Discrimination of particle masses in multivariant space-time geometry

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Abstract

Multivariance of geometry means that at the point P_0 there exist many vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$,... which are equivalent (equal) to the vector $\mathbf{Q}_0\mathbf{Q}_1$ at the point Q_0, but they are not equivalent between themselves. The discrimination capacity (zero-variance) of geometry appears, when at the point P_0 there are no vectors, which are equivalent to the vector $\mathbf{Q}_0\mathbf{Q}_1$ at the point Q_0. It is shown, that in some multivariant space-time geometries some particles of small mass may be discriminated (i.e. either they do not exist, or their evolution is impossible) . The possibility of some particle discrimination may appear to be important for explanation of the discrete character of mass spectrum of elementary particles.

1 Introduction

Geometrical dynamics is dynamics of elementary particles, generated by the space-time geometry. In the space-time of Minkowski the geometrical dynamics coincides with the conventional classical dynamics, and the geometrical dynamics may be considered to be a generalization of classical dynamics onto more general space-time geometries. However, the geometric dynamics has a more fundamental basis, and it may be defined in multivariant space-time geometries, where one cannot introduce the conventional classical dynamics. The fact is that, the classical dynamics has been introduced for the space-time geometry with unlimited divisibility, whereas the real space-time has a limited divisibility. The limited divisibility of the space-time is of no importance for dynamics of macroscopic bodies. However, when the size of moving bodies is of the order of the size of heterogeneity, one may not neglect the limited divisibility of the space-time geometry.

The geometric dynamics is developed in the framework of the program of the further physics geometrization, declared in [1]. The special relativity and the general relativity are steps in the development of this program. Necessity on the further development appeared in the thirtieth of the twentieth century, when diffraction of electrons has been discovered. The motion of electrons, passing through the slit, is multivariant. As far as the free electron motion depends only on the properties of the space-time, one needed to change the space-time geometry, making it to be multivariant. In multivariant geometry at the point Q_0 there are many vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$,..., which are equal to the given vector $\mathbf{P}_0\mathbf{P}_1$ at the point P_0, but they are not equal between themselves. Such a space-time geometry was not known in the beginning of the twentieth century. It is impossible in the framework of the Riemannian geometry. As a result the multivariance was prescribed to dynamics. Dynamic variables were replaced by matrices and operators. One obtains the quantum dynamics, which differs from the classical dynamics in its principles. Multivariant space-time geometry appeared only in the end of the twentieth century [2, 3]. The further geometrization of physics became to be possible.

Any geometry is constructed as a modification of the proper Euclidean geometry. But not all representations of the proper Euclidean geometry are convenient for modification. There are three representation of the proper Euclidean geometry [4]. They differ in the number of primary (basic) elements, forming the Euclidean geometry.

The Euclidean representation (E-representation) contains three basic elements (point, segment, angle). Any geometrical object (figure) can be constructed of these basic elements. Properties of the basic elements and the method of their application are described by the Euclidean axioms.

The vector representation (V-representation) of the proper Euclidean geometry contains two basic elements (point, vector). The angle is a derivative element, which is constructed of two vectors. A use of the two basic elements at the construction of geometrical objects is determined by the special structure, known as the linear vector space with the scalar product, given on it (Euclidean space). The scalar product of linear vector space describes interrelation of two basic elements (vectors), whereas other properties of the linear vector space associate with the displacement of vectors.

The third representation (σ-representation) of the proper Euclidean geometry contains only one basic element (point). Segment (vector) is a derivative element. It is constructed of points. The angle is also a derivative element. It is constructed of two segments (vectors). The σ-representation contains a special structure: world function σ, which describes interrelation of two basic elements (points). The world function $\sigma(P_0, P_1) = \frac{1}{2}\rho^2(P_0, P_1)$, where $\rho(P_0, P_1)$ is the distance between points P_0 and P_1. The concept of distance ρ, as well as the world function σ, is used in all representations of the proper Euclidean geometry. However, the world function forms a structure only in the σ-representation, where the world function σ describes interrelation of two basic elements (points). Besides, the world function of the proper Euclidean geometry satisfies a series of constraints, formulated in terms of σ and only in terms of σ. These conditions (the Euclideaness conditions) will be formulated below.

The Euclideaness conditions are equivalent to a use of the vector linear space with the scalar product on it, but formally they do not mention the linear vector space, because all concepts of the linear vector space, as well as all concepts of the proper Euclidean geometry are expressed directly via world function σ and only via it.

If we want to modify the proper Euclidean geometry, then we should use the σ-representation for its modification. In the σ-representation the special geometric structure (world function) has the form of a function of two points. Modifying the form of the world function, we automatically modify all concepts of the proper Euclidean geometry, which are expressed via the world function. It is very important, that the expression of geometrical concepts via the world function does not refer to the means of description (dimension, coordinate system, concept of a curve). The fact, that modifying the world function, one violates the Euclideaness conditions, is of no importance, because one obtains non-Euclidean geometry as a result of such a modification. A change of the world function means a change of the distance, which is interpreted as a deformation of the proper Euclidean geometry. The generalized geometry, obtained by a deformation of the proper Euclidean geometry is called the tubular geometry (T-geometry), because in the generalized geometry straight lines are tubes (surfaces), in general, but not one-dimensional lines. Another name of T-geometry is the physical geometry. The physical geometry is the geometry, described completely by the world function. Any physical geometry may be used as a space-time geometry in the sense, that the set of all T-geometries is the set of all possible space-time geometries.

Modification of the proper Euclidean geometry in V-representation is very restricted, because in this representation there are two basic elements. They are not independent, and one cannot modify them independently. Formally it means, that the linear vector space is to be preserved as a geometrical structure. It means, in particular, that the generalized geometry retains to be continuous, uniform and isotropic. The dimension of the generalized geometry is to be fixed. Besides, the generalized geometry cannot be multivariant. Such a property of the space-time geometry as multivariance can be obtained only in σ-representation. As far as the σ-representation of the proper Euclidean geometry was not known in the twentieth century, the multivariance of geometry was also unknown concept.

Transition from the V-representation to σ-representation is carried out as follows. All concepts of the linear vector space are expressed in terms of the world function σ. In reality, concepts of vector, scalar product of two vectors and linear dependence of n vectors are expressed via the world function σ_{E} of the proper Euclidean geometry. Such operations under vectors as equality of vectors, summation of vectors and multiplication of a vector by a real number are expressed by means of some formulae. The characteristic properties of these operations, which are given in V-representation by means of axioms, are given now by special properties of the Euclidean world function σ_{E}. After expression of the linear vector space via the world function the linear vector space may be not mentioned, because all its properties are described by the world function. We obtain the σ-representation of the proper Euclidean geometry, where some properties of the linear vector space are expressed in the form of formulae, whereas another part of properties is hidden in the specific form of the Euclidean world function σ_{E}. Modifying world function, we modify automatically the properties of the linear vector space (which is not mentioned in fact). At such a modification we are not to think about the way of modification of the linear vector space, which is the principal geometrical structure in the V-representation. In the σ-representation the linear vector space is a derivative structure, which may be not mention at all. Thus, at transition to σ-representation the concepts of the linear vector space (primary concepts in V-representation) become to be secondary concepts (derivative concepts of the σ-representation).

In σ-representation we have the following expressions for concepts of the proper Euclidean geometry. Vector $\mathbf{PQ} = \overrightarrow{PQ}$ is an ordered set of two points P and Q. The length $|\mathbf{PQ}|$ of the vector $\mathbf{PQ}$ is defined by the relation

$$|\mathbf{P}_0\mathbf{P}_1| = \sqrt{2\sigma\left(P_0, P_1\right)} \tag{1.1}$$

The scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma\left(P_0, Q_1\right) + \sigma\left(P_1, Q_0\right) - \sigma\left(P_0, Q_0\right) - \sigma\left(P_1, Q_1\right) \tag{1.2}$$

where the world function σ

$$\sigma: \qquad \Omega\times\Omega \rightarrow \mathbb{R}, \qquad \sigma\left(P,Q\right) = \sigma\left(Q,P\right), \qquad \sigma\left(P,P\right) = 0, \qquad \forall P,Q \in \Omega \tag{1.3}$$

is the world function σ_{E} of the Euclidean geometry.

In the proper Euclidean geometry n vectors $\mathbf{P}_0\mathbf{P}_k$, $k = 1, 2, ...n$ are linear dependent, if and only if the Gram's determinant

$$F_n\left(\mathcal{P}^n\right) = 0, \qquad \mathcal{P}^n = \{P_0, P_1, ..., P_n\} \tag{1.4}$$

where the Gram's determinant $F\left(\mathcal{P}^n\right)$ is defined by the relation

$$F_n\left(\mathcal{P}^n\right) \equiv \det||\left(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k\right)||, \qquad i, k = 1, 2, ...n \tag{1.5}$$

Using expression (1.2) for the scalar product, the condition of the linear dependence of n vectors $\mathbf{P}_0\mathbf{P}_k$, $k = 1, 2, ...n$ is written in the form

$$F_n\left(\mathcal{P}^n\right) \equiv \det||\sigma\left(P_0, P_i\right) + \sigma\left(P_0, P_k\right) - \sigma\left(P_i, P_k\right)|| = 0, \qquad i, k = 1, 2, ...n \tag{1.6}$$

Definition (1.2) of the scalar product of two vectors coincides with the conventional scalar product of vectors in the proper Euclidean space. (One can verify this easily). The relations (1.2), (1.6) do not contain a reference to the dimension of the Euclidean space and to a coordinate system in it. Hence, the relations (1.2), (1.6) are general geometric relations, which may be considered as a definition of the scalar product of two vectors and that of the linear dependence of vectors.

Equivalence (equality) of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined by the relations

$$\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1: \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1|\cdot|\mathbf{Q}_0\mathbf{Q}_1| \wedge |\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1| \tag{1.7}$$

where $|\mathbf{P}_0\mathbf{P}_1|$ is the length (1.1) of the vector $\mathbf{P}_0\mathbf{P}_1$

$$|\mathbf{P}_0\mathbf{P}_1| = \sqrt{(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_1)} = \sqrt{2\sigma(P_0, P_1)} \tag{1.8}$$

In the developed form the condition (1.7) of equivalence of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ has the form

$$\sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) = 2\sigma(P_0, P_1) \tag{1.9}$$

$$\sigma(P_0, P_1) = \sigma(Q_0, Q_1) \tag{1.10}$$

If the points P_0, P_1, determining the vector $\mathbf{P}_0\mathbf{P}_1$, and the origin Q_0 of the vector $\mathbf{Q}_0\mathbf{Q}_1$ are given, we can determine the vector $\mathbf{Q}_0\mathbf{Q}_1$, which is equivalent (equal) to the vector $\mathbf{P}_0\mathbf{P}_1$, solving two equations (1.9), (1.10) with respect to the position of the point Q_1.

In the case of the proper Euclidean space there is one and only one solution of equations (1.9), (1.10) independently of the space dimension n. In the case of arbitrary T-geometry one can guarantee neither existence nor uniqueness of the solution of equations (1.9), (1.10) for the point Q_1. Number of solutions depends on the form of the world function σ. This fact means a multivariance of the property of two vectors equivalence in the arbitrary T-geometry. In other words, the single-variance of the vector equality in the proper Euclidean space is a specific property of the proper Euclidean geometry, and this property is conditioned by the form of the Euclidean world function. In other T-geometries this property does not take place, in general.

The multivariance is a general property of a physical geometry. It is connected with a necessity of solution of algebraic equations, containing the world function. As far as the world function is different in different physical geometries, the solution of these equations may be not unique, or it may not exist at all. If there are many solutions of equations (1.9), (1.10) at fixed vector $\mathbf{P}_0\mathbf{P}_1$ and fixed point Q_0, we shall speak on the property of multivariance of the physical geometry. If there is no solution of equations (1.9), (1.10) at fixed vector $\mathbf{P}_0\mathbf{P}_1$ and fixed point Q_0, we shall speak, that the physical geometry has the property of discrimination (zero-variance).

If in the n-dimensional Euclidean space $F_n(\mathcal{P}^n) \neq 0$, the vectors $\mathbf{P}_0\mathbf{P}_k$, $k = 1, 2, ...n$ are linear independent. We may construct rectilinear coordinate system with basic vectors $\mathbf{P}_0\mathbf{P}_k$, $k = 1, 2, ...n$ in the n-dimensional Euclidean space. Covariant coordinates $x_k = (\mathbf{P}_0\mathbf{P})_k$ of the vector $\mathbf{P}_0\mathbf{P}$ in this coordinate system have the form

$$x_k = x_k(P) = (\mathbf{P}_0\mathbf{P})_k = (\mathbf{P}_0\mathbf{P}.\mathbf{P}_0\mathbf{P}_k), \qquad k = 1, 2, ...n \tag{1.11}$$

Now we can formulate the Euclideaness conditions. These conditions are conditions of the fact, that the T-geometry, described by the world function σ, is n-dimensional proper Euclidean geometry.

I. Definition of the dimension and introduction of the rectilinear coordinate system:

$$\exists \mathcal{P}^n \equiv \{P_0, P_1, ...P_n\} \subset \Omega, \qquad F_n(\mathcal{P}^n) \neq 0, \qquad F_k\left(\Omega^{k+1}\right) = 0, \qquad k > n \quad (1.12)$$

where $F_n(\mathcal{P}^n)$ is the Gram's determinant (1.5). Vectors $\mathbf{P}_0\mathbf{P}_i$, $i = 1, 2, ...n$ are basic vectors of the rectilinear coordinate system K_n with the origin at the point P_0. In K_n the covariant metric tensor $g_{ik}(\mathcal{P}^n)$, $i, k = 1, 2, ...n$ and the contravariant one $g^{ik}(\mathcal{P}^n)$, $i, k = 1, 2, ...n$ are defined by the relations

$$\sum_{k=1}^{k=n} g^{ik}(\mathcal{P}^n)\, g_{lk}(\mathcal{P}^n) = \delta_l^i, \qquad g_{il}(\mathcal{P}^n) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_l), \qquad i, l = 1, 2, ...n \quad (1.13)$$

$$F_n(\mathcal{P}^n) = \det ||g_{ik}(\mathcal{P}^n)|| \neq 0, \qquad i, k = 1, 2, ...n \quad (1.14)$$

II. Linear structure of the Euclidean space:

$$\sigma(P, Q) = \frac{1}{2}\sum_{i,k=1}^{i,k=n} g^{ik}(\mathcal{P}^n)\,(x_i(P) - x_i(Q))\,(x_k(P) - x_k(Q)), \qquad \forall P, Q \in \Omega \quad (1.15)$$

where coordinates $x_i = x_i(P)$, $i = 1, 2, ...n$ of the point P are covariant coordinates of the vector $\mathbf{P}_0\mathbf{P}$, defined by the relation (1.11).

III: The metric tensor matrix $g_{lk}(\mathcal{P}^n)$ has only positive eigenvalues

$$g_k > 0, \qquad k = 1, 2, ..., n \quad (1.16)$$

IV. The continuity condition: the system of equations

$$(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}) = y_i \in \mathbb{R}, \qquad i = 1, 2, ...n \quad (1.17)$$

considered to be equations for determination of the point P as a function of coordinates $y = \{y_i\}$, $i = 1, 2, ...n$ has always one and only one solution. All conditions I $\div$ IV contain a reference to the dimension n of the Euclidean space.

One can show that conditions I $\div$ IV are the necessary and sufficient conditions of the fact, that the world function σ, given on Ω, describes the n-dimensional Euclidean space [2].

2 Dynamics as a result of the space-time geometry

Construction of dynamics in the space-time on the basis of a physical geometry (T-geometry), is presented in [1]. Here we remind the statement of the problem of dynamics.

Geometrical object $\mathcal{O} \subset \Omega$ is a subset of points in the point set Ω. In the T-geometry the geometric object $\mathcal{O}$ is described by means of the skeleton-envelope method. It means that any geometric object $\mathcal{O}$ is considered to be a set of intersections and joins of elementary geometric objects (EGO).

The elementary geometrical object $\mathcal{E}$ is described by its skeleton $\mathcal{P}^n$ and envelope function $f_{\mathcal{P}^n}$. The finite set $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\} \subset \Omega$ of parameters of the envelope function $f_{\mathcal{P}^n}$ is the skeleton of elementary geometric object (EGO) $\mathcal{E} \subset \Omega$. The set $\mathcal{E} \subset \Omega$ of points forming EGO is called the envelope of its skeleton $\mathcal{P}^n$. The envelope function $f_{\mathcal{P}^n}$

$$f_{\mathcal{P}^n} : \qquad \Omega \to \mathbb{R}, \tag{2.1}$$

determining EGO is a function of the running point $R \in \Omega$ and of parameters $\mathcal{P}^n \subset \Omega$. The envelope function $f_{\mathcal{P}^n}$ is supposed to be an algebraic function of s arguments $w = \{w_1, w_2, ...w_s\}$, $s = (n+2)(n+1)/2$. Each of arguments $w_k = \sigma(Q_k, L_k)$ is the world function σ of two points $Q_k, L_k \in \{R, \mathcal{P}^n\}$, either belonging to skeleton $\mathcal{P}^n$, or coinciding with the running point R. Thus, any elementary geometric object $\mathcal{E}$ is determined by its skeleton $\mathcal{P}^n$ and its envelope function $f_{\mathcal{P}^n}$. Elementary geometric object $\mathcal{E}$ is the set of zeros of the envelope function

$$\mathcal{E} = \{R | f_{\mathcal{P}^n}(R) = 0\} \tag{2.2}$$

Definition. Two EGOs $\mathcal{E}_{\mathcal{P}^n}$ and $\mathcal{E}_{\mathcal{Q}^n}$ are equivalent, if their skeletons $\mathcal{P}^n$ and $\mathcal{Q}^n$ are equivalent and their envelope functions $f_{\mathcal{P}^n}$ and $g_{\mathcal{Q}^n}$ are equivalent. Equivalence ($\mathcal{P}^n \text{eqv} \mathcal{Q}^n$) of two skeletons $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\} \subset \Omega$ and $\mathcal{Q}^n \equiv \{Q_0, Q_1, ..., Q_n\} \subset \Omega$ means that

$$\mathcal{P}^n \text{eqv} \mathcal{Q}^n : \qquad \mathbf{P}_i\mathbf{P}_k \text{eqv} \mathbf{Q}_i\mathbf{Q}_k, \qquad i,k = 0,1,...n, \quad i \leq k \tag{2.3}$$

Equivalence of the envelope functions $f_{\mathcal{P}^n}$ and $g_{\mathcal{Q}^n}$ means, that they have the same set of zeros. It means that

$$f_{\mathcal{P}^n}(R) = \Phi(g_{\mathcal{P}^n}(R)), \qquad \forall R \in \Omega \tag{2.4}$$

where Φ is an arbitrary function, having the property

$$\Phi : \mathbb{R} \to \mathbb{R}, \qquad \Phi(0) = 0 \tag{2.5}$$

Evolution of EGO $\mathcal{O}_{\mathcal{P}^n}$ in the space-time is described as a world chain $\mathcal{C}_{\text{fr}}$ of equivalent connected EGOs. The point P_0 of the skeleton $\mathcal{P}^n = \{P_0, P_1, ...P_n\}$ is considered to be the origin of the geometrical object $\mathcal{O}_{\mathcal{P}^n}$. The EGO $\mathcal{O}_{\mathcal{P}^n}$ is considered to be placed at its origin P_0. Let us consider a set of equivalent skeletons $\mathcal{P}^n_{(l)} = \left\{P_0^{(l)}, P_1^{(l)}, ...P_n^{(l)}\right\}$, $l = ...0, 1, ...$which are equivalent in pairs

$$\mathbf{P}_i^{(l)}\mathbf{P}_k^{(l)} \text{eqv} \mathbf{P}_i^{(l+1)}\mathbf{P}_k^{(l+1)}, \qquad i,k = 0,1,...n; \qquad l = ...1,2,... \tag{2.6}$$

The skeletons $\mathcal{P}^n_{(l)}$, $l = ...0, 1, ...$are connected, and they form a chain in the direction of vector $\mathbf{P}_0\mathbf{P}_1$, if the point P_1 of one skeleton coincides with the origin P_0 of the adjacent skeleton

$$P_1^{(l)} = P_0^{(l+1)}, \qquad l = ...0,1,2,... \tag{2.7}$$

The chain $\mathcal{C}_{\text{fr}}$ describes evolution of the elementary geometrical object $\mathcal{O}_{\mathcal{P}^n}$ in the direction of the leading vector $\mathbf{P}_0\mathbf{P}_1$. The evolution of EGO $\mathcal{O}_{\mathcal{P}^n}$ is a temporal evolution, if the vectors $\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}$ are timelike $\left|\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}\right|^2 > 0, \quad l = ...0,1,...$ The evolution of EGO $\mathcal{O}_{\mathcal{P}^n}$ is a spatial evolution, if the vectors $\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}$ are spacelike $\left|\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}\right|^2 < 0, \quad l = ...0,1,...$

Note, that all adjacent links (EGOs) of the chain are equivalent in pairs, although two links of the chain may be not equivalent, if they are not adjacent. However, lengths of corresponding vectors are equal in all links of the chain

$$\left|\mathbf{P}_i^{(l)}\mathbf{P}_k^{(l)}\right| = \left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|, \qquad i,k = 0,1,...n; \qquad l,s = ...1,2,... \tag{2.8}$$

We shall refer to the vector $\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}$, which determines the form of the evolution and the shape of the world chain, as the leading vector. This vector determines the direction of 4-velocity of the physical body, associated with the link of the world chain.

If the relations

$$\begin{aligned} \mathcal{P}^n \text{eqv} \mathcal{Q}^n \quad &: \qquad (\mathbf{P}_i\mathbf{P}_k.\mathbf{Q}_i\mathbf{Q}_k) = |\mathbf{P}_i\mathbf{P}_k| \cdot |\mathbf{Q}_i\mathbf{Q}_k|, \qquad |\mathbf{P}_i\mathbf{P}_k| = |\mathbf{Q}_i\mathbf{Q}_k|, \\ i,k \quad &= \quad 0,1,2,...n \end{aligned} \tag{2.9}$$

$$\begin{aligned} \mathcal{Q}^n \text{eqv} \mathcal{R}^n \quad &: \qquad (\mathbf{Q}_i\mathbf{Q}_k.\mathbf{R}_i\mathbf{R}_k) = |\mathbf{Q}_i\mathbf{Q}_k| \cdot |\mathbf{R}_i\mathbf{R}_k|, \qquad |\mathbf{Q}_i\mathbf{Q}_k| = |\mathbf{R}_i\mathbf{R}_k|, \\ i,k \quad &= \quad 0,1,2,...n \end{aligned} \tag{2.10}$$

are satisfied, the relations

$$\begin{aligned} \mathcal{P}^n \text{eqv} \mathcal{R}^n \quad &: \qquad (\mathbf{P}_i\mathbf{P}_k.\mathbf{R}_i\mathbf{R}_k) = |\mathbf{P}_i\mathbf{P}_k| \cdot |\mathbf{R}_i\mathbf{R}_k|, \qquad |\mathbf{P}_i\mathbf{P}_k| = |\mathbf{R}_i\mathbf{R}_k|, \\ i,k \quad &= \quad 0,1,2,...n \end{aligned} \tag{2.11}$$

are not satisfied, in general, because the relations (2.11) contain the scalar products $(\mathbf{P}_i\mathbf{P}_k.\mathbf{R}_i\mathbf{R}_k)$. These scalar products contain the world functions $\sigma(P_i, R_k)$, which are not contained in relations (2.9), (2.10).

The world chain $\mathcal{C}_{\text{fr}}$, consisting of equivalent links (2.6), (2.7), describes a free motion of a physical body (particle), associated with the skeleton $\mathcal{P}^n$. We assume that *the motion of a physical body is free, if all points of the body move free* (i.e. without acceleration). If the external forces are absent, the physical body as a whole moves without acceleration. However, if the body rotates, one may not consider the motion of this body as a free motion, because not all points of this body move free (without acceleration). In the rotating body there are internal forces, which generate centripetal acceleration to some points of the body. As a result some points of the body do not move free. Motion of the rotating body may be free only on the average, but not exactly free.

Conception of non-free motion of a particle is rather indefinite, and we restrict ourselves only with consideration of a free motion.

Conventional conception of the motion of extensive (non-pointlike) particle, which is free on the average, contains a free displacement, described by the velocity 4-vector, and a spatial rotation, described by the angular velocity 3-pseudovector $\mathbf{\Omega}$. The velocity 4-vector is associated with the timelike leading vector $\mathbf{P}_0\mathbf{P}_1$. At the free on the average motion of a rotating body some of vectors $\mathbf{P}_0\mathbf{P}_2^{(\mathrm{s})}, \mathbf{P}_0\mathbf{P}_3^{(\mathrm{s})}$,...of the skeleton $\mathcal{P}^n$ are not in parallel with vectors $\mathbf{P}_0\mathbf{P}_2^{(\mathrm{s}+1)}, \mathbf{P}_0\mathbf{P}_3^{(\mathrm{s}+1)}$, ..., although at the free motion all vectors $\mathbf{P}_0\mathbf{P}_2^{(\mathrm{s})}, \mathbf{P}_0\mathbf{P}_3^{(\mathrm{s})}$, ... are to be in parallel with $\mathbf{P}_0\mathbf{P}_2^{(\mathrm{s}+1)}, \mathbf{P}_0\mathbf{P}_3^{(\mathrm{s}+1)}$, ... as it follows from (2.6). It means that the world chain $\mathcal{C}_{\mathrm{fr}}$ of a freely moving body can describe only translation of a physical body, but not its rotation.

If the leading vector $\mathbf{P}_0\mathbf{P}_1$ is spacelike, the body, described by the skeleton $\mathcal{P}^n$, evolves in the spacelike direction. It seems, that the spacelike evolution is prohibited. But it is not so. If the world chain forms a helix with the timelike axis, such a world chain may be considered as timelike on the average. In reality such world chains are possible. For instance, the world chain of the classical Dirac particle is a helix with timelike axis [5, 6, 7]. It is not quite clear, whether or not the links of this chain are spacelike, because internal degrees of freedom of the Dirac particle, responsible for helicity of the world chain, are described nonrelativistically.

Thus, consideration of a spatial evolution is not meaningless, especially if we take into account, that the spatial evolution may imitate rotation, which is absent at the free motion of a particle.

3 Discreteness and zero-variance of multivariant space-time geometry

Let us consider the flat homogeneous isotropic space-time $V_{\mathrm{d}} = \{\sigma_{\mathrm{d}}, \mathbb{R}^4\}$, described by the world function

$$\sigma_{\mathrm{d}} = \sigma_{\mathrm{M}} + d \cdot \mathrm{sgn}\,(\sigma_{\mathrm{M}}), \qquad d = \lambda_0^2 = \mathrm{const} > 0 \tag{3.1}$$

$$\mathrm{sgn}\,(x) = \begin{cases} 1, & \text{if } \ x > 0 \\ 0, & \text{if } \ x = 0 \\ -1, & \text{if } \ x < 0 \end{cases}, \tag{3.2}$$

where σ_{M} is the world function of the 4-dimensional space-time of Minkowski. λ_0 is some elementary length. In such a space-time geometry two connected equivalent timelike vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ are described as follows [1]

$$\mathbf{P}_0\mathbf{P}_1 \mathrm{eqv} \mathbf{P}_1\mathbf{P}_2 : \qquad \mathbf{P}_0\mathbf{P}_1 = \{\mu, 0, 0, 0\}, \qquad \mathbf{P}_1\mathbf{P}_2 = \left\{\mu + \frac{3\lambda_0^2}{\mu}, \lambda_0\sqrt{6 + \frac{9\lambda_0^2}{\mu^2}}\mathbf{n}\right\} \tag{3.3}$$

where $\mathbf{n} = \{n_1, n_2, n_3\}$ is an arbitrary unit 3-vector ($n_1^2 + n_2^2 + n_3^2 = 1$). The quantity μ is the length of the vector $\mathbf{P}_0\mathbf{P}_1$ (geometrical mass, associated with the particle, which is described by the vector $\mathbf{P}_0\mathbf{P}_1$). We see that the spatial part of the vector

$\mathbf{P}_1\mathbf{P}_2$ is determined to within the arbitrary 3-vector of the length $\lambda_0\sqrt{6+\frac{9\lambda_0^2}{\mu^2}}$. This multivariance generates wobbling of the links of the world chain, consisting of equivalent timelike vectors ...$\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_2\mathbf{P}_3$,... Statistical description of the chain with wobbling links coincides with the quantum description of the particle with the mass $m = b\mu$, if the elementary length $\lambda_0 = \hbar^{1/2}(2bc)^{-1/2}$, where c is the speed of the light, $\hbar$ is the quantum constant, and b is some universal constant, whose exact value is not determined [8], because the statistical description does not contain the quantity b. Thus, the characteristic wobbling length is of the order of λ_0.

Thus, to explain the quantum description of the particle motion as a statistical description of the multivariant classical motion, we should use the world function (3.1). However, the form of the world function (3.1) is determined by the coincidence of the two description only for the value $\sigma_{\mathrm{M}} > \sigma_0$, where the constant σ_0 is determined via the mass m_{L} of the lightest massive particle (electron) by means of the relation

$$\sigma_0 \leq \frac{\mu_{\mathrm{L}}^2}{2} - d = \frac{m_{\mathrm{L}}^2}{2b^2} - d = \frac{m_{\mathrm{L}}^2}{2b^2} - \frac{\hbar}{2bc} \tag{3.4}$$

where $\mu_{\mathrm{L}} = m_{\mathrm{L}}/b$ is the geometrical mass of the lightest massive particle (electron). The geometrical mass μ_{LM} of the same particle, considered in the space-time geometry of Minkowski, has the form

$$\mu_{\mathrm{LM}} = \sqrt{\mu_{\mathrm{L}}^2 - 2d}$$

As far as $\sigma_0 > 0$, and, hence, $m_{\mathrm{L}}^2 - b\hbar c^{-1} > 0$, we obtain the following estimation for the universal constant b

$$b < \frac{m_{\mathrm{L}}^2 c}{\hbar} \approx 2.4 \times 10^{-17}\mathrm{g/cm}. \tag{3.5}$$

Intensity of wobbling may be described by the multivariance vector $\mathbf{a}_{\mathrm{m}}$, which is defined as follows. Let $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_1\mathbf{P}_2'$ be to vectors which are equivalent to the vector $\mathbf{P}_0\mathbf{P}_1$. Let

$$\mathbf{P}_1\mathbf{P}_2 = \left\{\mu + \frac{3\lambda_0^2}{\mu}, \lambda_0\sqrt{6+\frac{9\lambda_0^2}{\mu^2}}\mathbf{n}\right\}, \qquad \mathbf{P}_1\mathbf{P}_2' = \left\{\mu + \frac{3\lambda_0^2}{\mu}, \lambda_0\sqrt{6+\frac{9\lambda_0^2}{\mu^2}}\mathbf{n}'\right\}$$

Let us consider the vector

$$\mathbf{P}_2\mathbf{P}_2' = \left\{0, \lambda_0\sqrt{6+\frac{9\lambda_0^2}{\mu^2}}(\mathbf{n}'-\mathbf{n})\right\} \tag{3.6}$$

which is a difference of vectors $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_1\mathbf{P}_2'$. We consider the length $|\mathbf{P}_2\mathbf{P}_2'|_{\mathrm{M}}$ of the vector $\mathbf{P}_2\mathbf{P}_2'$ in the Minkowski space-time. We obtain

$$|\mathbf{P}_2\mathbf{P}_2'|_{\mathrm{M}}^2 = -\lambda_0^2\left(6+\frac{9\lambda_0^2}{\mu^2}\right)(2-2\mathbf{n}\mathbf{n}') \tag{3.7}$$

The length of the vector (3.6) is minimal at $\mathbf{n}=-\mathbf{n}'$. At $\mathbf{n}=\mathbf{n}'$ the length of the vector (3.6) is maximal, and it is equal to zero. By definition the vector $\mathbf{P}_2\mathbf{P}_2'$ at $\mathbf{n}=-\mathbf{n}'$ is the multivariance 4-vector $\mathbf{a}_\mathrm{m}$, which describes the intensity of the multivariance. We have

$$\mathbf{a}_\mathrm{m}=\left\{0,\ 2\lambda_0\sqrt{6+\frac{9\lambda_0^2}{\mu^2}}\mathbf{n}\right\}\qquad |\mathbf{a}_\mathrm{m}|^2=(\mathbf{a}_\mathrm{m}.\mathbf{a}_\mathrm{m})=-4\lambda_0^2\left(6+\frac{9\lambda_0^2}{\mu^2}\right) \tag{3.8}$$

where $\mathbf{n}$ is an arbitrary unit 3-vector. The multivariance vector $\mathbf{a}_\mathrm{m}$ is spacelike

In the case, when $\mu\gg\lambda_0$, corresponding wobbling length

$$\lambda_\mathrm{w}=\frac{1}{2}\sqrt{|(\mathbf{a}_\mathrm{m}.\mathbf{a}_\mathrm{m})|}\approx\sqrt{6}\lambda_0=\sqrt{6}\sqrt{\frac{\hbar}{2bc}}>\sqrt{3}\frac{\hbar}{m_\mathrm{L}c}=\sqrt{3}\lambda_\mathrm{com}$$

where λ_com is the Compton wave length of electron.

The relation (3.5) means that

$$\sigma_\mathrm{d}=\sigma_\mathrm{M}+d,\qquad \text{if } \sigma_\mathrm{M}>\sigma_0 \tag{3.9}$$

For other values $\sigma_\mathrm{M}<\sigma_0$ the form of the world function σ_d may distinguish from the relation (3.9). However, $\sigma_\mathrm{d}=0$, if $\sigma_\mathrm{M}=0$.

The space-time (3.1) is a discrete space-time, $|\mathbf{P}_0\mathbf{P}_1|^2\geq\lambda_0^2$, if the vector $\mathbf{P}_0\mathbf{P}_1$ is timelike, and $|\mathbf{P}_0\mathbf{P}_1|^2\leq\lambda_0^2$, if the vector $\mathbf{P}_0\mathbf{P}_1$ is spacelike. It means that in the space-time there are no close points. Absence of close points on the continual set of points seems rather unexpected, because the space-time discreteness associates usually with a latticed space-time, but not with a continual space-time. Nevertheless, the fact of absence of close points means a discreteness of the space-time, and we see no other interpretation, than discreteness. If we consider equivalence of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ in some latticed space, we should expect a discrimination in some cases, because due to latticed character of the space one cannot always find at the point P_0 a vector $\mathbf{P}_0\mathbf{P}_1$, which is equivalent to a given vector $\mathbf{Q}_0\mathbf{Q}_1$.

Let us consider the space-time, described by the world function

$$\sigma=\sigma_\mathrm{M}+d\left(\sigma_\mathrm{M}\right) \tag{3.10}$$

$$d\left(\sigma_\mathrm{M}\right)=\lambda_0^2 f\left(\frac{\sigma_\mathrm{M}}{\sigma_0}\right)=\left\{\begin{array}{ll}\lambda_0^2\mathrm{sgn}\left(\frac{\sigma_\mathrm{M}}{\sigma_0}\right) & \text{if}\quad |\sigma_\mathrm{M}|>\sigma_0>0\\ \lambda_0^2\frac{\sigma_\mathrm{M}}{\sigma_0} & \text{if}\quad |\sigma_\mathrm{M}|\leq\sigma_0\end{array}\right. \tag{3.11}$$

If σ_0 is small, the world function is close to the world function (3.1). If $\sigma_0\to 0$, the world function (3.11) tends to (3.1). Strictly, the space-time geometry (3.11) is not discrete, however it is close to the discrete space-time geometry (3.1). This semidiscreteness manifests itself in a the capacity of discrimination, when evolution of some particles with small mass appears to be prohibited.

Let us consider two connected timelike vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$ of the world chain $\mathbf{P}_k\mathbf{P}_{k+1}$, $k=...0,1,...$ Let us use the inertial coordinate system, where the points P_0,P_1,P_2 have coordinates

$$P_0=\{0,0,0,0\}\,,\qquad P_1=\{s,0,0,0\}\,,\qquad P_2=\{2s+\alpha_0,\alpha_1,\alpha_2,\alpha_3\} \tag{3.12}$$

and coordinates of corresponding vectors are

$$\begin{aligned}
\mathbf{P}_0\mathbf{P}_1 &= \{s,0,0,0\}, \qquad \mathbf{P}_1\mathbf{P}_2 = \{s+\alpha_0,\alpha_1,\alpha_2,\alpha_3\} &(3.13)\\
\mathbf{P}_0\mathbf{P}_2 &= \{2s+\alpha_0,\alpha_1,\alpha_2,\alpha_3\} &(3.14)
\end{aligned}$$

In this case

$$\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{M}} = \left|\mathbf{P}_1\mathbf{P}_2\right|_{\mathrm{M}} = s \tag{3.15}$$

is the length of vectors in the space-time of Minkowski, whereas their true length in the space-time (3.10), (3.11) is

$$\left|\mathbf{P}_0\mathbf{P}_1\right| = \left|\mathbf{P}_1\mathbf{P}_2\right| = \sqrt{\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{M}}^2 + 2d\left(\sigma_{\mathrm{M}}\left(P_0,P_1\right)\right)} = \sqrt{s^2+2d\left(\sigma_{\mathrm{M}}\left(P_0,P_1\right)\right)} \tag{3.16}$$

According to (1.2), (3.10), we have

$$\left(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1\right) = \left(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1\right)_{\mathrm{M}} + w\left(P_0,P_1,Q_0,Q_1\right) \tag{3.17}$$

where $\left(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1\right)_{\mathrm{M}}$ is the scalar product in the space-time of Minkowski and

$$w\left(P_0,P_1,Q_0,Q_1\right) = d\left(\sigma_{\mathrm{M}}\left(P_0,Q_1\right)\right)+d\left(\sigma_{\mathrm{M}}\left(P_1,Q_0\right)\right)-d\left(\sigma_{\mathrm{M}}\left(P_0,Q_0\right)\right)-d\left(\sigma_{\mathrm{M}}\left(P_1,Q_1\right)\right) \tag{3.18}$$

Thus, using relations (3.16),(3.17), one may write the equivalence relations $\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{P}_1\mathbf{P}_2$ (1.9), (1.10) in terms of the Minkowskian world function σ_{M} and distortion d.

$$s\left(s+\alpha_0\right) + w\left(P_0,P_1,P_1,P_2\right) = s^2 \tag{3.19}$$

$$\left(s+\alpha_0\right)^2 - \boldsymbol{\alpha}^2 = s^2, \qquad \boldsymbol{\alpha}^2 = \alpha_1^2+\alpha_2^2+\alpha_3^2 \geq 0 \tag{3.20}$$

where

$$\begin{aligned}
w\left(P_0,P_1,P_1,P_2\right) &= d\left(\sigma_{\mathrm{M}}\left(P_0,P_2\right)\right) - d\left(\sigma_{\mathrm{M}}\left(P_0,P_1\right)\right) - d\left(\sigma_{\mathrm{M}}\left(P_1,P_2\right)\right)\\
&= \lambda_0^2 f\left(\frac{\left(2s+\alpha_0\right)^2-\boldsymbol{\alpha}^2}{2\sigma_0}\right) - 2\lambda_0^2 f\left(\frac{s^2}{2\sigma_0}\right) &(3.21)
\end{aligned}$$

Here four quantities $\alpha = \{\alpha_0,\boldsymbol{\alpha}\} = \{\alpha_0,\alpha_1,\alpha_2,\alpha_3\}$ are to be determined as a solution of the system of two equations (3.19), (3.20).

Equations (3.20) and (3.19) are written in the form

$$2s\alpha_0 + \alpha_0^2 - \boldsymbol{\alpha}^2 = 0 \tag{3.22}$$

$$s\alpha_0 + \lambda_0^2 f\left(\frac{\left(2s+\alpha_0\right)^2-\boldsymbol{\alpha}^2}{2\sigma_0}\right) - 2\lambda_0^2 f\left(\frac{s^2}{2\sigma_0}\right) = 0 \tag{3.23}$$

Resolving equation (3.22) in the form

$$\alpha_0 = -s+\sqrt{s^2+\boldsymbol{\alpha}^2} = \frac{\boldsymbol{\alpha}^2}{s+\sqrt{s^2+\boldsymbol{\alpha}^2}} \tag{3.24}$$

and eliminating α_0 from (3.23), we obtain

$$\frac{s\boldsymbol{\alpha}^2}{s+\sqrt{s^2+\boldsymbol{\alpha}^2}}+\lambda_0^2 f\left(\frac{s\left(s+\sqrt{s^2+\boldsymbol{\alpha}^2}\right)}{\sigma_0}\right)-2\lambda_0^2 f\left(\frac{s^2}{2\sigma_0}\right)=0 \tag{3.25}$$

After transformations we obtain

$$\boldsymbol{\alpha}^2=\frac{\lambda_0^2}{s}\left(s+\sqrt{s^2+\boldsymbol{\alpha}^2}\right)\left(2f\left(\frac{s^2}{2\sigma_0}\right)-f\left(\frac{s\left(s+\sqrt{s^2+\boldsymbol{\alpha}^2}\right)}{\sigma_0}\right)\right) \tag{3.26}$$

Introducing dimensionless quantities x, l , k

$$x=\frac{\boldsymbol{\alpha}^2}{\sigma_0},\qquad l=\frac{s}{\sqrt{\sigma_0}},\qquad k=\frac{\lambda_0^2}{\sigma_0} \tag{3.27}$$

we obtain

$$x=k\frac{l+\sqrt{l^2+x}}{l}\left(2f\left(\frac{l^2}{2}\right)-f\left(l\left(l+\sqrt{l^2+x}\right)\right)\right) \tag{3.28}$$

We are interested only in solutions $x=\frac{\boldsymbol{\alpha}^2}{\sigma_0}\geq 0$. These solutions are possible only for some values of l^2. Boundaries of the regions with the positive solutions of equation (3.28) are determined as zeros of the equation

$$k\left(2f\left(\frac{l^2}{2}\right)-f\left(2l^2\right)\right)=0 \tag{3.29}$$

If the function f is defined by the relation (3.11)

$$f\left(l^2\right)=\left\{\begin{array}{lll}1 & \text{if} & l^2>1\\ l^2 & \text{if} & l^2\leq 1\end{array}\right., \tag{3.30}$$

zeros of equation (3.29) are $\{0,1\}$. It means that values $l^2\in(0,1)$ are prohibited. The values

$$l^2\in(0,1) \tag{3.31}$$

are prohibited in the sense, that evolution of a particle with such a value of l^2 is impossible.

According to (3.16) and (3.27) the geometrical mass μ is expressed as follows

$$\mu=\left|\mathbf{P}_0\mathbf{P}_1\right|=\sqrt{\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{M}}^2+2\lambda_0^2 f\left(\frac{l^2}{2}\right)}=\sqrt{\sigma_0 l^2+2\lambda_0^2 f\left(\frac{l^2}{2}\right)} \tag{3.32}$$

Then it follows from (3.32), that the following values of the geometrical mass μ

$$\mu^2\in\left(0,\sigma_0\left(1+k\right)\right)=\left(0,\sigma_0+\lambda_0^2\right) \tag{3.33}$$

are prohibited, because for these values of μ^2 the quantity $\boldsymbol{\alpha}^2<0$. As one can see, the case $\sigma_0\to 0$ with fixed $\lambda_0>0$ corresponds to the discrete space-time geometry (3.1) with the minimal size λ_0. In this case the geometrical mass $\mu < \lambda_0$ is prohibited. Particles of the mass $\mu < \lambda_0$ are impossible in the discrete space-time geometry (3.1), and the geometric mass $\mu = \lambda_0$ is the minimal mass in the space-time geometry (3.1). Thus, the case $k = \lambda_0^2/\sigma_0 = \infty$ corresponds to the completely discrete geometry. It is reasonable to consider the case of finite $k = \lambda_0^2/\sigma_0$ as the case of partly discrete geometry. In this case the quantity $k = \lambda_0^2/\sigma_0$ may be considered as a degree of discreteness of the space-time geometry.

In general, the concept of partly discrete geometry is not conventional. But it is absent in the conventional approach, because the conception of a discrete geometry is not developed properly. However, it appears that one can introduce such a continuous parameter k, that at $k = 0$ we have a continuous space-time geometry (geometry of Minkowski), and at $k = \infty$ we have a completely discrete geometry. Investigating the space-time geometry with arbitrary k, we use general methods of T-geometry investigation. In such a situation it seems to be reasonable to introduce the concept of partly discrete geometry with the discreteness scale λ_0 and the discreteness degree k .

In the partly discrete multivariant geometry there are constraints on possible mass of a particle even in the case, when the discreteness is not complete. The discreteness of the space-time geometry is connected with the capacity of discrimination, generated by the fact of zero-variance, when the equations (1.9), (1.10), describing equivalence of two vectors, have no solution. Zero-variance and multivariance are two sides of one coin. Multivariance generates quantum effects, whereas the zero-variance generates discrete character of properties of elementary particles. Quantum effects are generated by large values of the world function ($\sigma > \sigma_0$), whereas zero-variance and discreteness are generated by small values of the world function ($\sigma < \sigma_0$).

Substituting (3.24) in (3.13), we obtain

$$P_0 = \{0,0,0,0\}, \qquad P_1 = \{s,0,0,0\}, \qquad P_2 = \left\{s + \sqrt{s^2 + \boldsymbol{\alpha}^2}, \alpha_1, \alpha_2, \alpha_3\right\} \tag{3.34}$$

$$\mathbf{P}_0\mathbf{P}_1 = \{s,0,0,0\}, \qquad \mathbf{P}_1\mathbf{P}_2 = \left\{\sqrt{s^2 + \boldsymbol{\alpha}^2}, \alpha_1, \alpha_2, \alpha_3\right\} \tag{3.35}$$

$$\mathbf{P}_0\mathbf{P}_2 = \left\{s + \sqrt{s^2 + \boldsymbol{\alpha}^2}, \alpha_1, \alpha_2, \alpha_3\right\} \tag{3.36}$$

4 Concluding remarks

Consideration of T-geometry as a space-time geometry admits one to obtain dynamics of a particle as corollary of the geometrical object structure. Evolution of the geometrical object in the space-time is determined by the skeleton $\{P_0, P_1, ..P_n\}$ of the geometrical object and by fixing of the leading vector $\mathbf{P}_0\mathbf{P}_1$. The skeleton and the leading vector determine the world chain, which describes the evolution completely. The world chain may be wobbling, it is a manifestation of the space-time geometry multivariance. Quantum effects are only one of manifestations of the multivariance. It is remarkable, that for determination of the world chain one does not need differential equations, which may be used only in the space-time manifold. One does not need space-time continuity (continual geometry). Of course, one may introduce the continual coordinate system and write dynamic differential equation there. One may, but it is not necessary. In general, geometrical dynamics (i.e. dynamics generated by the space-time geometry) is a discrete dynamics, where step of evolution is determined by the length of the leading vector. It is possible, that one will need a development of special mathematical technique for an effective use of the geometrical dynamics.

The real space-time geometry contains the quantum constant $\hbar$ as a parameter. As a result the geometric dynamics explains freely quantum effects, but not only them. The particle mass is geometrized (the particle mass is simply a length of some vector). As a result the problem of mass of elementary particles is simply a geometrical problem. It is a problem of the structure of elementary geometrical object and its evolution. One needs simply to investigate different forms of skeletons of simplest geometrical objects. Besides, the space-time geometry, looking as a continuous geometry, appears to be partly discrete. These discreteness of "continual" space-time geometry is a source of a discrete character of the elementary particle properties. In turn T-geometry (new conception of geometry) is a source of multivariance and of the discrimination capacity (zero-variance), which are responsible for quantum effects and the discrete character of the elementary particles properties.

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