Granular geometry of space-time as a result of Newtonian investigation strategy

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Abstract

The Newtonian investigation strategy declares "Hypotheses non fingo!" In practice it means that, having problems in the theory development, one looks for mistakes in papers of predecessors and corrects them. Sometimes such an investigation strategy admits one to solve the arising problems without a use of additional hypotheses.

The conventional method of a generalized geometry construction, based on deduction of all propositions of the geometry from axioms, appears to be imperfect (incomplete) in the sense, that multivariant geometries cannot be constructed by means of this method. Multivariant geometry is such a geometry, where at the point P there are many vectors $\mathbf{PP}'$, $\mathbf{PP}''$,... which are equivalent to the vector $\mathbf{QQ}'$ at the point Q, but they are not equivalent between themselves. In the conventional (Euclidean) method the equivalence relation is transitive, whereas in a multivariant geometry the equivalence relation is intransitive, in general. It is a reason, why the multivariant geometries cannot be deduced from a system of axioms. The space-time geometry in microcosm is multivariant. As a rule the multivariant geometry is a granular geometry, i.e. such a geometry, which is partly continuous and partly discrete. Multivariance is a mathematical method of the granularity description. The granularity (and multivariance) of the space-time geometry generates a multivariant (quantum) motion of particles in microcosm. Besides, the granular space-time generates some discrimination mechanism, responsible for discrete parameters (mass, charge, spin) of elementary particles. Dynamics of particles appears to be determined completely by properties of the granular space-time geometry. The quantum principles appear to be needless.

Investigation strategy is a very important matter, because, having a inefficient investigation strategy, one cannot obtain true results of investigation. Conventional investigation strategy uses the trial and error method, which traces back to the time of the quantum mechanics creation. The Newtonian investigation strategy traces back to the time of Sir Isaac Newton, who declared: "Hypotheses non fingo." This slogan means that having a problem in a theory, one should first of all to look for mistakes in papers of the predecessors. Finding mistakes, one should correct them. After such a correction invention of new hypotheses becomes to be needless in many cases. The Newtonian investigation strategy is safe, because there is no necessity to verify the correction. The found mistake must be corrected in any case, whereas the invented hypothesis must be verified. The Newtonian strategy has some defects. At first, the researcher is to have very high qualification to find a possible mistake, and such a high qualification is rather rare. Second, other researchers are rather sceptical with respect to researcher, who declares, that he uses the Newtonian investigation strategy. They prefer the strategy, which uses the trial and error method, because this method does not need high qualification. Besides, they do not like, when anybody find mistakes in papers of predecessors, because there are papers founded on these wrong statements, and discovery of mistakes depreciate these papers. Third, if anybody succeeded to find a very serious mistake and to correct it, such a situation arises, when papers of almost all theorists appear to be depreciated. In such a situation new papers of the adherer of the Newtonian strategy are not accepted. However, the third property of Newtonian strategy becomes to act, only if the mistake and its correction are sufficiently fundamental. Although defects of the Newtonian investigation strategy in the social relation are essential, This strategy is safe and reliable in the scientific relation.

In the end of nineteenth century the physics developed in the direction of its geometrization, i.e. the more properties of physical phenomena were explained by properties of the event space (space-time). Explanation of the conservation laws by means of isotropy and homogeneity of the event space, the special relativity, the general relativity, explanation of the electric charge discreteness by compactification of 5-dimensional Kaluza-Klein geometry are consequent stages of the physics geometrization. Geometrization of physics was a very effective program of the theoretical physics development.

However, attempts of this program applications to the microcosm physical phenomena failed. This failure was conditioned by the very sad circumstance, that our knowledge of geometry were poor. We can describe only continuous geometries with unlimited divisibility. We cannot work with granular geometries, i.e. with geometries, which are partly continuous and partly discrete. We did not know how one can describe a geometry with limited divisibility. We could not imagine, that there are multivariant geometries, where at the point P_0 there exist many vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$,$\mathbf{P}_0\mathbf{P}_3$,..., which are equivalent to the vector $\mathbf{Q}_0\mathbf{Q}_1$ at the point Q_0, but these vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$,$\mathbf{P}_0\mathbf{P}_3$,.. are not equivalent between themselves. We could not imagine, the geometry in itself may discriminate existence of some geometrical objects. In reality the space-time geometry of microcosm possessed such exotic properties, however we cannot describe these properties. Our knowledge of geometry were too poor. However, the multivariance is a very important property of the space-time geometry, which is responsible for quantum effects [1].

All generalized geometries are modifications of the proper Euclidean geometry, constructed by Euclid many years ago. Euclid presented two very important matters: (1) The Euclidean geometry and (2) the Euclidean method of the geometry construction.

Conventionally one uses the Euclidean method for construction of generalized geometries. This method is only a half-finished product (the product is the Euclidean geometry itself). Using the Euclidean method, one can construct only axiomatizable geometries. The axiomatizable geometries are such geometries, where all geometrical objects can be constructed of blocks. Euclid himself used three kinds of such blocks: point, segment of straight and angle. Formalization of the construction procedure leads to the statement, that all propositions of the proper Euclidean geometry may be deduced from a finite system of axioms. One supposes, that for construction of a generalized geometry one has to use another system of axioms (i.e. the Euclidean blocks are to be replaced by another series of blocks). Thus, the Euclidean method admits one to construct only axiomatizable geometries.

Another method of the generalized geometry construction admits one to construct only physical geometries, i.e. geometries, which can be described completely by the world function of the geometry in question. The world function σ is defined by the relation $\sigma(P,Q) = \frac{1}{2}\rho^2(P,Q)$, where $\rho(P,Q)$ is the distance between the points P and Q. This method uses the already constructed proper Euclidean geometry as follows. The proper Euclidean geometry $\mathcal{G}_{\rm E}$ is a physical geometry. All propositions $\mathcal{P}$ of the proper Euclidean geometry $\mathcal{G}_{\rm E}$ are presented in the form $\mathcal{P}(\sigma_{\rm E})$, where $\sigma_{\rm E}$ is the world function of $\mathcal{G}_{\rm E}$. Thereafter one deform the standard geometry $\mathcal{G}_{\rm E}$, replacing $\sigma_{\rm E}$ by the world function σ of some other physical geometry $\mathcal{G}$: $\mathcal{P}(\sigma_{\rm E}) \rightarrow \mathcal{P}(\sigma)$. One obtains the set $\mathcal{P}(\sigma)$ of all propositions of the physical geometry $\mathcal{G}$. The physical geometry $\mathcal{G}$, obtained from the standard (proper Euclidean) geometry by means of the deformation is not an axiomatizable geometry, in general, i.e. it cannot be constructed of any blocks.

Let us demonstrate this fact in a simple model. Let we have only one kind of cubic plasticine blocks. These blocks are painted by a red paint, in order one can distinguish boundaries of blocks in a building. Let us construct some building of these blocks, for instance, a cube. Let us deform this cube in an arbitrary way, for instance, into a circular cylinder. After such a deformation all cubic blocks, constituting the cube will be deformed. The deformation will be different for different blocks, and they cannot be used for construction of a new building. Of course, one can reconstruct the cylinder, but this cylinder will be reconstructed of blocks, having different shapes, which they have been obtained as a result of the deformation. These blocks are not suitable for construction of another buildings.

This model shows, how a deformation destroys the axiomatizability of the axiomatizable geometry.

In any axiomatizable geometry the equivalence relation is transitive. This transitivity is necessary, in order that any deduction leads to a definite result. Deformation destroys the transitivity of the equivalence relation, and the geometry becomes to be nonaxiomatizable. Let us demonstrate this in the example of two vector equivalence. In the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ the equivalence of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined as follows. Vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent ($\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\,\mathbf{Q}_0\mathbf{Q}_1$), if vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are in parallel ($\mathbf{P}_0\mathbf{P}_1 \uparrow\uparrow \mathbf{Q}_0\mathbf{Q}_1$) and their lengths $|\mathbf{P}_0\mathbf{P}_1|$ and $|\mathbf{Q}_0\mathbf{Q}_1|$ are equal. Mathematically these two conditions are written in the form

$$(\mathbf{P}_0\mathbf{P}_1 \uparrow\uparrow \mathbf{Q}_0\mathbf{Q}_1): \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \tag{1}$$

$$|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|, \qquad |\mathbf{P}_0\mathbf{P}_1| = \sqrt{2\sigma(P_0, P_1)} \tag{2}$$

where $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ is the scalar product of two vectors, defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) \tag{3}$$

Here σ is the world function of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. The length $|\mathbf{PQ}|$ of vector $\mathbf{PQ}$ is defined by the relation

$$|\mathbf{PQ}| = \rho(P, Q) = \sqrt{2\sigma(P, Q)} \tag{4}$$

Using relations (1) - (4), one can write the equivalence condition in the form

$$\begin{aligned} \mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1 \quad &: \quad \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) \\ &= \sigma(P_0, P_1) \wedge \sigma(P_0, P_1) = \sigma(Q_0, Q_1) \end{aligned} \tag{5}$$

The equivalence relation is used in any physical geometry. The definition of equivalence (5) is a satisfactory geometrical definition, because it does not contain a reference to the dimension of the space and to the coordinate system. It contains only points P_0, P_1, Q_0, Q_1, determining vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ and distances (world functions) between these points. The definition of equivalence (5) coincides with the conventional definition of two vectors equivalence in the proper Euclidean geometry. If one fixes points P_0, P_1, Q_0 in the relations (5) and solve them with respect to the point Q_1, one finds that these equations always have one and only one solution. This statement follows from the properties of the world function of the proper Euclidean geometry. It means that the proper Euclidean geometry is single-variant with respect any pairs of its points. It means also, that the equivalence relation is transitive in the proper Euclidean geometry.

In the arbitrary physical geometry the equivalence relation has the same form (5) with another world function σ, satisfying the constraints

$$\sigma: \quad \Omega \times \Omega \to \mathbb{R}, \qquad \sigma(P, P) = 0, \qquad \forall P, Q \in \Omega \tag{6}$$

Here Ω is the set of all points, where the geometry is given.

In the case of arbitrary world function one cannot guarantee, that equations (5) have always an unique solution. There may be many solutions. In this case one has a multivariant geometry. There may be no solution. In this case one has a zero-variant (discriminating) geometry. In both cases the equivalence relation is intransitive, and geometry is nonaxiomatizable. There may be such a situation, that the geometry is multivariant with respect to some points and vectors, and it is zero-variant with respect to other points and vectors.

One sets conventionally, that the world function of the space-time is symmetric

$$\sigma(P,Q)=\sigma(Q,P), \qquad \forall P,Q\in\Omega \tag{7}$$

This condition means that the future and the past are geometrically equivalent. However, the physical geometry can be constructed for asymmetric world function Σ [2]

$$\Sigma(P,Q)=G(P,Q)+A(P,Q), \tag{8}$$

$$G(P,Q)=G(Q,P), \qquad A(P,Q)=-A(Q,P) \tag{9}$$

The time is considered as an attribute of the event space (space-time). The time arrow can be taken into account in the technique of asymmetric space-time geometry.

The asymmetric geometry with asymmetric world function may appear in the microcosm, however, its application is especially interesting in the cosmology, where the future and the past of our universe may appear to be not equal. Besides, the gravitational law in asymmetric space-time geometry distinguishes from gravitational law in the symmetric one. Maybe, reasonable supposition on asymmetry of the space-time geometry will be able to explain deflection of astronomical observations from predictions of the general relativity. In this case the invention of the dark matter will be needless. However, such a possibility is not investigated yet properly.

The granular space-time geometry $\mathcal{G}_{\rm g}$, given on the manifold of Minkowski is described approximately by the world function $\sigma_{\rm g}$

$$\sigma_{\rm g}=\sigma_{\rm M}+\lambda_0^2\begin{cases}\operatorname{sgn}(\sigma_{\rm M}) & \text{if} \quad |\sigma_{\rm M}|>\sigma_0\\ \frac{\sigma_{\rm M}}{\sigma_0} & \text{if} \quad |\sigma_{\rm M}|\leq\sigma_0\end{cases}, \qquad \lambda_0^2,\sigma_0=\text{const}\geq 0 \tag{10}$$

where $\sigma_{\rm M}$ is the world function of the geometry $\mathcal{G}_{\rm M}$ of Minkowski, λ_0 is a elementary length. The world function $\sigma_{\rm M}$ of the Minkowski geometry $\mathcal{G}_{\rm M}$ is Lorentz-invariant, and the world function $\sigma_{\rm g}$ of the granular geometry $\mathcal{G}_{\rm g}$ is Lorentz-invariant also, because it is a function of $\sigma_{\rm M}$. If $\sigma_0=0$, the geometry $\mathcal{G}_{\rm g}$ is discrete, although it is given on the continuous manifold of Minkowski. Indeed, if $\sigma_0=0$, in the geometry $\mathcal{G}_{\rm g}$ there are no close points separated by a distance less, than $\sqrt{2}\lambda_0$. This statement follows from (10). Discrete Lorentz-invariant geometry on a continuous manifold! This fact seems to be very unexpected in the conventional approach to geometry, where discreteness of geometry depends on the structure of the point set Ω, where the geometry is given.

In the physical geometry discreteness and continuity of the geometry is determined by the world function and only by the world function, whereas the structure of the point set Ω is important only in such extent, in which it influences on the world function.

Granularity of the geometry $\mathcal{G}_{\rm g}$ becomes more clear, if one considers the relative density $\rho(\sigma_{\rm g})=\frac{d\sigma_{\rm M}(\sigma_{\rm g})}{d\sigma_{\rm g}}$ of points in $\mathcal{G}_{\rm M}$ with respect to the density of points in $\mathcal{G}_{\rm g}$.

One obtains from (10)

$$\rho\left(\sigma_{\mathrm{g}}\right)=\frac{d\sigma_{\mathrm{M}}\left(\sigma_{\mathrm{g}}\right)}{d\sigma_{\mathrm{g}}}=\left\{\begin{array}{lll}1 & \text{if} & \left|\sigma_{\mathrm{g}}\right|>\sigma_0+\lambda_0^2 \\ \frac{\sigma_0}{\sigma_0+\lambda_0^2} & \text{if} & \left|\sigma_{\mathrm{g}}\right|\leq\sigma_0+\lambda_0^2\end{array}\right. \tag{11}$$

One can see from (11), that at $\sigma_0=0$ there is no points in the interval $\sigma_g\in\left(-\lambda_0^2,\lambda_0^2\right)$. If $\sigma_0\neq 0$, one can see from (11), that the relative density of points in the interval $\sigma_g\in\left(-\lambda_0^2-\sigma_0,\lambda_0^2+\sigma_0\right)$ is less, than unity but it is not equal to zero. We have some intermediate situation between the continuity (when $\rho=1$) and discreteness (when $\rho=0$). Such a situation is treated as granularity.

In the granular space-time geometry the elementary particle is described by its skeleton $\mathcal{P}_n=\{P_0,P_1,..,P_n\}$, consisting of $n+1$ points. The pointlike particle is described by the skeleton $\mathcal{P}_1=\{P_0,P_1\}$, consisting of two points, or by the vector $\mathbf{P}_0\mathbf{P}_1$. The vector $\mathbf{P}_0\mathbf{P}_1$ represents the momentum of the pointlike particle, whereas its length $|\mathbf{P}_0\mathbf{P}_1|=\mu$ is the geometrical mass of the pointlike particle. The geometrical mass μ is connected with its usual mass m by means of the relation

$$m=b\mu \tag{12}$$

where b is some universal constant.

Evolution of the elementary particle is described by the world chain, consisting of connected skeletons $...\mathcal{P}_n^{(0)},\mathcal{P}_n^{(1)},...,\mathcal{P}_n^{(s)}...$

$$\mathcal{P}_n^{(s)}=\left\{P_0^{(s)},P_1^{(s)},..P_n^{(s)}\right\},\qquad s=...0,1,... \tag{13}$$

The adjacent skeletons $\mathcal{P}_n^{(s)},\mathcal{P}_n^{(s+1)}$ of the chain are connected by the relations $P_1^{(s)}=P_0^{(s+1)}$, $s=...0,1,...$ The vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}=\mathbf{P}_0^{(s)}\mathbf{P}_0^{(s+1)}$ is the leading vector, which determined the world chain direction.

Dynamics of free elementary particle is determined by the relations

$$\mathcal{P}_n^{(s)}\mathrm{eqv}\mathcal{P}_n^{(s+1)}:\qquad \mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\mathrm{eqv}\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)},\qquad i,k=0,1,..n;\qquad s=...0,1,... \tag{14}$$

which describe equivalence of adjacent skeletons.

Thus, dynamics of a free elementary particle is described by a system of algebraic equations (14). Specific of dynamics depends on the elementary particle structure (disposition of particles inside the skeleton) and on the space-time geometry.

In the simplest case, when the space-time geometry is the 5-dimensional Kaluza-Klein geometry, the dynamic equations (14) for the pointlike particle are reduced to conventional differential dynamic equations, describing motion of the charged pointlike particle in the given electromagnetic and gravitational fields. Thus, dynamic equations (14) can be considered as a generalization of classical differential dynamic equations for the particle motion on the case of the granular space-time geometry. It is quite reasonable, that the dynamic equations in the granular space-time geometry cannot be differential equations.

Let the elementary length λ_0 have the form

$$\lambda_0^2 = \frac{\hbar}{2bc} \tag{15}$$

where $\hbar$ is the quantum constant, c is the speed of the light and b is the universal constant, defined by (12). Let the constant σ_0 in (11) be small enough. Then the motion of a pointlike particle in the granular space-time geometry (11) appears to be multivariant (stochastic). Statistical description of this multivariant particle motion coincides with the quantum description in terms of the Schrödinger equation. Quantum constant appears in the description via elementary length (15), which is a parameter of the granular space-time geometry.

Motion of elementary particles, which are not pointlike is not yet investigated properly. There is only some information on the Dirac particle, whose skeleton consists of three points and leading vector is spacelike [3]. In this case the world chain is a spacelike helix with the timelike axis. Such a spacelike helix cannot exist in the granular geometry (10). However, if the world function (10) is modified slightly at small distances $\sigma_{\rm g} \to \sigma_{\rm gm}$

$$\sigma_{\rm gm} = \sigma_{\rm M} + \lambda_0^2 \begin{cases} \operatorname{sgn}(\sigma_{\rm M}) & \text{if} \quad |\sigma_{\rm M}| > \sigma_0 \\ \left(\frac{\sigma_{\rm M}}{\sigma_0}\right)^3 & \text{if} \quad |\sigma_{\rm M}| \leq \sigma_0 \end{cases}, \qquad \lambda_0^2, \sigma_0 = \text{const} \geq 0 \tag{16}$$

such a spacelike helix becomes possible. The spacelike helix is possible also for other space-time geometries, where the the world function in interval $(-\sigma_0, \sigma_0)$ has the form $f(\sigma_{\rm M}/\sigma_0)$, $|\sigma_{\rm M}| < |\sigma_0|$. It is to satisfy the condition $|f(\sigma_{\rm M}/\sigma_0)| < |\sigma_{\rm M}/\sigma_0|$.

Identification of the elementary particle with the world chain, having a shape of the spacelike helix, with the Dirac particle is founded on the following fact. In the classical limit the Dirac equation for a free particle describes a classical dynamic system having 10 degrees of freedom. Solution of dynamic equations describes a helical world line with the timelike axis [4]. It is not quite clear, whether this helix is spacelike, or timelike, because the internal degrees of freedom, responsible for circular motion, are described nonrelativistically (i.e. incorrectly), although external degrees of freedom are described relativistically [5].

Such an approach to dynamics of the elementary particles seems to be very reasonable, because the structure of the elementary particle is defined by its skeleton structure. Description does not contain wave functions, branes, strings and other exotic matters, which are very far from the space-time geometry. It is important also the fact, that the description in the granular geometry is described on the Kaluza-Klein manifold, and the granular space-time geometry can be reduced to the description in terms of the Kaluza-Klein geometry with addition of some force fields, which describe deflection of the granular geometry from the Kaluza-Klein one.

A use of the Kaluza-Klein geometry needs a compactification of the fifth coordinate, responsible for the electric charge of the particle. Compactification of the Kaluza-Klein geometry means a modification of its topology. However, in the physical geometry the topology is determined completely by the world function. One cannot change the topology independently of a corresponding change of the world function. The modification of the world function, which corresponds to compactification, leads to constraints, imposed on the electric charge of the particle [6]. This constraint has nothing to do with quantum principles.

The presented conception is completely orthodox, because it does not use any new principles. Introducing into consideration nonaxiomatizable geometries, one removes only incompleteness in description of the space-time geometry. Orthodoxy of the conception evidences in behalf of this conception.

In general, the geometric dynamics (14) is a classical dynamics in the granular space-time geometry. The granularity of the space-time generates two new properties, which are absent in the axiomatizable geometries: (1) multivariance, which is responsible for quantum properties, (2) zero-variance (discrimination mechanism), which is responsible for discreteness of the elementary particles parameters. The multivariance of the granular space-time geometry can be taken into account by means of the statistical description. Quantum theory can imitate multivariance (and statistical description) on the level of dynamics, but it cannot imitate the zero-variance (discrimination mechanism). As a result the contemporary theory of elementary particles has no key to explanation of discrete parameters of the elementary particles.

Let us note in conclusion that we did not use any new hypotheses. Our conception is not a conceptually new theory. It is simply a generalization of the classical dynamics onto the case of granular space-time geometry, which was ignored by contemporary mathematicians (and physicists). Using granular space-time, we do not use any new hypotheses or principles. We have overcame simply the preconception, that the space-time geometry may be only axiomatizable. Besides, we reduce the number of principles in the theory in the sense, that the quantum principles are not used. Quantum effects are described now by multivariance of the granular space-time geometry.

The generalization of classical physics on the case of the granular space-time geometry is not yet accomplished in the sense, that only generalization of dynamic equations for the particle motion in the given external fields has been obtained. Another part of the classical physics, which describes influence of the matter on the space-time geometry (gravitation equations and Maxwell equations) has not been generalized yet on the case of the granular space-time geometry.

The considered conception may be qualified as the point 3 in the program of the physics geometrization: (1) special relativity in the framework of axiomatizable geometries, (2) general relativity in the framework of axiomatizable geometries, (3) special relativity in the framework of granular geometries, (4) general relativity in the framework of granular geometries. The point 4 is not yet realized.

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