Geometrical dynamics: spin as a result of rotation with superluminal speed.

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Abstract

Dynamics is considered as a corollary of the space-time geometry. Evolution of a particle in the space-time is described as a chain of connected equivalent geometrical objects. Space-time geometry is determined uniquely by the world function σ. Proper modification of the Minkowskian world function for large space-time interval leads to wobbling of the chain, consisted of timelike straight segments. Statistical description of the stochastic world chain coincides with the quantum description by means of the Schrödinger equation. Proper modification of the Minkowskian world function for small space-time interval may lead to appearance of a world chain, having a shape of a helix with timelike axis. Links of the chain are spacelike straight segments. Such a world chain describes a spatial evolution of a particle. In other words, the helical world chain describes the particle rotation with superluminal velocity. The helical world chain associated with the classical Dirac particle, whose world line is a helix. Length of world chain link cannot be arbitrary. It is determined by the space-time geometry and, in particular, by the elementary length. There exists some discrimination mechanism, which can discriminate some world chains.

1 Introduction

Geometrical dynamics is a dynamics of elementary particles, generated by the space-time geometry. In the space-time of Minkowski the geometrical dynamics coincides with the conventional classical dynamics, and the geometrical dynamics may be considered to be a generalization of classical dynamics onto more general space-time geometries. However, the geometric dynamics has more fundamental basis, and it may be defined in multivariant space-time geometries, where one cannot introduce the conventional classical dynamics. The fact is that, the classical dynamics has been introduced for the space-time geometry with unlimited divisibility, whereas the real space-time has a limited divisibility. The limited divisibility of the space-time is of no importance for dynamics of macroscopic bodies. However, when the size of moving bodies is of the order of the size of heterogeneity, one may not neglect the limited divisibility of the space-time geometry.

The geometric dynamics is developed in the framework of the program of the further physics geometrization, declared in [1]. The special relativity and the general relativity are steps in the development of this program. Necessity of the further development appeared in the thirtieth of the twentieth century, when diffraction of electrons has been discovered. The motion of electrons, passing through the narrow slit, is multivariant. As far as the free electron motion depends only on the properties of the space-time, one needed to change the space-time geometry, making it to be multivariant. In multivariant geometry there are many vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$,...at the point Q_0, which are equal to the vector $\mathbf{P}_0\mathbf{P}_1$, given at the point P_0, but they are not equal between themselves, in general. Such a space-time geometry was not known in the beginning of the twentieth century. It is impossible in the framework of the Riemannian geometry. As a result the multivariance was prescribed to dynamics. To take into account multivariance, dynamic variables were replaced by matrices and operators. One obtains the quantum dynamics, which differs from the classical dynamics in its principles. But the space-time conception remains to be Newtonian (nonrelativistic). Multivariant space-time geometry appeared only in the end of the twentieth century [2, 3]. The further geometrization of physics became to be possible.

It should note that there were numerous attempts of further geometrization of physics. They were based on the Riemannian space-time geometry. Unfortunately, the true space-time geometry of microcosm does not belong to the class of Riemannian geometries, and approximation of real space-time geometry by a Riemannian geometry cannot be completely successful. In particular, the Riemannian geometry cannot describe such a property of real space-time geometry as multivariance. The multivariance of the space-time geometry was replaced by the multivariance of dynamics (quantum theory).

Understanding of nature of elementary particles is the aim of the further geometr\-ization of physics. This aim distinguishes from the aim of the conventional theory of elementary particles. Let us explain the difference of aims in the example of history the chemical elements investigation. Investigation of chemical elements reminds to some extent investigation of elementary particles. Chemical elements are investigated from two sides. Chemists systematized chemical elements, investigating their phenomenological properties. The results of these investigations were formulated in the form of the periodical system of chemical elements in 1870. Formulating this system, D.I.Mendeleev conceived nothing about the atom construction. Nevertheless the periodical system appears to be very useful from the practical viewpoint. Physicists did not aim to explain the periodical system of chemical elements, they tried to understand simply the atom structure and the discrete character of atomic spectra. After construction of the atomic theory it became clear, that the periodical system of chemical elements can be obtained and explained on the basis of the atomic theory. As a result the "physical" approach to investigation of chemical elements appeared to be more fundamental, deep and promising, than the "chemical" one. On the other hand, the way of the "physical" approach to explanation of the periodical system is very long and difficult. Explanation of the periodical system was hardly possible at the "physical" approach, i.e. without the intermediate aim (construction of atomic structure).

Thus, using geometrization of physics, we try to approach only intermediate aim: explanation of multivariance of particle motion (quantum motion) and capacity of discrimination of particle masses. Discrete character of masses of elementary particles can be understood, only if we understand the reason of the elementary particle discrimination. Contemporary approach to the elementary particle theory is the "chemical" (phenomenological) approach. It is useful from the practical viewpoint. However, it admits hardly to understand nature of elementary particles, because the nature of the discrimination mechanism, leading to discrete characteristics of elementary particle, remains outside the consideration.

The most general geometry is a physical geometry, which is called also the tubular geometry (T-geometry) [2, 3, 4], because straights in T-geometry are hallow tubes, in general. The T-geometry is determined completely by its world function $\sigma(P,Q) = \frac{1}{2}\rho^2(P,Q)$, where $\rho(P,Q)$ is interval between the points P and Q in space-time, described by the T-geometry. All concepts of T-geometry are expressed in terms of the world function σ. Dynamics of particles (geometric dynamics) is also described in terms of the world function. The elementary particle is considered as an elementary geometrical object (EGO) in the space-time. The elementary geometrical object $\mathcal{O}$ is described by its skeleton $\mathcal{P}^n = \{P_0, P_1, ...P_n\}$ and its envelope $\mathcal{E}$. The envelope $\mathcal{E}$ is defined as a set of zeros of the envelope function $f_{\mathcal{P}^n}$

$$\mathcal{O} = \{R | f_{\mathcal{P}^n}(R) = 0\} \tag{1.1}$$

The envelope function $f_{\mathcal{P}^n}$ is a real function of arguments $w = \{w_1, w_2, ...w_s\}$. Any argument w_k $k = 1, 2, ...s$ is a world function $w_k = \sigma(L_k, S_k)$, $L_k, S_k \in \{R, \mathcal{P}^n\}$. It is supposed that EGO with skeleton $\mathcal{P}^n = \{P_0, P_1, ...P_n\}$ is placed at the point P_0.

In T-geometry the vector $\overrightarrow{P_0P_1} \equiv \mathbf{P}_0\mathbf{P}_1$ is an ordered set of two points $\{P_0, P_1\}$. The length $|\mathbf{P}_0\mathbf{P}_1|$ of the vector $\mathbf{P}_0\mathbf{P}_1$ is defined via the world function by means of the relation

$$|\mathbf{P}_0\mathbf{P}_1|^2 = 2\sigma(P_0, P_1) \tag{1.2}$$

The scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) \tag{1.3}$$

Equivalence $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined as follows. Two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent (equal), if

$$\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1: \quad ((\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1|) \wedge (|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|) \tag{1.4}$$

In the developed form we have

$$\begin{aligned}
\sigma(P_0,Q_1)+\sigma(P_1,Q_0)-\sigma(P_0,Q_0)-\sigma(P_1,Q_1) &= 2\sigma(P_0,P_1) \\
\sigma(P_0,P_1) &= \sigma(Q_0,Q_1)
\end{aligned}$$

Skeletons $\mathcal{P}^n = \{P_0, P_1, ...P_n\}$ and $\mathcal{Q}^n = \{Q_0, Q_1, ...Q_n\}$ are equivalent ($\mathcal{P}^n \mathrm{eqv} \mathcal{Q}^n$), if corresponding vectors of both skeletons are equivalent

$$\mathcal{P}^n \mathrm{eqv} \mathcal{Q}^n: \quad \mathbf{P}_i\mathbf{P}_k \mathrm{eqv} \mathbf{Q}_i\mathbf{Q}_k, \qquad i,k=0,1,...n \tag{1.5}$$

The skeleton $\mathcal{P}^n = \{P_0, P_1, ...P_n\}$ of EGO at the point P_0 may exist as a skeleton of a physical body, if it may exist at any point $Q_0 \in \Omega$ of the space-time Ω. It means that there is a solution for system of equations

$$\mathbf{P}_i\mathbf{P}_k \mathrm{eqv} \mathbf{Q}_i\mathbf{Q}_k, \qquad i,k=0,1,...n \tag{1.6}$$

for any point $Q_0 \in \Omega$. Further for brevity we take, that an existence of a skeleton means an existence of corresponding geometrical object.

In the space-time of Minkowski the problem of the skeleton existence is rather simple, because at given $\mathcal{P}^n$ and Q_0 the system (1.6) of $n(n+1)$ algebraic equations has a unique solution, although the number of equations may distinguish from the number of variables to be determined. Indeed, in the four-dimensional space-time the number of coordinates of n points $Q_1, Q_2, ...Q_n$ is equal to $4n$ (the point Q_0 is supposed to be given). If $n > 3$, the number $n(n+1)$ of equations is larger than the number $(4n)$ of variables. In the case of an arbitrary space-time geometry (arbitrary world function σ) existence of solution of the system (1.6) is problematic, and the question of existence of the skeleton as a skeleton of a physical body is an essential problem. On the contrary, if $n < 3$, the number of coordinates to be determined is less, than the number of equations, and one may have many skeletons $\mathcal{Q}^n, \mathcal{Q}'^n, ...$placed at the point Q_0, which are equivalent to skeleton $\mathcal{P}^n$, but they are not equivalent between themselves. This property is a property of multivariance of the space-time geometry. This property is actual for simple skeletons, which contain less, than four points ($n < 3$). For instance, for the skeleton of two points $\{P_0, P_1\}$, which is described by the vector $\mathbf{P}_0\mathbf{P}_1$, the problem of multivariance is actual. In the space-time of Minkowski the equivalence of two vectors ($\mathbf{P}_0\mathbf{P}_1 \mathrm{eqv} \mathbf{Q}_0\mathbf{Q}_1$) is single-variant for the timelike vectors, however it is multivariant for spacelike vectors. In the general space-time the equivalence relation $\mathbf{P}_0\mathbf{P}_1 \mathrm{eqv} \mathbf{Q}_0\mathbf{Q}_1$ is multivariant for both timelike and spacelike vectors.

The problem of multivariance is essential for both existence and dynamics of elementary geometrical objects (elementary particles). Let us formulate dynamics of elementary particles in the coordinateless form. Dynamics of an elementary particle, having skeleton $\mathcal{P}^n = \{P_0, P_1, ...P_n\}$, is described by the world chain

$$\mathcal{C} = \bigcup_k \mathcal{P}^n_{(k)}, \qquad \mathcal{P}^n_{(s)} = \left\{P_0^{(s)}, P_1^{(s)}, ...P_n^{(s)}\right\}, \qquad \mathcal{P}^n_{(0)} = \mathcal{P}^n, \tag{1.7}$$

$$P_0^{(s+1)} = P_1^{(s)} \qquad s = ...0,1,2,... \tag{1.8}$$

Direction of evolution in the space-time is described by the leading vector $\mathbf{P}_0\mathbf{P}_1$. If the motion of the elementary particle is free, the adjacent links $\mathcal{P}_{(s)}^n$ and $\mathcal{P}_{(s+1)}^n$ are equivalent in the sense that

$$\mathcal{P}_{(s)}^n \mathrm{eqv} \mathcal{P}_{(s+1)}^n : \qquad \mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)} \mathrm{eqv} \mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}, \qquad i,k = 0,1,...n, \qquad s = ...0,1,2,... \tag{1.9}$$

Relations (1.7) - (1.9) realizes coordinateless description of the free elementary particle motion. In the simplest case, when the space-time is the space-time of Minkowski, and the skeleton consists of two points P_0, P_1 with timelike leading vector $\mathbf{P}_0\mathbf{P}_1$, the coordinateless description by means of relations (1.7) - (1.9) coincides with the conventional description. The conventional classical dynamics is well defined only in the Riemannian space-time. The coordinateless dynamic description (1.7) - (1.9) of elementary particles is a generalization of the conventional classical dynamics onto the case of arbitrary space-time geometry.

2 Representations of the proper Euclidean geometry

Any geometry is constructed as a modification of the proper Euclidean geometry. But not all representations of the proper Euclidean geometry are convenient for modification. There are three representation of the proper Euclidean geometry [5]. They differ in the number of primary (basic) elements, forming the Euclidean geometry.

The Euclidean representation (E-representation) contains three basic elements (point, segment, angle). Any geometrical object (figure) can be constructed of these basic elements. Properties of the basic elements and the method of their application are described by the Euclidean axioms.

The vector representation (V-representation) of the proper Euclidean geometry contains two basic elements (point, vector). The angle is a derivative element, which is constructed of two vectors. A use of the two basic elements at the construction of geometrical objects is determined by the special structure, known as the linear vector space with the scalar product, given on it (Euclidean space). The scalar product of linear vector space describes interrelation of two basic elements (vectors), whereas other properties of the linear vector space associate with the displacement of vectors.

The third representation (σ-representation) of the proper Euclidean geometry contains only one basic element (point). Segment (vector) is a derivative element. It is constructed of points. The angle is also a derivative element. It is constructed of two segments (vectors). The σ-representation contains a special structure: world function σ, which describes interrelation of two basic elements (points). The world function $\sigma(P_0, P_1) = \frac{1}{2}\rho^2(P_0, P_1)$, where $\rho(P_0, P_1)$ is the distance between points P_0 and P_1. The concept of distance ρ, as well as the world function σ, is used in all representations of the proper Euclidean geometry. However, the world function forms a structure only in the σ-representation, where the world function σ describes interrelation of two basic elements (points). Besides, the world function satisfies a series of constraints, formulated in terms of σ and only in terms of σ. These conditions (the Euclideaness conditions) will be formulated below.

The Euclideaness conditions are equivalent to a use of the vector linear space with the scalar product on it, but formally they do not mention the linear vector space, because all concepts of the linear vector space, as well as all concepts of the proper Euclidean geometry are expressed directly via world function σ and only via it.

If we want to modify the proper Euclidean geometry, then we should use the σ-representation for its modification. In the σ-representation the special geometric structure (world function) has the form of a function of two points. Modifying the form of the world function, we modify automatically all concepts of the proper Euclidean geometry, which are expressed via the world function. It is very important, that the expression of geometrical concepts via the world function does not refer to the means of description (dimension, coordinate system, concept of a curve). The fact, that modifying the world function, one violates the Euclideaness conditions, is of no importance, because one obtains non-Euclidean geometry as a result of such a modification. A change of the world function means a change of the distance, which is interpreted as a deformation of the proper Euclidean geometry. The generalized geometry, obtained by a deformation of the proper Euclidean geometry is called the tubular geometry (T-geometry), because in the generalized geometry straight lines are tubes (surfaces), in general, but not one-dimensional lines. Another name of T-geometry is the physical geometry. The physical geometry is the geometry, described completely by the world function. Any physical geometry may be used as a space-time geometry in the sense, that the set of all T-geometries is the set of all possible space-time geometries.

Modification of the proper Euclidean geometry in V-representation is very restricted, because in this representation there are two basic elements. They are not independent, and one cannot modify them independently. Formally it means, that the linear vector space is to be preserved as a geometrical structure. It means, in particular, that the generalized geometry retains to be continuous, uniform and isotropic. The dimension of the generalized geometry is to be fixed. Besides, the generalized geometry,obtained by such a way, cannot be multivariant. Such a property of the space-time geometry as multivariance can be obtained only in σ-representation. As far as the σ-representation of the proper Euclidean geometry was not known in the twentieth century, the multivariance of geometry was also unknown concept.

Transition from the V-representation to σ-representation is carried out as follows. All concepts of the linear vector space are expressed in terms of the world function σ. In reality, concepts of vector, scalar product of two vectors and linear dependence of n vectors are expressed via the world function σ_{E} of the proper Euclidean geometry. Such operations under vectors as equality of vectors, summation of vectors and multiplication of a vector by a real number are expressed by means of some formulae. The characteristic properties of these operations, which are given in V-representation by means of axioms, are given now by special properties of the Euclidean world function σ_{E}. After expression of the linear vector space via the world function the linear vector space may be not mentioned, because all its properties are described by the world function. We obtain the σ-representation of the proper Euclidean geometry, where some properties of the linear vector space are expressed in the form of formulae, whereas another part of properties is hidden in the specific form of the Euclidean world function σ_{E}. Modifying world function, we modify automatically the properties of the linear vector space (which is not mentioned in fact). At such a modification we are not to think about the way of modification of the linear vector space, which is the principal geometrical structure in the V-representation. In the σ-representation the linear vector space is a derivative structure, which may be not mentioned at all. Thus, at transition to σ-representation the concepts of the linear vector space (primary concepts in V-representation) become to be secondary concepts (derivative concepts of the σ-representation).

In σ-representation we have the following expressions for concepts of the proper Euclidean geometry. Vector $\mathbf{PQ}=\overrightarrow{PQ}$ is an ordered set of two points P and Q. The length $|\mathbf{PQ}|$ of the vector $\mathbf{PQ}$ is defined by the relation

$$|\mathbf{P}_0\mathbf{P}_1|=\sqrt{2\sigma\left(P_0,P_1\right)} \tag{2.1}$$

The scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)=\sigma\left(P_0,Q_1\right)+\sigma\left(P_1,Q_0\right)-\sigma\left(P_0,Q_0\right)-\sigma\left(P_1,Q_1\right) \tag{2.2}$$

where the world function σ

$$\sigma:\qquad \Omega\times\Omega\rightarrow\mathbb{R},\qquad \sigma\left(P,Q\right)=\sigma\left(Q,P\right),\qquad \sigma\left(P,P\right)=0,\qquad \forall P,Q\in\Omega \tag{2.3}$$

is the world function σ_{E} of the Euclidean geometry.

In the proper Euclidean geometry n vectors $\mathbf{P}_0\mathbf{P}_k$, $k=1,2,...n$ are linear dependent, if and only if the Gram's determinant

$$F\left(\mathcal{P}^n\right)=0,\qquad \mathcal{P}^n=\left\{P_0,P_1,...,P_n\right\} \tag{2.4}$$

where the Gram's determinant $F\left(\mathcal{P}^n\right)$ is defined by the relation

$$F\left(\mathcal{P}^n\right)\equiv\det\left|\left|\left(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k\right)\right|\right|,\qquad i,k=1,2,...n \tag{2.5}$$

Using expression (2.2) for the scalar product, the condition of the linear dependence of n vectors $\mathbf{P}_0\mathbf{P}_k$, $k=1,2,...n$ is written in the form

$$F\left(\mathcal{P}^n\right)\equiv\det\left|\left|\sigma\left(P_0,P_i\right)+\sigma\left(P_0,P_k\right)-\sigma\left(P_i,P_k\right)\right|\right|=0,\qquad i,k=1,2,...n \tag{2.6}$$

Definition (2.2) of the scalar product of two vectors coincides with the conventional scalar product of vectors in the proper Euclidean space. (One can verify this easily). The relations (2.2), (2.6) do not contain a reference to the dimension of the Euclidean space and to a coordinate system in it. Hence, the relations (2.2), (2.6) are general geometric relations, which may be considered as a definition of the scalar product of two vectors and that of the linear dependence of vectors.

Equivalence (equality) of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined by the relations

$$\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1: \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \wedge |\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1| \tag{2.7}$$

where $|\mathbf{P}_0\mathbf{P}_1|$ is the length (2.1) of the vector $\mathbf{P}_0\mathbf{P}_1$

$$|\mathbf{P}_0\mathbf{P}_1| = \sqrt{(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_1)} = \sqrt{2\sigma(P_0, P_1)} \tag{2.8}$$

In the developed form the condition (2.7) of equivalence of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ has the form

$$\sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) = 2\sigma(P_0, P_1) \tag{2.9}$$

$$\sigma(P_0, P_1) = \sigma(Q_0, Q_1) \tag{2.10}$$

Let the points P_0, P_1, determining the vector $\mathbf{P}_0\mathbf{P}_1$, and the origin Q_0 of the vector $\mathbf{Q}_0\mathbf{Q}_1$ be given. Let $\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1$. We can determine the vector $\mathbf{Q}_0\mathbf{Q}_1$, solving two equations (2.9), (2.10) with respect to the position of the point Q_1.

In the case of the proper Euclidean space there is one and only one solution of equations (2.9), (2.10) independently of the space dimension n. In the case of arbitrary T-geometry one can guarantee neither existence nor uniqueness of the solution of equations (2.9), (2.10) for the point Q_1. Number of solutions depends on the form of the world function σ. This fact means a multivariance of the property of two vectors equivalence in the arbitrary T-geometry. In other words, the single-variance of the vector equality in the proper Euclidean space is a specific property of the proper Euclidean geometry, and this property is conditioned by the form of the Euclidean world function. In other T-geometries this property does not take place, in general.

The multivariance is a general property of a physical geometry. It is connected with a necessity of solution of algebraic equations, containing the world function. As far as the world function is different in different physical geometries, the solution of these equations may be not unique, or it may not exist at all.

If in the n-dimensional Euclidean space $F(\mathcal{P}^n) \neq 0$, the vectors $\mathbf{P}_0\mathbf{P}_k$, $k = 1, 2, ...n$ are linear independent. We may construct rectilinear coordinate system with basic vectors $\mathbf{P}_0\mathbf{P}_k$, $k = 1, 2, ...n$ in the n-dimensional Euclidean space. Covariant coordinates $x_k = (\mathbf{P}_0\mathbf{P})_k$ of the vector $\mathbf{P}_0\mathbf{P}$ in this coordinate system have the form

$$x_k = x_k(P) = (\mathbf{P}_0\mathbf{P})_k = (\mathbf{P}_0\mathbf{P}.\mathbf{P}_0\mathbf{P}_k), \qquad k = 1, 2, ...n \tag{2.11}$$

Now we can formulate the Euclideaness conditions. These conditions are conditions of the fact, that the T-geometry, described by the world function σ, is n-dimensional proper Euclidean geometry.

I. Definition of the dimension and introduction of the rectilinear coordinate system:

$$\exists\mathcal{P}^n \equiv \{P_0, P_1, ...P_n\} \subset \Omega, \qquad F_n(\mathcal{P}^n) \neq 0, \qquad F_k\left(\Omega^{k+1}\right) = 0, \qquad k > n \tag{2.12}$$

where $F_n(\mathcal{P}^n)$ is the Gram's determinant (2.5). Vectors $\mathbf{P}_0\mathbf{P}_i,\quad i=1,2,...n$ are basic vectors of the rectilinear coordinate system K_n with the origin at the point P_0. In K_n the covariant metric tensor $g_{ik}(\mathcal{P}^n),\ i,k=1,2,...n$ and the contravariant one $g^{ik}(\mathcal{P}^n),\ i,k=1,2,...n$ are defined by the relations

$$\sum_{k=1}^{k=n} g^{ik}(\mathcal{P}^n)\, g_{lk}(\mathcal{P}^n)=\delta_l^i,\qquad g_{il}(\mathcal{P}^n)=(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_l),\qquad i,l=1,2,...n \tag{2.13}$$

$$F_n(\mathcal{P}^n)=\det\|g_{ik}(\mathcal{P}^n)\|\neq 0,\qquad i,k=1,2,...n \tag{2.14}$$

II. Linear structure of the Euclidean space:

$$\sigma(P,Q)=\frac{1}{2}\sum_{i,k=1}^{i,k=n} g^{ik}(\mathcal{P}^n)\,(x_i(P)-x_i(Q))\,(x_k(P)-x_k(Q)),\qquad \forall P,Q\in\Omega \tag{2.15}$$

where coordinates $x_i=x_i(P),\ i=1,2,...n$ of the point P are covariant coordinates of the vector $\mathbf{P}_0\mathbf{P}$, defined by the relation (2.11).

III: The metric tensor matrix $g_{lk}(\mathcal{P}^n)$ has only positive eigenvalues

$$g_k>0,\qquad k=1,2,...,n \tag{2.16}$$

IV. The continuity condition: the system of equations

$$(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P})=y_i\in\mathbb{R},\qquad i=1,2,...n \tag{2.17}$$

considered to be equations for determination of the point P as a function of coordinates $y=\{y_i\},\ i=1,2,...n$ has always one and only one solution. All conditions I $\div$ IV contain a reference to the dimension n of the Euclidean space.

One can show that conditions I $\div$ IV are the necessary and sufficient conditions of the fact that the set Ω together with the world function σ, given on $\Omega\times\Omega$, describes the n-dimensional Euclidean space [2].

Investigation of the Dirac particle (dynamic system, described by the Dirac equation) has shown, that the Dirac particle is a composite particle [6], whose internal degrees of freedom are described nonrelativistically [7]. The composite structure of the Dirac particle may be explained as a relativistic rotator, consisting of two (or more) particles, rotating around their inertia centre. The relativistic rotator explains existence of the Dirac particle spin, however, the problem of the rotating particles confinement appears. In this paper we try to explain the problem of spin in the framework of the program of the physics geometrization, when dynamics of physical bodies is determined by the space-time geometry.

Although the first stages of the physics geometrization (the special relativity and the general relativity) manifest themselves very well, the papers on further geometrization of physics, which ignore the quantum principles, are considered usually as dissident.

3 Dynamics as a result of the space-time geometry

Dynamics in the space-time, described by a physical geometry (T-geometry), is presented in [1]. Here we remind the statement of the problem of dynamics.

Geometrical object $\mathcal{O} \subset \Omega$ is a subset of points in the point set Ω. In the T-geometry the geometric object $\mathcal{O}$ is described by means of the skeleton-envelope method. It means that any geometric object $\mathcal{O}$ is considered to be a set of intersections and joins of elementary geometric objects (EGO).

The elementary geometrical object $\mathcal{E}$ is described by its skeleton $\mathcal{P}^n$ and envelope function $f_{\mathcal{P}^n}$. The finite set $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\} \subset \Omega$ of parameters of the envelope function $f_{\mathcal{P}^n}$ is the skeleton of elementary geometric object (EGO) $\mathcal{E} \subset \Omega$. The set $\mathcal{E} \subset \Omega$ of points forming EGO is called the envelope of its skeleton $\mathcal{P}^n$. The envelope function $f_{\mathcal{P}^n}$

$$f_{\mathcal{P}^n}: \qquad \Omega \rightarrow \mathbb{R}, \tag{3.1}$$

determining EGO is a function of the running point $R \in \Omega$ and of parameters $\mathcal{P}^n \subset \Omega$. The envelope function $f_{\mathcal{P}^n}$ is supposed to be an algebraic function of s arguments $w = \{w_1, w_2, ...w_s\}$, $s = (n+2)(n+1)/2$. Each of arguments $w_k = \sigma(Q_k, L_k)$ is the world function σ of two points $Q_k, L_k \in \{R, \mathcal{P}^n\}$, either belonging to skeleton $\mathcal{P}^n$, or coinciding with the running point R. Thus, any elementary geometric object $\mathcal{E}$ is determined by its skeleton $\mathcal{P}^n$ and its envelope function $f_{\mathcal{P}^n}$. Elementary geometric object $\mathcal{E}$ is the set of zeros of the envelope function

$$\mathcal{E} = \{R | f_{\mathcal{P}^n}(R) = 0\} \tag{3.2}$$

Definition. Two EGOs $\mathcal{E}_{\mathcal{P}^n}$ and $\mathcal{E}_{\mathcal{Q}^n}$ are equivalent, if their skeletons $\mathcal{P}^n$ and $\mathcal{Q}^n$ are equivalent and their envelope functions $f_{\mathcal{P}^n}$ and $g_{\mathcal{Q}^n}$ are equivalent. Equivalence ($\mathcal{P}^n \mathrm{eqv} \mathcal{Q}^n$) of two skeletons $\mathcal{P}^n \equiv \{P_0, P_1, ..., P_n\} \subset \Omega$ and $\mathcal{Q}^n \equiv \{Q_0, Q_1, ..., Q_n\} \subset \Omega$ means that

$$\mathcal{P}^n \mathrm{eqv} \mathcal{Q}^n: \qquad \mathbf{P}_i \mathbf{P}_k \mathrm{eqv} \mathbf{Q}_i \mathbf{Q}_k, \qquad i, k = 0, 1, ...n, \quad i \leq k \tag{3.3}$$

Equivalence of the envelope functions $f_{\mathcal{P}^n}$ and $g_{\mathcal{Q}^n}$ means, that they have the same set of zeros. It means that

$$f_{\mathcal{P}^n}(R) = \Phi\left(g_{\mathcal{P}^n}(R)\right), \qquad \forall R \in \Omega \tag{3.4}$$

where Φ is an arbitrary function, having the property

$$\Phi: \mathbb{R} \rightarrow \mathbb{R}, \qquad \Phi(0) = 0 \tag{3.5}$$

Evolution of EGO $\mathcal{O}_{\mathcal{P}^n}$ in the space-time is described as a world chain $\mathcal{C}_{\mathrm{fr}}$ of equivalent connected EGOs. The point P_0 of the skeleton $\mathcal{P}^n = \{P_0, P_1, ...P_n\}$ is considered to be the origin of the geometrical object $\mathcal{O}_{\mathcal{P}^n}$. The EGO $\mathcal{O}_{\mathcal{P}^n}$ is considered to be placed at its origin P_0. Let us consider a set of equivalent skeletons $\mathcal{P}_{(l)}^n = \left\{P_0^{(l)}, P_1^{(l)}, ...P_n^{(l)}\right\}$, $l = ...0, 1, ...$which are equivalent in pairs

$$\mathbf{P}_i^{(l)}\mathbf{P}_k^{(l)}\mathrm{eqv}\mathbf{P}_i^{(l+1)}\mathbf{P}_k^{(l+1)}, \qquad i,k = 0,1,...n; \qquad l = ...1,2,... \tag{3.6}$$

The skeletons $\mathcal{P}_{(l)}^n$, $l = ...0, 1, ...$are connected, and they form a chain in the direction of vector $\mathbf{P}_0\mathbf{P}_1$, if the point P_1 of one skeleton coincides with the origin P_0 of the adjacent skeleton

$$P_1^{(l)} = P_0^{(l+1)}, \qquad l = ...0,1,2,... \tag{3.7}$$

The chain $\mathcal{C}_{\mathrm{fr}}$ describes evolution of the elementary geometrical object $\mathcal{O}_{\mathcal{P}^n}$ in the direction of the leading vector $\mathbf{P}_0\mathbf{P}_1$. The evolution of EGO $\mathcal{O}_{\mathcal{P}^n}$ is a temporal evolution, if the vectors $\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}$ are timelike $\left|\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}\right|^2 > 0$, $l = ...0,1,...$ The evolution of EGO $\mathcal{O}_{\mathcal{P}^n}$ is a spatial evolution, if the vectors $\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}$ are spacelike $\left|\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}\right|^2 < 0$, $l = ...0,1,...$

Note, that all adjacent links (EGOs) of the chain are equivalent in pairs, although two links of the chain may be not equivalent, if they are not adjacent. However, lengths of corresponding vectors are equal in all links of the chain

$$\left|\mathbf{P}_i^{(l)}\mathbf{P}_k^{(l)}\right| = \left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|, \qquad i,k = 0,1,...n; \qquad l,s = ...1,2,... \tag{3.8}$$

We shall refer to the vector $\mathbf{P}_0^{(l)}\mathbf{P}_1^{(l)}$, which determines the form of the evolution and the shape of the world chain, as the leading vector. This vector determines the direction of 4-velocity of the physical body, associated with the link of the world chain.

If the relations

$$\mathcal{P}^n\mathrm{eqv}\mathcal{Q}^n \quad : \quad (\mathbf{P}_i\mathbf{P}_k.\mathbf{Q}_i\mathbf{Q}_k) = |\mathbf{P}_i\mathbf{P}_k| \cdot |\mathbf{Q}_i\mathbf{Q}_k|, \qquad |\mathbf{P}_i\mathbf{P}_k| = |\mathbf{Q}_i\mathbf{Q}_k|, \tag{3.9}$$
$$i,k = 0,1,2,...n \tag{3.10}$$

$$\mathcal{Q}^n\mathrm{eqv}\mathcal{R}^n \quad : \quad (\mathbf{Q}_i\mathbf{Q}_k.\mathbf{R}_i\mathbf{R}_k) = |\mathbf{Q}_i\mathbf{Q}_k| \cdot |\mathbf{R}_i\mathbf{R}_k|, \qquad |\mathbf{Q}_i\mathbf{Q}_k| = |\mathbf{R}_i\mathbf{R}_k|, \tag{3.11}$$
$$i,k = 0,1,2,...n \tag{3.12}$$

are satisfied, the relations

$$\mathcal{P}^n\mathrm{eqv}\mathcal{R}^n \quad : \quad (\mathbf{P}_i\mathbf{P}_k.\mathbf{R}_i\mathbf{R}_k) = |\mathbf{P}_i\mathbf{P}_k| \cdot |\mathbf{R}_i\mathbf{R}_k|, \qquad |\mathbf{P}_i\mathbf{P}_k| = |\mathbf{R}_i\mathbf{R}_k|, \tag{3.13}$$
$$i,k = 0,1,2,...n \tag{3.14}$$

are not satisfied, in general, because the relations (3.13) contain the scalar products $(\mathbf{P}_i\mathbf{P}_k.\mathbf{R}_i\mathbf{R}_k)$. These scalar products contain the world functions $\sigma(P_i, R_k)$, which are not contained in relations (3.9), (3.11).

The world chain $\mathcal{C}_{\mathrm{fr}}$, consisting of equivalent links (3.6), (3.7), describes a free motion of a physical body (particle), associated with the skeleton $\mathcal{P}^n$. We assume that *the motion of physical body is free, if all points of the body move free* (i.e. without acceleration). If the external forces are absent, the physical body as a whole moves without acceleration. However, if the body rotates, one may not consider a motion of this body as a free motion, because not all points of this body move free (without acceleration). In the rotating body there are internal forces, which generate centripetal acceleration of some points of the body. As a result some points of the body do not move free. Motion of the rotating body may be free only on the average, but not exactly free.

Conception of non-free motion of a particle is rather indefinite, and we restrict ourselves with consideration of a free motion only.

Conventional conception of the motion of extensive (non-pointlike) particle, which is free on the average, contains a free displacement, described by the velocity 4-vector, and a spatial rotation, described by the angular velocity 3-pseudovector $\boldsymbol{\omega}$. The velocity 4-vector is associated with the timelike leading vector $\mathbf{P}_0\mathbf{P}_1$. At the free on the average motion of a rotating body some of vectors $\mathbf{P}_0\mathbf{P}_2^{(\mathrm{s})}, \mathbf{P}_0\mathbf{P}_3^{(\mathrm{s})}$,...of the skeleton $\mathcal{P}^n$ are not in parallel with vectors $\mathbf{P}_0\mathbf{P}_2^{(\mathrm{s}+1)}, \mathbf{P}_0\mathbf{P}_3^{(\mathrm{s}+1)}$, ..., although at the free motion all vectors $\mathbf{P}_0\mathbf{P}_2^{(\mathrm{s})}, \mathbf{P}_0\mathbf{P}_3^{(\mathrm{s})}$, ... are to be in parallel with $\mathbf{P}_0\mathbf{P}_2^{(\mathrm{s}+1)}, \mathbf{P}_0\mathbf{P}_3^{(\mathrm{s}+1)}$, ... as follows from (3.6). It means that the world chain $\mathcal{C}_{\mathrm{fr}}$ of a freely moving body can describe only translation of a physical body, but not its rotation.

If the leading vector $\mathbf{P}_0\mathbf{P}_1$ is spacelike, the body, described by the skeleton $\mathcal{P}^n$, evolves in the spacelike direction. It seems, that the spacelike evolution is prohibited. But it is not so. If the world chain forms a helix with the timelike axis, such a world chain may be considered as timelike on the average. In reality such world chains are possible. For instance, the world chain of the classical Dirac particle is a helix with timelike axis. It is not quite clear, whether or not the links of this chain are spacelike, because internal degrees of freedom of the Dirac particle, responsible for helicity of the world chain, are described nonrelativistically.

Thus, consideration of a spatial evolution is not meaningless, especially if we take into account, that the spatial evolution may imitate rotation, which is absent at the free motion of a particle. Further we consider the problem of the spatial evolution.

4 Dynamics of classical Dirac particle

Dirac particle $\mathcal{S}_{\mathrm{D}}$ is the dynamic system, described by the Dirac equation. The free Dirac particle $\mathcal{S}_{\mathrm{D}}$ is described by the free Dirac equation

$$i\hbar\gamma^l\partial_l\psi - m\psi = 0 \tag{4.1}$$

where ψ is the four-component complex wave function, and γ^l, $l = 0, 1, 2, 3$ are 4×4 complex matrices, satisfying the relations

$$\gamma^i\gamma^k + \gamma^k\gamma^i = 2Ig^{ik}, \qquad i, k = 0, 1, 2, 3,$$

I is the 4×4 unit matrix, g^{ik} is the metric tensor. Expressions of physical quantities: the 4-flux j^k of particles and the energy-momentum tensor T_l^k have the form

$$j^k = \bar{\psi}\gamma^k\psi, \qquad T_l^k = \frac{i}{2}\left(\bar{\psi}\gamma^k\partial_l\psi - \partial_l\bar{\psi}\cdot\gamma^k\psi\right), \qquad k,l = 0,1,2,3 \tag{4.2}$$

where $\bar{\psi} = \psi^*\gamma^0$, ψ^* is the Hermitian conjugate to ψ. The classical Dirac particle is a dynamic system $\mathcal{S}_{\mathrm{Dcl}}$, which is obtained from the dynamic system $\mathcal{S}_{\mathrm{D}}$ in the classical limit.

To obtain the classical limit, one may not set the quantum constant $\hbar = 0$ in the equation (4.1), because in this case we do not obtain any reasonable description of the particle.

The Dirac particle $\mathcal{S}_{\mathrm{D}}$ is a quantum particle in the sense, that it is described by a system of partial differential equations (PDE), which contain the quantum constant $\hbar$. The classical Dirac particle $\mathcal{S}_{\mathrm{Dcl}}$ is described by a system of ordinary differential equations (ODE), which contain the quantum constant $\hbar$ as a parameter. May the system of ODE carry out the classical description, if it contains the quantum constant $\hbar$? The answer depends on the viewpoint of investigator. If the investigator believes that *the quantum constant is an attribute of quantum principles and only of quantum principles*, he supposes that, containing $\hbar$, the dynamic equations cannot realize a classical description, where the quantum principles are not used. However, if the investigator consider the classical description simply as method of investigation of the quantum dynamic equations, it is of no importance, whether or not the system of ODE contains the quantum constant. It is important only, that the system of PDE is approximated by a system of ODE. The dynamic system, described by PDE, contains infinite number of the freedom degrees. The dynamic system, described by ODE, contains several degrees of freedom. It is simpler and can be investigated more effectively.

Obtaining the classical approximation, we use the procedure of dynamic disquantization [8]. This procedure transforms the system of PDE into the system of ODE. The procedure of dynamic disquantization is a dynamical procedure, which has no relation to the process of quantization or disquantization in the sense, that it does not refer to the quantum principles. The dynamic disquantization means that all derivatives ∂_k in dynamic equations are replaced by the projection of vector ∂_k onto the current vector j^k

$$\partial_k \longrightarrow \frac{j_k}{j_l j^l} j^s\partial_s \tag{4.3}$$

This dynamical operation is called the dynamic disquantization, because, applying it to the Schrödinger equation, we obtain the dynamic equations for the statistical ensemble of classical nonrelativistic particles. These dynamic equations are ODE, which do not depend on the quantum constant $\hbar$.

Applying the operation (4.3), to the Dirac equation (4.1), we transform it to the form

$$i\hbar\gamma^l\frac{j_l}{j^k j_k} j^s\partial_s\psi - m\psi = 0, \qquad j^k = \bar{\psi}\gamma^k\psi \tag{4.4}$$

The equation (4.4) is the dynamic equation for the dynamic system system $\mathcal{E}_{\mathrm{Dqu}}$. The equation (4.4) contains only derivative $j^s\partial_s = \left(\bar{\psi}\gamma^s\psi\right)\partial_s$ in the direction of the current 4-vector j^k. In terms of the wave function ψ the dynamic equation (4.4) for $\mathcal{E}_{\mathrm{Dqu}}$ looks rather bulky. However, in the properly chosen variables the action for the dynamic system $\mathcal{E}_{\mathrm{Dqu}}$ has the form [8]

$$\mathcal{A}_{\mathrm{Dqu}}[x,\boldsymbol{\xi}] = \int \left\{ -\kappa_0 m\sqrt{\dot{x}^i\dot{x}_i} + \hbar\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{2(1+\boldsymbol{\xi}\mathbf{z})} + \hbar\frac{(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}}{2\sqrt{\dot{x}^s\dot{x}_s}(\sqrt{\dot{x}^s\dot{x}_s}+\dot{x}^0)} \right\} d^4\tau \quad (4.5)$$

where the dot means the total derivative $\dot{x}^s \equiv dx^s/d\tau_0$. The quantities $x = \{x^0, \mathbf{x}\} = \{x^i\}$, $i = 0,1,2,3$, $\boldsymbol{\xi} = \{\xi^\alpha\}$, $\alpha = 1,2,3$ are considered to be functions of the Lagrangian coordinates τ_0, $\boldsymbol{\tau} = \{\tau_1,\tau_2,\tau_3\}$. The variables x describe position of the Dirac particle. Here and in what follows the symbol $\times$ means the vector product of two 3-vectors. The quantity $\mathbf{z}$ is the constant unit 3-vector, κ_0 is a dichotomic quantity $\kappa_0 = \pm 1$, m is the constant (mass) taken from the Dirac equation (4.1). In fact, variables x depend on $\boldsymbol{\tau}$ as on parameters, because the action (4.5) does not contain derivatives with respect to τ_α, $\alpha = 1,2,3$. Lagrangian density of the action (4.5) does not contain independent variables τ explicitly. Hence, it may be written in the form

$$\mathcal{A}_{\mathrm{Dqu}}[x,\boldsymbol{\xi}] = \int \mathcal{A}_{\mathrm{Dcl}}[x,\boldsymbol{\xi}]d\boldsymbol{\tau}, \qquad d\boldsymbol{\tau} = d\tau_1 d\tau_2 d\tau_3 \quad (4.6)$$

where

$$\mathcal{S}_{\mathrm{Dcl}}: \qquad \mathcal{A}_{\mathrm{Dcl}}[x,\boldsymbol{\xi}] = \int \left\{ -\kappa_0 m\sqrt{\dot{x}^i\dot{x}_i} + \hbar\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{2(1+\boldsymbol{\xi}\mathbf{z})} + \hbar\frac{(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}}{2\sqrt{\dot{x}^s\dot{x}_s}(\sqrt{\dot{x}^s\dot{x}_s}+\dot{x}^0)} \right\} d\tau_0 \quad (4.7)$$

The action (4.6) is the action for the dynamic system $\mathcal{E}_{\mathrm{Dqu}}$, which is a set of similar independent dynamic systems $\mathcal{S}_{\mathrm{Dcl}}$. Such a dynamic system is called a statistical ensemble. Dynamic systems $\mathcal{S}_{\mathrm{Dcl}}$ are elements (constituents) of the statistical ensemble $\mathcal{E}_{\mathrm{Dqu}}$. Dynamic equations for each $\mathcal{S}_{\mathrm{Dcl}}$ form a system of ordinary differential equations. It may be interpreted in the sense, that the dynamic system $\mathcal{S}_{\mathrm{Dcl}}$ may be considered to be a classical one, although Lagrangian of $\mathcal{S}_{\mathrm{Dcl}}$ contains the quantum constant $\hbar$. The dynamic system $\mathcal{S}_{\mathrm{Dcl}}$ will be referred to as the classical Dirac particle.

The dynamic system $\mathcal{S}_{\mathrm{Dcl}}$ has ten degrees of freedom. It describes a composite particle [6]. External degrees of freedom are described relativistically by variables x. Internal degrees of freedom are described nonrelativistically [7] by variables $\boldsymbol{\xi}$. Solution of dynamic equations, generated by the action (4.7), gives the following result [6]. In the coordinate system, where the canonical momentum four-vector P_k has the form

$$P_k = \{p_0, \mathbf{p}\} = \left\{ -\left(2 - \frac{1}{\gamma}\right)\kappa_0 m, 0, 0, 0 \right\} \quad (4.8)$$

the world line of the classical Dirac particle is a helix, which is described by the relation

$$\{t,\mathbf{x}\} = \{t, R\sin(\Omega t), R\cos(\Omega t), 0\} \tag{4.9}$$

$$R = \frac{\hbar\gamma\sqrt{\gamma^2-1}}{2m}, \qquad \Omega = \frac{2m}{\hbar\gamma^2} \tag{4.10}$$

where the speed of the light $c = 1$, and γ is an arbitrary constant (Lorentz factor of the classical Dirac particle). The velocity $\mathbf{v} = d\mathbf{x}/dt$ of the classical Dirac particle is expressed as follows

$$\mathbf{v}^2 = 1 - \frac{1}{\gamma^2}, \qquad \gamma = \frac{1}{\sqrt{1-\mathbf{v}^2}} \tag{4.11}$$

Helical world line of the classical Dirac particle means a rotation of the particle around some point. On the one hand, such a rotation seems to be reasonable, because it explains freely the Dirac particle spin and magnetic moment. On the other hand, the description of this rotation is nonrelativistic. Besides, it seems rather strange, that the world line of a free classical particle is a helix, but not a straight line. Attempt of consideration of the Dirac particle as a rotator, consisting of two particles [6], meets the problem of confinement of the two particles.

Although the pure dynamical methods of investigation are more general and effective, than the investigation methods, based on quantum principles, the purely dynamical methods of investigation meet incomprehension of most investigators, who believe, that the Dirac particle must be investigated by quantum methods. The papers, devoted to investigation of the Dirac equation by the dynamic methods, are considered as dissident. They are rejected by the peer review journals (see discussion in [9, 10]).

Suddenly it is discovered that the helical world line, which is characteristic for the classical Dirac particle, can be obtained as a result of a spatial evolution of geometric objects in the framework of properly chosen space-time geometry.

5 Existence of such a space-time geometry, where a spatial evolution may look as world line of classical Dirac particle

Let us consider the flat homogeneous isotropic space-time $V_{\mathrm{d}} = \{\sigma_{\mathrm{d}}, \mathbb{R}^4\}$, described by the world function

$$\sigma_{\mathrm{d}} = \sigma_{\mathrm{M}} + d\cdot\mathrm{sgn}(\sigma_{\mathrm{M}}) \tag{5.1}$$

$$d = \lambda_0^2 = \mathrm{const} > 0 \tag{5.2}$$

$$\mathrm{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}, \tag{5.3}$$

where σ_{M} is the world function of the 4-dimensional space-time of Minkowski. λ_0 is some elementary length. In such a space-time geometry two connected equivalent timelike vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ are described as follows [1]

$$\mathbf{P}_0\mathbf{P}_1 \mathrm{eqv} \mathbf{P}_1\mathbf{P}_2 : \qquad \mathbf{P}_0\mathbf{P}_1 = \{\mu, 0, 0, 0\}, \qquad \mathbf{P}_1\mathbf{P}_2 = \left\{\mu + \frac{3\lambda_0^2}{\mu}, \lambda_0\sqrt{6 + \frac{9\lambda_0^2}{\mu^2}}\mathbf{n}\right\} \tag{5.4}$$

where $\mathbf{n}$ is an arbitrary unit 3-vector. The quantity μ is the length of the vector $\mathbf{P}_0\mathbf{P}_1$ (geometrical mass, associated with the particle, which is described by the vector $\mathbf{P}_0\mathbf{P}_1$). We see that the spatial part of the vector $\mathbf{P}_1\mathbf{P}_2$ is determined to within the arbitrary 3-vector of the length $\lambda_0\sqrt{6 + \frac{9\lambda_0^2}{\mu^2}}$. This multivariance generates wobbling of the links of the world chain, consisting of equivalent timelike vectors ...$\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_2\mathbf{P}_3$,... Statistical description of the chain with wobbling links coincides with the quantum description of the particle with the mass $m = b\mu$, if the elementary length $\lambda_0 = \hbar^{1/2}(2bc)^{-1/2}$, where c is the speed of the light, $\hbar$ is the quantum constant, and b is some universal constant, whose exact value is not determined [11], because the statistical description does not contain the quantity b. Thus, the characteristic wobbling length is of the order of λ_0.

To explain the quantum description of the particle motion as a statistical description of the multivariant classical motion, we should use the world function (5.3). However, the form of the world function (5.3) is determined by the coincidence of the two descriptions only for the value $\sigma_{\mathrm{M}} > \sigma_0$, where the constant σ_0 is determined via the mass m_{L} of the lightest massive particle (electron) by means of the relation

$$\sigma_0 \leq \frac{\mu_{\mathrm{L}}^2}{2} - d = \frac{m_{\mathrm{L}}^2}{2b^2} - d = \frac{m_{\mathrm{L}}^2}{2b^2} - \frac{\hbar}{2bc} \tag{5.5}$$

where $\mu_{\mathrm{L}} = m_{\mathrm{L}}/b$ is the geometrical mass of the lightest massive particle (electron). The geometrical mass μ_{LM} of the same particle, considered in the space-time geometry of Minkowski, has the form

$$\mu_{\mathrm{LM}} = \sqrt{\mu_{\mathrm{L}}^2 - 2d}$$

As far as $\sigma_0 > 0$, and, hence, $m_{\mathrm{L}}^2 - b\hbar c^{-1} > 0$, we obtain the following estimation for the universal constant b

$$b < \frac{m_{\mathrm{L}}^2 c}{\hbar} \approx 2.4 \times 10^{-17} \mathrm{g/cm}. \tag{5.6}$$

Intensity of wobbling may be described by the multivariance vector b_{m}, which is defined as follows. Let $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_1\mathbf{P}_2'$ be two vectors which are equivalent to the vector $\mathbf{P}_0\mathbf{P}_2$. Let

$$\mathbf{P}_1\mathbf{P}_2 = \left\{\mu + \frac{3\lambda_0^2}{\mu}, \lambda_0\sqrt{6 + \frac{9\lambda_0^2}{\mu^2}}\mathbf{n}\right\}, \qquad \mathbf{P}_1\mathbf{P}_2' = \left\{\mu + \frac{3\lambda_0^2}{\mu}, \lambda_0\sqrt{6 + \frac{9\lambda_0^2}{\mu^2}}\mathbf{n}'\right\}$$

Let us consider the vector

$$\mathbf{P}_2\mathbf{P}_2' = \left\{0, \lambda_0\sqrt{6+\frac{9\lambda_0^2}{\mu^2}}\left(\mathbf{n}'-\mathbf{n}\right)\right\} \tag{5.7}$$

which is a difference of vectors $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_1\mathbf{P}_2'$. We consider the length $|\mathbf{P}_2\mathbf{P}_2'|_{\mathrm{M}}$ of the vector $\mathbf{P}_2\mathbf{P}_2'$ in the Minkowski space-time. We obtain

$$|\mathbf{P}_2\mathbf{P}_2'|_{\mathrm{M}}^2 = -\lambda_0^2\left(6+\frac{9\lambda_0^2}{\mu^2}\right)(2-2\mathbf{n}\mathbf{n}') \tag{5.8}$$

The length of the vector (5.7) is minimal at $\mathbf{n} = -\mathbf{n}'$. At $\mathbf{n} = \mathbf{n}'$ the length of the vector (5.7) is maximal, and it is equal to zero. By definition the vector $\mathbf{P}_2\mathbf{P}_2'$ at $\mathbf{n} = -\mathbf{n}'$ is the multivariance 4-vector b_{m}, which describes the intensity of the multivariance. We have

$$b_{\mathrm{m}} = \left\{0,\ 2\lambda_0\sqrt{6+\frac{9\lambda_0^2}{\mu^2}}\mathbf{n}\right\} \qquad |b_{\mathrm{m}}|^2 = (b_{\mathrm{m}}.b_{\mathrm{m}}) = -4\lambda_0^2\left(6+\frac{9\lambda_0^2}{\mu^2}\right) \tag{5.9}$$

where $\mathbf{n}$ is an arbitrary unit 3-vector. The multivariance vector b_{m} is spacelike

In the case, when $\mu \gg \lambda_0$, the corresponding wobbling length

$$\lambda_{\mathrm{w}} = \frac{1}{2}\sqrt{|(b_{\mathrm{m}}.b_{\mathrm{m}})|} \approx \sqrt{6}\lambda_0 = \sqrt{6}\sqrt{\frac{\hbar}{2bc}} > \sqrt{3}\frac{\hbar}{m_{\mathrm{L}}c} = \sqrt{3}\lambda_{\mathrm{com}}$$

where λ_{com} is the electron Compton wave length.

The relation (5.6) means that

$$\sigma_{\mathrm{d}} = \sigma_{\mathrm{M}} + d, \qquad \text{if } \sigma_{\mathrm{M}} > \sigma_0 \tag{5.10}$$

For other values $\sigma_{\mathrm{M}} < \sigma_0$ the form of the world function σ_{d} may distinguish from the relation (5.10). However, $\sigma_{\mathrm{d}} = 0$, if $\sigma_{\mathrm{M}} = 0$.

Two equivalent connected spacelike vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_1\mathbf{Q}_2$ have the form [1]

$$\mathbf{Q}_0\mathbf{Q}_1 = \{0, l, 0, 0\}, \qquad \mathbf{Q}_1\mathbf{Q}_2 = \left\{\sqrt{\gamma_2^2+\gamma_3^2+6\lambda_0^2+\frac{9\lambda_0^4}{l^2}}, l, \gamma_2, \gamma_3\right\} \tag{5.11}$$

where constants γ_2 and γ_3 are arbitrary. The result is obtained for the space-time geometry (5.1). Arbitrariness of constants γ_2, γ_3 generates multivariance of the vector $\mathbf{Q}_1\mathbf{Q}_2$ even in the space-time geometry of Minkowski, where $\lambda_0 = 0$.

Vectors $\mathbf{Q}_1\mathbf{Q}_2$, $\mathbf{Q}_1\mathbf{Q}_2'$

$$\begin{aligned}
\mathbf{Q}_1\mathbf{Q}_2 &= \left\{\sqrt{\gamma_2^2+\gamma_3^2+6\lambda_0^2+\frac{9\lambda_0^4}{l^2}}, l, \gamma_2, \gamma_3\right\}, \\
\mathbf{Q}_1\mathbf{Q}_2' &= \left\{\sqrt{\gamma_2'^2+\gamma_3'^2+6\lambda_0^2+\frac{9\lambda_0^4}{l^2}}, l, \gamma_2', \gamma_3'\right\}
\end{aligned}$$

are equivalent to the vector $\mathbf{Q}_0\mathbf{Q}_1$. The difference $\mathbf{Q}_2\mathbf{Q}_2'$ of two vectors $\mathbf{Q}_1\mathbf{Q}_2$, $\mathbf{Q}_1\mathbf{Q}_2'$ has the form

$$\mathbf{Q}_2\mathbf{Q}_2' = \left\{ \sqrt{\gamma_2'^2 + \gamma_3'^2 + 6\lambda_0^2 + \frac{9\lambda_0^4}{l^2}} - \sqrt{\gamma_2^2 + \gamma_3^2 + 6\lambda_0^2 + \frac{9\lambda_0^4}{l^2}}, 0, \gamma_2' - \gamma_2, \gamma_3' - \gamma_3 \right\}$$

The vector $\mathbf{Q}_2\mathbf{Q}_2'$ may be spacelike and timelike. Its length has an extremum, if $\gamma_2' = \gamma_2$ and $\gamma_3' = \gamma_3$. In this case the length $|\mathbf{Q}_2\mathbf{Q}_2'|^2 = 0$

However, the length

$$\begin{aligned} |\mathbf{Q}_2\mathbf{Q}_2'|^2 &= \left(\sqrt{\gamma_2'^2 + \gamma_3'^2 + 6\lambda_0^2 + \frac{9\lambda_0^4}{l^2}} - \sqrt{\gamma_2^2 + \gamma_3^2 + 6\lambda_0^2 + \frac{9\lambda_0^4}{l^2}} \right)^2 \\ &\quad - (\gamma_2' - \gamma_2)^2 - (\gamma_3' - \gamma_3)^2 \end{aligned}$$

has neither maximum, nor minimum, and one cannot introduce the multivariance vector of the type (5.9). The multivariance of the spacelike vectors equality is not introduced by the distortion d, defined by (5.2). It takes place already in the space-time of Minkowski. In the conventional approach to the geometry of Minkowski one does not accept the multivariance of spacelike vectors equivalence. Furthermore, the concept of multivariance of two vectors parallelism (and equality) is absent at all in the conventional approach to the geometry. For instance, when in the Riemannian geometry the multivariance of parallelism of remote vectors appears, the mathematicians prefer to deny at all the fernparallelism (parallelism of two remote vectors), but not to introduce the concept of multivariance. This circumstance is connected with the fact, that the multivariance may not appear, if the geometry is constructed on the basis of a system of axioms .

The world chain, consisting of timelike equivalent vectors, imitates a world line of a free particle. This fact seems to be rather reasonable. Considering the vectors $\mathbf{Q}_0\mathbf{Q}_1$ and $\mathbf{Q}_1\mathbf{Q}_2$ in (5.11) from the viewpoint of the geometry of Minkowski, we see that the vector $\mathbf{Q}_1\mathbf{Q}_2$ is obtained from the vector $\mathbf{Q}_0\mathbf{Q}_1$ as a result of spatial rotation and of an addition of some temporal component. One should expect, that the world chain, consisting of spacelike equivalent vectors, imitates a world line of a free particle, moving with a superluminal velocity. The motion with the superluminal velocity seems to be unobservable. Such a motion is considered to be impossible. However, if the spacelike world line has a shape of a helix with timelike axis, such a situation may be considered as a free rotating particle. The fact, that the particle rotates with the superluminal velocity is not so important, if the helix axis is timelike. The world line of a classical Dirac particle is a helix. It is not very important, whether the rotation velocity is tardyon or not. Especially, if we take into account that the Dirac equation describes the internal degrees of freedom (rotation) nonrelativistically, (i.e. the description of internal degrees of the classical Dirac particle is incorrect from the viewpoint of the relativity theory).

We investigate now, whether a world chain of equivalent spacelike vectors may form a helix with timelike axis. If it is possible, then we try to investigate, under which world function such a situation is possible. We consider the world function $\sigma_{\rm d}$ of the form

$$\sigma_{\rm d}=\sigma_{\rm M}+d\cdot f\left(\frac{\sigma_{\rm M}}{\sigma_0}\right),\qquad f\left(x\right)=\mathrm{sgn}\left(x\right),\quad \text{if}\quad \left|x\right|>1,\qquad d=\lambda_0^2=\mathrm{const}>0 \tag{5.12}$$

where the function $f\left(\frac{\sigma_{\rm M}}{\sigma_0}\right)$ should be determined from the condition, that the world chain, consisting of spacelike links, forms a helix with timelike axis.

To estimate the form of $\sigma_{\rm d}$ as a function of $\sigma_{\rm M}$ at $\sigma_{\rm M}<\sigma_0$, we consider the chain, consisting of equivalent spacelike vectors ... $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_2\mathbf{P}_3$,... We suppose that the chain is a helix with timelike axis in the space-time. Let the points $...P_0,P_1,...$ have the coordinates

$$P_k=\left\{kl_0,R\cos\left(k\varphi-\varphi_0\right),R\sin\left(k\varphi-\varphi_0\right),0\right\},\qquad k=...0,1,2,... \tag{5.13}$$

All points (5.13) lie on a helix with timelike axis. In the space-time of Minkowski the step of helix is equal to $2\pi l_0/\varphi$, and R is the radius of the helix. The constants φ and φ_0 are parameters of the helix. All vectors $\mathbf{P}_k\mathbf{P}_{k+1}$ have the same length. Introducing designations

$$\phi=\frac{\varphi}{2},\qquad l_1=2R\sin\phi,\qquad \varphi_0=\phi-\frac{\pi}{2} \tag{5.14}$$

we obtain coordinates of vectors $\mathbf{P}_k\mathbf{P}_{k+1}$ in the form

$$\mathbf{P}_{k-1}\mathbf{P}_k=\left\{l_0,l_1\cos\left(2k\phi\right),l_1\sin\left(2k\phi\right),0\right\},\qquad k=...0,1,... \tag{5.15}$$

where l_0,l_1,ϕ are parameters of the helix.

Let us investigate, under which conditions the relation $\mathbf{P}_{k-1}\mathbf{P}_k\mathrm{eqv}\mathbf{P}_k\mathbf{P}_{k+1}$ takes place. We suppose that all vectors of the helix are spacelike $\left|\mathbf{P}_k\mathbf{P}_{k+1}\right|^2<0$. It is evident, that it is sufficient to investigate the situation for the case $k=1$, when $\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{P}_1\mathbf{P}_2$. Let coordinates of vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$ be

$$\mathbf{P}_0\mathbf{P}_1=\left\{l_0,l_1,0,0\right\},\qquad \mathbf{P}_1\mathbf{P}_2=\left\{l_0,l_1\cos\left(2\phi\right),l_1\sin\left(2\phi\right),0\right\} \tag{5.16}$$

In this case the coordinates of the points P_0,P_1,P_2 may be chosen in the form

$$P_0=\left\{0,0,0,0\right\},\qquad P_1=\left\{l_0,l_1,0,0\right\},\qquad P_2=\left\{2l_0,l_1\left(1+\cos\left(2\phi\right)\right),l_1\sin\left(2\phi\right),0\right\}, \tag{5.17}$$

and the vector $\mathbf{P}_0\mathbf{P}_2$ has coordinates

$$\mathbf{P}_0\mathbf{P}_2=\left\{2l_0,l_1\left(1+\cos\left(2\phi\right)\right),l_1\sin\left(2\phi\right),0\right\} \tag{5.18}$$

We choose the world function (5.12) in the form

$$\sigma_{\rm d}=\sigma_{\rm M}+\lambda_0^2\left\{\begin{array}{ll}\mathrm{sgn}\left(\sigma_{\rm M}\right) & \text{if}\quad \left|\sigma_{\rm M}\right|>\sigma_0>0\\ \left(\frac{\sigma_{\rm M}}{\sigma_0}\right)^3 & \text{if}\quad \left|\,\sigma_{\rm M}\right|<\sigma_0\end{array}\right. \tag{5.19}$$

and introduce the quantity

$$\varkappa = \frac{\sigma_0}{\lambda_0^2} \tag{5.20}$$

Thus, we have

$$\sigma_{\rm d} = \sigma_{\rm M} + d\left(\sigma_{\rm M}\right), \qquad d\left(\sigma_{\rm M}\right) = \lambda_0^2 f\left(\frac{\sigma_{\rm M}}{\sigma_0}\right), \qquad f\left(x\right) = \begin{cases} 1 & \text{if} \quad x \geq 1 \\ x^3 & \text{if} \quad -1 < x < 1 \\ -1 & \text{if} \quad x \leq -1 \end{cases} \tag{5.21}$$

The space-time geometry (5.21) is a special case of the space-time geometry (5.10). We do not pretend to the claim, that (5.21) is the world function of true space-time geometry. We shall show only that in the space-time geometry (5.21) the spacelike vectors (5.16) may be equivalent at some proper choice of parameters l_0, l_1 and φ.

In our calculations we shall use two geometries: the geometry $\mathcal{G}_{\rm M}$ of Minkowski and the space-time geometry $\mathcal{G}_{\rm d}$, described by the world function $\sigma_{\rm d}$, determined by (5.21). Then expressions of the geometry $\mathcal{G}_{\rm d}$ may be reduced to expressions of the geometry $\mathcal{G}_{\rm M}$ by means of relations

$$\left|\mathbf{P}_0\mathbf{P}_1\right|^2 = \left|\mathbf{P}_0\mathbf{P}_1\right|_{\rm M}^2 + 2d\left(\sigma_{\rm M}\left(P_0, P_1\right)\right) \tag{5.22}$$

$$\left(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1\right) = \left(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1\right)_{\rm M} + w\left(P_0, P_1, Q_0, Q_1\right) \tag{5.23}$$

$$\begin{aligned} w\left(P_0, P_1, Q_0, Q_1\right) &= d\left(\sigma_{\rm M}\left(P_0, Q_1\right)\right) + d\left(\sigma_{\rm M}\left(P_1, Q_0\right)\right) \\ &-d\left(\sigma_{\rm M}\left(P_0, Q_0\right)\right) - d\left(\sigma_{\rm M}\left(P_1, Q_1\right)\right) \end{aligned} \tag{5.24}$$

The geometrical relations in $\mathcal{G}_{\rm d}$ are expressed via the same relations, written in $\mathcal{G}_{\rm M}$ with additional terms, containing the distortion d. This additional terms in dynamic relations are interpreted as additional metric interactions, acting on a particle, when the real space-time geometry $\mathcal{G}_{\rm d}$ is considered to be the geometry $\mathcal{G}_{\rm M}$. Appearance of additional interactions reminds appearance of inertial forces at a use of accelerated coordinate systems instead of inertial ones.

Condition $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{P}_1\mathbf{P}_2$ of equivalence of vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$ is written in the form of two equations

$$\left(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2\right) = \left(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2\right)_{\rm M} + w\left(P_0, P_1, P_1, P_2\right) = \left|\mathbf{P}_0\mathbf{P}_1\right|_{\rm M}^2 + 2d\left(\sigma_{\rm M}\left(P_0, P_1\right)\right) \tag{5.25}$$

$$\left|\mathbf{P}_0\mathbf{P}_1\right|_{\rm M}^2 + 2d\left(\sigma_{\rm M}\left(P_0, P_1\right)\right) = \left|\mathbf{P}_1\mathbf{P}_2\right|_{\rm M}^2 + 2d\left(\sigma_{\rm M}\left(P_1, P_2\right)\right) \tag{5.26}$$

where index 'M' means, that the corresponding quantities are calculated in $\mathcal{G}_{\rm M}$. The function d is determined by the relation (5.21), and the quantity w is determined by the relation

$$w\left(P_0, P_1, P_1, P_2\right) = d\left(\sigma_{\rm M}\left(P_0, P_2\right)\right) - d\left(\sigma_{\rm M}\left(P_0, P_1\right)\right) - d\left(\sigma_{\rm M}\left(P_1, P_2\right)\right) \tag{5.27}$$

which follows from the definition of the scalar product (5.24). Using the conventional relations for the scalar product in $\mathcal{G}_{\rm M}$, we can rewrite the relations (5.25), (5.26) in the form

$$l_0^2 - l_1^2\cos\left(2\phi\right) + w\left(P_0, P_1, P_1, P_2\right) = l_0^2 - l_1^2 + 2d\left(\sigma_{\rm M}\left(P_0, P_1\right)\right) \tag{5.28}$$

$$l_0^2 - l_1^2 = l_0^2 - l_1^2 \left(\cos^2(2\phi) + \sin^2(2\phi)\right) \tag{5.29}$$

where

$$w(P_0, P_1, P_1, P_2) = d\left(2l_1^2 \sin^2\phi + 2\left(l_0^2 - l_1^2\right)\right) - 2d\left(\frac{l_0^2 - l_1^2}{2}\right) \tag{5.30}$$

To obtain the relation (5.30) from (5.27), we use the relations

$$|\mathbf{P}_0\mathbf{P}_1|_{\mathrm{M}}^2 = |\mathbf{P}_1\mathbf{P}_2|_{\mathrm{M}}^2 = 2\sigma_{\mathrm{M}}(P_0, P_1) = l_0^2 - l_1^2 \equiv l^2 \tag{5.31}$$

$$\frac{1}{2}|\mathbf{P}_0\mathbf{P}_2|_{\mathrm{M}}^2 = \sigma_{\mathrm{M}}(P_0, P_2) = 2l_0^2 - l_1^2(1 + \cos(2\phi)) = 2l_1^2 \sin^2\phi + 2l^2 \tag{5.32}$$

The equation (5.29) is the identity.

Let us introduce pure quantities ν, a, defining them by relations

$$l^2 = l_0^2 - l_1^2 = -2\nu\sigma_0, \qquad \nu > 0 \tag{5.33}$$

$$a = \frac{2l_1^2}{\sigma_0}\sin^2\phi, \qquad \varkappa = \frac{\sigma_0}{\lambda_0^2} \tag{5.34}$$

Then the equation (5.28) takes the form

$$\varkappa a + f(a - 4\nu) = -4f(\nu) \tag{5.35}$$

where the function f is defined by the relation (5.21)

$$f(\nu) = \frac{1}{\lambda_0^2} d(\sigma_0\nu) = \begin{cases} \operatorname{sgn}(\nu) & \text{if} \quad |\nu| > 1 \\ \nu^3 & \text{if} \quad |\nu| < 1 \end{cases} \tag{5.36}$$

and the constant $\varkappa$ is defined by the relation (5.20).

Let us note, that in the case, when $f(\nu)$ is a linear function $f(\nu) = \nu$, for $\nu \in [-1, 1]$, the equation (5.35) has the unique solution $a = 0$. The solution with $a = \frac{2l_1^2}{\sigma_0}\sin^2\phi = 0$ describes a straight but not a helix.

Considering solutions of equation (5.35) with respect to $a = a(\nu)$, we are interested in positive values of a, because the quantity a is nonnegative by definition (5.34). At $\varkappa = 1$ numerical solutions of equation (5.35) with respect to a are presented in the form

ν	$a(\nu)$	ν	$a(\nu)$	ν	$a(\nu)$	ν	$a(\nu)$
0	0	0.4	0.63701	−0.63	0	−0.956 47	2
0.1	0.04191	0.5	0.5	−0.7	0.372	−0.974 35	2.7
0.2	0.19236	0.6	0.136	−0.8	1.048	−0.991 60	2.9
0.3	0.40137	0.63	0	−0.9	1. 916	−1	3

According to (5.14), (5.33) and (5.34) we have the following relations for the helix radius R

$$\sin\phi = \sqrt{\frac{a\sigma_0}{2l_1^2}}, \qquad R = \frac{l_1}{2\sin\phi} = \frac{l_1^2}{\sqrt{2a\sigma_0}} \tag{5.37}$$

We obtain the helix step S in the form

$$S=\frac{\pi}{\phi}l_0=\frac{\pi l_0}{\arcsin\sqrt{\frac{a\sigma_0}{2l_1^2}}}=\frac{\pi\sqrt{l_1^2-2\sigma_0\nu}}{\arcsin\sqrt{\frac{a\sigma_0}{2l_1^2}}} \tag{5.38}$$

Negative values of ν correspond to helix with timelike vectors $\mathbf{P}_{k-1}\mathbf{P}_k$. Positive solutions of equation (5.35) take place only for $\nu\in(0,0.63)$ (spacelike vectors) and $\nu\in(-0.63,-1)$ (timelike vectors). The values of parameter a belong to intervals

$$a\in[0,0.695]\,,\qquad a\in(0,3) \tag{5.39}$$

for spacelike and timelike vectors correspondingly.

Thus, we see that in the space-time geometry with the world function (5.21) the spatial evolution, determined by the spacelike vectors $\mathbf{P}_k\mathbf{P}_{k+1}$, may lead to a helical world chain with timelike axis. However, equivalence of spacelike vectors $\mathbf{P}_k\mathbf{P}_{k+1}$ is multivariant even in the space-time of Minkowski. It is valid for the space-time geometry (5.21) also. As a result the wobbling of the spacelike vectors appears. It may lead to destruction of the helix.

In reality the conditions $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{P}_1\mathbf{P}_2$ determines vector $\mathbf{P}_1\mathbf{P}_2$ to within the vector $\alpha=\{\alpha_0,\boldsymbol{\alpha}\}$, and we have instead of equations (5.16)

$$\mathbf{P}_0\mathbf{P}_1=l,\qquad \mathbf{P}_1\mathbf{P}_2=q+\alpha \tag{5.40}$$

where l, q, α are 4-vectors with coordinates

$$l=\{l_0,l_1,0,0\}\,,\qquad q=\{l_0,l_1\cos(2\phi)\,,l_1\sin(2\phi)\,,0\}\,,\qquad \alpha=\{\alpha_0,\alpha_1,\alpha_2,\alpha_3\} \tag{5.41}$$

Instead of equations (5.28) – (5.30) we have the following equations

$$\alpha^2+2\,(q.\alpha)=0 \tag{5.42}$$

$$2l_1^2\sin^2\phi+(l.\alpha)+\lambda_0^2f\left(\frac{2l^2+2l_1^2\sin^2\phi+(l.\alpha)}{\sigma_0}\right)-4\lambda_0^2f\left(\frac{l^2}{2\sigma_0}\right)=0 \tag{5.43}$$

where $(l.\alpha)$ and $(q.\alpha)$ mean scalar products of vectors l,q,α in the space-time of Minkowski. The relation (5.35) is the necessary condition of the fact, that $\alpha=0$ is a solution of equations (5.42), (5.43). We obtain from (5.42)

$$\alpha_0=-q_0\pm\sqrt{q_0^2+\boldsymbol{\alpha}^2+2\mathbf{q}\boldsymbol{\alpha}}=-l_0\pm\sqrt{l_0^2+\boldsymbol{\alpha}^2+2\mathbf{q}\boldsymbol{\alpha}} \tag{5.44}$$

where $\mathbf{q}\boldsymbol{\alpha}$ means the scalar product of 3-vectors $\mathbf{q}$ and $\boldsymbol{\alpha}$.

Taking into account the relation (5.35), we obtain from relation (5.43)

$$(l.\alpha)+\lambda_0^2\left(f\left(\frac{2l^2+2l_1^2\sin^2\phi+(l.\alpha)}{\sigma_0}\right)-f\left(\frac{2l^2+2l_1^2\sin^2\phi}{\sigma_0}\right)\right)=0 \tag{5.45}$$

Supposing, that $(l.\alpha)$ is a small quantity we obtain from (5.45) by means of (5.44)

$$\left(l_0\left(-l_0\pm\sqrt{l_0^2+\boldsymbol{\alpha}^2+2\mathbf{q}\boldsymbol{\alpha}}\right)-\mathbf{l}\boldsymbol{\alpha}\right)\left(1+\frac{\lambda_0^2}{\sigma_0}f'\left(\frac{2l^2+2l_1^2\sin^2\phi}{\sigma_0}\right)\right)=0 \qquad (5.46)$$

The relation (5.46) may be transformed to the equation

$$\left(1-\frac{\mathbf{l}^2}{l_0^2}\right)\left(\boldsymbol{\alpha}_\parallel+\frac{\mathbf{q}_\parallel-2\mathbf{l}}{1-\frac{\mathbf{l}^2}{l_0^2}}\right)^2+(\boldsymbol{\alpha}_\perp+\mathbf{q}_\perp)^2=\frac{(\mathbf{q}_\parallel-2\mathbf{l})^2}{\left(1-\frac{\mathbf{l}^2}{l_0^2}\right)}+\mathbf{q}_\perp^2 \qquad (5.47)$$

where

$$\boldsymbol{\alpha}=\boldsymbol{\alpha}_\parallel+\boldsymbol{\alpha}_\perp,\qquad \boldsymbol{\alpha}_\parallel=\frac{\mathbf{l}\,(\mathbf{l}\boldsymbol{\alpha})}{\mathbf{l}^2},\qquad \mathbf{q}_\parallel=\frac{\mathbf{l}\,(\mathbf{l}\mathbf{q})}{\mathbf{l}^2},\qquad \mathbf{q}_\perp=\mathbf{q}-\mathbf{q}_\parallel \qquad (5.48)$$

As far as $\mathbf{l}^2>l_0^2$, we obtain, that $1-\mathbf{l}^2/l_0^2<0$, and the surface (5.47) is a hyperboloid in the 3-space of quantities $\alpha_1,\alpha_2,\alpha_3$. It means that solutions of the equations (5.44), (5.45) may deflect arbitrarily far from the helix solution (5.16). This deflection is a manifestation of the multivariance of the space-time geometry.

6 Stabilization of the spacelike world chain

Suppression of multivariance and stabilization of the world chain, consisting of spacelike vectors, can be achieved, if we consider the world chain with composed links, whose skeleton consists of three points $\{P_k,P_{k+1},Q_{k+1}\}$, $k=...1.2,...$ (see figure 1). Let $\mathbf{P}_k\mathbf{P}_{k+1}$ be a spacelike vector, whereas the vector $\mathbf{P}_k\mathbf{Q}_{k+1}$ be a timelike vector in $\mathcal{G}_\mathrm{M}$. To investigate the effect of stabilization, it is sufficient to consider the points P_0,P_1,P_2,Q_1,Q_2, having coordinates

$$\begin{aligned} P_0 &= \{0\}, \qquad P_1=\{l\}, \qquad P_2=\{l+q+\alpha\}, \\ Q_1 &= \{s\}, \qquad Q_2=\{s+\rho+l+\beta\} \end{aligned} \qquad (6.1)$$

The following vectors are associated with these points of the skeletons

$$\begin{aligned} \mathbf{P}_0\mathbf{P}_1 &= l, \qquad \mathbf{P}_1\mathbf{P}_2=q+\alpha, \qquad \mathbf{P}_0\mathbf{P}_2=l+q+\alpha, & (6.2)\\ \mathbf{P}_0\mathbf{Q}_1 &= s, \qquad \mathbf{P}_1\mathbf{Q}_2=s+\rho+\beta, \qquad \mathbf{P}_0\mathbf{Q}_2=s+\rho+l+\beta, & (6.3)\\ \mathbf{P}_1\mathbf{Q}_1 &= s-l, \qquad \mathbf{P}_2\mathbf{Q}_2=s+\rho-q+\gamma, \qquad \mathbf{Q}_1\mathbf{Q}_2=\rho+l+\beta, & (6.4)\\ \mathbf{Q}_1\mathbf{P}_2 &= l+q-s+\alpha, \qquad \gamma=\beta-\alpha & \end{aligned}$$

where the quantities l,q,s,ρ are the given 4-vectors, whereas the quantities $\alpha,\beta,\gamma=\beta-\alpha$ are 4-vectors, which are to be determined from the condition

$$\{P_0,P_1,Q_1\}\;\mathrm{eqv}\;\{P_1,P_2,Q_2\} \qquad (6.5)$$

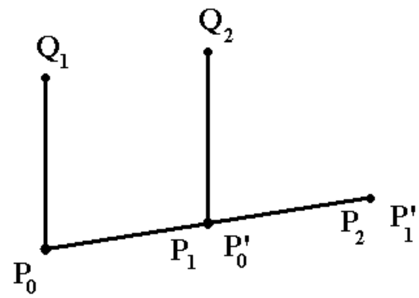

Fig. 1.

Expressions for 4-vectors q and ρ are chosen in such a way, that vectors $\mathbf{P}_1\mathbf{P}_2$ and $\mathbf{P}_1\mathbf{Q}_2$ (at $\alpha = \beta = 0$) were a result of rotation of vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_0\mathbf{Q}_1$ in the plane x^1x^2 by the angle 2ϕ. The quantities

$$s = \{s_0, s_\perp \cos\phi, s_\perp \sin\phi, s_3\} \qquad q = \{l_0, l_1 \cos(2\phi), l_1 \sin(2\phi), 0\} \tag{6.6}$$

$$\rho = \{0, -2s_\perp \sin\phi \sin(2\phi), 2s_\perp \sin\phi \cos(2\phi), 0\}, \qquad l = \{l_0, l_1, 0, 0\} \tag{6.7}$$

are supposed to be given. The 4-vectors

$$\alpha = \{\alpha_0, \alpha_1, \alpha_2, \alpha_3\} = \{\alpha_0, \boldsymbol{\alpha}\}, \qquad \beta = \{\beta_0, \beta_1, \beta_2, \beta_3\} = \{\beta_0, \boldsymbol{\beta}\} \tag{6.8}$$

are to be determined from the relations (6.5).

The 4-vectors l and q coincide with vectors (5.16). We are interested in the following question, whether the stabilizing vector $\mathbf{P}_0\mathbf{Q}_1 = s$ can be chosen in such a way, that equations (6.5) have the unique solution $\alpha = \beta = 0$. If such a stabilizing vector exists, the world chain will have a shape of a helix without wobbling. It may be, that the complete stabilization is impossible. Then, maybe, a partial stabilization is possible, when the quantities α, β are small, although they do not vanish. In any case the problem of the stabilizing vector existence is a pure mathematical problem.

Solving this problem, we shall use relations (5.22), (5.23) to reduce all geometrical relations to the geometrical relations in the space-time of Minkowski. Working in the space-time of Minkowski, we shall use the conventional covariant formalism, where the expressions of the type α^2 and $(\alpha.\beta)$ mean

$$\alpha^2 \equiv \alpha_0^2 - \boldsymbol{\alpha}^2 \equiv \alpha_0^2 - \alpha_1^2 - \alpha_2^2 - \alpha_3^2, \tag{6.9}$$

$$(\alpha.\beta) \equiv \alpha_0\beta_0 - \boldsymbol{\alpha}\boldsymbol{\beta} \equiv \alpha_0\beta_0 - \alpha_1\beta_1 - \alpha_2\beta_2 - \alpha_3\beta_3 \tag{6.10}$$

Index "M" will be omitted for brevity.

It follows from the condition $\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{P}_1\mathbf{P}_2$

$$\begin{aligned} l^2 &= (q+\alpha)^2 && (6.11) \\ (l.q+\alpha)+w\left(P_0,P_1,P_1,P_2\right) &= l^2+2d\left(\frac{s^2}{2}\right) && (6.12) \end{aligned}$$

where

$$\begin{aligned} & w\left(P_0,P_1,P_1,P_2\right) \\ = \; & d\left(\sigma_{\mathrm{M}}\left(P_0,P_2\right)\right)+d\left(\sigma_{\mathrm{M}}\left(P_1,P_1\right)\right)-d\left(\sigma_{\mathrm{M}}\left(P_0,P_1\right)\right)-d\left(\sigma_{\mathrm{M}}\left(P_1,P_2\right)\right) \\ = \; & \lambda_0^2 f\left(\frac{(l+q+\alpha)^2}{2\sigma_0}\right)-2\lambda_0^2 f\left(\frac{l^2}{2\sigma_0}\right) \qquad (6.13) \end{aligned}$$

After transformations we obtain

$$\alpha^2+2\,(q.\alpha)=0 \tag{6.14}$$

$$2l_1^2\sin^2\phi+(l.\alpha)+\lambda_0^2 f\left(\frac{2l^2+2l_1^2\sin^2\phi+(l.\alpha)}{\sigma_0}\right)-4\lambda_0^2 f\left(\frac{l^2}{2\sigma_0}\right)=0 \tag{6.15}$$

These equations coincide with (5.42), (5.43). If $\alpha=0$ the equations (6.14), (6.15) coincide with (5.29), (5.35) respectively.

We obtain from the condition $\mathbf{P}_0\mathbf{Q}_1\mathrm{eqv}\mathbf{P}_1\mathbf{Q}_2$

$$s^2=(s+\rho+\beta)^2 \tag{6.16}$$

$$(s+\rho+\beta.s)+w\left(P_0,Q_1,P_1,Q_2\right)=s^2+2d\left(\frac{s^2}{2}\right) \tag{6.17}$$

where

$$\begin{aligned} & w\left(P_0,Q_1,P_1,Q_2\right) \\ = \; & d\left(\sigma_{\mathrm{M}}\left(P_0,Q_2\right)\right)+d\left(\sigma_{\mathrm{M}}\left(Q_1,P_1\right)\right)-d\left(\sigma_{\mathrm{M}}\left(P_0,P_1\right)\right)-d\left(\sigma_{\mathrm{M}}\left(Q_1,Q_2\right)\right) \\ = \; & d\left(\frac{(s+\rho+l+\beta)^2}{2}\right)+d\left(\frac{(s-l)^2}{2}\right)-d\left(\frac{l^2}{2}\right)-d\left(\frac{(\rho+l+\beta)^2}{2}\right) \end{aligned}$$

The equations (6.16) and (6.17) are transformed to the form

$$\rho^2+2\,(\rho.s)+2\,(s+\rho.\beta)+\beta^2=0 \tag{6.18}$$

$$\begin{aligned} (\rho+\beta.s)+d\left(\frac{(s+\rho+l+\beta)^2}{2}\right)+d\left(\frac{(s-l)^2}{2}\right)-d\left(\frac{l^2}{2}\right) & \\ -d\left(\frac{(\rho+l+\beta)^2}{2}\right)-2d\left(\frac{s^2}{2}\right)=0 & \qquad (6.19) \end{aligned}$$

Let us suppose that the stabilizing vector s is long in the sense that

$$s^2 \gg \sigma_0 \tag{6.20}$$

Then in (6.19) the functions d, which contains s in its argument will be equal to λ_0^2 and all terms, containing s compensate each other. The necessary condition of the fact, that $\beta = 0$ is a solution of equations (6.18), (6.19), has the form

$$\rho^2 + 2\,(\rho.s) = 0 \tag{6.21}$$

$$(\rho.s) - d\left(\frac{l^2}{2}\right) - d\left(\frac{(\rho + l)^2}{2}\right) = 0 \tag{6.22}$$

The equation (6.21) is satisfied identically by the choice (6.6), (6.7) of vectors s and ρ.

We obtain from the condition $\mathbf{P}_1\mathbf{Q}_1\mathrm{eqv}\mathbf{P}_2\mathbf{Q}_2$

$$(s - l)^2 = (s + \rho - q + \gamma)^2 \tag{6.23}$$

$$(s - l.s + \rho - q + \gamma) + w\,(P_1, Q_1, P_2, Q_2) = (s - l)^2 + 2d\left(\frac{(s - l)^2}{2}\right) \tag{6.24}$$

$$w\,(P_1, Q_1, P_2, Q_2) = d\,(\sigma_{\mathrm{M}}\,(P_1, Q_2)) + d\,(\sigma_{\mathrm{M}}\,(Q_1, P_2)) - d\,(\sigma_{\mathrm{M}}\,(P_1, P_2)) - d\,(\sigma_{\mathrm{M}}\,(Q_1, Q_2))$$
$$= d\left(\frac{(s + \rho + \beta)^2}{2}\right) + d\left(\frac{(l + q - s + \alpha)^2}{2}\right) - d\left(\frac{l^2}{2}\right) - d\left(\frac{(\rho + l + \beta)^2}{2}\right) \tag{6.25}$$

The necessary conditions of the fact, that $\gamma = \beta - \alpha = 0$ is a solution of equations (6.23), (6.24), have the form

$$(s - l)^2 = (s + \rho - q)^2 \tag{6.26}$$

$$(s - l.s + \rho - q) - d\left(\frac{l^2}{2}\right) - d\left(\frac{(\rho + l)^2}{2}\right) = 0 \tag{6.27}$$

The equation (6.26) is satisfied identically by the relations (6.6), (6.7). The difference of equations (6.22) and (6.27) leads to the equation

$$(\rho.s) = (s - l.s + \rho - q) \tag{6.28}$$

Let us substitute expressions for ρ, s, l, q , determined by the relations (6.6), (6.7), in (6.28). After transformations we obtain the connection between the quantities $s_\perp, l_1$ and ϕ in the form

$$s_\perp = l_1 \frac{1 - 2\sin^2\phi}{\left(1 - 4\sin^2\phi\right)\cos\phi} \tag{6.29}$$

The equation (6.22) by means of (6.29) is reduced to the form

$$\frac{2l_1^2\left(1-2\sin^2\phi\right)^2}{\left(1-3\sin^2\phi\right)^2\left(1-\sin^2\phi\right)}\sin^2\phi-\lambda_0^2 f\left(\frac{l^2}{2\sigma_0}\right)$$
$$-\lambda_0^2 f\left(\frac{l^2+4l_1^2\frac{\left(1-2\sin^2\phi\right)}{\left(1-3\sin^2\phi\right)}\sin^2\phi-16l_1^2\frac{\left(1-2\sin^2\phi\right)^2}{\left(1-3\sin^2\phi\right)^2\left(1-\sin^2\phi\right)}\sin^2\phi}{2\sigma_0}\right)=0 \qquad (6.30)$$

where according to (5.21) the function $d(x)$ is substituted by $\lambda_0^2 f(x/\sigma_0)$.

Setting

$$y=\sin^2\phi \qquad (6.31)$$

and using designations (5.33), (5.34), we transform the equation (6.30) to the form

$$\varkappa\frac{a(1-2y)^2}{(1-3y)^2(1-y)}+f(\nu)+f\left(\nu+2a\frac{(1-2y)(3+y)}{(1-3y)(1-y)}\right)=0 \qquad (6.32)$$

The equations (5.35) and (6.32) form a system of two necessary conditions, imposed on parmeters of the helical world chain. Each link of the chain consists of two vectors: leading vector $\mathbf{P}_s\mathbf{P}_{s+1}$ and stabilizing vector $\mathbf{P}_s\mathbf{Q}_s$. Parameter $\varkappa=\sigma_0/\lambda_0^2$ is determined by the space-time geometry. The quantity $\nu=-2l^2/\sigma_0$ describes the length of the spacelike leading vector $\mathbf{P}_s\mathbf{P}_{s+1}$. Parameter $a/y=2l_1^2/\sigma_0$ describes the length of the projection of the leading vector $\mathbf{P}_s\mathbf{P}_{s+1}$ on the plane of rotation. Finally, $y=\sin^2\phi$ describes the angle 2ϕ of rotation of the leading vector in the plane of rotation.

Numerical solutions of equations (5.35) and (6.32) are presented for the parameter $\varkappa=1$

ν	a	y	$s_\perp^2/\sigma_0$
0.1	4.1915×10^{-2}	0.39241	0.12957
0.15	0.10661	0.44436	2.4098×10^{-2}
0.2	0.19236	0.46267	1.4324×10^{-2}
0.3	0.40137	0.47461	1.1553×10^{-2}
0.4	0.63701	0.47889	1.1931×10^{-2}
0.5	0.5	0.46809	2.5023×10^{-2}
		0.39899	1.0967
0.6	0.136	0.40667	0.20286
0.615	6.9567×10^{-2}	0.37528	0.58294

7 Estimation of wobbling of leading vector

Solutions of equations, which describe the necessary conditions of the fact, that the world chain may be a helix, are not unique. There may be solutions of (6.5), described by nonvanishing α and β, which generate wobbling and violate the helical character of world chain. We write six equation (6.5) as equation for α, β with parameters l, q, s, ρ, satisfying the necessary conditions (5.35) and (6.32). We obtain instead of equations (6.14), (6.15) the following two equations

$$\alpha^2 + 2\left(q.\alpha\right) = 0 \tag{7.1}$$

$$\left(l.\alpha\right)\left(1 + \frac{\lambda_0^2}{\sigma_0} f'\left(\frac{2l^2 + 2l_1^2 \sin^2\phi}{\sigma_0}\right)\right) = 0 \tag{7.2}$$

where the quantities l, q satisfy the necessary conditions (6.32) (5.35), and f' is a derivative of the function (5.36), which is always nonnegative. Then it follows from (7.2)

$$\left(l.\alpha\right) = l_0\alpha_0 - l_1\alpha_1 = 0 \tag{7.3}$$

Equations (7.1), (7.3) contain only the variable α (but not β) and coincide with the equations (5.42), (5.43). However, there are additional constraints, containing α. As a result the constraints on α distinguish from the relation (5.47), describing values of α without the stabilizing vector $\mathbf{P}_s\mathbf{Q}_s$.

In the developed form the relations (6.18), (6.19) have the form

$$\beta_0^2 - \boldsymbol{\beta}^2 + 2\left(s_0\beta_0 - \beta_1 s_\perp \cos\phi\left(1 - 4\sin^2\phi\right) - \beta_2 s_\perp \sin\phi\left(1 + 2\cos\left(2\phi\right)\right)\right) = 0 \tag{7.4}$$

$$\left(\beta_0 s_0 - \boldsymbol{\beta}\mathbf{s}\right)\left(1 + \frac{\lambda_0^2}{\sigma_0} f'\left(\frac{\left(\rho + l\right)^2}{2\sigma_0}\right)\right) - \frac{\lambda_0^2}{\sigma_0} f'\left(\frac{\left(\rho + l\right)^2}{2\sigma_0}\right)\left(l.\beta\right) = 0 \tag{7.5}$$

They contain only the variable β (but not α)

Finally the relations (6.23), (6.24) in the developed form can be written as follows

$$\begin{gathered}\gamma_0^2 - \boldsymbol{\gamma}^2 + 2\left(s_0 - l_0\right)\gamma_0 - \gamma_1\left(s_\perp \cos\phi\left(1 - 4\sin^2\phi\right) - l_1\cos\left(2\phi\right)\right) \\ -2\gamma_2\left(s_\perp \sin\phi\left(1 + 2\cos\left(2\phi\right)\right) + l_1 \sin\left(2\phi\right)\right) = 0\end{gathered} \tag{7.6}$$

$$l_0\beta_0 - l_1\beta_1 + s_0\alpha_0 - \mathbf{s}\boldsymbol{\alpha} = 0 \tag{7.7}$$

The relation (7.7) is a linear combination of equations (6.19) and (6.24), which does not contain the function f. Relations (7.6) and (7.7) contain both quantities α, β and $\gamma = \beta - \alpha$. The constraints (7.6) and (7.7) modify the constraints (5.47), transforming the hyperboloid into ellipsoid.

We suppose for simplicity, that the vector $\mathbf{P}_s\mathbf{Q}_s$ is very long ($s_0 \gg \sigma_0$). We suppose, that $s_0 \to \infty$. In this case we obtain from the relation (7.5), that $\beta_0 = 0$. It follows from (7.7), that $\alpha_0 = 0$. Besides, it follows from (7.3), that $\alpha_1 = 0$. Thus, solutions of the equations (7.5), (7.7) and (7.3) have the form

$$\beta_0 = \alpha_0 = 0, \qquad \alpha_1 = 0 \tag{7.8}$$

At the constraints (7.8) three other equations (7.1), (7.4) and (7.6) take the form

$$\alpha_2^2 + \alpha_3^2 + 2l_1 \sin\left(2\phi\right)\alpha_2 = 0 \tag{7.9}$$

$$\beta_1^2+\beta_2^2+\beta_3^2+2\beta_1 s_\perp \cos\phi\left(1-4\sin^2\phi\right)-2\beta_2 s_\perp \sin\phi\left(1+2\cos(2\phi)\right)=0 \quad (7.10)$$

$$\begin{aligned}&\gamma_1^2+\gamma_2^2+\gamma_3^2+2\gamma_1\left(s_\perp\cos\phi\left(1-4\sin^2\phi\right)-l_1\cos(2\phi)\right)\\&+2\gamma_2\left(s_\perp\sin\phi\left(1+2\cos(2\phi)\right)-l_1\sin(2\phi)\right)=0\end{aligned} \quad (7.11)$$

Solution of equation (7.9) has the form

$$\alpha_1=0,\qquad \alpha_2=-l_1\sin(2\phi)\left(1-\cos\eta\right),\qquad \alpha_3=l_1\sin(2\phi)\sin\eta \quad (7.12)$$

where η is an arbitrary angle.

Solution of equation (7.10) has the form

$$\begin{aligned}\beta_1 &= -s_\perp\cos\phi\left(1-4\sin^2\phi\right)+s_\perp\cos\xi_1\cos\xi_2 && (7.13)\\ \beta_2 &= -s_\perp\sin\phi\left(1+2\cos(2\phi)\right)+s_\perp\cos\xi_1\sin\xi_2 && (7.14)\\ \beta_3 &= s_\perp\sin\xi_1 && (7.15)\end{aligned}$$

where the quantities ξ_1, ξ_2 are arbitrary. and the quantity $s_\perp$ is determined by the relation (6.29).

Substituting (7.12) - (7.15) in (7.11), one obtains a constraint on the quantities η, ξ_1, ξ_2. Independently of this constraint the 3-vector

$$\mathbf{q}+\boldsymbol{\alpha}=\left\{l_1\cos(2\phi),l_1\sin(2\phi)\cos\eta,l_1\sin(2\phi)\sin\eta\right\} \quad (7.16)$$

has the same 3-length l_1, as the length of 3-vector $\mathbf{l}=\{l_1,0,0\}$. The angle between the 3-vectors $\mathbf{q}+\boldsymbol{\alpha}$ and $\mathbf{l}$ is equal to 2ϕ. If $\eta=0$, then $\alpha=0$, and vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$ are elements of the same helix.

We see that the stabilizing vector $\mathbf{P}_s\mathbf{Q}_s$ reduces wobbling of vector $\mathbf{P}_s\mathbf{P}_{s+1}$. In the case of equation (5.47) the spatial component $\boldsymbol{\alpha}$ of the 4-vector α may be infinite. In the case of the equation (7.9) the length $|\boldsymbol{\alpha}|$ of the spatial component $\boldsymbol{\alpha}$ of the 4-vector α is less, than $|l_1\sin(2\phi)|$. Thus, the stabilizing vector $\mathbf{P}_s\mathbf{Q}_s$ reduces the wobbling of the world chain. One cannot be sure, that this wobbling does not destroy the helical character of the world chain. However, The main question is, whether or not the evolution of the world chain in the spacelike direction lead to the world chain, which is timelike on the average.

Any next point P_l of the world chain jumps along the timelike direction at the distance l_0 and in the 3-space, which is orthogonal to this direction, the point jumps at the distance $l_1>l_0$. Direction of the jump in the 3-space is described by the vector $\mathbf{q}+\boldsymbol{\alpha}$, which is given by the relation (7.16). The length of $\mathbf{q}+\boldsymbol{\alpha}$ is l_1. If the direction of jump is completely random, the displacement L_n for n steps ($n\gg1$) is proportional to $\sqrt{n}l_1$, whereas displacement in the temporal direction is nl_0. It means that the mean velocity

$$\langle v\rangle=\frac{\sqrt{n}l_1}{nl_0}=\frac{l_1}{\sqrt{n}l_0}<1,\qquad n\gg1$$

tends to zero for $n\to\infty$, although $l_0<l_1$. In the case, if $\alpha=0$ and the 3-vector $\mathbf{q}+\boldsymbol{\alpha}$ determined by (7.16) is not random, the world chain form a helix with timelike

axis. In this case the mean velocity tends to zero also. It should expect that in the case, when the vector (7.16) is stochastic, but its stochasticity is restricted by the relation (7.16) (the angle η is completely random), the mean world chain will be also timelike on the average. We cannot prove this fact strictly now, but this result seems to be very probable.

8 Discussion

The obtained classical helical world chain (5.13) associates with the classical Dirac particle, which has alike world line (4.9), (4.10). The direction of the mean momentum distinguishes from the direction of the 4-velocity. This fact is characteristic for both particles (the Dirac particle, and the particle, described by the world chain (5.13)). Both particles have angular moment. For the Dirac particle the mass m, which enters in the Dirac equation, distinguishes from the mass M of the particle moving along the world line (4.9), (4.10) [6]. As to the mass of the particle, described by the world chain (5.13), it is not yet determined. For determination of the mass, one needs to consider the world chain (5.13) of charged particle with the skeleton $\{P_k, P_{k+1}, Q_{k+1}\}$ in the distorted space-time of Klein-Kaluza, containing electromagnetic field.

Existence of helical world chain with timelike axis seems to be rather unexpected, because leading vectors $\mathbf{P}_k\mathbf{P}_{k+1}$ of the chain are spacelike, and it corresponds to superluminal motion of a particle. Superluminal motion seems to be incompatible with the relativity principle, which admits only motion with the speed less, than the speed of the light. However, this constraint is valid only for continuous space-time geometry, which admits unlimited divisibility. In a discrete geometry there are no distances less, than some elementary length, and it is difficult to formulate the relativity principle statement on impossibility of superluminal motion. One needs another more adequate formulation of the relativity principle.

Is the space-time geometry (5.19) discrete? At $\sigma_0 \to 0$ the space-time geometry (5.19) turns to the space-time geometry

$$\sigma_{\mathrm{d}} = \sigma_{\mathrm{M}} + \lambda_0^2 \mathrm{sgn}\left(\sigma_{\mathrm{M}}\right) \tag{8.1}$$

which is certainly discrete, because in the space-time there no timelike intervals $|\mathbf{PQ}|$, which are less, than λ_0, and there are no spacelike intervals $\sqrt{-\left|\mathbf{PQ}\right|^2}$, which are less, than λ_0. In such a space-time geometry there are no particles, whose geometrical mass μ is less than λ_0.

However, if $\sigma_0 > 0$, is the space-time geometry discrete? To answer this question, we introduce the parameter of discreteness: the relative density of points in the space-time with respect to the point density in the space-time of Minkowski. Let us define the quantity $\rho\left(\sigma_{\mathrm{d}}\right)$ by means of the relation

$$\rho\left(\sigma_{\mathrm{d}}\right) = \frac{d\sigma_{\mathrm{M}}\left(\sigma_{\mathrm{d}}\right)}{d\sigma_{\mathrm{d}}} \tag{8.2}$$

In the case (5.19) we have for $\sigma_{\mathrm{M}} \in [-\sigma_0, \sigma_0]$

$$\sigma_{\mathrm{M}} + \lambda_0^2 \left(\frac{\sigma_{\mathrm{M}}}{\sigma_0}\right)^3 - \sigma_{\mathrm{d}} = 0 \tag{8.3}$$

Resolving (8.3) with respect to σ_{M}, we obtain

$$\sigma_{\mathrm{M}} = \sigma_0 g^{1/3} (\sigma_{\mathrm{d}}) - \frac{\sigma_0^2}{3\lambda_0^2} g^{-1/3} (\sigma_{\mathrm{d}}) \tag{8.4}$$

where

$$g (\sigma_{\mathrm{d}}) = \sqrt{\frac{\sigma_0^3}{27\lambda_0^6} + \frac{\sigma_d^2}{4\lambda_0^4}} + \frac{\sigma_d}{2\lambda_0^2} \tag{8.5}$$

Taking into account (8.4) we obtain the world function σ_{M} as a function of σ_{d}

$$\sigma_{\mathrm{M}} = \begin{cases} \sigma_{\mathrm{d}} - \lambda_0^2 \mathrm{sgn} (\sigma_{\mathrm{d}}) & \text{if} \quad |\sigma_{\mathrm{d}}| > \sigma_0 + \lambda_0^2 \\ \sigma_0 g^{1/3} (\sigma_{\mathrm{d}}) - \frac{\sigma_0^2}{3\lambda_0^2} g^{-1/3} (\sigma_{\mathrm{d}}) & \text{if} \quad |\sigma_{\mathrm{d}}| \leq \sigma_0 + \lambda_0^2 \end{cases} \tag{8.6}$$

The relative density of points in the space-time geometry $\mathcal{G}_{\mathrm{d}}$ with respect to the standard geometry $\mathcal{G}_{\mathrm{M}}$ of Minkowski is given by the relation (8.2). The expression for $\rho (\sigma_{\mathrm{d}})$ is given by the relation

$$\rho (\sigma_{\mathrm{d}}) = \begin{cases} 1 & \text{if} \quad |\sigma_{\mathrm{d}}| > \sigma_0 + \lambda_0^2 \\ g' (\sigma_{\mathrm{d}}) \left(\frac{\sigma_0}{3} g^{-2/3} (\sigma_{\mathrm{d}}) + \frac{\sigma_0^2}{9\lambda_0^2} g^{-4/3} (\sigma_{\mathrm{d}})\right) & \text{if} \quad |\sigma_{\mathrm{d}}| \leq \sigma_0 + \lambda_0^2 \end{cases} \tag{8.7}$$

where $g' (\sigma_{\mathrm{d}})$ is given by the relation

$$g' (\sigma_{\mathrm{d}}) = \frac{\sigma_d}{4\lambda_0^4 \sqrt{\frac{1}{27} \frac{\sigma_0^3}{\lambda_0^6} + \frac{1}{4\lambda_0^4} \sigma_d^2}} + \frac{1}{2\lambda_0^2} \tag{8.8}$$

If $\sigma_0 \to 0$ and $\sigma_0 \ll \lambda_0^2$, we have approximately

$$g (\sigma_{\mathrm{d}}) = \frac{\sigma_d}{\lambda_0^2}, \qquad g' (\sigma_{\mathrm{d}}) = \frac{1}{\lambda_0^2} \tag{8.9}$$

In the limit $\sigma_0 \to 0$, when the world function (5.19) turns into the world function (8.1) of the completely discrete geometry, we obtain for the relative density

$$\lim_{\sigma_0 \to 0} \rho (\sigma_{\mathrm{d}}) = \begin{cases} 1 & \text{if} \quad |\sigma_{\mathrm{d}}| > \lambda_0^2 \\ 0 & \text{if} \quad |\sigma_{\mathrm{d}}| \leq \lambda_0^2 \end{cases} \tag{8.10}$$

Thus, $\rho (\sigma_{\mathrm{d}}) = 0$ for $\sigma_{\mathrm{d}} \in \left(-\lambda_0^2, \lambda_0^2\right)$, and this fact correspond to the space-time geometry (8.1), where close points, for which $|\sigma_{\mathrm{d}}| \leq \lambda_0^2$, are absent. The relative density $\rho (\sigma_{\mathrm{d}})$ of points may serve as quantity, describing the discreteness of the space-time geometry and the character of this discreteness. The discreteness may be complete, when the density $\rho(\sigma_{\mathrm{d}})$ vanishes in some region as in the case (8.10). But the discreteness may be incomplete, as in the case (8.7). In this case for $\sigma_0 = \lambda_0^2$ we have

$$\rho(\sigma_{\mathrm{d}}) = \frac{1}{6\sqrt{\frac{4}{27}\lambda_0^4 + \sigma_d^2}} \left(\sqrt[3]{g_1(\sigma_{\mathrm{d}})} + \frac{1}{3}\frac{1}{\sqrt[3]{g_1(\sigma_{\mathrm{d}})}} \right), \qquad \sigma_{\mathrm{d}} \in \left(-2\lambda_0^2, 2\lambda_0^2\right) \quad (8.11)$$

where

$$g_1(\sigma_{\mathrm{d}}) = \frac{1}{2\lambda_0^2}\left(\sigma_d + \sqrt{\frac{4}{27}\lambda_0^4 + \sigma_d^2} \right) \quad (8.12)$$

The expression (8.11) is a symmetric function of σ_d, as one can see from (8.3). It is symmetric, indeed, although it does not look formally as a symmetric function of σ_{d}. Numerical values of $\rho(\sigma_{\mathrm{d}})$, $\sigma_{\mathrm{d}} \in \left(-2\lambda_0^2, 2\lambda_0^2\right)$ are presented in the table

σ_d	ρ	σ_d	ρ	σ_d	ρ
0	0.5	$1.0\lambda_0^2$	0.208 62	$1.8\lambda_0^2$	0.135 28
$0.2\lambda_0^2$	0.449 82	$1.2\lambda_0^2$	0.182 85	$2.0\lambda_0^2$	0.125
$0.4\lambda_0^2$	0.362 72	$1.4\lambda_0^2$	0.163 20	$-0.2\lambda_0^2$	0.449 82
$0.8\lambda_0^2$	0.243 63	$1.6\lambda_0^2$	0.147 74	$-0.4\lambda_0^2$	0.362 72

The relative density $\rho(\sigma_{\mathrm{d}})$ is less, than unity. It may be interpreted in the sense, that the space-time geometry is discrete only partly. Nevertheless the incompletely discrete space-time geometry discriminates most of world chains with spatial leading vector, remaining only some of them.

Multivariance of particle motion and discrimination of some states of motion play the crucial role in structure of elementary particles, as well as in the structure of atoms. Let us explain this circumstance in the example of the hydrogen atom. According to laws of the classical mechanics the electron of the hydrogen atom is to fall onto the nucleus due to the Coulomb attraction. Two reasons prevent from this falling: (1) multivariant (stochastic) motion of the electron, and (1) rotation of electron around the nucleus.

The multivariant motion of the electron leads to escape of the electron from the nuclear surface. This process has the same nature, as an escape of dust from the Earth surface. Moving multivariantly (as Brownian particles), the flecks of dust form a stationary distribution in the gravitational field of the Earth. If multivariance of their motion is cut out, the flecks of the dust fall onto the surface of the Earth. Statistical description of the electron distribution and the dust distribution are different, because the multivariant electron motion is conceptually relativistic, whereas the Brownian particles motion is nonrelativistic. One may describe Brownian particles by means of probabilistic statistical description, whereas one may use only dynamical conception of statistical description for statistical description of multivariant motion of relativistic particles.

Rotation of the electron around the nucleus creates the field of centrifugal force, which is added to the Coulomb force. As a result additional distributions of the electrons appear. If the obtained distribution of electrons is nonstationary, the electrons emanate the electromagnetic radiation until the electron distribution becomes to be stationary. Thus, the electromagnetic radiation carries out discrimination of non-stationary states (electron distributions). The multivariance of the electron motion and mechanism of discrimination of non-stationary states generates the structure of the hydrogen atom and discrete character of the radiation spectra. From the mathematical viewpoint the discrete character of the electron states is conditioned by procedure of the eigenstates determination. Only eigenstates of the Hamilton operator appear to be stationary and stable.

The multivariance of the particle motion and some mechanism of discrimination play also the crucial role in the understanding of the structure of elementary particles. However, in the case of the elementary particle structure the discrimination mechanism is conditioned by some metric (geometric) forces, which appear, when we use space-time geometry of Minkowski instead of the real multivariant space-time geometry. Formally these forces have the form of additional terms of the type (5.27) in dynamic equations. These additional terms are expressed via the space-time distortion d. They describe both multivariance of motion and the discrimination mechanism. The multivariance of motion is associated with the multivariance of the vector equivalence definition (2.7), whereas the discrimination mechanism is associated with the zero-variance of the same definition (2.7) for some vectors. Besides, as we have seen, the zero-variance (discrimination) is associated with the discreteness (or partial discreteness) of the space-time geometry.

It is very important, that consideration of multivariant space-time geometry *is not a hypothesis*, which needs an experimental test. Consideration of the multivariant space-time geometry is a corollary of correction of our imperfect conception of geometry. Conception of geometry, based on supposition that any space-time geometry may be axiomatized (i.e. may be concluded from some system of axioms), is imperfect, because it does not admit one to construct multivariant geometry conceptually. However, the motion of electrons and other elementary particles is multivariant. Multivariance of this motion is an experimental fact, which cannot be ignored. As far as the imperfect conception of geometry did not admit one to construct multivariant space-time geometry, investigators were forced to ascribe multivariance to dynamics, introducing quantum principles with all their attributes.

The quantum principles look enigmatic and artificial, because multivariance is ascribed to dynamics, whereas it should be ascribed to the space-time geometry. Multivariance and zero-variance as properties of the space-time geometry look as quite natural properties of the definition (2.7). Indeed, it does not follow from anywhere, that equations (2.7) are to have unique solution for arbitrary world function, which determines the form of these equations. Absence of any hypotheses is a very important property of the geometrical approach to the structure of elementary particles. Besides, the geometrical dynamics is very general and simple. Dynamic equations of the geometric dynamics do not use even differential equations. Formulation of dynamic equations does not contain a reference to the coordinate system. On the other hand, when the geometric dynamics in the real space-time is described in terms of the space-time of Minkowski, one obtains additional metric forces, which look rather exotic. They can be obtained hardly in the framework of the conventional approach.

The conventional approach to the theory of elementary particles contains a lot of secondary concepts and properties. One may not see any discrimination mechanism in wave functions, field equations, branes, symmetries and other remote corollaries of the unknown structure of elementary particles. But it is impossible to obtain and to understand the discrete properties of elementary particles without a reliable mechanism of discrimination.

Even if investigating and systematizing these remote corollaries, one succeeds to obtain a perfect systematization of elementary particles, one can obtain structure of elementary particles from the perfect systematization with the same success, as one can obtain the atomic structure from the periodical system of chemical elements.

9 Concluding remarks

Consideration of T-geometry as a space-time geometry admits one to obtain dynamics of a particle as corollary of its geometrical structure. Evolution of the geometrical object in the space-time is determined by the skeleton $\{P_0, P_1, ..P_n\}$ of the geometrical object and by fixing of the leading vector $\mathbf{P}_0\mathbf{P}_1$. The skeleton and the leading vector determine the world chain, which describes the evolution completely. The world chain may wobble, it is manifestation of the space-time geometry multivariance. Quantum effects are only one of manifestation of the multivariance. It is remarkable, that for determination of the world chain one does not need differential equations, which may be used only on the space-time manifold. One does not need space-time continuity (continual geometry). Of course, one may introduce the continual coordinate system and write dynamic differential equation there. One may, but it is not necessary. In general, the geometrical dynamics (i.e. dynamics generated by the space-time geometry) is a discrete dynamics, where step of evolution is determined by the length of the leading vector. It is possible, that one will need a development of special mathematical technique for the geometrical dynamics.

The real space-time geometry contains the quantum constant $\hbar$ as a parameter. As a result the geometric dynamics explains freely quantum effects, but not only them. The particle mass is geometrized (the particle mass is simply a length of some vector). As a result the problem of mass of elementary particles is simply a geometrical problem. It is a problem of the structure of elementary geometrical object and its evolution. One needs simply to investigate different forms of skeletons of simplest geometrical objects. In general, not all skeletons are possible, because at the spatial evolution the world chain is observable (helical) only for several skeletons. Additional points of skeleton lead to additional (sometimes unexpected) properties of corresponding elementary geometrical objects (elementary particles).

Note that the geometric dynamics does not contain a rotational motion. It contains only a shift. All vectors of the skeleton $\left\{P_0^{(s)}, P_1^{(s)}, ..P_n^{(s)}\right\}$ of the link L_s

are equivalent to vectors of the skeleton $\left\{P_0^{(s+1)}, P_1^{(s+1)}, ... \; P_n^{(s+1)}\right\}$ of the adjacent link L_{s+1}. Such a situation is quite reasonable, because the geometrical dynamics describes evolution of free particles. The rotating particle cannot be completely free, because in the rotating particle there is centripetal acceleration. However, acceleration of all parts of the body has to be absent for completely free motion. On the other hand, the geometric dynamics contains the spatial evolution, which absent in the conventional dynamics. From the geometrical viewpoint we may not discriminate spatial evolution on the basis, that the leading vector $\mathbf{P}_0\mathbf{P}_1$ is spacelike and its length is imaginary. In fact the spatial evolution discriminates itself, by the fact, that the corresponding world chain is unobservable, in general. It appears to be observable only for some complex skeletons, consisting of more, than two points. The world chain, describing the spatial evolution is observable only in the case, when it may be localized near the world chain of the observer. It takes place, when the world chain has a shape of a helix with timelike axis, or some other shape, which may be localized near the world chain of the observer. As a result not all skeletons appear to be observable.

Although the geometric dynamics does not contain a rotation, but the corollaries of the rotation (angular momentum, magnetic momentum) may be obtained as a result of the spatial evolution, when the world chain is a helix. Apparently, the fact, that such a "particle rotation" is a corollary of the spatial evolution, leads to the spin discreteness of the Dirac particle. Of course, such statements are to be tested by exact mathematical investigations of different types of skeletons and of different space-time geometries. However, such a statement of the problem is quite concrete and realizable.

Note, that the geometric dynamics in the real (non-Minkowskian) space-time contains additional terms with respect to dynamics in the space-time of Minkowski. From viewpoint of the space-time of Minkowski these additional terms may be interpreted as some (metric) interactions, which take place inside the elementary particles. From the conventional viewpoint these interactions look very exotic and strange. It is impossible (or very difficult) to guess at them, starting from conventional conception of the space-time and dynamics. In the geometric dynamics there are no additional interactions, if we use the true space-time geometry. However, additional interactions appear, if we use inadequate geometry (for instance, geometry of Minkowski, or Riemannian geometry). In other words, it is possible to compensate false space-time geometry by introduction of additional interactions. It is well known from the general relativity, that the motion of free body in the curved space-time looks as a motion in the gravitational field, if one interprets this motion as a motion in the space-time of Minkowski.

Description of conceptually new unknown phenomena by means of a change of the space-time geometry is simpler, than an introduction of additional interactions, because the space-time geometry is described by the world function, which is a function of two points. The form of the world function for large distances is determined by the necessity of obtaining the nonrelativistic quantum mechanics. Restrictions, imposed on the world function at small distances, are determined by the condition, that the spatial evolution may describe the Dirac particle. (Very many elementary particles are the Dirac particles). Besides, the condition of localization of the world chain (helical world chain) imposes restrictions on parameters of the particle. Not all parameters of particles appear to be possible. This condition is a condition of "peculiar quantization" of the particle parameters, which include the particle mass.

Let us note that the contemporary theory of elementary particles returns to geometrical considerations (strings, branes). However, these considerations are restricted by the framework of the Riemannian geometries and geometries close to the Riemannian geometry. For instance, the quantum geometry, which uses operators instead of the point coordinates. This is some way of introduction of multivariance in the geometry. However, this geometry is developed on the basis of the linear vector space, which is a restriction on the space-time geometry. In any case the conventional approach to the space-time geometry considers only a part of all possible space-time geometries. One cannot be sure, that the class of considered geometries contains true space-time geometry. Of course, if one uses a false space-time geometry, there is a possibility to correct the false space-time geometry by means of additional interaction, generated by difference with the true space-time geometry. But such a correction is difficult, especially if the true geometry is discrete or close to the discrete geometry.

Note, that the geometry (5.1) is discrete, although it is given on the continuous manifold of Minkowski. It is discrete, because the module of distance between any two points is more, than λ_0. It is very unexpected, because it is a common practice to consider any geometry on the manifold as a continuous geometry, although in reality the geometry is determined by the world function and only by the world function. A discrete geometry is associated with a grid. Of course, a geometry, given on a grid, cannot be continuous. However, a geometry, given on the continuous set of points (manifold), may be discrete.

Why the microcosm physics of the twentieth century did leave the successful program of the physics geometrization and choose the alternative program of quantum theory? Discovery of the electron diffraction need of multivariance of the microcosm physics. Multivariance may be taken into account either on the level of the space-time geometry, or on the level of dynamics. The multivariant space-time geometry was not known in the thirtieth, when the electron diffraction was discovered. The nonrelativistic quantum mechanics had been constructed already, and it was applied successfully for explanation of the electron diffraction.

The space-time geometry is a basis of dynamics. Introducing multivariance in dynamics, one can describe not only nonrelativistic phenomena of microcosm. One can describe also relativistic phenomena and that part of the microcosm physics, which is known as the theory of elementary particles. The principles of quantum mechanics, which introduce multivariance in the microcosm physics, were invented for the Newtonian conception of the space-time, and their extrapolation to the relativistic phenomena appeared to be problematic. Of course, some properties of the true space-time geometry may be taken into account by introduction of additional interactions. However, it is very difficult to invent and introduce additional interactions without understanding of these innovations. Capacities of the geometrical approach are very large, especially if one takes into account all possible space-time geometries. The theory of elementary particles returns to the geometrical description, but this description is burthened by such concepts as wave function, string, brane, which have very abstracted relation to the structure of elementary particles and microcosm physics.

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