Generalization of the relativity theory on the arbitrary space-time geometry

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Abstract

Contemporary relativity theory is restricted in two points: (1) a use of the Riemannian space-time geometry and (2) a use of inadequate (nonrelativistic) concepts. Reasons of these restrictions are analysed in [1]. Eliminating these restrictions the relativity theory is generalized on the case of non-Riemannian (nonaxiomatizable) space-time geometry.

Necessity of the general relativity generalization arises as a result of a geometry progress [2]. Now we know nonaxiomatizable (physical) geometries, which were unknown 20 years ago. Physical geometries are essentially the metric geometries, whose metric is free of almost all conventional restrictions. In a metric geometry there exists a problem, how one should define geometric concepts and rules of geometric objects construction. One can construct sphere and ellipsoid, which are defined in terms of metric (world function). However, one needs to impose constraints on metric (triangle axiom) even for construction of a straight line. It is unclear, how one can define the scalar product and linear dependence of vectors. The deformation principle [6] solves the problem of the geometrical concepts definition, without imposing any restrictions on the metric. The physical geometry is equipped by the deformation principle, which admits one to construct all definitions of the physical geometry as a deformation of corresponding definitions of the proper Euclidean geometry.

In the physical geometry the information on the geometry dimension and its topology appears to be redundant. It is determined by the metric (world function [3]), and one may not give it independently. A physical geometry is described completely by its world function. The geometry is multivariant and nonaxiomatizable in general. The world function describes uniformly continuous and discrete geometries. As a result the dynamic equations in physical space-time geometry are finite-difference (but not differential) equations. Besides, the particle dynamics can be described coordinateless in the physical space-time geometry. It is conditioned by a possibility of ignoring the linear vector space, whose properties are not used in the physical geometry. It is rather uncustomary for investigators dealing with the Riemannian geometry, which is based on usage of the linear vector space properties.

There is only one uniform isotropic geometry (geometry of Minkowski) in the class of Riemannian geometries, whereas there is a lot of uniform isotropic geometries among physical geometries. In particular, if the world function σ has the form

$$\sigma = \sigma_{\mathrm{M}} + \lambda_0 \mathrm{sgn}(\sigma_{\mathrm{M}}), \qquad \lambda_0 = \frac{\hbar}{2bc} \tag{1}$$

where σ_{M} is the world function of Minkowski, and $\hbar, c, b$ are respectively quantum constant, the speed of the light and some universal constant. The space-time geometry is discrete and multivariant. Free particle motion appears stochastic (multivariant). Its statistical description is equivalent to quantum description in terms of the Schrödinger equation [4].

Thus, application of the physical geometry in the microcosm admits one to give a statistical foundation of quantum mechanics and convert the quantum principles into appearance of the correctly chosen space-time geometry. One should expect, that a consideration of a more general space-time geometry and a refusal from the Riemanniance, which conditioned by our insufficient knowledge of geometry, will lead to a progress in our understanding of gravitation and cosmology.

An arbitrary space-time geometry is described completely by the world function $\sigma(P, P')$, given for all pairs of points P, P'. Information on dimension and on topology of the geometry is redundant, as far as it may be obtained from the world function. The Riemannian geometry, which is used in the contemporary theory of gravitation, is considered usually to be the most general possible space-time geometry. However, it cannot describe a discrete geometry, or a geometry having a restricted divisibility. The world function of the Riemannian geometry satisfies the equation

$$\frac{\partial \sigma}{\partial x^i} g^{ik}(x) \frac{\partial \sigma}{\partial x^k} = 2\sigma, \qquad \sigma(x, x') = \sigma(x', x) \tag{2}$$

It means, that in the expansion

$$\sigma(x, x') = \frac{1}{2} g_{ik}(x) \xi^i \xi^k + \frac{1}{6} g_{ikl}(x) \xi^i \xi^k \xi^l + ... \qquad \xi^k = x^k - x'^k$$

the metric tensor determines completely the whole world function.

Conventional gravitation equations determine only metric tensor. The world function and the space-time geometry are determined on the basis of supposition on the Riemannian geometry. Generalization of the gravitation equations admits one to obtain the world function directly (but not only the metric tensor).

The deformation principle admits one to construct all definitions of a physical geometry as a result of deformation of definitions of the proper Euclidean geometry.

One uses the fact, that proper Euclidean geometry is an axiomatizable geometry and a physical geometry simultaneously. It means, that all definitions of the Euclidean geometry, obtained in the framework of Euclidean axiomatics can be presented in terms and only in terms of the world function $\sigma_{\rm E}$ of the Euclidean geometry $\mathcal{G}_{\rm E}$. Replacing $\sigma_{\rm E}$ in all definitions of the Euclidean geometry $\mathcal{G}_{\rm E}$ by a world function σ of some other geometry $\mathcal{G}$, one obtains all definitions of the geometry $\mathcal{G}$. Definition of the scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ and their equivalency $(\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1)$ are the most used definitions

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma(P_0,Q_1) + \sigma(P_1,Q_0) - \sigma(P_0,Q_0) - \sigma(P_1,Q_1) \tag{3}$$

$$(\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1) \quad \text{if} \quad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1|\cdot|\mathbf{Q}_0\mathbf{Q}_1| \wedge |\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1| \tag{4}$$

They are defined in such a way in the Euclidean geometry. They are defined in the same way also in any physical geometry.

Solution of (4) is unique in the case of the proper Euclidean geometry, although there are only two equations, whereas the number of variables to be determined is larger, than two. For arbitrary physical geometry a solution is not unique, in general. As a result there are many vectors at the point P_0, which are equivalent to vector $\mathbf{Q}_0\mathbf{Q}_1$ at the point Q_0. Even geometry of Minkowski is multivariant with respect to spacelike vectors, although it is single-variant with respect to timelike vectors. Space-time geometry becomes to be multivariant with respect to timelike vectors only after proper deformation.

At the generalization of the general relativity on the case of arbitrary space-time geometry the two circumstances are important. (1) A use of the deformation principle, (2) A use of adequate relativistic concepts, in particular, a use of relativistic concept of the events nearness (See details in [5]). Two events A and B are near, if and only if

$$\sigma(A,B) = 0 \tag{5}$$

In the space-time of Minkowski a variation δg_{ik} of the metric tensor under influence of the matter have the form

$$\delta g_{ik}(x) = -\kappa \int G_{\rm ret}(x,x')\, T_{ik}(x')\sqrt{-g(x')}d^4x', \quad \kappa = \frac{8\pi G}{c^2} \tag{6}$$

where

$$G_{\rm ret}(x,x') = \frac{\theta(x^0 - x'^0)}{2\pi c}\delta(2\sigma_{\rm M}(x,x')) \tag{7}$$

and G is the gravitational constant.

Appearance of world function in the δ-function means, that the condition of the nearness $\sigma_{\rm M}(x,x') = 0$ leads to interpretation of gravitational (and electromagnetic) interactions as a direct collision of particles. Being presented in terms of world function these formulae have the same form in any physical geometry.

$$\begin{aligned}\delta\sigma(S_1,S_2) &= -G\sum_s m_s \frac{\theta((\mathbf{P}'_l\mathbf{P}.\mathbf{P}\mathbf{Q}_0))}{(\mathbf{P}'_l\mathbf{P}.\mathbf{P}'_l\mathbf{P}'_{l+1})}\frac{(\mathbf{P}'_l\mathbf{P}'_{l+1}.\mathbf{P}\mathbf{Q}_0)}{(\mathbf{P}'_l\mathbf{P}'_{l+1}.\mathbf{P}'_l\mathbf{P}'_{l+1})|\mathbf{P}\mathbf{Q}_0|} \\ &\quad\times\left((\mathbf{P}'_l\mathbf{P}'_{l+1}.\mathbf{P}\mathbf{S}_1) - (\mathbf{P}'_l\mathbf{P}'_{l+1}.\mathbf{P}\mathbf{S}_2)\right)^2 \end{aligned} \tag{8}$$

where S_1, S_2 are arbitrary points of the space-time. Summation is produced over all world lines of particles perturbing the space-time geometry. The segment $\mathbf{P}'_l\mathbf{P}'_{l+1}$ is infinitesimal element of the world line of one of perturbing particles. The point P'_l is near to the point P, which is a middle of the segment S_1S_2.

$$\sigma(P, P'_l) = 0, \qquad \mathbf{PS}_1 = -\mathbf{PS}_2 \tag{9}$$

The vectors $\mathbf{PQ}_i$, $i = 0, 1, 2, 3$ are basic vectors at the point P. Vector $\mathbf{PQ}_0$ is timelike. If σ_0 is the unperturbed world function of space-time geometry without particles, then $\sigma = \sigma_0 + \delta\sigma$ is the world function of the space-time geometry after appearance of perturbing particles. One should use the world function σ at calculation of scalar products in rhs of (8) by the formula (3). At first the world function σ is unknown and relation (8) is the equation for determination of σ.

Equation (8) is solved by the method of subsequent approximations. At the first step one calculates rhs of (8) by means of σ_0 and obtains $\sigma_1 = \sigma_0 + \delta\sigma_0$. At the second step one calculates rhs of (8) by means of σ_1 and obtains $\sigma_2 = \sigma_0 + \delta\sigma_1$ and so on.

Applying relation (8) to heavy pointlike particle, one obtains in the first approximation

$$\sigma_1 (t_1, \mathbf{y}_1; t_2, \mathbf{y}_2) = \frac{1}{2}\left(1 - \frac{4GM}{c^2 |\mathbf{y}_2 + \mathbf{y}_1|}\right) c^2 (t_2 - t_1)^2 - \frac{1}{2} (\mathbf{y}_2 - \mathbf{y}_1)^2 \tag{10}$$

where M is the mass of the particle.

Space-time geometry appears to be non-Riemannian already at the first approximation, although the metric tensor has the form, which it has for a slight gravitational field. The next approximations do not change the situatition.

Thus, the space-time geometry appears to be non-Riemannian. Furthermore, supposition on the Riemannian space-time leads to an ambiguity of the world function for large difference of times $(t_1 - t_2)$ even in the case of a gravitational field of a heavy particle. It is conditioned by the fact, that there are many free world lines, connecting two points. It is forbidden in a physical geometry, where the world function must be single-valued.

Thus, generalization of the relativity theory on the general case of the space-time geometry is generated by our progress in geometry and by a use of adequate relativistic concepts. The deformation principle is not a hypotheses, but it is principle, which lies in the basis of physical geometry. The uniform formalism, suitable for both continuous and discrete geometries, is characteristic for physical geometries. This formalism uses dynamic equations in the form of finite-difference equations. Sometimes these equations have a form of finite relations. The uniform formalism is formulated in coordinateless form. It gets rid of necessity to consider coordinate transformations and their invariants.

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