Generalization of relativistic particle dynamics on the case of non-Riemannian space-time geometry.

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Abstract

Conventional relativistic dynamics of a pointlike particle is generalized on the case of arbitrary non-Riemannian space-time geometry. Non-Riemannian geometry is an arbitrary physical geometry, i.e. a geometry, described completely by the world function of the space-time geometry. The physical geometry may be discrete, or continuous. It may be granular (partly continuous and partly discrete). As a rule the non-Riemannian geometry is nonaxiomatizable, because the equivalence relation is intransitive. The dynamic equations are the difference equations. They do not contain references to a dimension and to a coordinate system. The generalization is produced on the dynamics of composite particles, which may be identified with elementary particles. The granular space-time geometry generates multivariant motion, which is responsible for quantum effects. It generates a discrimination mechanism, which is responsible for discrete values of the elementary particles parameters. The quantum principles appear to be needless in such a dynamics.

1 Introduction

This paper is devoted to expanding of relativistic dynamics on the case of the non-Riemannian space-time geometry. It is a generalization of classical principles on the case of more general space-time geometries. Necessity of such a generalization appeared, when it became to be clear, that the space-time geometry may be not only a Riemannian one. The space-time geometry may be a physical geometry, i.e. a geometry, which is described by its world function completely [1, 2]. Another name of the physical geometry is the tubular geometry (or T-geometry). In general, straight lines in the tubular geometry are tubes (surfaces), but not one-dimensional lines.

In the approximation, when the influence of the matter on the space-time geometry is neglected, the space-time geometry is to be equal at all points of the space-time. It means, that the geometry is uniform and isotropic. In the class of Riemannian geometries there is only one uniform and isotropic geometry (appropriate for the space-time description): the geometry of Minkowski. In the class of physical geometries any geometry $\mathcal{G}$, described by the world function

$$\sigma = F\left(\sigma_{\mathrm{M}}\right), \qquad F\left(0\right) = 0 \tag{1.1}$$

is isotropic and uniform. Here

$$\sigma_{\mathrm{M}}\left(x, x'\right) = \frac{1}{2} g_{ik}\left(x^{i} - x'^{i}\right)\left(x^{k} - x'^{k}\right), \qquad g_{ik} = \mathrm{const} \tag{1.2}$$

is the world function of the geometry of Minkowski and F is an arbitrary function.

As far as the world function σ_{M} is invariant with respect to Lorentz transformations, any function (1.1) of σ_{M} is also invariant. Then the physical geometry, described by the world function of the form (1.1) is also Lorentz-invariant.

The classical principles of dynamic do not work in the microcosm, provided one uses the space-time geometry of Minkowski. It is supposed conventionally, that in the microcosm the principles of quantum dynamics are valid. We shall refer to such an approach (quantum principles of dynamics + geometry of Minkowski) as the *quantum paradigm*.

However, another approach (classical principles of dynamics + some space-time geometry of the form (1.1)) is possible also. This approach will be referred to as a *geometrical paradigm*. In the framework of the *geometrical paradigm* the space-time geometry is not determined. It must be chosen inside the class of physical geometries in such way, that the classical particle dynamics, based of the chosen space-time geometry, explains all experimental data in the microcosm physics. Thus, in the framework of the *geometrical paradigm* the classical principles of dynamics are fixed, whereas the space-time geometry is varied.

On the contrary, in the framework of *quantum paradigm* the space-time geometry is fixed, and principles of dynamics are varied. It is evident, that from a technical viewpoint a variation of dynamic principles is more complicated, than a variation of the space-time geometry, which is determined by the world function completely. Besides, the *quantum paradigm* is generated by our poor knowledge of geometry, whereas the *geometrical paradigm* takes into account our more perfect knowledge of geometry. Let me explain the situation in a simple example.

Let us imagine a person N (Nicola), who does not know, that the algebraic equation may have many roots. (He thinks, that the algebraic equation has only one root). Such a situation seems to be fantastic, but nevertheless, let us consider this situation. Nicola is a physicist-theorist, who creates fundamental physical theories. In general, Nicola does know algebra. He can add and multiply the numbers, he can even differentiate and integrate. In other words, he possesses the contemporary mathematical technique. However, Nicola does not know, that the algebraic equation may have many roots. Constructing theories, Nicola may meet such a situation, when one needs to use several roots of an algebraic equation. In this case he should think about his knowledge of algebra.

But Nicola is self-opinionated. He does not question his knowledge of algebra. He invents new hypotheses, which admit him to compensate his poor knowledge of algebra. These new hypotheses are simple fittings, but in some cases these fittings work successfully. In other situations these fittings cease to work, and Nicola is forced to search for other fittings (hypotheses).

Let us imagine Mike, who is a friend of Nicola. Mike knows very well, that the algebraic equation may have many roots, and he says to Nicola: "Nicola, try to use the fact, that the algebraic equation may have many roots. Maybe, some problems of your fundamental physical theory disappear by themselves. You will not need to invent your hypotheses, and your theory will contain less fundamental principles". Nicola exclaims: "Oh, Mike! This is a splendid idea! One needs to test this hypothesis! However, it is your idea. Test it yourself, please. If it will appear, that by means of your idea one can explain more experiments, than I can explain by my hypotheses, we create a new fundamental physical theory. However, if your idea helps to explain only those experiments, which are explained by my theory, I do not see any reason to use your idea in the fundamental physical theory, because my theory explains all known experimental data. Go ahead! Introduce your idea in the theory and test it!"

It remains to add, that Mike has suggested only to use mathematics correctly and nothing besides. Nicola perceived Mike's suggestion as a new theoretical conception. The presented story seems to be fantastic, however, the situation, when one chooses the geometry of Minkowski among many possible uniform isotropic geometries is a mathematical mistake of the same kind, as Nicola has made. This mistake is a source of the quantum paradigm. One may say, that we did not know another uniform isotropic geometries other, than the geometry of Minkowski. It is true. However, it is a mathematical mistake, which should be corrected.

Reaction of Nicola is a typical reaction of contemporary theorist, which does not distinguish between a hypothesis and a correction of a mistake, based on our poor knowledge of mathematics. Most contemporary theorists believe that a fundamental physical theory is a list of correctly solved problems, and the physics of microcosm progresses by means of invention of new physical hypotheses. They did not understand, that the fundamental theory deals with physical principles, but not with single physical phenomena. They understand the role of physical principles in usual physics. However, as it concerns the microcosm physical phenomena, they believe, that usual classical principles of physics are insufficient in the description of the microcosm physics, and the theorists retire from the classical principles. In other words, they believe in the *quantum paradigm*, founded on our poor knowledge of geometry.

Working in the framework of the *geometrical paradigm*, we are to choose the true space-time geometry in the microcosm. This choice is rather complicated, because in the true space-time geometry the dynamics of particle is to explain all experimental data. To test coincidence of the theory predictions with experiment, one is to construct particle dynamics in any possible space-time geometry. In reality, it is a generalization of the particle dynamics in the Riemannian space-time geometry (which is known) on the case of the general physical space-time geometry (which is not yet known). This problem is solved in the present paper.

Let us stress, that producing such a generalization, we shall not test our results by means of a comparison with the experiment. Such a comparison is impossible and useless, if we work with arbitrary space-time geometry (but not with the geometry, which is supposed to be a true space-time geometry). In order to find the true space-time geometry of microcosm, one needs to formulate principles of classical dynamics in arbitrary physical space-time geometry and to construct corresponding mathematical formalism. In this paper we construct the mathematical formalism of dynamics. However, we do not try to choose the true space-time geometry of microcosm. Concrete space-time geometries, considered in the paper are used only as an illustration of capacities of the *geometrical paradigm*. They do not pretend to be an example of a true space-time geometry.

In the end of nineteenth century the physics developed in the direction of its geometrization, i.e. the more properties of physical phenomena were explained by properties of the event space (space-time). Explanation of the conservation laws by means of isotropy and homogeneity of the event space, the special relativity, the general relativity, explanation of the electric charge discreteness by compactification of 5-dimensional Kaluza-Klein geometry are consequent stages of the physics geometrization. Geometrization of physics was a very effective program of the theoretical physics development.

However, attempts of this program applications to the microcosm physical phenomena failed. This failure was conditioned by the very sad circumstance, that our knowledge of geometry were poor. We could describe only continuous geometries with unlimited divisibility. We could not work with granular geometries, i.e. with geometries, which are partly continuous and partly discrete. We did not know, how one can describe a geometry with limited divisibility. We could not imagine, that there are multivariant geometries, where at the point P_0 there exist many vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$,$\mathbf{P}_0\mathbf{P}_3$,..., which are equivalent to the vector $\mathbf{Q}_0\mathbf{Q}_1$ at the point Q_0, but these vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$,$\mathbf{P}_0\mathbf{P}_3$,.. are not equivalent between themselves. We could not imagine, that the geometry in itself may discriminate existence of some geometrical objects. In reality the space-time geometry of microcosm possessed such exotic properties, however we could not describe these properties. Our knowledge of geometry were too poor. However, the multivariance is a very important property of the space-time geometry, which is responsible for quantum effects [3].

All generalized geometries are modifications of the proper Euclidean geometry, constructed by Euclid many years ago. Euclid presented two very important matters: (1) the Euclidean geometry and (2) the Euclidean method of the geometry construction.

Conventionally one uses the Euclidean method for construction of generalized geometries. This method is only a half-finished product (the product is the Euclidean geometry itself). Using the Euclidean method, one can construct only axiomatizable geometries. The axiomatizable geometries are such geometries, where all geometrical objects can be constructed of blocks. Euclid himself used three kinds of such blocks: point, segment of straight and angle. Formalization of the construction procedure leads to the statement, that all propositions of the proper Euclidean geometry may be deduced from a finite system of axioms. One supposes, that for construction of a generalized geometry one has to use another system of axioms (i.e. the Euclidean blocks are to be replaced by another series of blocks). Thus, the Euclidean method admits one to construct only axiomatizable geometries.

Another method of the generalized geometry construction admits one to construct only physical geometries, i.e. geometries, which can be described completely by the world function of the geometry in question. The world function σ is defined by the relation $\sigma(P,Q) = \frac{1}{2}\rho^2(P,Q)$, where $\rho(P,Q)$ is the distance between the points P and Q. This method uses the already constructed proper Euclidean geometry as a standard geometry. The proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ is a physical geometry. All propositions $\mathcal{P}$ of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ are presented in the form $\mathcal{P}(\sigma_{\mathrm{E}})$, where σ_{E} is the world function of $\mathcal{G}_{\mathrm{E}}$. Thereafter one deforms the standard geometry $\mathcal{G}_{\mathrm{E}}$, replacing σ_{E} by the world function σ of some other physical geometry $\mathcal{G}$: $\mathcal{P}(\sigma_{\mathrm{E}}) \rightarrow \mathcal{P}(\sigma)$. One obtains the set $\mathcal{P}(\sigma)$ of all propositions of the physical geometry $\mathcal{G}$. The physical geometry $\mathcal{G}$, obtained from the standard (proper Euclidean) geometry by means of the deformation is not an axiomatizable geometry, in general, i.e. it cannot be constructed of any blocks.

Let us demonstrate this fact in a simple model. Let we have only one kind of cubic plasticine blocks. These blocks are painted by a red paint, in order one can distinguish boundaries of blocks in a building. Let us construct some building of these blocks, for instance, a cube. Let us deform this cube in an arbitrary way, for instance, into a circular cylinder. After such a deformation all cubic blocks, constituting the cube will be deformed. The deformation will be different for different blocks, and they cannot be used for construction of a new building. Of course, one can reconstruct the cylinder, but this cylinder will be reconstructed of blocks, having different shapes, which they have been obtained as a result of the deformation. These blocks are not suitable for construction of another buildings. This model shows, how a deformation destroys the axiomatizability of the axiomatizable geometry.

Formally the axiomatizability is destroyed as follows. In any axiomatizable geometry the equivalence relation is transitive. This transitivity is necessary, in order that any deduction leads to a definite result. Deformation destroys the transitivity of the equivalence relation, and the geometry becomes to be nonaxiomatizable. Let us demonstrate this in the example of two vector equivalence. In the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ the equivalence of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined as follows. Vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent ($\mathbf{P}_0\mathbf{P}_1 \mathrm{eqv}\ \mathbf{Q}_0\mathbf{Q}_1$), if vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are in parallel ($\mathbf{P}_0\mathbf{P}_1 \uparrow\uparrow \mathbf{Q}_0\mathbf{Q}_1$) and their lengths $|\mathbf{P}_0\mathbf{P}_1|$ and $|\mathbf{Q}_0\mathbf{Q}_1|$ are equal. Mathematically these two conditions are written in the form

$$(\mathbf{P}_0\mathbf{P}_1 \uparrow\uparrow \mathbf{Q}_0\mathbf{Q}_1): \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \tag{1.3}$$

$$|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|, \qquad |\mathbf{P}_0\mathbf{P}_1| = \sqrt{2\sigma(P_0, P_1)} \tag{1.4}$$

where $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ is the scalar product of two vectors, defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) \tag{1.5}$$

Here σ is the world function of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. The length $|\mathbf{PQ}|$ of vector $\mathbf{PQ}$ is defined by the relation

$$|\mathbf{PQ}| = \rho(P, Q) = \sqrt{2\sigma(P, Q)} \tag{1.6}$$

Using relations (1.3) - (1.6), one can write the equivalence condition in the form

$$\begin{aligned} \mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1 \quad &: \quad \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) \\ &= \quad 2\sigma(P_0, P_1) \wedge \sigma(P_0, P_1) = \sigma(Q_0, Q_1) \end{aligned} \tag{1.7}$$

The equivalence relation is used in any physical geometry. The definition of equivalence (1.7) is a satisfactory geometrical definition, because it does not contain a reference to the dimension of the space and to the coordinate system. It contains only points P_0, P_1, Q_0, Q_1, determining vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ and distances (world functions) between these points. The definition of equivalence (1.7) coincides with the conventional definition of two vectors equivalence in the proper Euclidean geometry. If one fixes points P_0, P_1, Q_0 in the relations (1.7) and solve these equations with respect to the point Q_1, one finds that these equations always have one and only one solution. This statement follows from the properties of the world function of the proper Euclidean geometry. It means that the proper Euclidean geometry is single-variant with respect any pairs of its points. It means also, that the equivalence relation is transitive in the proper Euclidean geometry. By definition the transitivity of the equivalence relation means, that

$$\text{if} \quad \mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1 \wedge \mathbf{Q}_0\mathbf{Q}_1\mathrm{eqv}\mathbf{R}_0\mathbf{R}_1, \quad \text{then} \quad \mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{R}_0\mathbf{R}_1 \tag{1.8}$$

In the arbitrary physical geometry the equivalence relation has the same form (1.7) with another world function σ, satisfying the constraints

$$\sigma: \quad \Omega \times \Omega \to \mathbb{R}, \qquad \sigma(P, P) = 0, \qquad \forall P, Q \in \Omega \tag{1.9}$$

Here Ω is the set of all points, where the geometry is given.

In the case of arbitrary world function one cannot guarantee, that equations (1.7) have always a unique solution. There may be many solutions. In this case one has a multivariant geometry. There may be no solution. In this case one has a zero-variant (discriminating) geometry. In both cases the equivalence relation is intransitive, and geometry is nonaxiomatizable. There may be such a situation, that the geometry is multivariant with respect to some points and vectors, and it is zero-variant with respect to other points and vectors. Such a geometry will be also qualified as a multivariant geometry.

One sets conventionally, that the world function of the space-time is symmetric

$$\sigma(P,Q)=\sigma(Q,P), \qquad \forall P,Q\in\Omega \tag{1.10}$$

This condition means that the future and the past are geometrically equivalent. However, the physical geometry can be constructed for asymmetric world function Σ [4]

$$\Sigma(P,Q)=G(P,Q)+A(P,Q), \tag{1.11}$$

$$G(P,Q)=G(Q,P), \qquad A(P,Q)=-A(Q,P) \tag{1.12}$$

The time is considered as an attribute of the event space (space-time). The time arrow can be taken into account in the technique of asymmetric space-time geometry.

The asymmetric geometry with asymmetric world function may appear in the microcosm, however, its application is especially interesting in the cosmology, where the future and the past of our universe may appear to be not equal. Besides, the gravitational law in asymmetric space-time geometry distinguishes from the gravitational law in the symmetric one. Maybe, reasonable supposition on asymmetry of the space-time geometry will be able to explain deflection of astronomical observations from predictions of the general relativity. In this case the invention of the dark matter may appear to be needless. However, such a possibility is not yet investigated properly.

The granular space-time geometry $\mathcal{G}_{\mathrm{g}}$, given on the manifold of Minkowski is described approximately by the world function σ_{g}

$$\sigma_{\mathrm{g}}=\sigma_{\mathrm{M}}+\lambda_0^2\begin{cases}\operatorname{sgn}(\sigma_{\mathrm{M}}) & \text{if} \quad |\sigma_{\mathrm{M}}|>\sigma_0\\ \frac{\sigma_{\mathrm{M}}}{\sigma_0} & \text{if} \quad |\sigma_{\mathrm{M}}|\leq\sigma_0\end{cases}, \qquad \lambda_0^2,\sigma_0=\text{const}\geq 0 \tag{1.13}$$

where σ_{M} is the world function of the geometry $\mathcal{G}_{\mathrm{M}}$ of Minkowski, λ_0 is a elementary length. The world function σ_{M} of the Minkowski geometry $\mathcal{G}_{\mathrm{M}}$ is Lorentz-invariant, and the world function σ_{g} of the granular geometry $\mathcal{G}_{\mathrm{g}}$ is Lorentz-invariant also, because it is a function of σ_{M}. If $\sigma_0=0$, the geometry $\mathcal{G}_{\mathrm{g}}$ is discrete, although it is given on the continuous manifold of Minkowski. Indeed, if $\sigma_0=0$, in the geometry $\mathcal{G}_{\mathrm{g}}$ there are no close points separated by a distance less, than $\sqrt{2}\lambda_0$. This statement follows from (1.13). Discrete Lorentz-invariant geometry on a continuous manifold! This fact seems to be very unexpected at the conventional approach to geometry, where discreteness of geometry depends on the structure of the point set Ω, where the geometry is given, and where the geometry is formulated in some coordinate system.

In the physical geometry a discreteness and a continuity of the geometry are determined by the world function and only by the world function, whereas the structure of the point set Ω is important only in such extent, in which it influences on the world function.

Granularity of the geometry $\mathcal{G}_{\rm g}$ becomes more clear, if one considers the relative density $\rho(\sigma_{\rm g}) = \frac{d\sigma_{\rm M}(\sigma_{\rm g})}{d\sigma_{\rm g}}$ of points in $\mathcal{G}_{\rm M}$ with respect to the density of points in $\mathcal{G}_{\rm g}$. Such a density can be introduced, if both geometries $\mathcal{G}_{\rm g}$ and $\mathcal{G}_{\rm M}$ are uniform, and $\sigma_{\rm g}$ is a function of $\sigma_{\rm M}$. One obtains from (1.13)

$$\rho(\sigma_{\rm g}) = \frac{d\sigma_{\rm M}(\sigma_{\rm g})}{d\sigma_{\rm g}} = \begin{cases} 1 & \text{if} \quad |\sigma_{\rm g}| > \sigma_0 + \lambda_0^2 \\ \frac{\sigma_0}{\sigma_0+\lambda_0^2} & \text{if} \quad |\sigma_{\rm g}| \leq \sigma_0 + \lambda_0^2 \end{cases} \tag{1.14}$$

One can see from (1.14), that at $\sigma_0 = 0$ there is no points in the interval $\sigma_g \in \left(-\lambda_0^2, \lambda_0^2\right)$. It means, that in the case $\sigma_0 = 0$ the space-time geometry is discrete. If $\sigma_0 \neq 0$, one can see from (1.14), that the relative density of points in the interval $\sigma_g \in \left(-\lambda_0^2 - \sigma_0, \lambda_0^2 + \sigma_0\right)$ is less, than unity but it is not equal to zero. We have some intermediate situation between the continuity (when $\rho = 1$) and discreteness (when $\rho = 0$). Such a situation is treated as a granularity. Note, that the geometry $\mathcal{G}_{\rm M}$ may be considered as a special case of the granular geometry.

In the granular space-time geometry the elementary particle is described by its skeleton $\mathcal{P}_n = \{P_0, P_1, .., P_n\}$, consisting of $n+1$ points. The pointlike particle is described by the skeleton $\mathcal{P}_1 = \{P_0, P_1\}$, consisting of two points, or by the vector $\mathbf{P}_0\mathbf{P}_1$. The vector $\mathbf{P}_0\mathbf{P}_1$ represents the momentum of the pointlike particle, whereas its length $|\mathbf{P}_0\mathbf{P}_1| = \mu$ is the geometrical mass of the pointlike particle. The geometrical mass μ is connected with its usual mass m by means of the relation

$$m = b\mu \tag{1.15}$$

where b is some universal constant.

Evolution of the elementary particle is described by the world chain, consisting of connected skeletons $...\mathcal{P}_n^{(0)}, \mathcal{P}_n^{(1)}, ..., \mathcal{P}_n^{(s)}...$

$$\mathcal{P}_n^{(s)} = \left\{P_0^{(s)}, P_1^{(s)}, ..P_n^{(s)}\right\}, \qquad s = ...0, 1, ... \tag{1.16}$$

The adjacent skeletons $\mathcal{P}_n^{(s)}, \mathcal{P}_n^{(s+1)}$ of the chain are connected by the relations $P_1^{(s)} = P_0^{(s+1)}$, $s = ...0, 1, ...$ The vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)} = \mathbf{P}_0^{(s)}\mathbf{P}_0^{(s+1)}$ is the leading vector, which determined the world chain direction.

Dynamics of free elementary particle is determined by the relations

$$\mathcal{P}_n^{(s)}\text{eqv}\mathcal{P}_n^{(s+1)}: \qquad \mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\text{eqv}\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}, \qquad i, k = 0, 1, ..n; \qquad s = ...0, 1, ... \tag{1.17}$$

which describe equivalence of adjacent skeletons.

Thus, dynamics of a free elementary particle is described by a system of algebraic equations (1.17). Specific of dynamics depends on the elementary particle structure (disposition of particles inside the skeleton) and on the space-time geometry.

In the simplest case, when the space-time geometry is the 5-dimensional Kaluza-Klein geometry [5, 6], the dynamic equations (1.17) for the pointlike particle are reduced to conventional differential dynamic equations, describing motion of the charged pointlike particle in the given electromagnetic and gravitational fields. Thus, dynamic equations (1.17) can be considered as a generalization of classical differential dynamic equations for the particle motion on the case of the granular space-time geometry. It is a very important fact, which shows, that description of free particles by means of a world chain, consisting of connected skeletons, is simply a generalization of conventional relativistic dynamics of particles, which do not interact between themselves. This generalization does not contain any new principles. It is simply a generalization of the particle dynamics onto the case of the granular space-time geometry.

Formally it is a dynamics of free particles, moving in a very deformed and curved space-time geometry. However, dynamics of free particles can be considered as a motion of particles, interacting with some given fields (electromagnetic, gravitational and others) in the space-time geometry of Minkowski. In other words, the motion of a charged particle in the given gravitational and electromagnetic and some other fields can be considered as a motion of a free particle in the granular space-time geometry. This property was known for the case the Riemannian space-time geometry of Kaluza-Klein [5, 6]. Now this property is generalized on the case of arbitrary granular space-time geometry.

Note, that according to definition of dynamics (1.17) all vectors of the skeleton are transported along the chain in parallel with itself (translation), i.e. without a rotation. It means a stronger definition of a free particle, than that, which is used conventionally. Usually the rotating particle, moving in the absence of external fields, is considered to be free, although some parts of the particle move with acceleration, generated by the rotation. In the free motion, defined by the relation (1.17), all points of the skeleton move without an acceleration and all vectors of the skeleton do not rotate. The particle rotation appears as a special kind of motion with superlight speed (with space-like leading vector of the world chain). This property seems rather unexpected from conventional viewpoint [8, 9, 7]. However, this property may take place in some special form of the granular geometry. Then the composite particle rotation is realized in the helical shape of the world chain.

It is quite reasonable, that the dynamic equations in the granular space-time geometry cannot be differential equations. The dynamic equations can be only difference equations.

Let the elementary length λ_0 have the form

$$\lambda_0^2 = \frac{\hbar}{2bc} \tag{1.18}$$

where $\hbar$ is the quantum constant, c is the speed of the light and b is the universal constant, defined by (1.15). Let the constant σ_0 in (1.14) be small enough. Then the motion of a pointlike particle in the granular space-time geometry (1.14) appears to be multivariant (stochastic). Statistical description of this multivariant particle motion coincides with the quantum description in terms of the Schrödinger equation [3]. Quantum constant appears in the description via elementary length (1.18), which is a parameter of the granular space-time geometry.

2 Difference dynamic equations as a generalization of differential dynamic equations for pointlike particle in the Riemannian space-time

Let us show that dynamic difference equations (1.17) for the pointlike particle, described by the vector $\mathbf{P}_s\mathbf{P}_{s+1}$ can be transformed to dynamic equations for a geodesic, if the the space-time geometry is Riemannian. In the Kaluza-Klein space-time geometry, which is a 5-dimensional Riemannian geometry, the geodesic describes the pointlike charged particle motion in the given gravitational and electromagnetic fields.

At first, we consider the case of the Riemannian space-time without external fields, i.e. the pseudo-Euclidean space of index 1. For the pointlike particle the dynamic equations (1.17) take the form

$$(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2}) = |\mathbf{P}_s\mathbf{P}_{s+1}| \cdot |\mathbf{P}_{s+1}\mathbf{P}_{s+2}|, \qquad s = ...0, 1, ... \tag{2.1}$$

$$|\mathbf{P}_s\mathbf{P}_{s+1}|^2 = |\mathbf{P}_{s+1}\mathbf{P}_{s+2}|^2, \qquad s = ...0, 1, ... \tag{2.2}$$

Using relation (1.5) for the scalar product in the relation (2.1), one obtains from (2.2), (2.1) for the case $s = 0$

$$\sigma(P_0, P_1) = \sigma(P_1, P_2), \qquad \sigma(P_0, P_2) = 4\sigma(P_0, P_1) \tag{2.3}$$

Let us choose the coordinate system in such a way, that

$$\begin{aligned} P_0 &= \{0, 0, , ...0\}, \qquad P_1 = \left\{x^0, x^1, ..., x^n\right\} \\ P_2 &= \left\{2x^0 + \alpha^0, 2x^1 + \alpha^1, ..., 2x^n + \alpha^n\right\} \end{aligned} \tag{2.4}$$

$$\mathbf{P}_0\mathbf{P}_1 = \left\{x^0, x^1, ..., x^n\right\}, \qquad \mathbf{P}_1\mathbf{P}_2 = \left\{x^0 + \alpha^0, x^1 + \alpha^1, ..., x^n + \alpha^n\right\} \tag{2.5}$$

$$\mathbf{P}_0\mathbf{P}_2 = \left\{2x^0 + \alpha^0, 2x^1 + \alpha^1, ..., 2x^n + \alpha^n\right\} \tag{2.6}$$

Let us introduce designations

$$x = \left\{x^0, \mathbf{x}\right\} = \left\{x^0, x^1, ..., x^n\right\}, \qquad \alpha = \left\{\alpha^0, \boldsymbol{\alpha}\right\} = \left\{\alpha^0, \alpha^1, ..., \alpha^n\right\} \tag{2.7}$$

where x and α are $(n + 1)$-vectors, whereas $\mathbf{x}$ and $\boldsymbol{\alpha}$ are n-vectors. The equations (2.3) are reduced to the relations

$$(x.\alpha) = x^0\alpha^0 - \mathbf{x}\boldsymbol{\alpha} \equiv x^0\alpha^0 - \sum_{\mu=1}^{\mu=n} x^\mu\alpha^\mu = 0 \tag{2.8}$$

$$(\alpha.\alpha) = \alpha^0\alpha^0 - \boldsymbol{\alpha}\boldsymbol{\alpha} \equiv \alpha^0\alpha^0 - \sum_{\mu=1}^{\mu=n} \alpha^\mu\alpha^\mu = 0 \tag{2.9}$$

If the vector $\mathbf{P}_0\mathbf{P}_1$ is timelike ($|\mathbf{P}_0\mathbf{P}_1|^2 > 0$), then there is the unique solution $\alpha = \{\alpha^0, \boldsymbol{\alpha}\} = 0$ of equations (2.8), (2.9), and the points P_0, P_1, P_2 lie on the same timelike straight (geodesic). If the vector $\mathbf{P}_0\mathbf{P}_1$ is null ($|\mathbf{P}_0\mathbf{P}_1|^2 = 0$), then $\alpha = kx = \{k\alpha^0, k\mathbf{x}\}$, where k is arbitrary real number, and the points P_0, P_1, P_2 lie on the same null straight (geodesic). If the vector $\mathbf{P}_0\mathbf{P}_1$ is spacelike ($|\mathbf{P}_0\mathbf{P}_1|^2 < 0$), the solution is not unique. It has the form

$$\alpha = \{a, a\mathbf{n}\}, \qquad \mathbf{n}^2 = 1, \qquad \mathbf{nx} = -x^0 \tag{2.10}$$

where a is an arbitrary real number, $\mathbf{n}$ is a unite n-vector. The points P_0, P_1, P_2 lie on the same spacelike straight (geodesic), only if $a = 0$.

In general case $a \neq 0$, the spacelike vector $\mathbf{P}_1\mathbf{P}_2$ is multivariant, if the vector $\mathbf{P}_0\mathbf{P}_1$ is spacelike. As far as the quantity a may be infinitely large, the world chain with the spacelike leading vector $\mathbf{P}_s\mathbf{P}_{s+1}$ appears to be impossible. This fact is known. At the conventional approach it is postulated. In the dynamic difference equations (1.17) the impossibility of the spacelike leading vector is a corollary of dynamic equations.

Let us consider the case, when the space-time geometry is the pseudo-Riemannian geometry of index 1. Then supposition, that the vector $\mathbf{P}_0\mathbf{P}_1$ is timelike, and the points P_0, P_1, P_2 lie on the same timelike geodesic, is compatible with equations (2.3). According to (2.3) we have

$$\rho(P_0, P_1) + \rho(P_1, P_2) = \rho(P_0, P_2), \qquad \rho(P_1, P_2) = \sqrt{2\sigma(P_1, P_2)} \tag{2.11}$$

The timelike geodesic in the pseudo-Riemannian space of index 1 is the longest line. In other words, in the pseudo-Riemannian space of index 1 for any three points P_0, P_1, P_2, divided by timelike intervals "the triangle axiom" takes place

$$\rho(P_0, P_1) + \rho(P_1, P_2) \leq \rho(P_0, P_2) \tag{2.12}$$

If one supposes, that the point P_1 does not belong to the geodesic $\mathcal{L}_{P_0P_2}$, passing through points P_0 and P_2, then

$$P_1 \notin \mathcal{L}_{P_0P_2}: \qquad \rho(P_0, P_1) + \rho(P_1, P_2) < \rho(P_0, P_2) \tag{2.13}$$

Condition (2.13) is not compatible with dynamic equations (2.3) for timelike vector $\mathbf{P}_0\mathbf{P}_2$. Hence, for $|\mathbf{P}_0\mathbf{P}_2|^2 > 0$ the points P_0, P_1, P_2 lie on the same timelike geodesic. This statement is valid for any three points P_s, P_{s+1}, P_{s+2}, $s = ...0, 1, ...$

Thus, if the space-time is the pseudo-Riemannian space of index 1, the dynamic equations (1.17) for pointlike particle describe timelike geodesics of the space-time, if the leading vector $\mathbf{P}_s\mathbf{P}_{s+1}$ is timelike. On the other side, timelike geodesics in the 5-dimensional Kaluza-Klein space-time describe motion of charged pointlike particle in the given gravitational and electromagnetic fields. Thus, the dynamic equations (1.17) are a generalization of conventional relativistic dynamics on the granular space-time geometry.

3 Composite particles

If the skeleton $\mathcal{P}_1 = \{P_0, P_1\}$ consists of two points, it describes a pointlike particle. If the skeleton $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$ consists of more, than two points ($n \geq 2$) it describes a composite particle. Let the space-time dimension be N. Then the number of coordinates, describing evolution of the skeleton $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$, is equal to nN, whereas the number of dynamic equations (1.17) describing the skeleton evolution is equal to $n(n+1)$. If complexity n of the particle composition increases, the number $n(n+1)$ of dynamic equations increases faster, than the number of dependent dynamic variables nN. At $n \geq N$ the number of dynamic equations becomes larger, than the number of dependent variables.

For pointlike particle ($n = 1$) the number of dynamic equations is equal to two, whereas the number of dynamic variables is equal to five in the 5D Kaluza-Klein space-time geometry, and it is equal to four in the 4D space-time geometry of Minkowski. In both cases the impossibility of spacelike world chain is conditioned by the fact, that the number of dynamic variables is larger, than the number of dynamic equations. One should expect, that for sufficiently complex particles with sufficiently large n, the world chains with spacelike leading vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ appear to be possible. In order that the world chain with the spacelike leading vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ be observable, the world chain is to have the shape of a helix with timelike axis.

Motion of elementary particles, which are not pointlike, is not yet investigated properly. There is only some information on the Dirac particle, whose skeleton consists of three points ($n = 2$), and the leading vector is spacelike [7]. In this case the world chain is a spacelike helix with the timelike axis. Such a spacelike helix cannot exist in the granular geometry (1.13). However, if the world function (1.13) is modified slightly at small distances $\sigma_{\rm g} \rightarrow \sigma_{\rm gm}$

$$\sigma_{\rm gm} = \sigma_{\rm M} + \lambda_0^2 \left\{ \begin{array}{lll} \operatorname{sgn}(\sigma_{\rm M}) & \text{if} & |\sigma_{\rm M}| > \sigma_0 \\ \left(\frac{\sigma_{\rm M}}{\sigma_0}\right)^3 & \text{if} & |\sigma_{\rm M}| \leq \sigma_0 \end{array} \right. , \qquad \lambda_0^2, \sigma_0 = \text{const} \geq 0 \tag{3.1}$$

such a spacelike helix becomes possible. The spacelike helix is possible also for other space-time geometries, where the the world function in interval $(-\sigma_0, \sigma_0)$ has the form $f(\sigma_{\rm M}/\sigma_0)$, $|\sigma_{\rm M}| < |\sigma_0|$. It is to satisfy the condition $|f(\sigma_{\rm M}/\sigma_0)| < |\sigma_{\rm M}/\sigma_0|$.

Identification of the elementary particle with the helical world chain and the Dirac particle is founded on the following fact. In the classical limit the Dirac equation for a free particle describes a classical dynamic system having 10 degrees of freedom. Solution of dynamic equations leads to a helical world line with the timelike axis [8]. It is not quite clear, whether this helix is spacelike, or timelike, because the internal degrees of freedom, responsible for circular motion, are described nonrelativistically (i.e. incorrectly), although external degrees of freedom are described relativistically [9].

Note, that dynamic equations (1.17) could not describe the skeleton rotation directly. They describe only the parallel transport of all vectors $\mathbf{P}_i\mathbf{P}_k$, $i, k = 0, 1, ..n$. The phenomenon of rotation appears only in the form of helical world chain, and this circumstance generates restricted values of spin, which is supposed to be connected with the composite particle rotation.

Supposition, that the elementary particle structure is determined by mutual disposition of points in its skeleton seems to be natural and reasonable. Such parameters of the elementary particle as mass and electric charge are geometrized already for pointlike particle. In the case of pointlike particle its mass m is defined by the relation (1.15), whereas the electric charge is determined by the projection of the particle momentum onto the direction, chosen by the space-time compactification of the Kaluza-Klein space-time. One should expect, that in the case of composite particles all parameters of the particle, including the mass and the electric charge, will be determined by disposition of points in the particle skeleton. Such an idea of complete geometrization of the elementary particles, when dynamics and parameters of the elementary particle are determined completely by the particle skeleton, seems to be attractive. At such an approach the particle skeleton is the only characteristic of the elementary particle. Such a description does not contain wave functions, branes, strings and other exotic matters, which are very far from the space-time geometry. The observed symmetries of elementary particles may be interpreted as symmetries of the points inside the particle skeleton.

4 Multivariance and discrimination

The granular geometry has two important properties, which absent in the conventional description of the space-time as a Riemannian space. The multivariance is conditioned by the fact, that the equivalence relation (1.7) has many solutions. Multivariance of timelike vectors is introduced in granular geometries (1.13) and (3.1) by the term with coefficient λ_0^2, where λ_0 is some elementary length. Multivariance of timelike vectors is responsible for quantum effects. Multivariance of spacelike vectors does not depend on λ_0. It is connected by the fact that the space-time geometry is close to the pseudo-Riemannian geometry of index 1.

Discrimination (or zero-variance) is conditioned by the fact, that the equivalence relation (1.7) has no solutions. The discrimination effect is responsible for discrete values of the elementary particle parameters. The most clear manifestation of this effect appears at the compactification of the Kaluza-Klein geometry. Compactification means that the coordinate x^5, responsible for the electric charge, can change inside some finite interval $x^5 \in (-L, L]$, and the end points of this interval are identical. It means that all physical quantities (and wave functions) are periodic function of x^5 with the period $2L$. Compactification of the Kaluza-Klein geometry means a modification of its topology. However, in the physical geometry the topology is determined completely by the world function. One cannot change the topology independently of a corresponding change of the world function.

In the framework of the quantum paradigm the eigenvalues of the momentum operator $p_5 = -i\hbar\partial/\partial x^5$ are $(h/2L)$-fold, and there are many geodesics, connecting any two points of the space-time. The electric charge appears to be multiple to some elementary charge, and this property is conditioned by quantum principles. At the conventional approach the world function is defined as a derivative quantity

$$\sigma(P,Q)=\frac{1}{2}\left(\int\limits_{\mathcal{L}_{PQ}}\sqrt{g_{ik}(x)\,dx^i dx^k}\right)^2 \tag{4.1}$$

where $\mathcal{L}_{PQ}$ is a geodesic, connecting the points P and Q. As far as there are many geodesic, connecting two points P and Q, the world function $\sigma(P,Q)$ appears to be multivalued. At the conventional approach, where the world function is a derivative (not fundamental) quantity, it may be multivalued.

In the physical geometry, where the world function is a fundamental quantity, it must be single-valued. If one defines the world function using the relation (4.1), it is necessary to use only one geodesic, removing another ones. Using the shortest geodesic, one obtains the single-valued world function. The situation looks as follows. One modifies the world function, and compactification is a corollary of this modification. Dynamic equations (1.17) for a pointlike particle impose restrictions on the electric charge of the pointlike particle [10]. These constraints have nothing to do with the quantum principles. They are purely geometrical constraints, conditioned by zero-variance of the space-time geometry compactification.

5 Transformation of space-time geometry to a standard geometry by means of introduction of geometric force fields.

The difference dynamic equations (1.17) can be written in the form, which is close to the conventional description in the Kaluza-Klein space-time. Let σ_{K_0} be the world function in the space-time geometry $\mathcal{G}_{\mathrm{K}_0}$. The geometry $\mathcal{G}_{\mathrm{K}_0}$ is the 5D pseudo-Euclidean geometry of the index 1 with the compactificated coordinate x^5. In other words, the space-time geometry $\mathcal{G}_{\mathrm{K}_0}$ is the Kaluza-Klein geometry with vanishing gravitational and electromagnetic fields. Let us represent the world function σ of the space-time geometry $\mathcal{G}$ in the form

$$\sigma(P,Q)=\sigma_{\mathrm{K}_0}(P,Q)+d(P,Q) \tag{5.1}$$

where the function d describes the difference between the true world function σ of the real space-time geometry and the world function σ_{K_0} of the standard geometry $\mathcal{G}_{\mathrm{K}_0}$, where the description will be produced. Then one obtains

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)=(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{K}_0}+d(P_0,Q_1)+d(P_1,Q_0)-d(P_0,Q_0)-d(P_1,Q_1) \tag{5.2}$$

$$|\mathbf{P}_0\mathbf{P}_1|^2=|\mathbf{P}_0\mathbf{P}_1|^2_{\mathrm{K}_0}+2d(P_0,P_1) \tag{5.3}$$

where index "K_0" means, that the corresponding quantities are calculated in the geometry $\mathcal{G}_{\mathrm{K}_0}$ by means of the world function σ_{K_0}.

By means of (5.2), (5.3) the dynamic equations (1.17) can be written in the form

$$\left(\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}.\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}\right)_{\mathrm{K}_0}-\left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|_{\mathrm{K}_0}^2=w\left(P_i^{(s)},P_k^{(s)},P_i^{(s+1)},P_k^{(s+1)}\right),\qquad i,k=0,1,...n \tag{5.4}$$

$$\left|\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}\right|_{\mathrm{K}_0}^2-\left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|_{\mathrm{K}_0}^2=2d\left(P_i^{(s)},P_k^{(s)}\right)-2d\left(P_i^{(s+1)},P_k^{(s+1)}\right),\qquad i,k=0,1,...n \tag{5.5}$$

where

$$\begin{aligned} w\left(P_i^{(s)},P_k^{(s)},P_i^{(s+1)},P_k^{(s+1)}\right) &= 2d\left(P_i^{(s)},P_k^{(s)}\right)-d\left(P_i^{(s)},P_k^{(s+1)}\right)\\ &-d\left(P_k^{(s)},P_i^{(s+1)}\right)+d\left(P_i^{(s)},P_i^{(s+1)}\right)\\ &+d\left(P_k^{(s)},P_k^{(s+1)}\right) \end{aligned} \tag{5.6}$$

Equations (5.4), (5.5) are dynamic difference equations, written in the geometry $\mathcal{G}_{\mathrm{K}_0}$. Rhs of these equations can be interpreted as some geometric force fields, generated by the fact that the space-time geometry $\mathcal{G}$ is described in terms of some standard geometry $\mathcal{G}_{\mathrm{K}_0}$. These force fields describe deflection of the granular geometry from the Kaluza-Klein one. Such a possibility is used at the description of the gravitational field, which can be described as generated by the curvature of the curved space-time, or as a gravitational field in the space-time geometry of Minkowski. In dynamic equations (5.4), (5.5) such a possibility is realized for arbitrary granular space-time geometry.

Evolution of the leading vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ is of most interest. These equations are obtained from equations (5.4), (5.5) at $i=0,k=1$. One obtains from equations (5.4), (5.5)

$$\left|\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right|_{\mathrm{K}_0}^2-\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{K}_0}^2=2d\left(P_0^{(s)},P_1^{(s)}\right)-2d\left(P_1^{(s)},P_1^{(s+1)}\right) \tag{5.7}$$

$$\begin{aligned} &\left(\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}.\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right)_{\mathrm{K}_0}-\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{K}_0}^2\\ =\ &3d\left(P_0^{(s)},P_1^{(s)}\right)-d\left(P_0^{(s)},P_1^{(s+1)}\right)+d\left(P_1^{(s)},P_1^{(s+1)}\right) \end{aligned} \tag{5.8}$$

where one uses, that $P_1^{(s)}=P_0^{(s+1)}$.

In the case, when the space-time is uniform, and the function

$$d\left(P,Q\right)=D\left(\sigma_{\mathrm{K}_0}\left(P,Q\right)\right) \tag{5.9}$$

the equations (5.7), (5.8) take the from

$$\left|\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right|_{\mathrm{K}_0}^2-\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{K}_0}^2=0 \tag{5.10}$$

$$\left(\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}.\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right)_{\mathrm{K}_0}-\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{K}_0}^2=4d\left(P_0^{(s)},P_1^{(s)}\right)-d\left(P_0^{(s)},P_1^{(s+1)}\right) \tag{5.11}$$

In the case, when the leading vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ is timelike, one can introduce the angle $\phi_{01}^{(s)}$ between the vectors $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ and $\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}$ in the standard geometry $\mathcal{G}_{\mathrm{K}_0}$. By means of (5.10) it is defined by the relation

$$\cosh\phi_{01}^{(s)}=\frac{\left(\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}.\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right)_{\mathrm{K}_0}}{\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{K}_0}^2} \tag{5.12}$$

Then in the uniform geometry $\mathcal{G}$ the equation (5.11) has the form

$$\sinh\frac{\phi_{01}^{(s)}}{2}=\frac{\sqrt{4d\left(P_0^{(s)},P_1^{(s)}\right)-d\left(P_0^{(s)},P_1^{(s+1)}\right)}}{\sqrt{2}\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{K}_0}} \tag{5.13}$$

Thus, relativistic dynamics of particles can be generalized on the case of the granular space-time geometry.

6 Concluding remarks

The presented conception is completely orthodox, because it does not use any new principles. Introducing into consideration nonaxiomatizable geometries, one removes only incompleteness in description of the space-time geometry. Orthodoxy of the conception evidences in behalf of this conception.

In general, the geometric dynamics (1.17) is a classical dynamics in the granular space-time geometry. The granularity of the space-time generates two new properties, which are absent in the axiomatizable geometries: (1) multivariance, which is responsible for quantum properties, (2) zero-variance (discrimination mechanism), which is responsible for discreteness of the elementary particles parameters. The multivariance of the granular space-time geometry can be taken into account by means of the statistical description. Quantum theory can imitate multivariance (and statistical description) on the level of dynamics, but it cannot imitate the zero-variance (discrimination mechanism). As a result the contemporary theory of elementary particles has no key to explanation of discrete parameters of the elementary particles.

Let us note in conclusion that we did not use any new hypotheses. Our conception is not a conceptually new theory. It is simply a generalization of the classical relativistic dynamics onto the case of granular space-time geometry, which was ignored by contemporary mathematicians (and physicists). Using granular space-time, we do not use any new hypotheses or principles. We have overcame simply the preconception, that the space-time geometry may be only axiomatizable. Besides, we reduce the number of principles in the theory in the sense, that the quantum principles are not used. Quantum effects are described now by multivariance of the granular space-time geometry. Impossibility of the spacelike world chain appears to be only a corollary of the fact, that such a world chain cannot be observed, but not a matter of principle.

The generalization of classical physics on the case of the granular space-time geometry is not yet accomplished in the sense, that only generalization of dynamic equations for the particle motion in the given external fields has been obtained. Another part of the classical physics, which describes influence of the matter on the space-time geometry (gravitation equations and Maxwell equations) has not been generalized yet on the case of the granular space-time geometry.

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