Deformation principle and further geometrization of physics

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Abstract

The space-time geometry is considered to be a physical geometry, i.e. a geometry described completely by the world function. All geometrical concepts and geometric objects are taken from the proper Euclidean geometry. They are expressed via the Euclidean world function $\sigma_{\rm E}$ and declared to be concepts and objects of any physical geometry, provided the Euclidean world function $\sigma_{\rm E}$ is replaced by the world function σ of the physical geometry in question. The set of physical geometries is more powerful, than the set of Riemannian geometries, and one needs to choose a true space-time geometry. In general, the physical geometry is multivariant (there are many vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$,... which are equivalent to vector $\mathbf{P}_0\mathbf{P}_1$, but are not equivalent between themselves). The multivariance admits one to describe quantum effects as geometric effects and to consider existence of elementary particles as a geometrical problem, when the possibility of the physical existence of an elementary geometric object in the form of a physical body is determined by the space-time geometry. Multivariance admits one to describe discrete and continuous geometries, using the same technique. A use of physical geometry admits one to realize the geometrical approach to the quantum theory and to the theory of elementary particles.

1 Introduction

Geometrization is the principal direction of the contemporary theoretical physics development. It began in the nineteenth century. One can list the following stages of the physics geometrization:

1. Conservation laws of energy-momentum and angular momentum

2. The first modification of the space-time geometry (geometrization of the space-time, the concept of simultaneity, geometrization of particle motion, problem of high velocities)

3. The second modification of the space-time geometry (existence of nonhomogeneous space-time geometry, influence of the matter distribution on the space-time geometry)

4. Geometrization of charge and electromagnetic field. (Kaluza, O. Klein)

5. The third modification of the space-time geometry (the new space-time geometry of microcosm, the concept of multivariance, geometrization of mass, existence of geometrical objects in the form of physical bodies, geometrical approach to the elementary particles theory).

Now the theoretical physics stands before the third modification of the space-time geometry, connected with investigation of microcosm. The conventional space-time geometry is insensitive to structure of the microcosm. From viewpoint of the conventional geometry it is of no importance, whether the microcosm geometry is discrete or continuous. It is of no importance, whether geometrical objects may be divided into parts at no allowance, or their divisibility is restricted. The mathematical technique of contemporary theoretical physics is based on a use of the infinitesimal calculus, which supposes continuity of space-time and unlimited divisibility of geometrical objects. Furthermore, there exists no effective method, which admits one to construct a discrete geometry, or a geometry with a limited divisibility. This is connected with the fact that the contemporary geometry ignores the concept of multivariance and ejects the concept of multivariance, if it meets accidentally. The third modification of the space-time geometry is connected with appearance of a new method of the geometry construction, which describes multivariant geometries. The multivariant geometry may describe both discrete and continuous geometries, as well as geometries with a limited divisibility of geometrical objects [1]. These properties appear to be important in the space-time geometry of microcosm.

Necessity of the third modification appeared in the thirtieth of the twentieth century, when diffraction of electrons on the small hole was discovered. Motion of a free particle depends only on the space-time geometry, and one needs such a space-time geometry, where the free particle motion be multivariant, and the multivariance intensity depend on the particle mass. Neither physicists, nor mathematicians could imagine such a space-time geometry. As a result the problem of the particle motion multivariance has been solved in the framework of dynamics (but not on the level of geometry). Classical principles of dynamics in microcosm were replaced by quantum ones. The problem of multivariant motion of microparticles has been solved on the level of dynamics. W.Heisenberg suggested to replace conventional dynamic variables by matrices. Introduction of matrices is an introduction of a multivariance. However, it is a multivariance on the level of dynamics. The space-time geometry remained to be former.

Impossibility of the multivariance problem solution on the geometric level was connected with imperfection of the method of the geometry construction. It does not admit one to construct multivariant geometries, which possess properties, necessary for explanation quantum effects and other properties of microcosm. Besides, in that time the researchers were under the impression of advances of quantum mechanics, and multivariance of the electron motion had been explained as a quantum effect.

In the end of the twentieth century a more perfect method of the space-time geometry construction has been suggested [2]. This method is known as the deformation principle. Geometries, constructed by this method, are known as tubular geometries (T-geometries) This method is simpler and more general, than the conventional Euclidean method, because it does not use such a constraint of the conventional method, as absence of multivariance. In particular, in the framework of the Riemannian geometry there is only one plane uniform isotropic space-time geometry: the Minkowski geometry, whereas in the framework of T-geometries there is a set of plane uniform isotropic space-time geometries, labelled by a function of one argument. All geometries of this set (except for the Minkowski one) are multivariant with respect to timelike vectors. Multivariance of the space-time geometry with respect to timelike vectors means that there exist vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$,... which are equivalent to timelike vector $\mathbf{P}_0\mathbf{P}_1$, but not equivalent between themselves.

As far as there exist many uniform isotropic space-time geometries, we are to choose the true space-time geometry from this set. We may not choose the Minkowski space-time geometry on the ground, that this geometry was used before. We are to make the best of agreement of the space-time geometry with the experimental data. It appears that the parameters of the space-time geometry can be chosen in such a way, that the classical principles of dynamics describe correctly both quantum and classical motion of a free particle. Of course, the parameters of the true space-time geometry contain the quantum constant $\hbar$. In this case we do not need the quantum principles, which in the conventional theory compensate influence of the incorrectly chosen space-time geometry of microcosm.

Such an expansion of the space-time geometry capacities is connected with the non-Euclidean method of the geometry construction. This method of the geometry construction may be qualified as the deformation principle, because any physical geometry can be obtained as a result of a deformation of the proper Euclidean geometry. Capacities of the space-time geometries constructed by means of the deformation principle do not exhausted by explanation of quantum effects. Structure of elementary particles, their masses, appearance of short-range force fields in microcosm and such an enigmatic phenomenon as confinement can be easily explained in terms of the space-time geometry and its particularity. At any rate the mathematical technique of T-geometry admits this. In this paper we shall not try to determine the concrete form of the microcosm geometry. To choose the concrete microcosm geometry one needs very careful analysis of experimental data. It is a very difficult problem. We shall show only that mathematical capacity of microcosm geometry are larger, than that of the contemporary theory of elementary particles.

The main advantage of the microcosm geometry is the circumstance that it does not use any hypotheses. Of course, to obtain a concrete space-time geometry of microcosm, we are to use experimental data and make some suppositions on the space-time geometry. However, these suppositions will be made in framework of fixed principles. One may choose only the form of the world function, which determines the space-time geometry. The principles of the geometry construction remain to be changeless. They are not a result of a fitting. They are obtained by means of logic reasonings. This is an essential difference from the contemporary methods of the elementary particle theory, where the unprincipled fitting dominates.

Note, that probabilistic and noncommutative geometries are not geometries in the exact sense of this word. These "geometries" are fortified geometries, i.e. geometries, equipped with some additional structures (probabilistic and matrix), given on the Minkowski manifold. In other words in the framework of these "geometries" the physical geometry, as a science on mutual disposition of geometric objects remains to be the former geometry of Minkowski. One adds to this geometry additional structures, introducing multivariance, which is necessary for the microcosm description. However, this multivariance is introduced on the dynamic level, but not on the level of geometry/

Thus, in this paper we demonstrate only mathematical capacity of space-time geometry in explanation of the microcosm phenomena.

2 Approaches to geometry

There are two approaches to geometry. According to the conventional approach a geometry (axiomatic one) is constructed on a basis of some axiomatics. All propositions of the axiomatic geometry are obtained from several primordial propositions (axioms) by means of logical reasonings. Examples of axiomatic geometries: Euclidean geometry, affine geometry, projective geometry etc. The main defect of the axiomatic geometry: impossibility of axiomatization of nonhomogeneous geometries.

Axiomatization of geometry means that from the set $\mathcal{S}$ of all geometry propositions one can separate several primordial propositions $\mathcal{A}$ (axioms) in such a way that all propositions $\mathcal{S}$ can be obtained from axioms $\mathcal{A}$ by means of logical reasonings. Possibility of axiomatization is a hypothesis. Its validity has been proved only for the proper Euclidean geometry [3]. For nonhomogeneous geometries (for instance, for the Riemannian one) a possibility of axiomatization was not proved. In general, a possibility of axiomatization for nonhomogeneous geometries seems to be doubtful. Felix Klein [4] assumed that the Riemannian geometry (nonhomogeneous one) is rather a geography or a topography, than a geometry.

According to another approach a geometry (physical geometry) is a science on mutual position of geometrical objects in the space or in the space-time. (Euclidean geometry, metric geometry). All relations of the metric geometry are finite, but not differential, and the metric geometry may be given on an arbitrary set of points, but not necessarily on a manifold. It is supposed that the mutual position of geometrical objects is determined, if the distance (metric) between any two points of the set is given. The simplicity of the geometry characteristic and absence of constraints on the set of points (on the space) is the principal advantage of the metric geometry. The main defect of metric geometry is its poverty. Such important concepts of

Euclidean geometry as scalar product of vectors and concept of linear dependence are absent in the metric geometry. However, the proper Euclidean geometry is a special case of the metric geometry. It means that in the case of Euclidean geometry the scalar product, concept of linear dependence of vectors and other concepts and objects of Euclidean geometry can be expressed via Euclidean metric. These expressions of the Euclidean concepts via metric are declared to be valid for any metric geometry. Replacing Euclidean metric by the metric of the metric geometry in question, we obtain a system of geometrical concepts in any metric geometry. As a result we obtain the metric geometry, equipped by all concepts of Euclidean geometry [5].

Besides, one removes such constraints on the metric, as the triangle axiom and positivity of metric ρ. They are not necessary, if concepts of the Euclidean geometry are introduced in the metric geometry. Instead of metric ρ we use the world function $\sigma = \frac{1}{2}\rho^2$, which is real even in the geometries with indefinite metric (for instance, in the geometry of Minkowski). We shall refer to such geometries as the tubular geometries (T-geometry). Such a name of geometry is connected with the fact, that the straight in T-geometry is a tube, but not a one-dimensional line. The tubular character of straights in T-geometry is conditioned by the property of multivariance. Multivariance of a T-geometry means, that there exist many vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$, $\mathbf{Q}_0\mathbf{Q}_1''$, ...which are equivalent (equal) to vector $\mathbf{P}_0\mathbf{P}_1$, but are not equivalent between themselves.

At first sight the multivariance is unexpected and undesirable property of geometry. However, multivariance is very important in application of geometry to physics. For instance, the real space-time geometry appears to be multivariant. In particular, multivariance of the space-time geometry explains freely quantum effects as geometrical effects. Besides, the multivariance admits one to set the problem of existence of geometrical objects in the form of physical bodies. This problem cannot be set in framework of the Riemannian geometry. (More exact, this problem can be set, but its solution leads to the statement, that any geometric object can be realized in the form of a physical body). The statement of this problem is important, to obtain a simple geometrical approach to the elementary particles theory.

In T-geometry all geometrical propositions have ready-made form. In T-geometry there are no theorems, all theorems have been proved in the Euclidean geometry. Statements of the Euclidean geometry turn into definitions of T-geometry. All this is rather unusual and is difficult for perception, because some axioms of Euclidean geometry are not valid in T-geometry (for instance, the Euclidean axiom: "the straight has no thickness" is not valid in T-geometry, in general).

To overcome defects of physical and axiomatic geometries we use the fact, that the proper Euclidean geometry is the axiomatic and physical geometry simultaneously. The proper Euclidean geometry has been constructed as an axiomatic geometry, and consistency of its axioms has been proved. On the other side, the proper Euclidean geometry is a physical geometry and, hence, it is to be described completely in terms of the metric ρ. Indeed, such a theorem has been proved [5].

It is more convenient to introduce the world function $\sigma = \frac{1}{2}\rho^2$ instead of the metric ρ, because in this case one may describe the Minkowski geometry and other geometries with indefinite metric tensor in terms of real world function σ.

As soon as the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ is a known geometry, all propositions P_{E} of the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ can be presented in terms of the world function σ_{E} of the proper Euclidean geometry: $P_{\mathrm{E}} = P_{\mathrm{E}}(\sigma_{\mathrm{E}})$. Replacing the world function σ_{E} of $\mathcal{G}_{\mathrm{E}}$ by the world function σ of another physical geometry $\mathcal{G}$ in all propositions $P_{\mathrm{E}}(\sigma_{\mathrm{E}})$ of the Euclidean geometry: $P_{\mathrm{E}}(\sigma_{\mathrm{E}}) \rightarrow P_{\mathrm{E}}(\sigma)$, one obtains all propositions of the physical geometry $\mathcal{G}$. Replacement of the world function σ_{E} by other world function σ means a deformation of the Euclidean geometry (Euclidean space). It may be interpreted in the sense, that any physical geometry is a result of a deformation of the proper Euclidean geometry.

It is very important that all expressions of concepts of the Euclidean geometry via the world function have a finite (but not differential) form. The differential form of relations needs an additional information (initial or boundary conditions). The finite expressions do not need such an additional information. Besides, using the finite form of relations, we need to solve some algebraic equations, whereas, using a differential form of relations we are forced to solve differential equations.

Thus, to construct a physical geometry one needs to express all propositions of the proper Euclidean geometry in terms of the Euclidean world function σ_{E}.

3 Non-Euclidean method of the physical geometry construction (deformation principle)

Any physical geometry is described by the world function and obtained as a result of deformation of the proper Euclidean geometry. The world function is described by the relation

$$\sigma: \qquad \Omega \times \Omega \rightarrow \mathbb{R}, \qquad \sigma(P,Q) = \sigma(Q,P), \qquad \sigma(P,P) = 0, \qquad \forall P,Q \in \Omega \tag{3.1}$$

The vector $\mathbf{PQ} \equiv \overrightarrow{PQ}$ is the ordered set of two points $\{P,Q\}$, $P,Q \in \Omega$. The length $|\mathbf{PQ}|_{\mathrm{E}}$ of the vector $\mathbf{PQ}$ is defined by the relation

$$|\mathbf{PQ}|_{\mathrm{E}}^2 = 2\sigma_{\mathrm{E}}(P,Q) \tag{3.2}$$

where index "E" means that the length of the vector is taken in the proper Euclidean space.

The scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2)_{\mathrm{E}}$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_0\mathbf{P}_2$ having the common origin P_0 is defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2)_{\mathrm{E}} = \sigma_{\mathrm{E}}(P_0,P_1) + \sigma_{\mathrm{E}}(P_0,P_2) - \sigma_{\mathrm{E}}(P_1,P_2) \tag{3.3}$$

which is obtained from the Euclidean relation

$$|\mathbf{P}_1\mathbf{P}_2|^2 = |\mathbf{P}_0\mathbf{P}_2 - \mathbf{P}_0\mathbf{P}_1|^2 = |\mathbf{P}_0\mathbf{P}_2|^2 + |\mathbf{P}_0\mathbf{P}_1|^2 - 2(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_2)_{\mathrm{E}} \tag{3.4}$$

The scalar product of two remote vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ has the form

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{E}} = \sigma_{\mathrm{E}}(P_0, Q_1) + \sigma_{\mathrm{E}}(P_1, Q_0) - \sigma_{\mathrm{E}}(P_0, Q_0) - \sigma_{\mathrm{E}}(P_1, Q_1) \qquad (3.5)$$

The necessary and sufficient condition of linear dependence of n vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$,...$\mathbf{P}_0\mathbf{P}_n$ is defined by the relation

$$F_n(\mathcal{P}^n) = 0, \qquad \mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\} \qquad (3.6)$$

where $F_n(\mathcal{P}^n)$ is the Gram's determinant

$$\begin{aligned} F_n(\mathcal{P}^n) &\equiv \det||(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)_{\mathrm{E}}|| = \det||\sigma_{\mathrm{E}}(P_0, P_i) + \sigma_{\mathrm{E}}(P_0, P_k) - \sigma_{\mathrm{E}}(P_i, P_k)|| \\ i, k &= 1, 2, ...n \end{aligned} \qquad (3.7)$$

Collinearity $\mathbf{P}_0\mathbf{P}_1 \parallel \mathbf{Q}_0\mathbf{Q}_1$ of two vectors is a special case of the linear dependence. It described by the relation

$$\mathbf{P}_0\mathbf{P}_1 \parallel \mathbf{Q}_0\mathbf{Q}_1 : \qquad \left\| \begin{array}{cc} |\mathbf{P}_0\mathbf{P}_1|_{\mathrm{E}}^2 & (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{E}} \\ (\mathbf{Q}_0\mathbf{Q}_1.\mathbf{P}_0\mathbf{P}_1)_{\mathrm{E}} & |\mathbf{Q}_0\mathbf{Q}_1|_{\mathrm{E}}^2 \end{array} \right\| = 0 \qquad (3.8)$$

Two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are parallel, if

$$\mathbf{P}_0\mathbf{P}_1 \upuparrows_{\mathrm{E}} \mathbf{Q}_0\mathbf{Q}_1 : \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{E}} = |\mathbf{P}_0\mathbf{P}_1|_{\mathrm{E}} \cdot |\mathbf{Q}_0\mathbf{Q}_1|_{\mathrm{E}} \qquad (3.9)$$

Two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are antiparallel, if

$$\mathbf{P}_0\mathbf{P}_1 \uparrow\downarrow_{\mathrm{E}} \mathbf{Q}_0\mathbf{Q}_1 : \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{E}} = -|\mathbf{P}_0\mathbf{P}_1|_{\mathrm{E}} \cdot |\mathbf{Q}_0\mathbf{Q}_1|_{\mathrm{E}} \qquad (3.10)$$

Two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent (equal) $\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1$, if

$$\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1 : \qquad (\mathbf{P}_0\mathbf{P}_1 \upuparrows \mathbf{Q}_0\mathbf{Q}_1) \wedge (|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|) \qquad (3.11)$$

or

$$\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1 : \qquad \left((\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1|^2\right) \wedge (|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|) \qquad (3.12)$$

The property of the equivalence of two vectors in the proper Euclidean geometry is reversible and transitive.

$$\text{if} \quad \mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1, \quad \text{then} \quad \mathbf{Q}_0\mathbf{Q}_1\mathrm{eqv}\mathbf{P}_0\mathbf{P}_1 \qquad (3.13)$$

$$(\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1) \wedge (\mathbf{Q}_0\mathbf{Q}_1\mathrm{eqv}\mathbf{R}_0\mathbf{R}_1) \implies \mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{R}_0\mathbf{R}_1 \qquad (3.14)$$

In general case of physical geometry the equivalence property is intransitive. The intransitivity of the equivalence property is connected with its multivariance, when there are many vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$, $\mathbf{Q}_0\mathbf{Q}_1''$,...which are equivalent to the vector $\mathbf{P}_0\mathbf{P}_1$, but they are not equivalent between themselves. Multivariance of the equivalence property is conditioned by the fact, that the system of equations for determination of the point Q_1 (at fixed points P_0, P_1, Q_0) has, many solutions, in general. It is possible also such a situation, when these equations have no solution.

4 Construction of geometrical objects in T-geometry

Geometrical object $\mathcal{O} \subset \Omega$ is a subset of points in the point set Ω. In the T-geometry the geometric object $\mathcal{O}$ is described by means of the skeleton-envelope method. It means that any geometric object $\mathcal{O}$ is considered to be a set of intersections and joins of elementary geometric objects (EGO).

The finite set $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\} \subset \Omega$ of parameters of the envelope function $f_{\mathcal{P}^n}$ is the skeleton of elementary geometric object (EGO) $\mathcal{E} \subset \Omega$. The set $\mathcal{E} \subset \Omega$ of points forming EGO is called the envelope of its skeleton $\mathcal{P}^n$. The envelope function $f_{\mathcal{P}^n}$

$$f_{\mathcal{P}^n}: \qquad \Omega \rightarrow \mathbb{R}, \tag{4.1}$$

determining EGO is a function of the running point $R \in \Omega$ and of parameters $\mathcal{P}^n \subset \Omega$. The envelope function $f_{\mathcal{P}^n}$ is supposed to be an algebraic function of s arguments $w = \{w_1, w_2, ...w_s\}$, $s = (n+2)(n+1)/2$. Each of arguments $w_k = \sigma(Q_k, L_k)$ is the world function σ of two points $Q_k, L_k \in \{R, \mathcal{P}^n\}$, either belonging to skeleton $\mathcal{P}^n$, or coinciding with the running point R. Thus, any elementary geometric object $\mathcal{E}$ is determined by its skeleton $\mathcal{P}^n$ and its envelope function $f_{\mathcal{P}^n}$. Elementary geometric object $\mathcal{E}$ is the set of zeros of the envelope function

$$\mathcal{E} = \{R | f_{\mathcal{P}^n}(R) = 0\} \tag{4.2}$$

Characteristic points of the EGO are the skeleton points $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\}$. The simplest example of EGO is the segment $\mathcal{T}_{[P_0 P_1]}$ of the straight line between the points P_0 and P_1, which is defined by the relation

$$\mathcal{T}_{[P_0 P_1]} = \{R | f_{P_0 P_1}(R) = 0\}, \tag{4.3}$$

$$f_{P_0 P_1}(R) = \sqrt{2\sigma(P_0, R)} + \sqrt{2\sigma(R, P_1)} - \sqrt{2\sigma(P_0, P_1)} \tag{4.4}$$

Another example is the cylinder $\mathcal{C}(P_0, P_1, Q)$ with the points P_0, P_1 on the cylinder axis and the point Q on its surface. The cylinder $\mathcal{C}(P_0, P_1, Q)$ is determined by the relation

$$\begin{aligned} \mathcal{C}(P_0, P_1, Q) &= \{R | f_{P_0 P_1 Q}(R) = 0\}, \\ f_{P_0 P_1 Q}(R) &= F_2(P_0, P_1, Q) - F_2(P_0, P_1, R) \end{aligned} \tag{4.5}$$

$$F_2(P_0, P_1, Q) = \begin{vmatrix} (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_1) & (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{Q}) \\ (\mathbf{P}_0\mathbf{Q}.\mathbf{P}_0\mathbf{P}_1) & (\mathbf{P}_0\mathbf{Q}.\mathbf{P}_0\mathbf{Q}) \end{vmatrix} \tag{4.6}$$

Here $\sqrt{F_2(P_0, P_1, Q)}$ is the area of the parallelogram, constructed on the vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_0\mathbf{Q}$, and $\frac{1}{2}\sqrt{F_2(P_0, P_1, Q)}$ is the area of triangle with vertices at the points P_0, P_1, Q. The equality $F_2(P_0, P_1, Q) = F_2(P_0, P_1, R)$ means that the distance between the point Q and the axis, determined by the vector $\mathbf{P}_0\mathbf{P}_1$, is equal to the distance between R and the axis. Here the points P_0, P_1, Q form the skeleton of the cylinder, whereas the function $f_{P_0 P_1 Q}$ is the envelope function.

In the proper Euclidean geometry the cylinder depends only on the axis $\mathcal{T}_{[P_0P_1]}$ (or $\mathcal{T}_{P_0P_1}$), passing through the points P_0 and P_1. It means, that if the point $P_1' \in \mathcal{T}_{[P_0P_1]}$ and $P_1' \neq P_1 \wedge P_1' \neq P_0$, the cylinders $\mathcal{C}(P_0, P_1, Q)$ and $\mathcal{C}(P_0, P_1', Q)$ coincide in the proper Euclidean geometry. However, the cylinders $\mathcal{C}(P_0, P_1, Q)$ and $\mathcal{C}(P_0, P_1', Q)$ do not coincide, in general, in an arbitrary T-geometry. It is a result of the multivariance of the T-geometry, where the straight lines $\mathcal{T}_{P_0P_1}$ and $\mathcal{T}_{P_0P_1'}$ are in general different, even if $P_1' \in \mathcal{T}_{P_0P_1}$.

Definition. Two EGOs $\mathcal{E}(\mathcal{P}^n)$ and $\mathcal{E}(\mathcal{Q}^n)$ are equivalent, if their skeletons are equivalent and their envelope functions $f_{\mathcal{P}^n}$ and $g_{\mathcal{Q}^n}$ are equivalent. Equivalence of two skeletons $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\} \subset \Omega$ and $\mathcal{Q}^n \equiv \{Q_0, Q_1, ..., Q_n\} \subset \Omega$ means that

$$\mathbf{P}_i\mathbf{P}_k \mathrm{eqv} \mathbf{Q}_i\mathbf{Q}_k, \qquad i,k = 0,1,...n, \quad i<k \tag{4.7}$$

Equivalence of the envelope functions $f_{\mathcal{P}^n}$ and $g_{\mathcal{Q}^n}$ means that

$$f_{\mathcal{P}^n}(R) = \Phi\left(g_{\mathcal{P}^n}(R)\right), \qquad \forall R \in \Omega \tag{4.8}$$

where Φ is an arbitrary function, having the property

$$\Phi : \mathbb{R} \to \mathbb{R}, \qquad \Phi(0) = 0 \tag{4.9}$$

Equivalence of shapes of two EGOs $\mathcal{E}(\mathcal{P}^n)$ and $\mathcal{E}(\mathcal{Q}^n)$ is determined by equivalence of shapes of their skeletons $\mathcal{P}^n$ and $\mathcal{Q}^n$, which is described by the relations

$$|\mathbf{P}_i\mathbf{P}_k| = |\mathbf{Q}_i\mathbf{Q}_k|, \qquad i,k = 0,1,...n, \quad i<k \tag{4.10}$$

and equivalence of their envelope functions $f_{\mathcal{P}^n}$ and $g_{\mathcal{Q}^n}$ (4.8).

Equivalence of orientations of skeletons $\mathcal{P}^n$ and $\mathcal{Q}^n$ in the point space Ω is described by the relations

$$\mathbf{P}_i\mathbf{P}_k \upuparrows \mathbf{Q}_i\mathbf{Q}_k, \qquad i,k = 0,1,...n, \quad i<k \tag{4.11}$$

Equivalence of shapes and orientations of skeletons is equivalence of skeletons, described by the relations (4.7).

5 Existence of geometrical objects as physical objects

By definition an elementary geometric object $\mathcal{O}_{\mathcal{P}^n}$ exists at the point $P_0 \in \Omega$ in the space-time as a physical object, if it exists at any time moment at any place of the space-time. Mathematically it means, that at any point $Q_0 \in \Omega$ there exists a geometrical object $\mathcal{O}_{\mathcal{Q}^n}$ with the skeleton $\mathcal{Q}^n \mathrm{eqv} \mathcal{P}^n$. The relation $\mathcal{Q}^n \mathrm{eqv} \mathcal{P}^n$ means that

$$\mathbf{P}_i\mathbf{P}_k \mathrm{eqv} \mathbf{Q}_i\mathbf{Q}_k, \qquad i,k = 0,1,...n, \qquad i<k \tag{5.1}$$

According to definition of equivalence (3.12) the equivalence equation $\mathbf{P}_i\mathbf{P}_k\text{eqv}\mathbf{Q}_i\mathbf{Q}_k$ means two relations

$$(\mathbf{P}_i\mathbf{P}_k.\mathbf{Q}_i\mathbf{Q}_k) = |\mathbf{P}_i\mathbf{P}_k|^2, \qquad |\mathbf{P}_i\mathbf{P}_k| = |\mathbf{Q}_i\mathbf{Q}_k| \tag{5.2}$$

There are $n(n+1)$ equations for determination of $4n$ coordinates of points $Q_1, Q_2, ...Q_n$ in the 4-dimensional space-time. The skeleton $\mathcal{P}^n$ and the point Q_0 are supposed to be given.

In the case of Minkowski space-time we have only $2n$ relations (instead of $n\,(n+1)$) for determination of $4n$ coordinates

$$(\mathbf{P}_0\mathbf{P}_k.\mathbf{Q}_0\mathbf{Q}_k) = |\mathbf{P}_0\mathbf{P}_k| \cdot |\mathbf{Q}_0\mathbf{Q}_k|, \qquad |\mathbf{P}_0\mathbf{P}_k| = |\mathbf{Q}_0\mathbf{Q}_k|, \qquad k = 1, 2, ...n \tag{5.3}$$

because in the Minkowski space-time
$(\mathbf{P}_0\mathbf{P}_i\text{eqv}\mathbf{Q}_0\mathbf{Q}_i) \wedge (\mathbf{P}_0\mathbf{P}_k\text{eqv}\mathbf{Q}_0\mathbf{Q}_k) \Longrightarrow \mathbf{P}_i\mathbf{P}_k\text{eqv}\mathbf{Q}_i\mathbf{Q}_k$

The structure of equivalence constraints in the Minkowski space-time is such, that *two* relations

$$(\mathbf{P}_0\mathbf{P}_k.\mathbf{Q}_0\mathbf{Q}_k) = |\mathbf{P}_0\mathbf{P}_k|^2, \qquad |\mathbf{P}_0\mathbf{P}_k| = |\mathbf{Q}_0\mathbf{Q}_k| \tag{5.4}$$

determine uniquely four coordinates of the point Q_k, provided the vector $\mathbf{P}_0\mathbf{P}_k$ is timelike, i.e. $|\mathbf{P}_0\mathbf{P}_k|^2 > 0$. Thus, in the Minkowski space-time any geometrical object exists always, as a physical object. If the number of the skeleton points increases, the number $n\,(n+1)$ of constraints increases faster, than the number $4n$ of coordinates, to be determined.

In the simplest case, when all $n\,(n+1)\,/2$ vectors $\mathbf{P}_i\mathbf{P}_k$ of the skeleton are timelike, the relation between $n\,(n+1)$ constraints and $4n$ coordinates is given by the following table

n	$n\,(n+1)$	$4n$	diff
2	6	8	2
3	12	12	0
4	20	16	-4

We see that for $n > 3$, the number constraints is larger, than the number of variables to be determined. It appears that existence of complicated elementary geometrical objects is impossible.

6 Evolution of geometrical object

In some cases skeletons of equivalent geometrical objects may form a chain of identical skeletons. In such cases we shall speak on temporal evolution of the geometrical object. For instance, let skeletons $\left\{P_0^{(l)}, P_1^{(l)}, ...P_n^{(l)}\right\}$, $l = ...0, 1, ...$are equivalent in pairs

$$\mathbf{P}_i^{(l)}\mathbf{P}_k^{(l)}\text{eqv}\mathbf{P}_i^{(l+1)}\mathbf{P}_k^{(l+1)}, \qquad i, k = 0, 1, ...n; \qquad l = ...1, 2, ... \tag{6.1}$$

and besides

$$P_1^{(l)} = P_0^{(l+1)}, \qquad l = ...1, 2, ... \tag{6.2}$$

If vectors $\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}$ are timelike $\left|\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}\right| > 0$, one may speak on the temporal evolution of the geometrical object $\mathcal{O}(\mathcal{P}^n)$, which is described by the chain, consisting of equivalent skeletons $\mathcal{P}^n$. In some cases the temporal evolution arises, even if the vectors $\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}$ are spacelike. However, one may not speak on a temporal evolution of a geometrical object, if skeletons of the chain are not equivalent.

7 Temporal evolution of two-point objects

We consider some simple examples of temporal evolution of the skeleton, consisting of two points, in the flat homogeneous isotropic space-time $V_\mathrm{d} = \{\sigma_\mathrm{d}, \mathbb{R}^4\}$, described by the world function

$$\sigma_\mathrm{d} = \sigma_\mathrm{M} + d \cdot \mathrm{sgn}\,(\sigma_\mathrm{M}), \qquad d = \lambda_0^2 = \mathrm{const} > 0 \tag{7.1}$$

$$\mathrm{sgn}\,(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}, \tag{7.2}$$

where σ_M is the world function of the 4-dimensional space-time of Minkowski. λ_0 is some elementary length.

The distorted space V_d describes the real space-time better, than the Minkowski space-time. Description by means of V_d is better in the sense, that the space-time (7.1) describes quantum effects, if the distortion constant d is chosen in the form [6]

$$d = \frac{\hbar}{2bc} \tag{7.3}$$

where $\hbar$ is the quantum constant, c is the speed of the light and b is the universal constant, coupling the geometrical length μ of the vector $\mathbf{P}_i\mathbf{P}_{i+1}$ in the chain of skeletons with the conventional mass m of the particle, described by this chain

$$m = b\mu \tag{7.4}$$

Consideration of distortion taken, in the form (7.3) means a consideration of the quantum constant as a parameter of the space-time.

The space-time is discrete in the space-time model (7.1). The space-time is discrete in the sense that there are no timelike vectors $\mathbf{P}_0\mathbf{P}_1$ with $|\mathbf{P}_0\mathbf{P}_1|^2 \in (0, \lambda_0^2)$ and there are no spacelike vectors $\mathbf{P}_0\mathbf{P}_1$ with $|\mathbf{P}_0\mathbf{P}_1|^2 \in (-\lambda_0^2, 0)$ However, the space-time model (7.1) is not a final space-time geometry [6]. The fact is that the relation

$$\sigma_\mathrm{d} = \sigma_\mathrm{M} + \frac{\hbar}{2bc} \tag{7.5}$$

may be not valid for all $\sigma_\mathrm{M} > 0$. For explanation of quantum effects, it is sufficient, that the relation (7.5) be satisfied for $\sigma_\mathrm{M} > \sigma_0$, where the constant σ_0 is determined by the geometrical mass of the lightest massive particle (electron) by means of relation

$$\sqrt{2\sigma_{\mathrm{d}}}=\sqrt{2\sigma_0+\frac{\hbar}{bc}}\leq\mu_{\mathrm{e}}=\frac{m_{\mathrm{e}}}{b} \tag{7.6}$$

where m_{e} is the electron mass.

For $\sigma_{\mathrm{M}}<\sigma_0$ the form of the distorted world function may distinguish from (7.1) and have, for instance, the form

$$\sigma_{\mathrm{di}}=\sigma_{\mathrm{M}}+\frac{2d}{\pi}\arctan\left(\sigma_{\mathrm{M}}\right),\qquad d=\lambda_0^2=\mathrm{const}>0 \tag{7.7}$$

The space-time V_{di} with world function (7.7) takes intermediate position between the Minkowski space-time and space-time V_{d}, described by (7.1). Space-time V_{di} describes quantum effects as V_{d}, however, the space-time is not discrete as V_{d}.

If the world function $\sigma\left(x,x'\right)$ is given on a manifold and have derivatives with respect to arguments x and x' at the coinciding points $x=x'$, the metric tensor $g_{ik}\left(x\right)$ is defined via derivatives of the world function in the form

$$g_{ik}\left(x\right)=\left[-\frac{\partial^2\sigma\left(x,x'\right)}{\partial x^i\partial x'^k}\right]_{x'=x},\qquad i,k=0,1,2,3 \tag{7.8}$$

Evaluation of the infinitesimal space-time interval in the space-time (7.7) in the inertial coordinate system gives the result

$$dS^2=\left(1+\frac{2d}{\pi\sigma_0}\right)\left(c^2dt^2-d\mathbf{x}^2\right) \tag{7.9}$$

which means space-time interval between two close points x and $x+dx$ appears to be finite at $\sigma_0\rightarrow 0$, even if dx is infinitesimal. The metric tensor, defined by the relation (7.9), coincides with the metric tensor of the Minkowski space-time to within a constant factor.

7.1 Two connected timelike vectors

Let we have two connected timelike vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$. If $\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{P}_1\mathbf{P}_2$ and vector $\mathbf{P}_0\mathbf{P}_1$ is given, the vector $\mathbf{P}_1\mathbf{P}_2$ can be determined. Let coordinates of points P_0,P_1,P_2 in the inertial coordinate system be

$$P_0=\{0,0,0,0\},\qquad P_0=\{s,0,0,0\},\qquad P_2=\{2s+\alpha_0,\gamma_1,\gamma_2,\gamma_3\} \tag{7.10}$$

where the quantity s is given and the quantities $\alpha_0,\gamma_1,\gamma_2,\gamma_3$ are to be determined.

Vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ have coordinates

$$\mathbf{P}_0\mathbf{P}_1=\{s,0,0,0\},\qquad \mathbf{P}_1\mathbf{P}_2=\{s+\alpha_0,\gamma_1,\gamma_2,\gamma_3\} \tag{7.11}$$

According to (3.5) in the space-time V_{d}

$$\begin{aligned}\left(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2\right) &= \sigma\left(P_0,P_2\right)-\sigma\left(P_0,P_1\right)-\sigma\left(P_1,P_2\right)\\ &= \sigma_{\mathrm{M}}\left(P_0,P_2\right)-\sigma_{\mathrm{M}}\left(P_0,P_1\right)-\sigma_{\mathrm{M}}\left(P_1,P_2\right)+w\\ &= \left(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2\right)_{\mathrm{M}}+w\end{aligned} \tag{7.12}$$

where

$$w=\lambda_0^2\left(\operatorname{sgn}\left(\sigma_{\mathrm{M}}\left(P_0,P_2\right)\right)-\operatorname{sgn}\left(\sigma_{\mathrm{M}}\left(P_0,P_1\right)\right)-\operatorname{sgn}\left(\sigma_{\mathrm{M}}\left(P_1,P_2\right)\right)\right) \tag{7.13}$$

and σ_{M} means the world function of the Minkowski space-time.

Note that in the space-time (7.1)

$$\operatorname{sgn}\left(\sigma_{\mathrm{M}}\left(P_0,P_2\right)\right)=\operatorname{sgn}\left(\sigma\left(P_0,P_2\right)\right) \tag{7.14}$$

If the vector $\mathbf{P}_0\mathbf{P}_1$ is timelike, $\left|\mathbf{P}_0\mathbf{P}_1\right|^2=s^2+2\lambda_0^2>0$, The equivalence equations of vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ take the form

$$\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{M}}^2=\left|\mathbf{P}_1\mathbf{P}_2\right|_{\mathrm{M}}^2, \qquad \left(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2\right)_{\mathrm{M}}+w=\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{M}}^2+2\lambda_0^2 \tag{7.15}$$

where

$$w=\lambda_0^2\left(\operatorname{sgn}\left(\sigma_{\mathrm{M}}\left(P_0,P_2\right)\right)-2\right) \tag{7.16}$$

Scalar products with a subscript "M" are usual scalar products in the Minkowski space-time. In the coordinate form the relations (7.15) are written as follows

$$\left(s+\alpha_0\right)^2-\gamma_1^2-\gamma_2^2-\gamma_3^2=s^2 \tag{7.17}$$

$$s\left(s+\alpha_0\right)+\lambda_0^2\left(\operatorname{sgn}\left(\sigma_{\mathrm{M}}\left(P_0,P_2\right)\right)-2\right)=s^2+2\lambda_0^2 \tag{7.18}$$

The vector $\mathbf{P}_0\mathbf{P}_2=\{2s+\alpha_0,\gamma_1,\gamma_2,\gamma_3\}$ have the length

$$\left|\mathbf{P}_0\mathbf{P}_2\right|_{\mathrm{M}}^2=\left(2s+\alpha_0\right)^2-\gamma_1^2-\gamma_2^2-\gamma_3^2 \tag{7.19}$$

Using relations (7.17) and (7.18), we eliminate α_0 and $\gamma_1^2+\gamma_2^2+\gamma_3^2$ from the relation (7.19). We obtain

$$\left|\mathbf{P}_0\mathbf{P}_2\right|_{\mathrm{M}}^2=4s^2+8\lambda_0^2-2\lambda_0^2\operatorname{sgn}\left(\sigma_{\mathrm{M}}\left(P_0,P_2\right)\right)>0 \tag{7.20}$$

It means that $\operatorname{sgn}\left(\sigma_{\mathrm{M}}\left(P_0,P_2\right)\right)=1$, and the relation (7.18) has the form

$$s\alpha_0=3 \tag{7.21}$$

Solution of equations (7.17) and (7.21) has the form

$$\alpha_0=\frac{3\lambda_0^2}{s}, \qquad \gamma_\alpha=\frac{\lambda_0}{s}\sqrt{6s^2+9\lambda_0^2}\frac{\beta_\alpha}{\sqrt{\beta_1^2+\beta_2^2+\beta_3^2}}, \qquad \alpha=1,2,3 \tag{7.22}$$

where β_1,β_2,β_3 are arbitrary real quantities.

Representing coordinates $\left(\mathbf{P}_1\mathbf{P}_2\right)_i$ of the vector $\mathbf{P}_1\mathbf{P}_2$ in the form

$$\left(\mathbf{P}_1\mathbf{P}_2\right)_i=\left(\mathbf{P}_0\mathbf{P}_1\right)_i+a_i \tag{7.23}$$

where 4-vector a_i describes the difference between the vectors $\mathbf{P}_1\mathbf{P}_2$ and $\mathbf{P}_0\mathbf{P}_1$ in the Minkowski space-time. We obtain

$$a_i=\left(\frac{3\lambda_0^2}{s},\frac{\lambda_0}{s}\sqrt{6s^2+9\lambda_0^2}\frac{\mathbf{q}}{|\mathbf{q}|}\right) \tag{7.24}$$

where $\mathbf{q}$ is an arbitrary 3-vector. The 4-vector a_i is spacelike

$$a_ia^i=\left(a_ia^i\right)_{\mathrm{M}}+2\lambda_0^2\operatorname{sgn}\left(\left(a_ia^i\right)_{\mathrm{M}}\right)=-6\lambda_0^2 \tag{7.25}$$

If the elementary length $\lambda_0\rightarrow 0$, the vector a_i tends to zero.

7.2 Two connected null vectors

Let us consider now two null connected equivalent vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ ($|\mathbf{P}_0\mathbf{P}_1|^2 = |\mathbf{P}_1\mathbf{P}_2|^2 = 0$). Using the coordinate representation for vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$

$$\mathbf{P}_0\mathbf{P}_1 = (s, s, 0, 0)\,, \qquad \mathbf{P}_1\mathbf{P}_2 = (s + \alpha_0, s + \alpha_1, \gamma_2, \gamma_3) \tag{7.26}$$

we obtain the following conditions of equivalence

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2)_{\mathrm{M}} + w = 0, \qquad |\mathbf{P}_1\mathbf{P}_2|^2 = 0 \tag{7.27}$$

$$w = \lambda_0^2 \mathrm{sgn}\left(\sigma_{\mathrm{M}}\left(P_0, P_2\right)\right) \tag{7.28}$$

The relations (7.27) and (7.28) are written in terms of coordinates

$$s\left(s + \alpha_0\right) - s\left(s + \alpha_1\right) = -\lambda_0^2 \mathrm{sgn}\left(\sigma_{\mathrm{M}}\left(P_0, P_2\right)\right) \tag{7.29}$$

$$\left(s + \alpha_0\right)^2 - \left(s + \alpha_1\right)^2 - \gamma_2^2 - \gamma_3^2 = 0 \tag{7.30}$$

After simplification we obtain

$$s\left(\alpha_0 - \alpha_1\right) = -\lambda_0^2 \mathrm{sgn}\left(\sigma_{\mathrm{M}}\left(P_0, P_2\right)\right) \tag{7.31}$$

$$\left(\alpha_0 - \alpha_1\right)\left(2s + \alpha_0 + \alpha_1\right) - \gamma_2^2 - \gamma_3^2 = 0 \tag{7.32}$$

For the length of the vector $\mathbf{P}_0\mathbf{P}_2$ we obtain

$$\begin{aligned} |\mathbf{P}_0\mathbf{P}_2|_{\mathrm{M}}^2 &= \left(2s + \alpha_0\right)^2 - \left(2s + \alpha_1\right)^2 - \gamma_2^2 - \gamma_3^2 \\ &= \left(\alpha_0 - \alpha_1\right)\left(4s + \alpha_0 + \alpha_1\right) - \gamma_2^2 - \gamma_3^2 \end{aligned} \tag{7.33}$$

By means of relations (7.31), (7.32) the relation (7.33) takes the form

$$|\mathbf{P}_0\mathbf{P}_2|_{\mathrm{M}}^2 = -2\lambda_0^2 \mathrm{sgn}\left(\sigma_{\mathrm{M}}\left(P_0, P_2\right)\right) \tag{7.34}$$

The relation (7.34) is fulfilled, only if

$$|\mathbf{P}_0\mathbf{P}_2|_{\mathrm{M}}^2 = 2\sigma_{\mathrm{M}}\left(P_0, P_2\right) = 0 \tag{7.35}$$

In this case the solution of equations has the form

$$\alpha_1 = \alpha_0, \qquad \gamma_2 = \gamma_3 = 0 \tag{7.36}$$

Thus, in the case of two connected equivalent null vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ we have

$$\mathbf{P}_0\mathbf{P}_1 = (s, s, 0, 0)\,, \qquad \mathbf{P}_1\mathbf{P}_2 = (s + \alpha_0, s + \alpha_0, 0, 0) \tag{7.37}$$

where α_0 is an arbitrary real number. The result does not depend on the elementary length λ_0. It takes place in the Minkowski space-time also.

7.3 Two connected spacelike vectors

Let we have two connected spacelike equivalent vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$. If $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{P}_1\mathbf{P}_2$ and vector $\mathbf{P}_0\mathbf{P}_1$ is given, the vector $\mathbf{P}_1\mathbf{P}_2$ can be determined. Let coordinates points P_0, P_1, P_2 in the inertial coordinate system be

$$P_0 = \{0,0,0,0\}, \qquad P_1 = \{0,l,0,0\}, \qquad P_2 = \{\alpha_0, 2l+\gamma_1, \gamma_2, \gamma_3\} \tag{7.38}$$

where the quantity l is given and the quantities $\alpha_0, \gamma_1, \gamma_2, \gamma_3$ are to be determined.

Vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ have coordinates

$$\mathbf{P}_0\mathbf{P}_1 = \{0,l,0,0\}, \qquad \mathbf{P}_1\mathbf{P}_2 = \{\alpha_0, l+\gamma_1, \gamma_2, \gamma_3\} \tag{7.39}$$

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2) = (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2)_{\mathrm{M}} + w \tag{7.40}$$

where

$$w = \lambda_0^2\left(\operatorname{sgn}\left(\sigma_{\mathrm{M}}\left(P_0, P_2\right)\right) + 2\right) \tag{7.41}$$

In the coordinate representation the equivalence equations take the form

$$-l\left(l+\gamma_1\right) + w = -l^2 - 2\lambda_0^2, \qquad \alpha_0^2 - \left(l+\gamma_1\right)^2 - \gamma_2^2 - \gamma_3^2 = -l^2 \tag{7.42}$$

For the vector $\mathbf{P}_0\mathbf{P}_2$ we have

$$|\mathbf{P}_0\mathbf{P}_2|_{\mathrm{M}}^2 = \alpha_0^2 - \left(2l+\gamma_1\right)^2 - \gamma_2^2 - \gamma_3^2 \tag{7.43}$$

Eliminating γ_1 and $\alpha_0^2 - \gamma_2^2 - \gamma_3^2$ from (7.43) by means of (7.42), we obtain

$$-l\gamma_1 + w = -2\lambda_0^2, \qquad |\mathbf{P}_0\mathbf{P}_2|_{\mathrm{M}}^2 = -2l\left(2l+\gamma_1\right) \tag{7.44}$$

$$|\mathbf{P}_0\mathbf{P}_2|_{\mathrm{M}}^2 = -2l\left(2l + \frac{w+2\lambda_0^2}{l}\right) = -4l^2 - 2\lambda_0^2\left(\operatorname{sgn}\left(\sigma_{\mathrm{M}}\left(P_0, P_2\right)\right) + 2\right) \tag{7.45}$$

It follows from (7.45) that the vector $\mathbf{P}_0\mathbf{P}_2$ is always spacelike, and, hence, $w = \lambda_0^2$. It follows from (7.42), that

$$\gamma_1 = \frac{3\lambda_0^2}{l}, \qquad \alpha_0 = \sqrt{\gamma_2^2 + \gamma_3^2 + 6\lambda_0^2 + \frac{9\lambda_0^4}{l^2}} \tag{7.46}$$

where γ_1 and γ_2 are arbitrary real numbers. Thus

$$\mathbf{P}_0\mathbf{P}_1 = \{0,l,0,0\}, \qquad \mathbf{P}_1\mathbf{P}_2 = \left\{\sqrt{\gamma_2^2 + \gamma_3^2 + 6\lambda_0^2 + \frac{9\lambda_0^4}{l^2}}, l, \gamma_2, \gamma_3\right\} \tag{7.47}$$

Representing coordinates of the vector $\mathbf{P}_1\mathbf{P}_2$ in the form

$$\left(\mathbf{P}_1\mathbf{P}_2\right)_i = \left(\mathbf{P}_0\mathbf{P}_1\right)_i + a_i \tag{7.48}$$

we obtain for the 4-vector a_i

$$a_i = \left\{\sqrt{\gamma_2^2 + \gamma_3^2 + 6\lambda_0^2 + \frac{9\lambda_0^4}{l^2}}, 0, \gamma_2, \gamma_3\right\}, \qquad \left(a_i a^i\right)_{\mathrm{M}} = 6\lambda_0^2 + \frac{9\lambda_0^4}{l^2} \tag{7.49}$$

Let us imagine now that there is an infinite chain of connected equivalent vectors $...\mathbf{P}_0\mathbf{P}_1, \mathbf{P}_1\mathbf{P}_2, ...\mathbf{P}_k\mathbf{P}_{k+1}, ...$If the vectors are timelike, the chain may be interpreted as a multivariant "world line" of a free particle. The vector $\mathbf{P}_k\mathbf{P}_{k+1}$ may be interpreted as the geometric particle momentum and $|\mathbf{P}_k\mathbf{P}_{k+1}|$ may be interpreted as the geometric mass μ. To obtain the conventional particle mass m, one needs to use the relation (7.4), where b is some universal constant. Statistical description of the multivariant particle motion leads to quantum description in terms of the Schrödinger equation [6]. However, this correspondence between the geometrical description and the quantum one admits one to determine only production $\lambda_0^2 b = \hbar/(2c)$. The universal constants λ_0 and b are not determined asunder from this relation. Thus, in the case of timelike vector $\mathbf{P}_k\mathbf{P}_{k+1}$ we obtain dynamics of free particle from the pure geometrical consideration (dynamics is a corollary of geometry).

If the vectors $\mathbf{P}_k\mathbf{P}_{k+1}$ of the chain are null, it is difficult to speak about temporal evolution. Although the chain of null vectors is single-variant, but vectors of the chain may change their direction, because the constant α_0 in (7.37) may have any sign and module.

At first sight, any temporal evolution in the chain of spacelike vectors $\mathbf{P}_k\mathbf{P}_{k+1}$ is impossible. It is true, provided there are no additional constraints on the chain of spacelike vectors. However, if the skeleton contains more than two points, for instance, $P_0, P_1, Q_1, ...$ the chain, determined by the points P_0, P_1, may contain additional constraints, generated by additional points $Q_1, ...$ of the skeleton. These constraints may be such, that the spacelike "world line" forms a helix with a timelike axis. If so, the helix may be interpreted as a world line of a particle, moving with the superlight velocity along some circle. In this case the mean 4-momentum of the particle is timelike and directed along the helix axis. The direction of mean 4-momentum does not coincide with that of the instantaneous 4-velocity, which is spacelike. We see a similar situation in the case of the Dirac particle, where 4-momentum is a usual timelike vector, whereas the velocity is always equal to the speed of the light. In the classical approximation the world line of the Dirac particle has the shape of a helix [7, 8]. The world line of a free particle, having a shape of a helix, may be explained by the circumstance, that the particle is composite, and in reality there are two connected particles, rotating around their common center of inertia. However, in this case one needs to explain the nature of the interaction, connecting two particles. Such a confinement cannot be explained by means of dynamics, but it can be explained geometrically as a temporal evolution, generated by the spatial evolution.

We are not sure, that such a situation may appear in the distorted space-time (7.1). However, there may exist such a space-time geometry, where the spatial evolution leads to the temporal evolution. Such a case is rather unexpected from the viewpoint of the conventional Riemannian space-time geometry.

8 Metric force fields

It is well known, that contorting the Minkowski space-time, we obtain the curved space-time. The space-time curvature generates the gravitational field, which is connected with the form of the metric tensor. The curvature is a special form of the space-time deformation, which does not generate multivariance of the space-time geometry. The world function σ_{R} of a Riemannian space satisfies the equation [9]

$$\sigma_{\mathrm{R},i}g^{ik}\left(x\right)\sigma_{\mathrm{R},k}=2\sigma_{\mathrm{R}},\qquad \sigma_{\mathrm{R},k}\equiv\frac{\partial\sigma_{\mathrm{R}}\left(x,x'\right)}{\partial x^{k}} \tag{8.1}$$

The two-point world function of the space-time is determined by the equation (8.1) and by the metric tensor $g^{ik}\left(x\right)$, given at any point x of the space-time.

However, if the space-time geometry is multivariant, the world function does not satisfy the equation (8.1), in general. In this case the metric tensor $g^{ik}\left(x\right)$ does not determine the world function, in general. For instance, in the case of world function (7.7) the metric tensor is given by the relation (7.9). It coincides with the metric tensor of the Minkowski space-time to within a constant factor. However, in this case the metric tensor does not determine the world function, because in this case the world function does not satisfy the equation (8.1). For $|\sigma|\gg|\sigma_0|$ the world function (7.7) satisfies the equation of the type (8.1), which has the form

$$\sigma_{,i}g_{\mathrm{M}}^{ik}\left(x\right)\sigma_{,k}=\left(1+\frac{2d}{\pi\sigma_{0}\left(1+\left(\frac{\sigma}{\sigma_{0}+d}\right)^{2}\right)}\right)^{2}\frac{2\sigma}{1+\frac{d}{\sigma_{0}}} \tag{8.2}$$

where $g_{\mathrm{M}}^{ik}\left(x\right)$ is the metric tensor of the Minkowski space-time.

In the case of the single-variant Riemannian space-time geometry, the influence of the space-time geometry can be imitated by means of a gravitational field in the Minkowski space-time. In the case of the space-time geometry (7.7) one also may say, that the space-time geometry is imitated by means of some metric fields in the Minkowski space-time. However, in this case the metric fields form a complex of fields, which cannot be imitated by a one-point field of the type of metric tensor. It remains to be unknown, how these fields are described and how they act on the matter. Formally one may speak about such metric fields and discuss to what extent such metric fields can imitate interaction between the elementary particles in microcosm.

Classification of these metric fields and their investigation is very difficult, if we do not take into account their geometrical origin. Properties of these fields and their description appear to be very exotic, because they are generated by the multivariant space-time geometry. For instance, respectively simple space-time geometry (7.7) is imitated by principles of quantum mechanics, but not by some force fields, because the quantum principles take into account the property of multivariance, whereas the conventional force fields ignore multivariance.

In reality, there exists a very important strategic problem. What is the starting point for investigation of microcosm phenomena? Now this investigation is produced on the basis of supposition, that elementary particles are described by some enigmatic wave functions. They move and interact in accordance with principles of quantum mechanics. Nobody understand, what does it mean. Nevertheless the researchers try different approaches and test them, comparing calculations with experimental data. No principles, only fitting! A use of space-time geometry is very restricted, because of imperfection of our knowledge on geometry.

After construction of multivariant geometries it became possible to explain quantum properties as an appearance of the multivariance of the space-time geometry. Besides, it becomes possible to set the problem of existence of geometrical objects in the form of physical bodies. At such conditions it seems more reasonable at first to investigate capacities of the geometric approach to the elementary particle theory. At geometric approach we also have fitting. However, this fitting concerns only a choice of proper space-time geometry (proper world function). As soon as the world function is chosen, the fitting ceases. One uses only logical reasonings and mathematical calculations. High-handedness of the theorist imagination is restricted.

References

[1] Yu. A. Rylov, Tubular geometry construction as a reason for new revision of the space-time conception. (Printed in *What is Geometry?* polimetrica Publisher, Italy, pp.201-235

[2] Yu. A. Rylov, Extremal properties of Synge's world function and discrete geometry. *J. Math. Phys.* **31**, 2876-2890, (1990).

[3] D. Hilbert, *Grundlagen der Geometrie*. 7 Auflage, B.G.Teubner, Leipzig, Berlin, 1930.

[4] F. Klein, *Vorlesungen über die Entwicklung die Mathematik im 19. Jahrhundert* teil 1, Berlin, Springer 1926.

[5] Yu.A. Rylov, Geometry without topology as a new conception of geometry. *Int. Jour. Mat. & Mat. Sci.* **30**, iss. 12, 733-760, (2002), (*math.MG*/0103002).

[6] Yu.A. Rylov:, Non-Riemannian model of the space-time responsible for quantum effects. *Journ. Math. Phys.* **32**(8), 2092-2098, (1991).

[7] Yu. A. Rylov, Is the Dirac particle composite? *physics* /0410045.

[8] Yu. A. Rylov, Is the Dirac particle completely relativistic? *physics*/0412032.

[9] J. L. Synge, *Relativity: The General Theory*, North-Holland, Amsterdam, 1960.