Hydrodynamical interpretation of quantum mechanics: the momentum distribution.

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Abstract

The quantum mechanics is considered to be a partial case of the stochastic system dynamics. It is shown that the wave function describes the state of statistically averaged system $\langle \mathcal{S}_{\text{st}} \rangle$, but not that of the individual stochastic system $\mathcal{S}_{\text{st}}$. It is a common practice to think that such a construction of quantum mechanics contains hidden variables, and it is incompatible with the von Neumann's theorem on hidden variables. It is shown that the original conditions of the von Neumann's theorem are not satisfied. In particular, the quantum mechanics cannot describe the particle momentum distribution. The distribution $w(\mathbf{p}) = |\psi_{\mathbf{p}}|^2$ is not a particle momentum distribution at the state ψ, because it cannot be attributed to a wave function. It is closer to the mean momentum distribution, although the two distributions do not coincide exactly.

1 Introduction

In the present paper we consider the quantum mechanics as a special case of the stochastic systems dynamics [1]. The dynamics of stochastic systems describes any stochastic systems, but not only the quantum ones, and dynamics of stochastic systems cannot be founded on the principles of quantum mechanics, which are specific only for description of the nonrelativistic quantum phenomena. In the stochastic system dynamics the quantum principles appear only on the dynamical level as *dynamic properties* of quantum systems. The stochastic system dynamics is a *usual statistical conception*, which contains two sorts of objects: (1) the individual stochastic system $\mathcal{S}_{\rm st}$ and (2) the statistical average system $\langle\mathcal{S}_{\rm st}\rangle$, which is the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$ of stochastic systems $\mathcal{S}_{\rm st}$ normalized to one system. Dynamic equations describe the state evolution of $\langle\mathcal{S}_{\rm st}\rangle$ and have no relation to $\mathcal{S}_{\rm st}$.

The conventional quantum mechanics is also a statistical conception, but it is an exotic statistical conception, because it contains only one sort of objects: the quantum system $\mathcal{S}_{\rm q}$. What is the quantum system $\mathcal{S}_{\rm q}$? Is it $\mathcal{S}_{\rm st}$, or $\langle\mathcal{S}_{\rm st}\rangle$? Different authors answer this question differently. The conventional (Copenhagen) interpretation of quantum mechanics supposes that $\mathcal{S}_{\rm q}$ is an individual system (i.e. $\mathcal{S}_{\rm st}$). But all authors consider that the quantum mechanical conception contains only one sort of objects: $\mathcal{S}_{\rm q}$. Dynamic equations describe the state evolution of $\mathcal{S}_{\rm q}$.

Model conception of quantum phenomena (MCQP) is the name of the quantum mechanics considered to be a special case of the stochastic systems dynamics. The conventional quantum mechanics will be referred to as the axiomatic conception of quantum phenomena (ACQP). These abbreviations will be used for brevity.

In MCQP the hydrodynamic interpretation of quantum mechanics is considered to be a primary interpretation, and the conventional interpretation in terms of the wave function appears to be a secondary (derivative) interpretation. Such an approach has a logical foundation, and we shall present it. Besides, MCQP and the hydrodynamic interpretation of quantum mechanics can be easily expanded onto the relativistic quantum phenomena, because MCQP takes into account the quantum specific only on the dynamical level. To describe the relativistic phenomena, it is sufficient to consider relativistic Lagrangian of $\langle\mathcal{S}_{\rm st}\rangle$.

In ACQP the quantum specific is taken into account on the conceptual level. The wave function (the main object of ACQP) is nonrelativistic, and transition to the relativistic phenomena description is connected with a revision of the conceptual structure of ACQP. Attempts of unification of the nonrelativistic QM technique with the relativity principles failed. At any rate, the expansion of conventional interpretation in terms of the wave function meets with difficulties. As a result of these difficulties the collision of relativistic particles is described only in terms of the S-matrix, because the detailed description of the collision process in terms of the wave function appears to be impossible. In particular, one cannot describe in terms of the wave function the details of the pair production mechanism [2].

Note that MCQP may be considered to be a theory with hidden variables. It is believed that the quantum theory with hidden variables is impossible, because according to the von Neumann's theorem on hidden variables [3] (chp.4, sec. 1,2) it is incompatible with the conventional conception of quantum mechanics. The mathematical formalism of ACQP and, in particular, the theorem of von Neumann on hidden variables are founded on the following statement.

For any observable quantity $\mathcal{R}$ and for any state φ of the considered quantum system the relation

$$\mathrm{Erv}\,(\mathcal{R},\varphi) = (R\varphi,\varphi) \tag{1.1}$$

takes place. Here $\mathrm{Erv}\,(\mathcal{R},\varphi)$ is a mathematical expectation of the quantity $\mathcal{R}$ at the state φ. The quantity R is the operator of the observable quantity $\mathcal{R}$, and $(R\varphi,\varphi)$ is the scalar product of two vectors $R\varphi$ and φ of the Hilbert space. It is supposed that any observable quantity $\mathcal{R}$ can be measured instantaneously and attributed to the state (wave function) φ at this moment. The formula (1.1) is supposed to be valid *for all observable quantities* $\mathcal{R}$ *and for all states* φ (wave functions). If the formula (1.1) is valid not for all quantities $\mathcal{R}$, the mathematical formalism of ACQP cannot be founded, and the theorem on hidden variables appears to be unproved.

We are going to show that the formula (1.1) is not valid in the case, when $R = F(\mathbf{p})$, where F is an arbitrary function and $\mathbf{p} = -i\hbar\boldsymbol{\nabla}$ is the momentum operator. In particular, the measurement of the momentum of a free particle needs a long time. During this time the wave function ψ of the particle changes essentially, and the measured value $\mathbf{p}'$ of the momentum cannot be attributed to any definite state (wave function). As a result the formula (1.1) is not satisfied for such a quantity as the particle momentum.

Indeed, the wave function for the free particle of the mass m has the form

$$\psi(t,\mathbf{x}) = \frac{1}{(2\pi\hbar)^{3/2}}\int |\psi_{\mathbf{p}}|\, e^{-i\zeta(\mathbf{p})}\exp\left(i\frac{\mathbf{p}^2}{2m}t + i\frac{\mathbf{px}}{\hbar}\right)d\mathbf{p} \tag{1.2}$$

where $|\psi_{\mathbf{p}}|$ and $\zeta(\mathbf{p})$ are arbitrary real functions of $\mathbf{p}$. The quantity $|\psi_{\mathbf{p}}|\exp\left(i\frac{\mathbf{p}^2}{2m}t\right)\exp(-i\zeta(\mathbf{p}))$ is the Fourier component of the wave function $\psi(t,\mathbf{x})$. The module $|\psi_{\mathbf{p}}|$ of the Fourier component of the wave function is conserved. It can be determined by means of the diffraction grating. We obtain the momentum distribution $w(\mathbf{p})$ by the relation

$$w(\mathbf{p}) = A\,|\psi_{\mathbf{p}}|^2, \qquad A = \left(\int |\psi_{\mathbf{p}}|^2\, d\mathbf{p}\right)^{-1} \tag{1.3}$$

The diffraction experiment needs a long time. During this time the phase $i\frac{\mathbf{p}^2}{2m}t - i\zeta(\mathbf{p})$ of the wave function changes essentially, and one cannot determine to what time and to what wave function the measured distribution (1.3) should be attributed. Other methods of the momentum measurement need also a long time [4].

In fact, it means that the particle momentum is not always described by the operator $-i\hbar\boldsymbol{\nabla}$. Operator $-i\hbar\boldsymbol{\nabla}$ is the momentum operator only in the case, when the wave function has the form of the wave packet (1.2), whose space width is large enough, and $|\psi_{\mathbf{p}}|$ does not vanish only in a very small domain of values of $\mathbf{p}$. In other cases the question whether the operator $-i\hbar\boldsymbol{\nabla}$ is the particle momentum operator is open.

In the present paper we show two points:

1. The quantum mechanical technique describes dynamics of statistical ensembles

2. The distribution (1.2) disagrees with the von Neumann's postulate (1.1).

2 Whose state does the wave function describe?

The fact, that the Schrödinger equation for the free quantum particle may be written in the form of the hydrodynamic equations, describing an irrotational flow of some fluid, was known long ago [5, 6]. Representation of the Schrödinger equation in the form of hydrodynamic equation for the ideal (quantum) fluid with the internal energy

$$E=\frac{\hbar^2}{8m\rho}\left(\mathbf{\nabla}\rho\right)^2 \tag{2.1}$$

known as the hydrodynamic interpretation of quantum mechanics, was used by many authors [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Here ρ is the fluid density, m is the mass of the quantum particle, $E=E\left(\rho,\mathbf{\nabla}\rho\right)$ is the internal energy of the fluid per unit mass, and $\hbar$ is the quantum constant.

Hydrodynamic description was used mainly as an interpretation of quantum mechanics, but not as a method of the quantum particle description, because the hydrodynamic equations are nonlinear and more difficult for solution, than the Schrödinger equation. On the other hand, the hydrodynamic interpretation is more demonstrative, than the conventional interpretation in terms of the wave function.

Although the connection between the Schrödinger equation and the hydrodynamic description was known for many years, all this time it was the one-way connection. One could obtain the irrotational fluid flow from the dynamic equation for the wave function (Schrödinger equation), but one did not know how to transform dynamic equations for a fluid to the dynamic equation for a wave function. In other words, we did not know how to describe rotational fluid flow in terms of the wave function. In terms of the wave function we could describe only irrotational fluid flow.

To describe an arbitrary fluid flow in terms of a wave function, one needs *to integrate conventional dynamic equations for a fluid* (Euler equations). Indeed, the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\mathbf{\nabla}^2\psi=0 \tag{2.2}$$

may be reduced to the hydrodynamic equations for the density ρ and the velocity $\mathbf{v}$ of some ideal (quantum) fluid. Substituting $\psi=\sqrt{\rho}\exp\left(i\hbar\varphi\right)$ in (2.2) and separating real and imaginary parts of the equation, we obtain expressions for the time derivatives $\partial_0\rho$ and $\partial_0\varphi$. To obtain expression for the time derivative $\partial_0\mathbf{v}$ of the velocity $\mathbf{v}=\frac{\hbar}{m}\mathbf{\nabla}\varphi$, we need to differentiate the dynamic equation for $\partial_0\varphi$, forming the combination $\partial_0\mathbf{v}=\mathbf{\nabla}\left(\frac{\hbar}{m}\partial_0\varphi\right)$. The reverse transition from the hydrodynamic equations to the dynamic equations for the wave function (Schrödinger equation) needs a general integration of hydrodynamic equations. This integration is simple in the partial case of the irrotational flow, but it is a rather complicated mathematical problem in the general case, when the result of integration has to contain three arbitrary functions of three arguments. Without producing this integration, one cannot derive a description of a fluid in terms of the wave function, and one cannot manipulate the dynamic equations, transforming them from the representation in terms of ρ, $\mathbf{v}$ to the representation in terms of the wave function and back. This problem has not been solved for years. Now it has been solved. It has been shown that any ideal fluid can be described in terms of a many-component complex wave function. The irrotational flow of the ideal fluid is described by the one-component wave function, whereas the rotational flow is described by the two-component wave function (or even by the many-component one) [16]. It means that the wave function and the spin are attributes of the ideal fluid description (but not those of the quantum principles). In other words, the wave function is only a way of description of the quantum fluid, whereas the properties of the quantum phenomena are determined by the quantum fluid itself, whose properties are determined in turn by its internal energy. It means that the quantum fluid is a real vehicle of quantum properties, but not the wave function, which is simply a way of the ideal fluid description.

Any statistical description contains two different objects: individual stochastic object $\mathcal{S}_{\rm st}$ (individual particle) and statistical average object (particle) $\langle\mathcal{S}_{\rm st}\rangle$. The individual stochastic particle is a stochastic system $\mathcal{S}_{\rm st}$. The state of the system $\mathcal{S}_{\rm st}$ is described by its position $\mathbf{x}$ and its velocity $\mathbf{v}$. By definition, the stochasticity of $\mathcal{S}_{\rm st}$ means that a single measurement (S-measurement) of a state of $\mathcal{S}_{\rm st}$ is irreproducible, i.e. preparing the particle $\mathcal{S}_{\rm st}$ in the same way and repeating the S-measurement (of the particle position), we obtain, in general, another result, because there are no dynamic equations for $\mathcal{S}_{\rm st}$. But the evolution of the individual system $\mathcal{S}_{\rm st}$ is not stochastic completely. It contains some regular evolution component $\mathcal{C}_{\rm reg}$.

To obtain $\mathcal{C}_{\rm reg}$, we consider a set $\mathcal{E}\left[N,\mathcal{S}_{\rm st}\right]$ of N independent identical stochastic systems $\mathcal{S}_{\rm st}$. The set $\mathcal{E}\left[N,\mathcal{S}_{\rm st}\right]$, known as the statistical ensemble, is a stochastic system, having $6N$ degrees of freedom (it is supposed that $\mathcal{S}_{\rm st}$ has 6 degrees of freedom). If N is large enough, the stochastic components $\mathcal{C}_{\rm st}$ of the state evolution compensate each other, whereas the regular components $\mathcal{C}_{\rm reg}$ are accumulated, and the statistical ensemble $\mathcal{E}\left[N,\mathcal{S}_{\rm st}\right]$, $(N\rightarrow\infty)$ becomes to be a dynamic system. In the limit $N\rightarrow\infty$ the set $\mathcal{E}\left[N,\mathcal{S}_{\rm st}\right]$ turns to the statistical ensemble $\mathcal{E}\left[\infty,\mathcal{S}_{\rm st}\right]$, which is a continuous dynamic system, having infinite number of the freedom degrees. There are dynamic equations for $\mathcal{E}\left[\infty,\mathcal{S}_{\rm st}\right]$, which can be obtained as a result of variation of the action functional $\mathcal{A}_{\mathcal{E}[\infty,\mathcal{S}_{\rm st}]}$. All essential characteristics of the statistical ensemble $\mathcal{E}\left[N,\mathcal{S}_{\rm st}\right]$ do not depend on the number N of elements $\mathcal{S}_{\rm st}$ of the statistical ensemble $\mathcal{E}\left[N,\mathcal{S}_{\rm st}\right]$, if N is large enough, and one can normalize the statistical ensemble to one system and introduce the statistical average system $\langle\mathcal{S}_{\rm st}\rangle$, which is the dynamical system, whose action functional $\mathcal{A}_{\langle\mathcal{S}_{\rm st}\rangle}$ is defined by the relation

$$\mathcal{A}_{\langle\mathcal{S}_{\rm st}\rangle}=\lim_{N\rightarrow\infty}\frac{1}{N}\mathcal{A}_{\mathcal{E}[N,\mathcal{S}_{\rm st}]}\tag{2.3}$$

Thus, the statistical average system $\langle\mathcal{S}_{\mathrm{st}}\rangle$ is a continuous dynamic system with the infinite number of the freedom degrees. Investigating $\langle\mathcal{S}_{\mathrm{st}}\rangle$, we study the regular evolution component $\mathcal{C}_{\mathrm{reg}}$ of the stochastic system $\mathcal{S}_{\mathrm{st}}$, and the hydrodynamic description of $\langle\mathcal{S}_{\mathrm{st}}\rangle$ is a way of the investigation of $\mathcal{S}_{\mathrm{st}}$.

Although ACQP is a statistical conception, it contains only one sort of objects: quantum particle (system) $\mathcal{S}_{\mathrm{q}}$. The quantum system $\mathcal{S}_{\mathrm{q}}$ is a continuous dynamic system, whose state (wave function ψ) is a point in the infinite-dimensional Hilbert space. ACQP pretends to be a special kind of statistical conception, which contains only one sort of physical objects: quantum particle $\mathcal{S}_{\mathrm{q}}$. According to conventional interpretation of the quantum mechanics [3] the quantum particle $\mathcal{S}_{\mathrm{q}}$ is an individual particle $\mathcal{S}_{\mathrm{st}}$, but not the statistical average particle $\langle\mathcal{S}_{\mathrm{st}}\rangle$. Such an identification generates the puzzling question: " The state of the deterministic classical particle $\mathcal{S}_{\mathrm{d}}$ is a point in six-dimensional phase space. The state of the statistical ensemble $\mathcal{E}\left[N,\mathcal{S}_{\mathrm{d}}\right]$ of N $(N\rightarrow\infty)$ deterministic particles $\mathcal{S}_{\mathrm{d}}$ is described as a point in $6N$-dimensional phase space. Why is the state of the individual quantum system $\mathcal{S}_{\mathrm{q}}$ described as a point in the infinite-dimensional Hilbert space?"

From the viewpoint of the quantum mechanical technique it would be better to identify the individual quantum particle $\mathcal{S}_{q}$ with the statistical average particle $\langle\mathcal{S}_{\mathrm{st}}\rangle$, whose state is also a point in the infinite-dimensional space. But the answer of ACQP is as follows: "The quantum mechanics is a special kind of the statistical conception, which contains only one sort of objects $\mathcal{S}_{\mathrm{q}}$, but not a partial case of the general statistical conception, containing two sorts of objects $\mathcal{S}_{\mathrm{st}}$ and $\langle\mathcal{S}_{\mathrm{st}}\rangle$. The state of individual quantum particle $\mathcal{S}_{\mathrm{q}}$ is described by the wave function ψ, and it is a postulate of the quantum mechanics."

Another question is as follows. Let us set $\hbar=0$ in the description of the quantum particle $\mathcal{S}_{\mathrm{q}}$. We must obtain a description of the deterministic classical particle. What is this description? A description of individual particle $\mathcal{S}_{\mathrm{d}}$, or a description of the statistical ensemble $\mathcal{E}\left[\infty,\mathcal{S}_{\mathrm{d}}\right]$ of classical particles $\mathcal{S}_{\mathrm{d}}$? At first, let us note that one cannot set $\hbar=0$ in the Schrödinger equation (2.2), because in this case we do not obtain any description. Before setting $\hbar=0$, we are to transform the phase of the wave function, setting

$$\psi=\sqrt{\rho}\exp\left(i\varphi\right)=\sqrt{\rho}\exp\left(\frac{i}{\hbar}S\right) \tag{2.4}$$

Substituting (2.4) in (2.2) and separating real and imaginary parts of the equation, we obtain

$$\frac{i}{2\rho}\left(\frac{\partial\rho}{\partial t}+\frac{\rho}{m}\nabla^{2}S+\frac{\nabla\rho}{m}\nabla S\right)=0 \tag{2.5}$$

$$\frac{\partial S}{\partial t}-\frac{(\boldsymbol{\nabla}S)^{2}}{2m}+\frac{\hbar^{2}}{2m}\left(\boldsymbol{\nabla}\frac{\boldsymbol{\nabla}\rho}{2\rho}+\left(\frac{\boldsymbol{\nabla}\rho}{2\rho}\right)^{2}\right)=0 \tag{2.6}$$

Now let us introduce designation $\mathbf{v}=m^{-1}\boldsymbol{\nabla}S$, take the gradient of the equation (2.6) and set $\hbar=0$ in (2.5) and (2.6). We obtain

$$\frac{\partial\rho}{\partial t}+\boldsymbol{\nabla}\left(\rho\mathbf{v}\right)=0,\qquad\frac{\partial\mathbf{v}}{\partial t}+\left(\mathbf{v}\boldsymbol{\nabla}\right)\mathbf{v}=0 \tag{2.7}$$

Thus, we obtain dynamic equations for the ideal fluid without a pressure. This fluid is the statistical ensemble $\mathcal{E}\left[\infty, \mathcal{S}_{\mathrm{d}}\right]$ of classical particles $\mathcal{S}_{\mathrm{d}}$. We see that at $\hbar = 0$ the quantum system $\mathcal{S}_{\mathrm{q}}$ turns to a statistical ensemble, but not to an individual system. It means that from the formal viewpoint the quantum particle $\mathcal{S}_{\mathrm{q}}$ should be regarded as the statistical ensemble $\mathcal{E}\left[\infty, \mathcal{S}_{\mathrm{st}}\right]$, or as the statistical average particle $\langle \mathcal{S}_{\mathrm{st}} \rangle$, but not as an individual particle.

Another interesting question: "Why does one need to transform the scale of the wave function phase φ, introducing factor $\hbar^{-1}$ in the exponent of (2.4)?" At $\hbar \to 0$ this factor tends to infinity. The answer is rather curious. The action functional, describing the irrotational flow of the ideal fluid with the internal energy (2.1) has the form [16, 17]

$$\mathcal{A}[\psi,\psi^*] = \int \left\{ \frac{ib_0}{2}(\psi^*\partial_0\psi - \partial_0\psi^*\cdot\psi) - \frac{b_0^2}{2m}\boldsymbol{\nabla}\psi^*\cdot\boldsymbol{\nabla}\psi + \frac{b_0^2}{8m}\frac{(\boldsymbol{\nabla}\rho)^2}{\rho} - \frac{\hbar^2}{8m}\frac{(\boldsymbol{\nabla}\rho)^2}{\rho} \right\} \mathrm{d}^4x, \tag{2.8}$$

where $\rho = \psi^*\psi$, ψ is a complex one-component wave function, ψ^* is the quantity complex conjugate to ψ. The quantity b_0 is an arbitrary (integration) constant ($b_0 \neq 0$), describing the scale of the wave function phase. The last term of (2.8) describes the internal energy of the fluid. Only this term contains the quantum constant $\hbar$. All other terms are dynamical terms which are present at the description of any nonrelativistic statistical ensemble. Dynamic equation generated by the action (2.8) is equivalent to the Schrödinger equation for any choice of the constant b_0. If we set $\hbar = 0$, the last term in (2.8) vanishes, the internal energy of the fluid becomes $E(\rho, \boldsymbol{\nabla}\rho) = 0$. In this case the action (2.8) describes the statistical ensemble of classical (deterministic) particles $\mathcal{S}_{\mathrm{d}}$. Thus, in the action (2.8) the transition to the classical case is obtained by setting $\hbar = 0$.

Formal variation of the action (2.8) with respect to ψ^* leads to the dynamic equation for ψ, which is nonlinear because of two last terms in (2.8). If we chose the integration constant $b_0 = \hbar$, two last terms compensate each other and the action (2.8) turns to the action

$$\mathcal{A}[\psi,\psi^*] = \int \left\{ \frac{i\hbar}{2}(\psi^*\partial_0\psi - \partial_0\psi^*\cdot\psi) - \frac{\hbar^2}{2m}\boldsymbol{\nabla}\psi^*\cdot\boldsymbol{\nabla}\psi \right\} \mathrm{d}^4x \tag{2.9}$$

which generates the Schrödinger equation (2.2) directly. But now all terms of the action (2.9) are quantum in the sense that they contain the quantum constant $\hbar$ as a factor. Setting $\hbar = 0$, we cannot transit now to the classical case, because at $\hbar = 0$ the action (2.9) vanishes. However, we can transform the phase scale of ψ by means of the transformation [17]

$$\psi \to \tilde{\psi} = |\psi| \exp\left(\frac{b_0}{\hbar}\log\frac{\psi}{|\psi|}\right), \tag{2.10}$$

After this transformation the action (2.9) turns to the action (2.8). If we now set $\hbar = 0$, we obtain the action for the statistical ensemble of classical particles. Making the change of the phase φ (2.4) in the Schrödinger equation (2.2), we produce the transformation (2.10) with $b_0 = 1$. Thereafter setting $\hbar = 0$, we can obtain dynamic equations for the statistical ensemble of classical particles.

Thus, the formal consideration shows that the quantum particle $\mathcal{S}_{\text{q}}$ is in reality the statistical average particle $\langle \mathcal{S}_{\text{st}} \rangle$, or the statistical ensemble $\mathcal{E}[\mathcal{S}_{\text{st}}]$, but not an individual particle. If we consider $\mathcal{S}_{\text{q}}$ as an individual particle, we destroy the continuous connection between the classical physics and the quantum mechanics.

Some authors consider the quantum particle $\mathcal{S}_{\text{q}}$ as the statistical ensemble $\mathcal{E}[\mathcal{S}_{\text{st}}]$ of stochastic particles $\mathcal{S}_{\text{st}}$ (for instance [18]). Such an interpretation is better, because it reestablishes the continuous connection between the classical physics and the quantum mechanics. Unfortunately, the quantum mechanics is presented not always consistently, because only one sort of objects (particle $\mathcal{S}_{\text{q}} = \mathcal{E}[\mathcal{S}_{\text{st}}]$) is considered. Such an interpretation is in accordance with the quantum mechanical technique, which deals with only one sort of particles $\mathcal{S}_{\text{q}} = \mathcal{E}[\mathcal{S}_{\text{st}}]$, and all quantum mechanical predictions concern only with $\mathcal{S}_{\text{q}} = \mathcal{E}[\mathcal{S}_{\text{st}}]$. But there are two sorts of measurements: (1) the single measurement (S-measurement) produced under an individual particle and (2) the massive measurement (M-measurement) produced under a statistical ensemble. Properties of S-measurement and those of M-measurement are discrepant and incompatible. The result of the S-measurement is a definite value of the measured quantity, whereas the result of the M-measurement is a distribution of the measured quantity. The result of the S-measurement is random and irreproducible, whereas the result of the M-measurement is regular and reproducible. The S-measurement does not change the state (wave function), whereas the M-measurement replaces the wave function ψ by the density matrix. If we do not distinguish between the M-measurement and S-measurement and use one term "measurement" for the two kinds of measurement, it is a source of possible paradoxes [1].

Unfortunately, we have not met such a presentation of the quantum mechanics, where two kinds of measurements be considered. Such a distinction between two kinds of measurements is necessary, if we consider the quantum mechanics as a special case of the general statistical conception (dynamics of stochastic systems). We have not met such a distinction between S-measurement and M-measurement even in the presentation of those authors, who believe that the wave function describes the state of the statistical ensemble (but not of a single particle). The presentation of quantum mechanics cannot be consistent without this distinction. Besides, any M-measurement is a set of many S-measurements, and one may not ignore S-measurement, although its results are not described by the QM formalism.

The quantum mechanics is a logical construction which contains axioms (primary statements) and corollaries of these axioms. The primary statements can be chosen in different way, and interpretation of quantum mechanics depends on this choice. This choice is unessential for the nonrelativistic quantum mechanics in itself, but it is essential, if we are going to expand the quantum mechanics onto the relativistic quantum phenomena. For instance, if the quantum principles are primary statements of the quantum mechanics, then dynamic equations in terms of the wave function must be linear, and the internal energy of the quantum fluid must have the form (2.1), in order the last term of (2.8) may be compensated with the antecedent term at $\hbar = b_0$. Any other form of the internal energy fluid is unallowable. Compensation of the dynamical term with the term describing some special fluid property looks rather artificial. It is justified only by the intention to obtain the linear dynamic equation. From the point of view of the stochastic system dynamics, where all ideal fluids are admissible, the consideration of the dynamic equation linearity as a principle of the logical construction seems to be rather doubtful. Of course, if for some special quantum fluid we can obtain the linear dynamic equation, be it even an artificial identification of the terms of different nature, we should accept and use this identification. But it seems doubtful to consider the linearity connected with such an identification as a principle of the quantum mechanics.

Besides, the action for the quantum fluid with internal energy (2.1) has the form (2.8) only for irrotational flow of the fluid. The wave function ψ is two-component at the rotational flow of the same fluid, and the action has the form [16]

$$\begin{aligned} \mathcal{A}[\psi,\psi^*] &= \int \left\{ \frac{ib_0}{2}(\psi^*\partial_0\psi - \partial_0\psi^*\cdot\psi) - \frac{b_0^2}{2m}\nabla\psi^*\cdot\nabla\psi \right. \\ &\left. + \frac{b_0^2}{8m}(\nabla s_\alpha)(\nabla s_\alpha)\rho + \frac{b_0^2-\hbar^2}{8m}\frac{(\nabla\rho)^2}{\rho} \right\} \mathrm{d}^4x \end{aligned} \tag{2.11}$$

$$\psi = \binom{\psi_1}{\psi_2}, \qquad \psi^* = (\psi_1^*, \psi_2^*), \qquad \rho \equiv \psi^*\psi, \qquad s_\alpha \equiv \frac{\psi^*\sigma_\alpha\psi}{\rho}, \qquad \alpha = 1,2,3, \tag{2.12}$$

where σ_α are the Pauli matrices. Dynamic equation for the rotational fluid flow is nonlinear, even if $b_0 = \hbar$. It has the form

$$i\hbar\partial_0\psi + \frac{\hbar^2}{2m}\partial_\alpha\partial_\alpha\psi + \frac{\hbar^2}{8m}(\nabla s_\alpha)(\nabla s_\alpha)\psi = \hbar^2\frac{\nabla(\rho\nabla s_\alpha)}{4\rho m}(\sigma_\alpha - s_\alpha)\psi \tag{2.13}$$

This equation is nonlinear, and one can hardly reduce it to a linear equation. Then the equation (2.13) is incompatible with principles of the quantum mechanics.

Being considered as a principle, the linearity of the dynamic equation restricts strongly capacity of the stochastic system dynamics. In particular, attempts of expansion of the quantum mechanics on relativistic phenomena of microcosm failed, and the constraints imposed on the dynamics of the stochastic systems by the quantum mechanics principles may play an essential role in this failure.

In the conventional (Copenhagen) interpretation of quantum mechanics the quantum particle motion is described in terms of the wave function ψ and related concepts: interference, diffraction, coherence. Such concepts as momentum, energy, angular momentum are used also, but they are expressed via the wave function ψ, and their sense is sometimes another, than in the classical mechanics. But the wave function ψ is a nonrelativistic concept, and the conventional interpretation meets difficulties at its expansion onto the relativistic motion of quantum particles.

It is the main defect of the conventional interpretation. Besides, the wave function is only a way of the quantum fluid description. It is unreliable to construct the interpretation on the basis of some method of a description. Consideration of the quantum fluid (that is $\langle \mathcal{S}_{\rm st} \rangle$) in itself is a more reliable interpretation. Being a fluidlike continuous dynamic system, the statistical average particle $\langle \mathcal{S}_{\rm st} \rangle$ can be described in terms of usual hydrodynamic concepts: the flux 4-vector $j^i = \{\rho, \rho\mathbf{v}\}$ and the energy-momentum tensor T^{ik}.

Integrating the system of dynamic equations

$$\frac{dx^i}{d\tau} = j^i(x), \qquad i = 0, 1, 2, 3 \tag{2.14}$$

where

$$j^i = \{\rho, \mathbf{j}\}, \qquad j^0 = \rho = \psi^* \psi, \qquad \mathbf{j} = \frac{\hbar}{2m}\left(\psi^* \boldsymbol{\nabla} \psi - \boldsymbol{\nabla} \psi^* \cdot \psi\right) \tag{2.15}$$

we obtain the mean world lines $\mathcal{L} : x^i = x^i(\tau)$, associated with the statistical average particle $\langle \mathcal{S}_{\rm st} \rangle$. Considering the energy-momentum tensor T^{ik} along $\mathcal{L}$, we can evaluate the energy-momentum characteristics associated with the mean world line $\mathcal{L}$. Thus, the hydrodynamic interpretation describes all quantum phenomena in terms of the mean world lines and other attributes of hydrodynamics.

Properties of the quantum fluid are determined by the internal energy E of the fluid, which depends on the properties of some 4-vector field u^k, $k = 0, 1, 2, 3$. This vector describes the mean value of the stochastic component of the particle velocity. Choice of the field of this vector u^k means the procedure of the classical particle quantization. To obtain the action $\mathcal{A}_{\langle \mathcal{S}_{\rm st} \rangle}$ for the quantum fluid $\langle \mathcal{S}_{\rm st} \rangle$, it is necessary to take the action $\mathcal{A}_{\mathcal{E}_{\rm d}[\mathcal{S}_{\rm d}(P)]}$ for the statistical ensemble $\mathcal{E}_{\rm d}[\mathcal{S}_{\rm d}(P)]$ of deterministic classical particles $\mathcal{S}_{\rm d}(P)$, where P is a set of parameters of the classical particle. For the free classical particle there is only one parameter P: its mass m. The action for the ensemble $\mathcal{E}_{\rm d}[\mathcal{S}_{\rm d}]$ of free deterministic particles $\mathcal{S}_{\rm d}$ has the form

$$\mathcal{E}_{\rm d}[\mathcal{S}_{\rm d}] : \qquad \mathcal{A}_{\mathcal{E}_{\rm d}[\mathcal{S}_{\rm d}(m)]}[\mathbf{x}] = \int L\left(\mathbf{x}, \frac{d\mathbf{x}}{dt}\right) dt d\boldsymbol{\xi}, \tag{2.16}$$

where the Lagrangian function density is described by the relation

$$L\left(\mathbf{x}, \frac{d\mathbf{x}}{dt}\right) = -mc^2 + \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 \tag{2.17}$$

$\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$, $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$. Here variables $\boldsymbol{\xi}$ label the particles $\mathcal{S}_{\rm d}$ of the statistical ensemble $\mathcal{E}_{\rm d}[\mathcal{S}_{\rm d}]$. The constant c is the speed of the light. The action (2.16) describes some fluid without pressure. Let us replace now the value of m in (2.17), (2.16) by its effective value $m_{\rm eff}$

$$m \rightarrow m_{\rm eff} = m\left(1 - \frac{\mathbf{u}^2}{2c^2} + \frac{\hbar}{2mc^2} \boldsymbol{\nabla} \mathbf{u}\right) \tag{2.18}$$

The change (2.18) is nonrelativistic. In this case the component u^0 of the 4-vector u^k is small and it can be neglected [17]. In the nonrelativistic approximation we obtain the action $\mathcal{E}_{\rm st}\left[\mathcal{S}_{\rm st}\right]$ for the statistical ensemble of stochastic particles $\mathcal{S}_{\rm st}$

$$\mathcal{E}_{\rm st}\left[\mathcal{S}_{\rm st}\right]: \qquad \mathcal{A}_{\mathcal{E}_{\rm d}\left[\mathcal{S}_{\rm d}\left(m_{\rm eff}\right)\right]}\left[\mathbf{x},\mathbf{u}\right]=\int L\left(\mathbf{x},\frac{d\mathbf{x}}{dt}\right)+L_{\rm st}\left(\mathbf{u},\boldsymbol{\nabla}\mathbf{u}\right)dtd\boldsymbol{\xi}, \tag{2.19}$$

$$L_{\rm st}\left(\mathbf{u},\boldsymbol{\nabla}\mathbf{u}\right)=\frac{m}{2}\mathbf{u}^2-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u} \tag{2.20}$$

Here $\mathbf{u}=\mathbf{u}\left(t,\mathbf{x}\right)$ is the mean value of the stochastic velocity component. The first term in (2.20) describes the energy of the stochastic component of the velocity. The quantum constant $\hbar$ appears here as a coupling constant, describing connection between the regular and stochastic components of the particle motion. The velocity $\mathbf{u}$ is supposed to be small with respect to c. Dynamic equations for the quantum fluid with the pressure are obtained as a result of variation of the action (2.19) with respect to variables $\mathbf{x}$ and $\mathbf{u}$. It follows from the dynamic equation for $\mathbf{u}$ that

$$\mathbf{u}=-\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho, \qquad E_{\rm int}=\frac{m\mathbf{u}^2}{2}\rho=\frac{\hbar^2}{8m}\frac{\left(\boldsymbol{\nabla}\rho\right)^2}{\rho}, \qquad \rho=\left(\frac{\partial\left(x^1,x^2,x^3\right)}{\partial\left(\xi_1,\xi_2,\xi_3\right)}\right)^{-1} \tag{2.21}$$

where ρ is the fluid density, and $E_{\rm int}$ is the energy density connected with the mean value $\mathbf{u}$ of the stochastic velocity component. One can show that the irrotational flow of the quantum fluid is described by the Schrödinger equation (2.2) [17]. Thus, the quantization of the classical particle is carried out by an addition of a supplemental term (2.20) to the action $\mathcal{E}_{\rm d}\left[\mathcal{S}_{\rm d}\right]$ for the statistical ensemble of free classical particles $\mathcal{S}_{\rm d}$. The statistical ensemble $\mathcal{E}_{\rm d}\left[\mathcal{S}_{\rm d}\right]$ ceases to be a statistical ensemble of classical particles $\mathcal{S}_{\rm d}$, because now its elements interact between themselves. This interaction generates a pressure in the quantum fluid.

Quantization of free relativistic classical particles is carried out by means of the relativistic version of the change (2.18) [16, 19]. Sometimes a puzzling question arises: "How can the quantum fluid describe such wave phenomena as interference and diffraction?" The fact is that the pressure and the internal energy of the quantum fluid depend on the density ρ and on the density gradient $\boldsymbol{\nabla}\rho$, whereas in the usual fluid they depend only on the density ρ. Appearance of additional spatial derivatives in the hydrodynamic equations generates the wave properties of the quantum fluid. Formally it follows from the description of the irrotational flow of quantum fluid in terms of the Schrödinger equation.

3 The momentum distribution

Hydrodynamic interpretation of quantum mechanics cannot give distribution $w\left(\mathbf{p}\right)$ over momenta $\mathbf{p}$ at the state, described by the wave function ψ. It can give only the mean momentum distribution. The conventional (Copenhagen) interpretation gives the momentum distribution by the relation

$$w(\mathbf{p}) = A\psi_{\mathbf{p}}^{*}\psi_{\mathbf{p}}, \qquad \psi_{\mathbf{p}} = \frac{1}{(2\pi\hbar)^{3}}\int \psi(\mathbf{x})\, e^{-i\frac{\mathbf{px}}{\hbar}} d\mathbf{x} \qquad \mathbf{A} = \int \psi_{\mathbf{p}}^{*}\psi_{\mathbf{p}} d\mathbf{p}, \tag{3.1}$$

where $\psi_{\mathbf{p}}$ is the Fourier component of the wave function ψ. But this statement cannot be tested experimentally, because the measured distribution cannot be attributed to the state $\psi(\mathbf{x})$. On the contrary, the mean momentum distribution, which is given by the hydrodynamic interpretation, can be tested experimentally and attributed to the wave function ψ, but this distribution does not coincide with (3.1).

To realize the difference between the momentum and the mean momentum, we consider the measurement of the molecule momentum in the example of the stationary flow of an ideal gas. Let the gas flow be isothermal, and the temperature kT =const. Distribution of the gas molecules over velocities is described by the Maxwell distribution

$$F(\mathbf{x},\mathbf{v})\, d\mathbf{v} = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left\{-\frac{m(\mathbf{v}-\mathbf{u}(\mathbf{x}))^{2}}{2kT}\right\} d\mathbf{v} \tag{3.2}$$

where $\mathbf{u}(\mathbf{x})$ is the gas velocity at the point $\mathbf{x}$ (the mean velocity of the gas molecules at the point $\mathbf{x}$). The distribution over momenta $\mathbf{p} = m\mathbf{v}$ has the form

$$F_1(\mathbf{x},\mathbf{p})\, d\mathbf{p} = \frac{1}{(2\pi mkT)^{3/2}} \exp\left\{-\frac{(\mathbf{p}-m\mathbf{u}(\mathbf{x}))^{2}}{2mkT}\right\} d\mathbf{p} \tag{3.3}$$

Let us divide the volume V of the gas flow into similar cubic cells $V_1, V_2, ...V_N$, $N \gg 1$. Let the following conditions be satisfied

$$l_{\mathrm{c}} \ll L, \qquad |v_{\mathrm{t}}\tau_{c}| \ll L, \qquad |\mathbf{u}(\mathbf{x})| \ll v_{\mathrm{t}} = \sqrt{\frac{3kT}{m}} \tag{3.4}$$

where L is the linear size of the cell, l_{c} is the mean path between the molecule collisions, τ_{c} is the mean time between the collisions and v_{t} is the mean thermal velocity of molecules.

Let us calculate the mean momentum $\langle\mathbf{p}_i\rangle$ of the gas molecule in the cell V_i. We obtain $\langle\mathbf{p}_i\rangle = m\mathbf{u}(\mathbf{x})$, $\mathbf{x} \in V_i$, $i = 1, 2, ...N$. The set of all $\langle\mathbf{p}_i\rangle$, $i = 1, 2, ...N$ forms the mean momentum distribution. This distribution is determined completely by the gas flow, and it has nothing to do with the Maxwell momentum distribution (3.3), which describes both the regular and random components of the molecule momenta. Under conditions (3.4) the mean momentum distribution is much narrower, than the Maxwell distribution, because the Maxwell distribution takes into account the random component of the molecule velocity, and in the given case the random component is much larger, than the regular one. Let us imagine that we measure the momentum of the accidentally taken molecule. We measure its position $\mathbf{x}$ at the time t and its position $\mathbf{x}_1$ at the time $t + t_\mathrm{m}$. The molecule momentum in the time interval $(t, t + t_\mathrm{m})$ is determined by the relation

$$\mathbf{p} = m \frac{\mathbf{x}_1 - \mathbf{x}}{t_\mathrm{m}} \tag{3.5}$$

Producing many such measurements of the molecule momentum, we obtain the momentum distribution. What distribution do we obtain? The mean momentum distribution, or the Maxwell distribution? The result depends on the measurement time t_m. If $t_\mathrm{m} \ll \tau_\mathrm{c}$, we obtain the Maxwell distribution (3.3), averaged over the volume V, occupied by the gas flow. If $\tau_\mathrm{c} \ll t_\mathrm{m} \ll L / |\mathbf{u}(\mathbf{x})|$, we obtain the mean momentum distribution. If $t_\mathrm{m} \approx \tau_\mathrm{c}$, we obtain some intermediate result, which is close to the Maxwell distribution, because the characteristic velocity of the Maxwell distribution $v_\mathrm{t} = \sqrt{\frac{3kT}{m}} \gg |\mathbf{u}(\mathbf{x})|$. Finally, if $\tau_\mathrm{c} \ll L / |\mathbf{u}(\mathbf{x})| \ll t_\mathrm{m}$, we obtain the mean momentum distribution, containing some additional averaging over the spatial cells.

If the gas flow is nonstationary, the measurement time t_m must be shorter, than the characteristic time τ of the gas flow change. If the mean gas velocity of the non-stationary flow is $\mathbf{u}(t, \mathbf{x})$, the distribution over the mean momenta $\langle \mathbf{p} \rangle$ of molecules is given by the relation

$$\begin{aligned} w(\langle \mathbf{p} \rangle) &= A \int \delta(\langle \mathbf{p} \rangle - m\mathbf{u}(t, \mathbf{x})) \rho(t, \mathbf{x}) \, d\mathbf{x}, \\ A &= \left(\int w(\langle \mathbf{p} \rangle) \, d\langle \mathbf{p} \rangle \right)^{-1} = \left(\int \rho(t, \mathbf{x}) \, d\mathbf{x} \right)^{-1} \end{aligned} \tag{3.6}$$

where $\rho(t, \mathbf{x})$ is the gas density. The mean momentum $\langle \mathbf{p} \rangle$ is a single-valued function of the position $\mathbf{x}$. It is useless to speak on simultaneous distribution $F(\mathbf{x}, \langle \mathbf{p} \rangle)$ over position and mean momenta, because variables $\mathbf{x}$ and $\langle \mathbf{p} \rangle$ are not independent, and this distribution is a function $\Phi(\mathbf{x})$ of only position $\mathbf{x}$.

Is it possible to obtain distribution over the mean momenta $\langle \mathbf{p} \rangle$ of molecules and to associate it with the state of the nonstationary gas flow in the case, when $\tau \approx \tau_\mathrm{c}$? Yes, it is possible. We must measure two subsequent positions $\mathbf{x}$ and $\mathbf{x}_1$ of a molecule at the times t and $t + t_\mathrm{m}$ respectively ($t_\mathrm{m} \ll \tau_\mathrm{c} \approx \tau$, $\mathbf{x} \in V_i$) and determine the momentum $\mathbf{p}$ by means of the relation (3.5). Producing many such measurements at the time t and taking the mean value of the measured momenta $\mathbf{p}$, we obtain the mean momentum $\langle \mathbf{p}_i \rangle$ in the cell V_i. Producing such measurements at all cells at the moment t, we obtain the distribution over the mean momenta $\langle \mathbf{p} \rangle$ at the time t. This mean momentum distribution can be associated with the state $\rho, \mathbf{u}$ of the gas flow at the time t, because the measurement time $t_\mathrm{m} \ll \tau \approx \tau_\mathrm{c}$. On the other hand, this distribution has nothing to do with the Maxwell distribution over the molecular momenta. Thus, the mean momentum distribution (3.6) can be measured and attributed to the gas flow state at the time t.

Let us now apply the above consideration to the quantum fluid, describing the free quantum particle. Let the fluid state be described by the wave function ψ.

According to (3.6) we obtain for the mean momentum distribution

$$w(\langle \mathbf{p}\rangle) = A\int \delta(\langle \mathbf{p}\rangle - m\mathbf{v}(t,\mathbf{x}))\rho(t,\mathbf{x})\,d\mathbf{x}, \qquad \mathbf{v} = \frac{\hbar}{2m\rho}\left(\psi^*\boldsymbol{\nabla}\psi - \boldsymbol{\nabla}\psi^*\cdot\psi\right), \tag{3.7}$$

$$\rho = \psi^*\psi, \qquad \mathbf{A} = \left(\int w(\langle \mathbf{p}\rangle)\,d\langle \mathbf{p}\rangle\right)^{-1} = \left(\int \psi^*\psi d\mathbf{x}\right)^{-1} \tag{3.8}$$

After integration over $\mathbf{x}$ the distribution (3.7) can be written in the form

$$w(\mathbf{p}) = A\sum_l \left[\frac{\rho(t,\mathbf{x})}{m^3\left|D(\mathbf{x})\right|}\right]_{\mathbf{x}=\mathbf{x}_l(t,\mathbf{p})}, \qquad D(\mathbf{x}) = \frac{\partial\left(v^1(\mathbf{x}), v^2(\mathbf{x}), v^3(\mathbf{x})\right)}{\partial\left(x^1,x^2,x^3\right)} \tag{3.9}$$

where summation is produced over all roots $\mathbf{x}_l(t,\mathbf{p})$ of the equation

$$\mathbf{v}(t,\mathbf{x}) = \frac{\mathbf{p}}{m} \tag{3.10}$$

The distribution (3.7) can be measured in a very short measurement time t_{m} and attributed to some state ψ of the fluid, but this distribution has not the form (1.1), because it is not bilinear with respect to the wave function ψ. The distribution (3.1) also has not the form (1.1), because it cannot be attributed to the wave function ψ.

In general, the distribution (3.7) distinguishes from the conventional momentum distribution (3.1). Let us compare the two distributions for the state, described by the wave function

$$\psi(x) = A_1 e^{-\frac{1}{2}\frac{(x-X)^2}{a^2}+\frac{i}{\hbar}k(x-X)} = \frac{A_1\left|a\right|}{\sqrt{2\pi}\hbar}\int\limits_{-\infty}^{\infty} e^{-\frac{a^2(k-p)^2}{2\hbar^2}}e^{\frac{i}{\hbar}p(x-X)}dp, \qquad A_1 = \mathrm{const} \tag{3.11}$$

This wave function describes one-dimensional wave packet of the characteristic width a, moving with the momentum k. The center of the wave packet is placed at the point X.

Calculation gives for the distribution (3.1)

$$w(p) = A\psi_p^*\psi_p = \frac{a}{\sqrt{\pi}\hbar}e^{-\frac{a^2}{\hbar^2}(k-p)^2} \tag{3.12}$$

For ρ and j we obtain

$$\rho(x) = \left|\psi\right|^2 = \left|A_1\right|^2\exp\left(-\frac{(x-X)^2}{\left|a\right|^2}\right)dx \tag{3.13}$$

$$j(x) = -\frac{i\hbar}{2m}\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right) = \frac{k}{m}\left|A_1\right|^2\exp\left(-\frac{(x-X)^2}{\left|a\right|^2}\right) \tag{3.14}$$

Then the mean momentum distribution (3.7) is described by the relation

$$w(\langle p\rangle) = \int \delta(\langle p\rangle - k)\rho(x)\,dx = \left|A_1\right|^2\frac{\sqrt{\pi}}{\left|a\right|}\delta(\langle p\rangle - k) \tag{3.15}$$

Expressions (3.15) and (3.12) coincide in the limit $a \to \infty$. They are close, if $a \gg \hbar/k$. Thus, distributions (3.15) and (3.12) are close for the wide wave packets. In other cases the distributions (3.15) and (3.12) may be close or not. Both interpretations (conventional and hydrodynamic) assume that simultaneous distribution over position and momenta is impossible. (Formally, such a distribution is possible, but in this distribution the momentum $\mathbf{p}$ is a function of the position $\mathbf{x}$). But reasons of this impossibility are different. The conventional interpretation supposes that such a distribution is impossible, because the position operator and the momentum operator do not commute. In the hydrodynamic interpretation such a distribution is impossible, because the position and the mean momentum are not independent quantities.

Let us consider the influence of the measuring device on the momentum measurement in the stationary state described by the wave function ψ. We measure the component p_1 of the momentum $\mathbf{p}$ in the interval $\{x, x+\Delta x\}$.

At first, we present the viewpoint of ACQP, which does not distinguish between the M-measurement and S-measurement. We consider the momentum measurement as an measurement under individual particle. We measure the particle coordinate x twice with the time interval t_m between them. For determination of the particle position, we use a photon of frequency ω with the momentum $\hbar\omega/c$. At the collision with the particle the photon transmits a portion of its momentum to the particle. Indeterminacy Δp of the momentum is connected with the indeterminacy Δx of the particle position by means of the indeterminacy relation

$$\Delta x \Delta p \geq \hbar/2 \tag{3.16}$$

Measurement of the first position of the particle is carried out by the indeterminacy Δx. The momentum is determined by the relation (3.5), and the indeterminacy δp of the momentum measurement is a function of Δx, defined by the relation

$$\delta p = \frac{m\Delta x}{t_\mathrm{m}} + \frac{\hbar}{\Delta x} \tag{3.17}$$

Minimum of (3.17) is attained at $\Delta x = \sqrt{m\hbar/t_\mathrm{m}}$. The minimal value of the momentum indeterminacy is given by the relation

$$\delta p_{\mathrm{min}} = 2\sqrt{\frac{m\hbar}{t_\mathrm{m}}} \tag{3.18}$$

Thus, the exact measurement of the momentum is possible only at $t_\mathrm{m} \to \infty$, when the measured value of the momentum cannot be attributed to any value of the wave function. Impossibility of the exact measurement of particle momentum is connected with influence of the measuring device (in the given case with the photon influence).

Now we consider the momentum measurement from the viewpoint of MCQP, which distinguishes between the S-measurement and the M-measurement. Consideration of the S-measurement is useless, because the result of the S-measurement is random and irreproducible. Besides, the result of the S-measurement cannot be predicted by the statistical conception (quantum mechanics).

The M-measurement is a set (statistical ensemble) of N similar independent S-measurements: M_1, $M_2, ..., M_N$ $(N \to \infty)$. Anyone of S-measurements is produced under a single particle. All particles are prepared in the same way, and all N particles form the statistical ensemble $\mathcal{E}_N$, whose state is described by the wave function ψ. Any S-measurement M_i consists of two S-measurements $\{F_i, S_i\}$, where F_i means the measurement of the first position of the ith particle at the time t, and S_i means the measurement of the position of the same particle at the time $t + t_{\mathrm{m}}$. Let the S-measurements $F_{\alpha_1}, F_{\alpha_2}, ...F_{\alpha_K}$, $(K \gg 1)$ of the position x give the result in interval $L_k = (k\Delta x, (k+1)\,\Delta x)$, where L is some space interval. We select these particles with numbers $\alpha_1, \alpha_2, ...\alpha_K$ and form a new statistical ensemble $\mathcal{E}'_K$. The K particles of the statistical ensemble $\mathcal{E}'_K$ have the position at the time t in the interval L_k. If $N \to \infty$ and $K \to \infty$, both statistical ensembles $\mathcal{E}_N$ and $\mathcal{E}'_K$ are dynamic systems, whose states are described respectively by the wave functions ψ and ψ'. The wave function ψ' of the statistical ensemble $\mathcal{E}'_K$ does not coincide, in general, with the state ψ of the statistical ensemble $\mathcal{E}_N$. This change of the ensemble state is mainly a result of the selection, although the dynamic interaction with the measuring device (photon) also contributes to the state of the statistical ensemble $\mathcal{E}'_K$.

As a result of the produced selection the coordinate x of all systems of the statistical ensemble $\mathcal{E}'_K$ is concentrated in the interval $L_k = (k\Delta x, (k+1)\,\Delta x)$. Then $|\partial\rho'/\partial x| \geq \hbar\rho'/\Delta x$, $\rho' = |\psi'|^2$ and the internal energy (2.1) of the statistical ensemble $\mathcal{E}'_K$ increases. After a lapse of time this increase of the internal energy transforms into the x-component of the particle momentum. Let us now produce the S-measurements $S_{\alpha_1}, S_{\alpha_1}, ...S_{\alpha_K}$ of coordinate x of all K particles of the statistical ensemble $\mathcal{E}'_K$ at the time $t + t_{\mathrm{m}}$. Using the relation (3.5), we obtain distribution $w_{t_{\mathrm{m}},\Delta x}(x, p_x)$ over the x-component of the momentum in the vicinity of the point $x = k\Delta x$. Form of this distribution depends on the measurement time t_{m} and on the influence of the measuring device. The distribution takes into account also the stochastic component of the particle $\mathcal{S}_{\mathrm{st}}$ motion. If the numbers N and K of elements of the statistical ensembles $\mathcal{E}_N$ and $\mathcal{E}'_K$ tend to ∞, the obtained distribution $w_{t_{\mathrm{m}},\Delta x}(x, p_x)$ appears to be reproducible. It is reproduced at repeated M-measurements of the x-component of the momentum.

The obtained distribution $w_{t_{\mathrm{m}},\Delta x}(x, p_x)$ over momenta depends on $x = k\Delta x$, on t_{m} and on Δx which are chosen arbitrarily. We can consider the limit

$$w_{\Delta x}(x, p_x) = \lim_{t_{\mathrm{m}} \to 0} w_{t_{\mathrm{m}},\Delta x}(x, p_x) \tag{3.19}$$

We cannot be sure that such a limit exists, because we know nothing about the stochastic evolution component $\mathcal{C}_{\mathrm{st}}$ of the particle $\mathcal{S}_{\mathrm{st}}$ motion. As far as all particles of the statistical ensemble $\mathcal{E}'_K$ are independent, we should expect, that influence of the quantum stochasticity and influence of the measuring device on the mean value $\langle p_x \rangle$ of the momentum x-component is compensated. One should expect that $w_{\Delta x}(x, p_x)$ depends only on $x = k\Delta x$, but not on Δx, i.e. $w_{\Delta x}(x, p_x) =$

$w(x, p_x)$. Then the mean value of the particle momentum $\langle p_x(x)\rangle$ on the interval $L_k = (k\Delta x, (k+1)\Delta x)$

$$\langle p_x(x)\rangle = \int p_x w(x, p_x)\, dp_x \tag{3.20}$$

exists and coincides with the value of mv_x, determined by the second relation of (3.7). The measured value $\langle p_x(x)\rangle$ of the mean momentum may be attributed to the state ψ (but not to ψ'), because the time of the measurement can be made to be very short. The state ψ' may be considered to be an intermediate state, which appears in the process of the M-measurement. The quantum mechanics formalism predicts the value of (3.20) and attributes it to the wave function ψ. However, the QM formalism cannot predict the form of the distribution (3.19), as well as the gas dynamics can describe the gas velocity (the mean molecular velocity), but the gas dynamics formalism does not describe the Maxwell distribution and its evolution. In this sense the parameters of the distribution (3.19) may be considered to be hidden parameters of ACQP. They are also hidden parameters of MCQP.

In a sense the momentum distribution (3.19) is an analog of the Maxwell momentum distribution (3.3) in the kinetic gas theory. Is it possible to measure this momentum distribution experimentally? Is it possible to eliminate influence of the measuring device on the measurement of the momentum distribution? It is an interesting question, which is open now.

4 Concluding remarks

At first sight the conventional interpretation seems to be more informative, than the hydrodynamic interpretation, because the ACQP predicts the momentum distribution, whereas MCQP cannot. We show in this paper that the momentum distribution (1.3) is not a momentum distribution in reality. It may be considered to be the Fourier component module distribution, or even the mean momentum distribution, but it is not the momentum distribution. If it is so, the hydrodynamic interpretation, which cannot predict the momentum distribution, appears to be as informative as the conventional interpretation of quantum mechanics.

Besides it was shown that the wave function describes the state of the statistical ensemble of stochastic particles $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$, or the state of the statistical average particle $\langle\mathcal{S}_{\mathrm{st}}\rangle$, but not the state of an individual stochastic particle $\mathcal{S}_{\mathrm{st}}$. It is a serious argument against the conventional interpretation of quantum mechanics.

Thus, we have shown that the quantum system $\mathcal{S}_{\mathrm{q}}$ is the statistical average system $\langle\mathcal{S}_{\mathrm{st}}\rangle$, but not an individual system $\mathcal{S}_{\mathrm{st}}$. The quantum mechanics formalism predicts only results of the M-measurements. The S-measurements are important, because any M-measurement is an ensemble of independent S-measurements.

It has been shown that some of original basic statements (1.1), which are necessary for the proof of the von Neumann's theorem on hidden variables cannot be tested experimentally, because the measured values cannot be attributed to a definite state (wave function). All these arguments attest in favour of hydrodynamic interpretation, which can be freely expanded to the case of relativistic phenomena.

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