mechanics, *one does not need quantum principles*, because the interpretation is carried out directly via the particles of the statistical ensemble. Besides, the quantum theory turns into a consistent statistical theory, where there are two sorts of particles: (1) individual stochastic particle $\mathcal{S}_{\rm st}$ and (2) statistical average particle $\langle\mathcal{S}_{\rm st}\rangle$, which is the statistical ensemble, normalized to one particle. Formalism of quantum mechanics deals only with the statistical average particle $\langle\mathcal{S}_{\rm st}\rangle$. All predictions of QM relate only to $\langle\mathcal{S}_{\rm st}\rangle$. In accordance with two sorts of particles we have two sorts of measurements: (1) S-measurement produced over $\mathcal{S}_{\rm st}$ and (2) M-measurement produced over $\langle\mathcal{S}_{\rm st}\rangle$. The two kinds of measurements have different properties and one may not confuse them (see for details [5]).

The Copenhagen interpretation meets the difficulties, when it tries to test predictions of mathematical formalism in single experiments. For instance, there exists the problem of the mechanism of the wave function reduction at a single experiment. Another problem concerns the two-slit experiment. How can an individual particle pass through two slits at once? The physical journals publish discussions concerning problems of quantum measurements. For instance, such a discussion was declared in 2002 by the journal Uspekhi Fizicheskich Nauk. These problems cannot be solved in the framework of the Copenhagen interpretation, which does not distinguish between the individual particle $\mathcal{S}$ and the statistically average particle $\langle\mathcal{S}\rangle$. Confusion of two different objects, having different properties generates difficulties and paradoxes. The wave function does not describe the state of individual particle $\mathcal{S}$, and it is meaningless to ask, how the wave function changes at a single measurement (S-measurement). At the massive experiment (M-measurement) we obtain a distribution $F(R')$ of the measured quantity $\mathcal{R}$, but not a single value R' of the measured quantity. At such a situation it is useless to ask, how the obtained result R' influences on the the state of the statistical ensemble (statistical average particle). Finally, we may define the third type of measurement (SM-measurement): the massive measurement of the quantity $\mathcal{R}$ leading to a definite value R' of the measured quantity $\mathcal{R}$. The SM-measurement is the M-measurement leading to a distribution $F(R')$, accompanied by a selection of those particles, where result of S-measurement is R'. Uniting all particles with the measured value R' in one statistical ensemble $\mathcal{E}_{R'}$, we can put the question about the wave function of $\mathcal{E}_{R'}$. Of course, the wave function $\psi_{R'}$ of $\mathcal{E}_{R'}$ does not coincide, in general, with the initial wave function ψ, and this change of the wave function is considered as a reduction of the wave function. The origin of the reduction is quite transparent. It is the selection, which is produced to obtain the same value R' of the measured quantity for all particles of the statistical ensemble. Thus, the problems of reduction are conditioned by the confusion of concepts of the individual particle $\mathcal{S}$ and the statistical average particle $\langle\mathcal{S}\rangle$, which takes place at the Copenhagen interpretation (see for details [5]).

As concerns the particle, passing through two slits simultaneously, it is a reasonable property of the statistical average object. It is a pure statistical property, which has nothing to do with quantum properties. Individual particle $\mathcal{S}$ can pass either through one slit, or through another, whereas the statistical average particle $\langle\mathcal{S}\rangle$ can pass through both slits simultaneously. (Compare, individual person is either a man, or a woman, whereas the statistical average person is a hermaphrodite (half-man half-woman), and there are no quantum mechanical properties here).

Using statistical ensemble as a starting point, we can test validity the quantum principles. The quantum principles or their conventional interpretation appear to be wrong in some cases. For instance, the momentum distribution $w(\mathbf{p}) = |\langle\mathbf{p}|\psi\rangle|^2$ is in reality a distribution over mean momenta $\langle\mathbf{p}\rangle$. Let us illustrate the difference between the two distribution in the example of ideal gas, whose state may be described by the wave function. In the case, when the gas moves with the constant velocity $\mathbf{u}$, the wave function has the form

$$\psi(\mathbf{x}) = A_1 e^{\frac{i}{\hbar}m\mathbf{ux}}, \qquad A_1 = \text{const} \tag{4.7}$$

where m is the mass of a molecule. Corresponding momentum distribution has the form

$$w(\mathbf{p}) = |\langle\mathbf{p}|\psi\rangle|^2 = A\delta(\mathbf{p} - m\mathbf{u}), \qquad A = \text{const} \tag{4.8}$$

This distribution does not coincides with the Maxwell momentum distribution

$$F(\mathbf{x},\mathbf{p})\,d\mathbf{p} = \frac{1}{(2\pi mkT)^{3/2}}\exp\left\{-\frac{(\mathbf{p} - m\mathbf{u})^2}{2mkT}\right\}d\mathbf{p} \tag{4.9}$$

which depends on the gas temperature kT.

Let us divide the space into small cells V_i and calculate the mean momentum $\langle\mathbf{p}\rangle_i$ of molecule in each cell V_i. In the given case we obtain in the ith cell $\langle\mathbf{p}\rangle_i = m\mathbf{u}$. All mean momenta $\langle\mathbf{p}\rangle_i$ form the mean momenta distribution (4.8). The mean momenta $\langle\mathbf{p}\rangle_i$ correlate with the position $\mathbf{x}_i$ of the cell V_i, and mutual distribution over position $\mathbf{x}$ and momentum $\langle\mathbf{p}\rangle$ appears to be impossible. More detail consideration one can find in [8].

Description of "classical particle " $\mathcal{S}_{\text{Scl}} = \mathcal{E}[\mathcal{S}_{\text{d}}]$ in the "quantum language" is realized by the action.

$$\mathcal{A}_{\text{Scl}}[\psi,\psi^*] = \int\left\{\frac{i\hbar}{2}(\psi^*\partial_0\psi - \partial_0\psi^*\cdot\psi) - \frac{\hbar^2}{2m}\boldsymbol{\nabla}\psi^*\boldsymbol{\nabla}\psi + \frac{\hbar^2}{2m}(\boldsymbol{\nabla}|\psi|)^2\right\}dt\,d\mathbf{x} \tag{4.10}$$

where we use the quantum constant $\hbar$ instead of arbitrary constant b.

$$i\hbar\partial_0\psi + \frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi - \frac{\hbar^2}{8m}\left(2\frac{\boldsymbol{\nabla}^2\rho}{\rho} - \frac{(\boldsymbol{\nabla}\rho)^2}{\rho^2}\right)\psi = 0 \qquad \rho = \psi^*\psi \tag{4.11}$$

Description of "quantum particle" $\mathcal{S}_{\text{S}} = \mathcal{E}[\mathcal{S}_{\text{st}}]$ in the "classical language" is realized by equations

$$\frac{d\mathbf{p}}{dt} = -\boldsymbol{\nabla}U(\rho,\boldsymbol{\nabla}\rho), \qquad \frac{d\mathbf{x}}{dt} = \frac{\mathbf{p}}{m}, \tag{4.12}$$

where U is defined by the second relation (4.6).

Dynamic equations (4.12) are the *partial* differential equations, because ρ contains derivatives with respect to ξ_α, $\alpha = 1, 2, 3$.

Description of "classical particle" $\mathcal{S}_{\rm Scl} = \mathcal{E}[\mathcal{S}_{\rm d}]$ in the "classical language" has the form

$$\frac{d\mathbf{p}}{dt} = 0, \qquad \frac{d\mathbf{x}}{dt} = \frac{\mathbf{p}}{m} \tag{4.13}$$

where $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$, $\mathbf{p} = \mathbf{p}(t, \boldsymbol{\xi})$. Dynamic equations (4.13) are *ordinary* differential equations.

Describing quantum system $\mathcal{S}$ and corresponding classical system $\mathcal{S}_{\rm cl}$ in the classical language, we recognize that dynamic equations for the quantum system $\mathcal{S}$ are *partial* differential equations, whereas dynamic equations for its classical approximation $\mathcal{S}_{\rm cl}$ are *ordinary* differential equations.

The difference between the ordinary and partial differential equations is dynamical (but not quantum). This invariant difference is a foundation of the procedure of the dynamical disquantization (transition to classical approximation).

Dynamic disquantization is the procedure of projecting of derivatives onto the direction of 4-current j^k

$$\partial^l \to \partial_{||}^l = \frac{j^l j^k}{j^s j_s}\partial_k, \qquad l = 0, 1, 2, 3,$$

To obtain ordinary differential equations, one needs to project derivatives onto one direction.

The dynamic disquantization admits one to *formalize* the transition to the classical description. Being applied to Schrödinger particle $\mathcal{S}_{\rm S}$, the dynamic disquantization transform $\mathcal{S}_{\rm S} = \mathcal{E}[\mathcal{S}_{\rm st}]$ into the statistical ensemble $\mathcal{E}[\mathcal{S}_{\rm d}]$ of free classical particles $\mathcal{S}_{\rm d}$.

The procedure of dynamic disquantization is determined by the dynamic system *completely* and does not refer to the quantum constant and the quantum principles.

5 Concluding remarks

We have analysed application of dynamical methods to the Schrödinger particle and have obtained results, which cannot be obtained by means of conventional axiomatic methods. Analogous results are obtained at application of dynamical methods to other quantum systems. Application of dynamical methods to investigation of quantum systems admits one to eliminate quantum principles. The number of fundamental propositions is reduced essentially. The quantum theory ceases to be a list of prescriptions and becomes a logical structure.

In the conventional quantum theory the allness of the quantum constant $\hbar$ is explained by the quantum nature of all physical phenomena. In particular, it means that all physical fields should be quantized. Application of dynamical methods and elimination of quantum principles suppose some primordial stochastic motion of free particles with dependence of the stochasticity intensity on the particle mass. The universal character of the quantum constant and the particle motion stochasticity may be explained freely by the space-time properties, when the quantum constant is a parameter of the space-time geometry [9, 10]. In this case not all physical fields have the quantum nature. Dynamic equations for the metric fields (gravitational and electromagnetic) do not contain the quantum constant $\hbar$. There is no necessity to quantize these fields.

It is well known that the electromagnetic field is absorbed and emitted in the form of quanta of energy $\hbar\omega$. However, this fact does not mean that the electromagnetic field exists in the form of quanta. There are no experiments, which show directly or indirectly that the electromagnetic field exists in the form of quanta. (At least, such experiments are unknown for us). Absorbtion and emission of the electromagnetic field in the form of quanta may be easily explained by the quantum properties of the absorbtion devices and the emission ones. The same relates to the gravitational field. All attempts of its quantization appear to be unsuccessful.

Quantum and classical dynamic systems distinguish *only* dynamically, i.e. by their dynamic equations, but not by their enigmatic quantum properties. Linearity of the conventional axiomatic methods is simple and attractive, but these methods are founded on nonrelativistic quantum principles, and we cannot be sure that these methods may be applied in the relativistic case.

There is a hope that application of dynamical methods in the relativistic case will be successful. First, extension to the relativistic case is produced on the logical basis, but not by means of incident hypotheses. Second, in the relativistic case the diffusion velocity $\mathbf{u}_{\mathrm{df}}$ turns to the relativistic field, responsible for pair production, whereas neither quantum principles nor classical relativistic dynamics can describe effect of pair production, which is the principle effect of the high energy physics.

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