Induced antigravitation in the extended general relativity

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Abstract

Extension of the general relativity on the case of non-Riemannian geometries, suitable for description of the space-time geometry, leads to integral dynamic equations, which are valid for continuous and discrete space-times. Gravitational field of homogeneous heavy non-rotating sphere is calculated inside the sphere. The space-time geometry appears to be non-Riemannian. In the case, when the gravitational radius of the sphere is of the order of the sphere radius, the induced antigravitation appears inside the sphere. In other words, the gravitational force inside the sphere appears to be directed from the sphere center. The antigravitation resists to collapsing of the sphere and to formation of a black hole.

1 Introduction

The extended general relativity is the general relativity, extended to arbitrary space-time geometry (Riemannian and non-Riemannian). The question: "Why does one consider extended general relativity (instead of the general relativity)?" is an example of an incorrect question. The correct question looks as follows. Why was the general relativity considered usually in the Riemannian space-time? The answer is as follows. In the twentieth century the Riemannian geometry was considered as a most general geometry, suitable for the space-time description. Physical geometry [1, 2], described completely by the world function σ [3] (or by distance) was not known. The physical geometry is nonaxiomatizable geometry, in general. The scientists thought, that there exist only axiomatizable geometries, which are logical constructions. Thus, the extended general relativity (EGR) realizes a natural approach to investigation of the space-time properties, whereas the general relativity

(GR) realizes such an approach, which is constrained by our imperfect knowledge of geometry. Presentation of EGR can be found in [4].

From formal mathematical viewpoint the Riemannian geometry is described by the world function σ, satisfying the equation

$$\frac{\partial \sigma\left(x,x'\right)}{\partial x^i} g^{ik}\left(x\right) \frac{\partial \sigma\left(x,x'\right)}{\partial x^k} = 2\sigma\left(x,x'\right), \qquad \sigma\left(x,x'\right) = \sigma\left(x',x\right) \tag{1.1}$$

where $g^{ik}\left(x\right)$ is the metric tensor at the point x, defined via world function by means of relations

$$g^{ik}\left(x\right) g_{kl}\left(x\right) = \delta_l^i, \qquad g_{kl}\left(x\right) = \left[\frac{\partial \sigma\left(x,x'\right)}{\partial x^k \partial x^l}\right]_{x'=x} \tag{1.2}$$

Because of constraint (1.1) the set of all Riemannian geometries is a small part of the set of all physical geometries. It is quite obscure, why one should consider only Riemannian geometries, constrained by the condition (1.1).

The gravitational field, calculated in the framework of EGR distinguishes from the gravitational field, calculated in the framework of GR. In particular, the space-time geometry, generated by a heavy sphere of radius R and of mass M is different in EGR and GR, because the space-time geometry appears to be non-Riemannian in EGR. The difference is small, if the parameter $\varepsilon = r_g/R = 2GM/\left(Rc^2\right)$ is small. Here r_g is the gravitational radius of the sphere. Nevertheless a new gravitational effect (induced antigravitation) appears inside the sphere, considered in the framework of EGR. In other words, the gravitational force, directed from the sphere center, appears inside the sphere at some values of ε. This force imitates antigravitation, although it does not mean, that the particles begin to repulse instead of attraction. The repulsion from the sphere center of a particle inside the sphere is induced by attraction of particles, which are located farther from the center, than the considered particle.

Let us consider a hallow uniform heavy sphere of the mass M with internal radius R_1 and external radius R. Let the origin of the Cartesian coordinate system is at the center of the sphere. According to the Newtonian gravitational theory the gravitational potential φ has the form

$$\varphi\left(\mathbf{x}\right) = \left\{\begin{array}{ccc} \frac{3}{2}\frac{GM}{R^2}\left(1 - \frac{R_1^2}{R^2} - \frac{R_1^3}{R^3}\right) & \text{if} & \left|\mathbf{x}\right| < R_1 \\ -\frac{GM}{R^3}\left(\frac{1}{2}\left|\mathbf{x}\right|^2 + \frac{R_1^3}{\left|\mathbf{x}\right|}\right) + \frac{3}{2}\frac{GM}{R}\left(1 - \frac{R_1^3}{R^3}\right) & \text{if} & R_1 < \left|\mathbf{x}\right| < R \\ \frac{GM}{\left|\mathbf{x}\right|}\left(1 - \frac{R_1^3}{R^3}\right) & \text{if} & \left|\mathbf{x}\right| > R \end{array}\right. \tag{1.3}$$

The gravitational force is proportional to the quantity $\mathbf{F} = \boldsymbol{\nabla}\varphi(\mathbf{x})$

$$\mathbf{F} = \boldsymbol{\nabla}\varphi(\mathbf{x}) = \left\{\begin{array}{ccc} 0 & \text{if} & \left|\mathbf{x}\right| < R_1 \\ -\frac{GM}{R^3}\left(1 - \frac{R_1^3}{\left|\mathbf{x}\right|^3}\right)\left|\mathbf{x}\right|\mathbf{x} & \text{if} & R_1 < \left|\mathbf{x}\right| < R \\ -\frac{GM}{\left|\mathbf{x}\right|^2}\left(1 - \frac{R_1^3}{R^3}\right)\mathbf{x} & \text{if} & \left|\mathbf{x}\right| > R \end{array}\right. \tag{1.4}$$

It follows from (1.3), (1.4), that the gravitational potential is maximal in the region $|\mathbf{x}| < R_1$, whereas the gravitational force is minimal in this region. It means that the variation of the metric tensor $\delta g_{00} = -2\varphi/c^2$ is maximal near the sphere center, where the gravitational force $\mathbf{F} = 0$. The gravitational force vanishes, because the force is a vector, and the gravitational influence of different parts of the sphere compensate each other, but not because the gravitational influence on the space-time geometry vanishes near the sphere center.

Let us imagine that the real gravitational force distinguishes from the Newtonian gravitational law. In this case there is possible such a situation, that the gravitational force near the sphere center is not compensated completely. The resulting gravitational force may be directed from the sphere center. It will be the induced antigravitation, generated by the attraction of different parts of the sphere.

We are going to calculate correction to the Newtonian gravitational law (and to the gravitation law of GR). It appears in EGR, that if the parameter ε is not too small, the corrections to the Newtonian gravitational law dominate over the Newtonian gravitational force. The induced antigravitation, generated by attraction, appears inside the gravitating matter, distributed in the space. This antigravitation prevents from collapsing (when parameter ε increases) of a physical body. As a result the black hole formation appears to be impossible. Thus, corrections to the gravitational law of Newton (and GR) may be essential at construction of cosmological models.

A physical geometry is a monistic conception [5], described completely by the only fundamental geometric quantity σ (world function [3]). All other geometric quantities are some known functions of the world function σ. To modify a physical geometry, it is sufficient to modify the world function σ. Other geometrical quantities are modified automatically, because they are the same known functions of σ. A form of these functions can be determined at consideration of the proper Euclidean geometry.

At construction of a Riemannian geometry one uses the pluralistic approach [5], when there are several independent fundamental geometric quantities (dimension, vector, scalar product, etc.). There exist connections between these fundamental quantities, but not all these connections are taken into account at the pluralistic approach.

Any geometry is a result of modification (generalization) of the proper Euclidean geometry. The Riemannian geometry is obtained as a result of a modification of the proper Euclidean geometry, considered as a pluralistic conception. At such a modification all fundamental quantities are modified independently. As a result the obtained geometry may appear to be inconsistent. The Riemannian geometry appears to be inconsistent, indeed [6, 5].

The physical geometry is a result of a modification of the Euclidean geometry, considered as a monistic conception, described by the only fundamental quantity (world function). As a result the physical geometry appears to be consistent. Furthermore, the physical geometry cannot be inconsistent, because, in general, it is a nonaxiomaitzable geometry, whose formalism does not use the formal logic. Instead of rules of the formal logic the physical geometry uses the rules of such a logical construction as the proper Euclidean geometry [7].

Extension of the general relativity to the case of physical space-time geometry has been produced [4, 8]. As a result of such a generalization one obtains a coordinateless formalism, which is suitable for any space-time geometry (continuous and discrete), because it does not use differential equations. This formalism is rather uncustomary, but it is very simple and effective.

The general relativity, extended to the arbitrary space-time geometry is a rather radical conception, which is free of defects of the conventional general relativity theory. The conventional general relativity uses inconsistent Riemannian geometry and ignores most part of possible space-time geometries. These (physical) space-time geometries are consistent and nonaxiomatizable, in general. However, scientists (even mathematicians) are not able to work with nonaxiomatizable geometries, and they prefer to ignore them. Besides, the general relativity, which is *essentially a geometrization of physics*, does not realizes this geometrization completely. In particular, the general relativity does not take into account the relativistic concept of nearness [4]. Instead of the relativistic concept of nearness the conventional general relativity uses the nonrelativistic concept of nearness, which disagrees with the relativity principles. Formalism of the extended theory is more perfect. Description of the space-time geometry is non-local. It is realized by means of integral equations, which are insensitive to continuity of the space-time.

Especially one should stress, that *the extended theory does not use any new hypotheses. It only corrects inconsequences of the conventional conception of general relativity.* It means, that if one trusts to the conventional general relativity, one is to trust to expanded general relativity, because it does not contain anything, which is outside the relativity principles. All new results of the expanded general relativity are results of overcoming of the conventional theory defects and preconceptions.

In this paper we apply the formalism of the extended general relativity theory for calculation of the space-time geometry, generated by a heavy non-rotating sphere. Practically we calculate corrections to the Newtonian gravitational potential. We are going to show, that correction to the Newtonian potential takes place. The gravitational field component, connected with this correction, corresponds to repulsion of particles from the sphere center. This induced antigravitation hinders from collapsing of the sphere. As a result the Schwarzchild sphere existence becomes to be impossible.

2 Statement of the problem

Let there be the space-time geometry, described by the world function σ_0. Let us add a particle in the space-time. The space-time geometry changes. It becomes to be described by the world function $\sigma = \sigma_0 + \delta\sigma$. We need to determine $\delta\sigma$.

The particle is described by its world chain

$$\mathcal{C} = \sum_l \mathcal{T}_{[P_l P_{l+1}]} \tag{2.1}$$

where $\mathcal{T}_{[P_l P_{l+1}]}$, $l = ... - 1, 0, 1, ...$are segments of the timelike straight line of the same length. Any link of the world chain is the segment $\mathcal{T}_{[P_l P_{l+1}]}$, defined as a set of points R by means of the relation

$$\mathcal{T}_{[P_l P_{l+1}]} = \left\{ R | \sqrt{2\sigma\left(P_l, R\right)} + \sqrt{2\sigma\left(P_{l+1}, R\right)} = \sqrt{2\sigma\left(P_l, P_{l+1}\right)} \right\} \tag{2.2}$$

Here $\sqrt{2\sigma\left(P_l, R\right)}$ is a distance between the points P_l and R. In the proper Euclidean geometry a segment of straight is defined by the relation (2.2). In any physical geometry a segment of the straight is defined by the same relation (2.2), where σ is the world function of the physical geometry in question. If the world chain describes a free motion of a particle, the vectors $\mathbf{P}_l\mathbf{P}_{l+1}$ and $\mathbf{P}_{l+1}\mathbf{P}_{l+2}$, describing adjacent segments $\mathcal{T}_{[P_l P_{l+1}]}$ and $\mathcal{T}_{[P_{l+1} P_{l+2}]}$, are equivalent ($\mathbf{P}_l\mathbf{P}_{l+1}\text{eqv}\mathbf{P}_{l+1}\mathbf{P}_{l+2}$). It means by definition, that vectors $\mathbf{P}_l\mathbf{P}_{l+1}$ and $\mathbf{P}_{l+1}\mathbf{P}_{l+2}$ are in parallel ($\mathbf{P}_l\mathbf{P}_{l+1}\upuparrows\mathbf{P}_{l+1}\mathbf{P}_{l+2}$) and their modules $|\mathbf{P}_l\mathbf{P}_{l+1}|$ and $|\mathbf{P}_{l+1}\mathbf{P}_{l+2}|$ are equal.

In the proper Euclidean geometry vectors $\mathbf{P}_l\mathbf{P}_{l+1}$ and $\mathbf{P}_{l+1}\mathbf{P}_{l+2}$ are in parallel, if

$$\left(\mathbf{P}_l\mathbf{P}_{l+1}\upuparrows\mathbf{P}_{l+1}\mathbf{P}_{l+2}\right): \qquad \left(\mathbf{P}_l\mathbf{P}_{l+1}.\mathbf{P}_{l+1}\mathbf{P}_{l+2}\right) = |\mathbf{P}_l\mathbf{P}_{l+1}| \cdot |\mathbf{P}_{l+1}\mathbf{P}_{l+2}| \tag{2.3}$$

where scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma\left(P_0, Q_1\right) + \sigma\left(P_1, Q_0\right) - \sigma\left(P_0, Q_0\right) - \sigma\left(P_1, Q_1\right) \tag{2.4}$$

In the proper Euclidean geometry the definition of the scalar product coincides with the conventional definition in terms of the linear vector space, which can be introduced in the proper Euclidean geometry. The definition of the scalar product in terms of the world function is useful in the sense, that it does not contain a reference to a linear vector space and to a coordinate system. It can be used in any physical geometry, which is described completely by its world function. It is of no importance, whether or not a linear vector space can be introduced in this geometry.

The equivalence condition $(\mathbf{P}_l\mathbf{P}_{l+1}\text{eqv}\mathbf{P}_{l+1}\mathbf{P}_{l+2})$ of two vectors $\mathbf{P}_l\mathbf{P}_{l+1}$ and $\mathbf{P}_{l+1}\mathbf{P}_{l+2}$ is written in the form

$$\begin{aligned} \left(\mathbf{P}_l\mathbf{P}_{l+1}\text{eqv}\mathbf{P}_{l+1}\mathbf{P}_{l+2}\right) \quad &: \qquad \left(\mathbf{P}_l\mathbf{P}_{l+1}.\mathbf{P}_{l+1}\mathbf{P}_{l+2}\right) = |\mathbf{P}_l\mathbf{P}_{l+1}| \cdot |\mathbf{P}_{l+1}\mathbf{P}_{l+2}| & (2.5) \\ \wedge |\mathbf{P}_l\mathbf{P}_{l+1}| \quad &= \quad |\mathbf{P}_{l+1}\mathbf{P}_{l+2}| & (2.6) \end{aligned}$$

Using (2.4) and

$$|\mathbf{P}_l\mathbf{P}_{l+1}| = \sqrt{2\sigma\left(P_l, P_{l+1}\right)} \tag{2.7}$$

one can rewrite two conditions (2.5), (2.6) in the form

$$\left(\mathbf{P}_l\mathbf{P}_{l+1}\text{eqv}\mathbf{P}_{l+1}\mathbf{P}_{l+2}\right): \qquad 4\sigma\left(P_l, P_{l+1}\right) = \sigma\left(P_l, P_{l+2}\right) \wedge \sigma\left(P_l, P_{l+1}\right) = \sigma\left(P_{l+1}, P_{l+2}\right) \tag{2.8}$$

Thus, the world chain of a free particle is described geometrically (in terms of the world function). The particle mass is described also geometrically as a length of a link of the world chain. $\mu = |\mathbf{P}_l\mathbf{P}_{l+1}|$ is the geometric mass of the particle. The usual mass m is connected with the geometric mass μ by means of the relation

$$m = b\mu = b\,|\mathbf{P}_l\mathbf{P}_{l+1}| \tag{2.9}$$

where b is some universal constant. Such a presentation of the particle mass admits one to reduce quantum effects to pure geometrical effects. Such a reduction is possible, if the space-time geometry in microcosm is described by the world function σ_d [9]

$$\sigma_\mathrm{d} = \sigma_\mathrm{M} + \lambda^2 \mathrm{sgn}\sigma_\mathrm{M}, \qquad \lambda^2 = \frac{\hbar}{2bc} \tag{2.10}$$

where σ_M is the world function of the Minkowski geometry, $\hbar$ is the quantum constant and c is the speed of the light.

Geometrization of the particle mass admits one to obtain completely geometrical description of the particle motion, when the particle mass is described by the length of links of its world chain (but not as some external parameter, attributed to the particle). The space-time geometry, described by the world function σ_d appears to be a discrete geometry, although it is given on a continuous manifold of Minkowski. From the conventional viewpoint, when the discrete geometry is given on a lattice point set, such a situation seems to be impossible. Discreteness of the geometry (2.10) manifests itself in the fact, that in the geometry (2.10) there are no vectors $\mathbf{P}_0\mathbf{P}_1$, whose length $|\mathbf{P}_0\mathbf{P}_1|$ lies in the interval $\left(0, \sqrt{2}\lambda\right)$. The geometry (2.10) is invariant with respect to space-time rotations, what is impossible for a geometry on a lattice point set.

Note, that the electrical charge is also geometrized. It is the component of the particle momentum along the fifth direction in the 5D geometry of Kaluza-Klein. Spin of elementary particle can be geometrized also [10]. However, in this case the elementary particle should be considered as a geometrical object, described by its skeleton $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$, consisting of $n+1$ space-time points $P_0, P_1, ...P_n$. In this case the motion of an elementary particle is described as a world chain, consisting of connected skeletons [11]. Such an approach admits one to geometrize completely the particle description and to reduce the fundamental particle physics to consideration of space-time points and distances (world functions) between them. At the description of a particle by its skeleton $\mathcal{P}_n$ such a problem appears. What reasons (or fields) do connect different points of the skeleton between themselves? Such a question appears, because we are used to consider the space-time geometry as an infinitely divisible geometry. However, the space-time geometry of microcosm may be a restrictedly divisible geometry. In this case there is no necessity to explain connections between the points of a skeleton. The restrictedly divisible space-time geometry is completely described by the world function as well as the infinitely divisible one.

For instance, for explanation of the quarks confinement there is no necessity to introduce gluons. It is sufficient to refer to the restrictedly divisible space-time geometry in microcosm.

In this connection it is worth to note, that the relativity theory is essentially a geometrization of physics. However, this geometrization is incomplete. Particle mass, particle charge and some other particle characteristics remain to be not geometrized. This incomplete geometrization was connected with insufficient knowledge of geometry, when nonaxiomatizable geometries were unknown. Now, having a more perfect knowledge of geometry, we hope to succeed in a complete geometrization of physics, when on the fundamental level all physical phenomena can be described in terms of the particle skeletons and world function of the space-time. Realization of the physics geometrization program admits one to construct a monistic conception of fundamental physics, when all physical phenomena are described in terms of one fundamental quantity (world function). All other physical quantities will appear to be derivative quantities of the world function. In particular, the force fields (electromagnetic and gravitational and other) appear to be attributes of the space-time geometry (world function), but not independent essences. Such an approach is very uncustomary (Einstein dreamed on united *field* theory, where the fields were some fundamental essences). We shall speak on the physics geometrization program instead of the united field theory. The geometrization program is a monistic conception, or at least, it is less pluralistic conception, than the united field theory.

If the space-time geometry is described by the world function σ, the space-time geometry with additional particles is described by the world function $\sigma = \sigma_0 + \delta\sigma$, where the variation $\delta\sigma$ of the world function is described by the relation (see details in [4])

$$\begin{aligned}\delta\sigma\left(S_1, S_2\right) &= -\frac{G}{c^2}\sum_s m_{(s)}\frac{\theta\left(\left(\mathbf{P}_l'\mathbf{P}.\mathbf{PQ}_0\right)\right)\left(\mathbf{P}_l'\mathbf{P}_{l+1}'.\mathbf{PQ}_0\right)}{\left(\mathbf{P}_l'\mathbf{P}.\mathbf{P}_l'\mathbf{P}_{l+1}'\right)\left|\mathbf{PQ}_0\right|} \\ &\quad\times\frac{\left(\left(\mathbf{P}_l'\mathbf{P}_{l+1}'.\mathbf{PS}_1\right)-\left(\mathbf{P}_l'\mathbf{P}_{l+1}'.\mathbf{PS}_2\right)\right)^2}{\left(\mathbf{P}_l'\mathbf{P}_{l+1}'.\mathbf{P}_l'\mathbf{P}_{l+1}'\right)} \qquad (2.11)\end{aligned}$$

where S_1 and S_2 are arbitrary points of the space-time, $m_{(s)}$ is the mass of sth particle, G is the gravitational constant. Summation is produced over all additional particles. All scalar products in rhs of (2.11) are calculated by means of the relation (2.4) with the world function $\sigma = \sigma_0 + \delta\sigma$, which is unknown at first. As a result the relation (2.11) is an equation for determination of $\delta\sigma$ (or $\sigma = \sigma_0 + \delta\sigma$).

The point P is the middle of the segment S_1S_2, determined by the relations

$$4\sigma\left(P, S_1\right) = \sigma\left(S_1, S_2\right), \qquad 4\sigma\left(P, S_2\right) = \sigma\left(S_1, S_2\right) \qquad (2.12)$$

The point P_l' is the point of the world line of sth particle $P_l' \in \mathcal{L}_{(s)}$, which is determined by the relation

$$\sigma\left(P, P_l'\right) = 0, \qquad P_l' \in \mathcal{L}_{(s)} \qquad (2.13)$$

The point $P_{l+1}' \in \mathcal{L}_{(s)}$ is infinitely close to the point P_l', lying on the same world line $\mathcal{L}_{(s)}$.

In the case of continuous distribution of particles the summation in (2.11) is to be substituted by integration over Lagrangian coordinates $\boldsymbol{\xi}$, labelling the perturbing particles. One obtains

$$\begin{aligned}\delta\sigma\left(S_1,S_2\right) &= -\frac{G}{c^2}\int_V \rho\left(\boldsymbol{\xi}\right)d\boldsymbol{\xi}\frac{\theta\left(\left(\mathbf{P}_l'\mathbf{P}.\mathbf{PQ}_0\right)\right)\left(\mathbf{P}_l'\mathbf{P}_{l+1}'.\mathbf{PQ}_0\right)}{\left(\mathbf{P}_l'\mathbf{P}.\mathbf{P}_l'\mathbf{P}_{l+1}'\right)\left|\mathbf{PQ}_0\right|} \\ &\quad\times\frac{\left(\left(\mathbf{P}_l'\mathbf{P}_{l+1}'.\mathbf{PS}_1\right)-\left(\mathbf{P}_l'\mathbf{P}_{l+1}'.\mathbf{PS}_2\right)\right)^2}{\left(\mathbf{P}_l'\mathbf{P}_{l+1}'.\mathbf{P}_l'\mathbf{P}_{l+1}'\right)}\end{aligned} \tag{2.14}$$

where $\rho\left(\boldsymbol{\xi}\right)$ is mass density of additional particles. V is the volume in the space of Lagrangian coordinates $\boldsymbol{\xi}$, labelling the additional particles. The total mass M of additional particles is defined by the relation

$$\int_V \rho\left(\boldsymbol{\xi}\right)d\boldsymbol{\xi} = M \tag{2.15}$$

The points S_1 and S_2 are arbitrary points of the space-time. The point P is to be some function $P = P\left(S_1,S_2\right)$ of points S_1 and S_2, which determined by relations (2.12). The function $P\left(S_1,S_2\right)$ is symmetric

$$P\left(S_1,S_2\right) = P\left(S_2,S_1\right) \tag{2.16}$$

The point P is the origin of a coordinate system with basic vectors $\mathbf{PQ}_i$, $i = 0,1,2,3$. Vector $\mathbf{PQ}_0$ is timelike.

It is supposed, that the motion of additional particles is fixed (their world lines are fixed), and one needs to determine the world function, generated by them. We know only one method of the equation (2.14) solution. It is the method of successive approximations. One gives the zero approximation σ_0 of the world function and calculate $\delta\sigma_1$, using σ_0 in rhs of (2.14). One obtains $\sigma_1 = \sigma_0 + \delta\sigma_1$. It is the first approximation of the world function. One calculates rhs of (2.14), using σ_1, and obtains $\delta\sigma_2$. The world function $\sigma_2 = \sigma_0 + \delta\sigma_2$ is the second approximation. In the same way one calculates the third approximation $\sigma_3 = \sigma_0 + \delta\sigma_3$, where $\delta\sigma_3$ is calculated by means of σ_2 and so on, When σ_{n+1} coincides with σ_n, one obtains solution of the equations (2.14), (2.13).

3 World function of non-rotating body

In the case of non-rotating physical body, which is at rest, one can obtain three integral equations for calculation of the world function for the points $S_1 = \{t_1,\mathbf{y}_1\}$, $S_2 = \{t_2,\mathbf{y}_2\}$. The world function is taken in the form of the second order polynomial [4] of $(t_2 - t_1)$

$$\sigma\left(t_1,\mathbf{y}_1;t_2,\mathbf{y}_2\right) = \frac{1}{2}A\left(\mathbf{y}_1,\mathbf{y}_2\right)c^2\left(t_2-t_1\right)^2 + B\left(\mathbf{y}_1,\mathbf{y}_2\right)c\left(t_2-t_1\right) + C\left(\mathbf{y}_1,\mathbf{y}_2\right) \tag{3.1}$$

Functions

$$A(\mathbf{y}_1,\mathbf{y}_2) = 1 - V(\mathbf{y}_1,\mathbf{y}_2), \qquad B(\mathbf{y}_1,\mathbf{y}_2), \qquad C(\mathbf{y}_1,\mathbf{y}_2) = -\frac{1}{2}(\mathbf{y}_1-\mathbf{y}_2)^2 + \delta C(\mathbf{y}_1,\mathbf{y}_2) \tag{3.2}$$

should be determined from the integral equations, written for variables $V, B, \delta C$

The form of the world function in the form of the second order polynomial of $(t_2 - t_1)$ is conserved after a use of the equation (2.14) [4]. As a result one can obtain equations for the quantities $V, B, \delta C$ in the form

$$V(\mathbf{y}_1,\mathbf{y}_2) = \frac{2G}{c^2}\int_V \frac{\rho(\boldsymbol{\xi}) A(\boldsymbol{\xi},\mathbf{x}) \left(1 - \frac{1}{2}\left(V(\boldsymbol{\xi},\mathbf{y}_2) + V(\boldsymbol{\xi},\mathbf{y}_1)\right)\right)^2}{A(\boldsymbol{\xi},\boldsymbol{\xi})\sqrt{A(\mathbf{x},\mathbf{x})}\sqrt{B^2(\boldsymbol{\xi},\mathbf{x}) + A(\boldsymbol{\xi},\mathbf{x})\left((\mathbf{x}-\boldsymbol{\xi})^2 - 2\delta C(\boldsymbol{\xi},\mathbf{x})\right)}} d\boldsymbol{\xi} \tag{3.3}$$

$$\begin{aligned} B(\mathbf{y}_1,\mathbf{y}_2) = -2\frac{G}{c^2}\int_V & \frac{\rho(\boldsymbol{\xi}) A(\boldsymbol{\xi},\mathbf{x}) \left(1 - \frac{1}{2}\left(V(\boldsymbol{\xi},\mathbf{y}_2) + V(\boldsymbol{\xi},\mathbf{y}_1)\right)\right)}{A(\boldsymbol{\xi},\boldsymbol{\xi})\sqrt{A(\mathbf{x},\mathbf{x})}\sqrt{B^2(\boldsymbol{\xi},\mathbf{x}) + A(\boldsymbol{\xi},\mathbf{x})\left((\mathbf{x}-\boldsymbol{\xi})^2 - 2\delta C(\boldsymbol{\xi},\mathbf{x})\right)}} d\boldsymbol{\xi} \\ & \times \left(\left(V(\boldsymbol{\xi},\mathbf{y}_2) - V(\boldsymbol{\xi},\mathbf{y}_1)\right) r + \left(B(\boldsymbol{\xi},\mathbf{y}_1) - B(\boldsymbol{\xi},\mathbf{y}_2)\right)\right) \end{aligned} \tag{3.4}$$

$$\delta C(\mathbf{y}_1,\mathbf{y}_2) = -\frac{G}{c^2}\int_V \frac{\rho(\boldsymbol{\xi}) A(\boldsymbol{\xi},\mathbf{x}) \left(\left(V(\boldsymbol{\xi},\mathbf{y}_2) - V(\boldsymbol{\xi},\mathbf{y}_1)\right) r + \left(B(\boldsymbol{\xi},\mathbf{y}_1) - B(\boldsymbol{\xi},\mathbf{y}_2)\right)\right)^2}{A(\boldsymbol{\xi},\boldsymbol{\xi})\sqrt{A(\mathbf{x},\mathbf{x})}\sqrt{B^2(\boldsymbol{\xi},\mathbf{x}) + A(\boldsymbol{\xi},\mathbf{x})\left((\mathbf{x}-\boldsymbol{\xi})^2 - 2\delta C(\boldsymbol{\xi},\mathbf{x})\right)}} d\boldsymbol{\xi} \tag{3.5}$$

where

$$\mathbf{x} = \frac{\mathbf{y}_1 + \mathbf{y}_2}{2}, \tag{3.6}$$

$$r = \frac{-B(\boldsymbol{\xi},\mathbf{x}) + \sqrt{B^2(\boldsymbol{\xi},\mathbf{x}) + A(\boldsymbol{\xi},\mathbf{x})\left((\mathbf{x}-\boldsymbol{\xi})^2 - 2\delta C(\boldsymbol{\xi},\mathbf{x})\right)}}{A(\boldsymbol{\xi},\mathbf{x})} \tag{3.7}$$

In the special case, when $\mathbf{y}_1 = \mathbf{y}_2$, one obtains from (3.3) - (3.6), that

$$\mathbf{y}_1 = \mathbf{y}_2 = \mathbf{x}, \qquad B(\mathbf{y}_1,\mathbf{y}_1) = 0, \qquad \delta C(\mathbf{y}_1,\mathbf{y}_1) = 0 \tag{3.8}$$

The equation (3.3) takes the form

$$\begin{aligned} & \left(1 - A(\mathbf{x},\mathbf{x})\right)\sqrt{A(\mathbf{x},\mathbf{x})} \\ = & \frac{2G}{c^2}\int_V \frac{\rho(\boldsymbol{\xi}) A(\boldsymbol{\xi},\mathbf{x}) \left(1 - V(\boldsymbol{\xi},\mathbf{x})\right)^2}{A(\boldsymbol{\xi},\boldsymbol{\xi})\sqrt{B^2(\boldsymbol{\xi},\mathbf{x}) + A(\boldsymbol{\xi},\mathbf{x})\left((\mathbf{x}-\boldsymbol{\xi})^2 - 2\delta C(\boldsymbol{\xi},\mathbf{x})\right)}} d\boldsymbol{\xi} \end{aligned} \tag{3.9}$$

It follows from (3.9) that the component $g_{00}(t,\mathbf{x}) = c^2 A(\mathbf{x},\mathbf{x})$ of the metric tensor g_{ik} cannot be negative because of the factor $\sqrt{A(\mathbf{x},\mathbf{x})}$.

It is well known, that the Schwarzchield surface, determining existence of a black hole, is defined by the relation

$$g_{00}(t,\mathbf{x}) = c^2 A(\mathbf{x},\mathbf{x}) = 0 \tag{3.10}$$

It means, that any non-rotating body, consisting of motionless particles cannot generate a black hole.

What is a reason of such an unexpected result, different from the result of the general relativity? Apparently, it is the induced antigravitation, which has been discussed in Introduction. The antigravitation hinders from collapsing of the body. We are going to show, that the antigravitation takes place, indeed.

4 World function of the homogeneous heavy sphere

We consider homogeneous sphere of radius R and of the mass M. Two first approximations of solution of the equations (3.3) - (3.7) will be calculated. Integrals in these equations are transformed into integrals of the form

$$\int_0^R \xi^2 d\xi \int_{-\pi/2}^{\pi/2} \sin\theta d\theta \int_0^{2\pi} \rho_0\left(.\right) d\varphi \tag{4.1}$$

where

$$\rho\left(\boldsymbol{\xi}\right) = \rho_0 = \frac{3M}{4\pi R^3} = \text{const} \tag{4.2}$$

The quantity $\varepsilon = r_g/R = 2GM/Rc^2$ is considered to be a small quantity.

$$\varepsilon = \frac{2GM}{c^2 R} = \frac{8\pi G R^2}{3c^2}\rho_0 \ll 1 \tag{4.3}$$

As a zeroth approximation one takes the empty space-time, described by the geometry of Minkowski. In this case

$$A_0\left(\mathbf{y}_1, \mathbf{y}_2\right) = 1, \qquad V_0\left(\mathbf{y}_1, \mathbf{y}_2\right) = 0, \qquad B_0\left(\mathbf{y}_1, \mathbf{y}_2\right) = 0, \tag{4.4}$$

$$C_0\left(\mathbf{y}_1, \mathbf{y}_2\right) = -\frac{1}{2}\left(\mathbf{y}_1 - \mathbf{y}_2\right)^2, \qquad \delta C_0\left(\mathbf{y}_1, \mathbf{y}_2\right) = 0 \tag{4.5}$$

It follows from (3.3) - (3.5), that

$$V_1\left(\mathbf{y}_1, \mathbf{y}_2\right) = \mathcal{O}\left(\varepsilon\right), \qquad B_1\left(\mathbf{y}_1, \mathbf{y}_2\right) = \mathcal{O}\left(\varepsilon^2\right), \qquad \delta C_1\left(\mathbf{y}_1, \mathbf{y}_2\right) = \mathcal{O}\left(\varepsilon^3\right) \tag{4.6}$$

The equation (3.3) has the form

$$V_1\left(\mathbf{y}_1, \mathbf{y}_2\right) = \frac{2G}{c^2}\int_0^R \xi^2 d\xi \int_{-\pi/2}^{\pi/2} \sin\theta d\theta \int_0^{2\pi} \frac{3M}{4\pi R^3}\frac{d\varphi}{\sqrt{\left(\mathbf{x} - \boldsymbol{\xi}\right)^2}} \tag{4.7}$$

As a result in the first approximation the world function has the form

$$\sigma_1\left(t_1, \mathbf{y}_1; t_2, \mathbf{y}_2\right) = \frac{1}{2}\left(1 - V_1\left(\mathbf{y}_1, \mathbf{y}_2\right)\right)c^2\left(t_2 - t_1\right)^2 - \frac{1}{2}\left(\mathbf{y}_2 - \mathbf{y}_1\right)^2 \tag{4.8}$$

where

$$V_1\left(\mathbf{y}_1, \mathbf{y}_2\right) = \begin{cases} \frac{2GM}{c^2|\mathbf{x}|} & \text{if } |\mathbf{x}| > R \\ \frac{3GM}{c^2 R} - \frac{GM}{c^2 R^3}|\mathbf{x}|^2 & \text{if } |\mathbf{x}| < R \end{cases}, \qquad \mathbf{x} = \frac{\mathbf{y}_1 + \mathbf{y}_2}{2} \tag{4.9}$$

One can see, that already in the first approximation the space-time geometry is non-Riemannian, although component $g_{00}(\mathbf{x}) = c^2(1 - V_1(\mathbf{x},\mathbf{x}))$ of the metric tensor coincides with the Newtonian approximation $c^2 - 2\varphi(\mathbf{x})$, where φ is determined by relation (1.3). Other components of the metric tensor coincide also.

In the second approximation equations (3.3) -(3.7) take the form

$$V_2(\mathbf{y}_1,\mathbf{y}_2) = \frac{2G}{c^2}\int_V \frac{\rho_0\sqrt{A_1(\boldsymbol{\xi},\mathbf{x})}\left(1 - \frac{1}{2}\left(V_1(\boldsymbol{\xi},\mathbf{y}_2) + V_1(\boldsymbol{\xi},\mathbf{y}_1)\right)\right)^2}{A_1(\boldsymbol{\xi},\boldsymbol{\xi})\sqrt{A_1(\mathbf{x},\mathbf{x})}\sqrt{(\mathbf{x}-\boldsymbol{\xi})^2}}d\boldsymbol{\xi} \tag{4.10}$$

$$B_2(\mathbf{y}_1,\mathbf{y}_2) = -2\frac{G}{c^2}\int_V \frac{\rho_0\sqrt{A_1(\boldsymbol{\xi},\mathbf{x})}\left(1 - \frac{1}{2}\left(V_1(\boldsymbol{\xi},\mathbf{y}_2) + V_1(\boldsymbol{\xi},\mathbf{y}_1)\right)\right)}{A_1(\boldsymbol{\xi},\boldsymbol{\xi})\sqrt{A_1(\mathbf{x},\mathbf{x})}\sqrt{(\mathbf{x}-\boldsymbol{\xi})^2}}d\boldsymbol{\xi}$$
$$\times\left(\left(V_1(\boldsymbol{\xi},\mathbf{y}_2) - V_1(\boldsymbol{\xi},\mathbf{y}_1)\right) r_1\right) \tag{4.11}$$

$$\delta C_2(\mathbf{y}_1,\mathbf{y}_2) = -\frac{G}{c^2}\int_V \frac{\rho_0\sqrt{A_1(\boldsymbol{\xi},\mathbf{x})}\left(\left(V_1(\boldsymbol{\xi},\mathbf{y}_2) - V_1(\boldsymbol{\xi},\mathbf{y}_1)\right) r_1\right)^2}{A_1(\boldsymbol{\xi},\boldsymbol{\xi})\sqrt{A_1(\mathbf{x},\mathbf{x})}\sqrt{(\mathbf{x}-\boldsymbol{\xi})^2}}d\boldsymbol{\xi} \tag{4.12}$$

$$r_1 = \sqrt{\frac{(\mathbf{x}-\boldsymbol{\xi})^2}{A_1(\boldsymbol{\xi},\mathbf{x})}} \tag{4.13}$$

The quantity $V_1(\mathbf{y}_1,\mathbf{y}_2)$ is a small quantity of the order ε. The quantity $A_1 = 1 - V_1 = 1 + \mathcal{O}(\varepsilon)$. The quantities $V_2, B_2, \delta C_2$ are calculated to within ε^2. Expanding (4.11), (4.12) over powers of ε and taking into account that, $2\frac{G}{c^2}\int_V \rho_0 d\boldsymbol{\xi}$ is a quantity of the order ε, one obtains to within ε^2

$$B_2(\mathbf{y}_1,\mathbf{y}_2) = -2\frac{G}{c^2}\int_V \rho_0\left(V_1(\boldsymbol{\xi},\mathbf{y}_2) - V_1(\boldsymbol{\xi},\mathbf{y}_1)\right)d\boldsymbol{\xi} + \mathcal{O}\left(\varepsilon^3\right) \tag{4.14}$$

$$\delta C_2(\mathbf{y}_1,\mathbf{y}_2) = -\frac{G}{c^2}\int_V \frac{\rho_0\sqrt{(\mathbf{x}-\boldsymbol{\xi})^2}\left(V_1(\boldsymbol{\xi},\mathbf{y}_2) - V_1(\boldsymbol{\xi},\mathbf{y}_1)\right)^2}{A_1(\boldsymbol{\xi},\boldsymbol{\xi})\sqrt{A_1(\mathbf{x},\mathbf{x})A_1(\boldsymbol{\xi},\mathbf{x})}}d\boldsymbol{\xi} = \mathcal{O}\left(\varepsilon^3\right) \tag{4.15}$$

$$\begin{aligned} V_2(\mathbf{y}_1,\mathbf{y}_2) &= V_1(\mathbf{y}_1,\mathbf{y}_2) + \frac{G}{c^2}\int_V \frac{\rho_0\left(2V_1(\boldsymbol{\xi},\boldsymbol{\xi}) - V_1(\boldsymbol{\xi},\mathbf{x}) + V_1(\mathbf{x},\mathbf{x})\right)}{\sqrt{(\mathbf{x}-\boldsymbol{\xi})^2}}d\boldsymbol{\xi} \\ &\quad - \frac{2G}{c^2}\int_V \frac{\rho_0\left(V_1(\boldsymbol{\xi},\mathbf{y}_2) + V_1(\boldsymbol{\xi},\mathbf{y}_1)\right)}{\sqrt{(\mathbf{x}-\boldsymbol{\xi})^2}}d\boldsymbol{\xi} + \mathcal{O}\left(\varepsilon^3\right) \end{aligned} \tag{4.16}$$

where according to (4.7)

$$V_1(\mathbf{y}_1,\mathbf{y}_2) = \frac{2G}{c^2}\int_V \frac{\rho_0}{\sqrt{\left(\frac{|\mathbf{y}_1+\mathbf{y}_2|^2}{4} - \boldsymbol{\xi}\right)^2}}d\boldsymbol{\xi} \tag{4.17}$$

Let us consider the case, when the points $\mathbf{y}_1, \mathbf{y}_2$ lie inside the sphere, i.e. $|\mathbf{y}_1|, |\mathbf{y}_2|, |\mathbf{x}| < R$. Then, using

$$V_1(\mathbf{y}_1, \mathbf{y}_2) = \frac{3GM}{c^2 R} - \frac{GM}{c^2 R^3}\left|\frac{\mathbf{y}_1+\mathbf{y}_2}{2}\right|^2 = \varepsilon\left(\frac{3}{2} - \frac{1}{2}\frac{\mathbf{x}^2}{R^2}\right) \tag{4.18}$$

the relation (4.16) is reduced to the form

$$V_2(\mathbf{y}_1, \mathbf{y}_2) = V_1(\mathbf{y}_1, \mathbf{y}_2) - 6\frac{G^2 M}{c^4 R}\int_V \frac{\rho_0 d\boldsymbol{\xi}}{\sqrt{|\mathbf{x}-\boldsymbol{\xi}|^2}}$$

$$+\frac{G^2 M}{4c^4 R^3}\int_V \frac{\rho_0\left(10\mathbf{x}\boldsymbol{\xi} - 3\boldsymbol{\xi}^2 - 3\mathbf{x}^2 + 2\mathbf{y}_2^2 + 2\mathbf{y}_1^2\right)}{\sqrt{|\mathbf{x}-\boldsymbol{\xi}|^2}} d\boldsymbol{\xi} + \mathcal{O}\left(\varepsilon^3\right) \tag{4.19}$$

This relation is transformed to the form

$$V_2(\mathbf{y}_1, \mathbf{y}_2) = V_1(\mathbf{y}_1, \mathbf{y}_2)\left(1 - 3\frac{GM}{c^2 R} + \frac{GM\left(2\left(\mathbf{y}_2^2+\mathbf{y}_1^2\right) - 3\mathbf{x}^2\right)}{8c^2 R^3}\right)$$

$$+\frac{5G^2 M}{2c^4 R^3}\int_V \frac{\rho_0 \mathbf{x}\boldsymbol{\xi}}{\sqrt{|\mathbf{x}-\boldsymbol{\xi}|^2}} d\boldsymbol{\xi} - \frac{3G^2 M}{4c^4 R^3}\int_V \frac{\rho_0 \boldsymbol{\xi}^2}{\sqrt{|\mathbf{x}-\boldsymbol{\xi}|^2}} d\boldsymbol{\xi} + \mathcal{O}\left(\varepsilon^3\right) \tag{4.20}$$

Calculation of integrals in (4.20) gives

$$\int_V \frac{\rho_0 \mathbf{x}\boldsymbol{\xi}}{\sqrt{|\mathbf{x}-\boldsymbol{\xi}|^2}} d\boldsymbol{\xi} = \frac{2\pi}{3}\rho_0 |\mathbf{x}|^2 R^2 - \frac{2\pi}{5}\rho_0 |\mathbf{x}|^4, \qquad |\mathbf{x}| < R \tag{4.21}$$

$$\int_V \frac{\rho_0 \boldsymbol{\xi}^2}{\sqrt{|\mathbf{x}-\boldsymbol{\xi}|^2}} d\boldsymbol{\xi} = \pi\rho_0 |\mathbf{x}|^4 \left(\left(\frac{R}{|\mathbf{x}|}\right)^4 - \frac{1}{5}\right), \qquad |\mathbf{x}| < R \tag{4.22}$$

Substituting (4.21), (4.22) in (4.20) and expressing M and ρ_0 via ε by means of (4.3), one obtains for (4.20)

$$\begin{aligned} & V_2(\mathbf{y}_1, \mathbf{y}_2) \\ = \ & \varepsilon\left(\frac{3}{2} - \frac{1}{2}\frac{\mathbf{x}^2}{R^2}\right) - \varepsilon^2\frac{153}{64} + \varepsilon^2\left(\frac{17}{16}\frac{\mathbf{x}^2}{R^2} + \frac{3}{2}\frac{\left(5\mathbf{x}^2 - 4\left(\mathbf{y}_2\mathbf{y}_1\right)\right)}{16R^2}\right) \\ & -\varepsilon^2\frac{51}{320}\frac{|\mathbf{x}|^4}{R^4} - \varepsilon^2\frac{1}{32}\frac{\mathbf{x}^2}{R^2}\frac{\left(5\mathbf{x}^2 - 4\left(\mathbf{y}_2\mathbf{y}_1\right)\right)}{R^2} + \mathcal{O}\left(\varepsilon^3\right), \qquad |\mathbf{x}|, |\mathbf{y}_1|, |\mathbf{y}_2| < R \end{aligned} \tag{4.23}$$

If besides, $\mathbf{y}_1 = \mathbf{y}_2 = \mathbf{x}$, then

$$V_2(\mathbf{x}, \mathbf{x}) = \varepsilon\left(\frac{3}{2} - \frac{1}{2}\frac{\mathbf{x}^2}{R^2}\right) - \varepsilon^2\frac{153}{64} + \varepsilon^2\frac{37}{32}\frac{\mathbf{x}^2}{R^2} - \varepsilon^2\frac{61}{320}\frac{|\mathbf{x}|^4}{R^4} + \mathcal{O}\left(\varepsilon^3\right) \tag{4.24}$$

The gravitational force inside the region $|\mathbf{x}| < R$ has the form

$$\mathbf{F} = \boldsymbol{\nabla} V_2(\mathbf{x}, \mathbf{x}) = -\frac{\varepsilon}{R^2}\mathbf{x} + \frac{\varepsilon^2}{R^2}\frac{37}{16}\mathbf{x} - \frac{61}{80}\frac{\varepsilon^2}{R^2}\frac{|\mathbf{x}|^2}{R^2}\mathbf{x}, \qquad |\mathbf{x}| < R \tag{4.25}$$

If $\varepsilon > \frac{16}{37} \approx 0.43$, the region, where the gravitational force is directed from the center, appears near the point $\mathbf{x} = 0$. If $\varepsilon \geq 0.65$, the gravitational force is directed from the center of the sphere in the whole region $|\mathbf{x}| < R$.

It is interesting to calculate the external gravitational potential V_{ext2}, generated by the hallow sphere with internal radius R_1 and external radius R. It is obtained by modifying the expression (4.10) by means of the replacement of the volume V ($|\boldsymbol{\xi}| < R$) by the volume V_1 ($R_1 < |\boldsymbol{\xi}| < R$). One obtains

$$V_{\text{ext2}}(\mathbf{y}_1, \mathbf{y}_2) = \frac{2G}{c^2} \int_{V_1} \frac{\rho_0 \sqrt{A_1(\boldsymbol{\xi}, \mathbf{x})} \left(1 - \frac{1}{2}\left(V_1(\boldsymbol{\xi}, \mathbf{y}_2) + V_1(\boldsymbol{\xi}, \mathbf{y}_1)\right)\right)^2}{A_1(\boldsymbol{\xi}, \boldsymbol{\xi}) \sqrt{A_1(\mathbf{x}, \mathbf{x})} \sqrt{(\mathbf{x} - \boldsymbol{\xi})^2}} d\boldsymbol{\xi} \qquad (4.26)$$

Calculation of (4.26) for $\mathbf{y}_1 = \mathbf{y}_2 = \mathbf{x}$, $|\mathbf{x}| < R_1$ gives the following result.

$$V_{\text{ext2}}(\mathbf{x}, \mathbf{x}) = \varepsilon \left(1 - \frac{R_1^2}{R^2}\right) \left(\frac{3}{2}\left(1 - \frac{3}{2}\varepsilon\right) - \frac{9}{64}\varepsilon\left(1 + \frac{R_1^2}{R^2}\right)\right)$$
$$+ \frac{13}{32}\varepsilon^2 \left(1 - \frac{R_1^2}{R^2}\right) \frac{|\mathbf{x}|^2}{R^2} + \mathcal{O}\left(\varepsilon^3\right), \qquad |\mathbf{x}| < R_1 \qquad (4.27)$$

Corresponding gravitational force $\mathbf{F}_{\text{ext}}$ has the form

$$\mathbf{F}_{\text{ext2}} = \frac{13}{16}\varepsilon^2 \left(1 - \frac{R_1^2}{R^2}\right) \frac{\mathbf{x}}{R^2} \qquad (4.28)$$

One can see from (4.28), that the external gravitational force appears only in the second approximation. It is always directed from the center of the sphere. The total gravitational force (4.25) contains a component, directed from the center, and this component is larger, than the external gravitational force (4.28)

Calculation of the potential V_2 for the case, when $|\mathbf{x}|$, $|\mathbf{y}_1|$, $|\mathbf{y}_2| \gg R$, leads to the following result

$$V_2(\mathbf{y}_1, \mathbf{y}_2) = V_1(\mathbf{y}_1, \mathbf{y}_2) + \frac{6}{5}\varepsilon^2 \frac{R}{|\mathbf{x}|} - \varepsilon^2 \frac{R^2}{2|\mathbf{x}|^2} \left(1 + \frac{2|\mathbf{x}|}{|\mathbf{y}_1|} + \frac{2|\mathbf{x}|}{|\mathbf{y}_2|}\right) + \mathcal{O}\left(\varepsilon^3\right) \qquad (4.29)$$

$$V_2(\mathbf{x}, \mathbf{x}) = V_1(\mathbf{x}, \mathbf{x}) + \frac{6}{5}\varepsilon^2 \frac{R}{|\mathbf{x}|} - \frac{5}{2}\varepsilon^2 \frac{R^2}{|\mathbf{x}|^2} + \mathcal{O}\left(\varepsilon^3\right) \qquad (4.30)$$

$$\mathbf{F} = \boldsymbol{\nabla} V_2(\mathbf{x}, \mathbf{x}) = -\varepsilon \frac{R}{|\mathbf{x}|^3}\mathbf{x} - \frac{6}{5}\varepsilon^2 \frac{R}{|\mathbf{x}|^3}\mathbf{x} + 5\varepsilon^2 \frac{R^2}{|\mathbf{x}|^4}\mathbf{x} \qquad (4.31)$$

Two last terms in (4.31) describe the second order correction to the gravitational force outside the sphere. The antigravitation (repulsion) dominates in this correction only for $|\mathbf{x}| < 4.17R$. Influence of antigravitation is less, than that of gravitation for $|\mathbf{x}| \gg R$.

Calculation of $B_2(\mathbf{y}_1, \mathbf{y}_2)$ gives the following result

$$B_2(\mathbf{y}_1, \mathbf{y}_2) = -2\frac{G^2 M^2}{c^4 R} \frac{\mathbf{y}_1^2 - \mathbf{y}_2^2}{R^2} = -2\varepsilon^2 \frac{\mathbf{y}_1^2 - \mathbf{y}_2^2}{R} + \mathcal{O}\left(\varepsilon^3\right) \qquad (4.32)$$

$$B_2(\mathbf{x}, \mathbf{x}) = \mathcal{O}\left(\varepsilon^3\right), \qquad \delta C(\mathbf{y}_1, \mathbf{y}_2) = \mathcal{O}\left(\varepsilon^3\right) \qquad (4.33)$$

5 Discussion

Let us imagine a sphere of radius R, filled uniformly by gravitating dust. Let the total mass of the dust be M. Under influence of gravitation the radius of the sphere is reduced, and the parameter $\varepsilon = 2GM/c^2R$ increases. When the parameter approaches the value $\varepsilon = 0.43$, a region of antigravitation arises near the center of the sphere. In this region there is a gravitational force, which is directed from the center of the sphere. Collapsing of the dust cloud will be reduced, and at the value $\varepsilon \geq 0.65$ antigravitation will take place inside the whole sphere.

Of course, it is a simplified consideration. In reality a collapsing of the dust cloud will be not uniform, and this nonhomogeneity should be taken into account at the calculation of the gravitational field inside the sphere. Besides, one should take into account the dust motion, which can influence on the gravitational field also.

However, in the Newtonian theory of gravitation, as well as in the general relativity the region of antigravitation does not appear inside the spherical cloud of dust under any circumstances. If a gravitation theory predicts a possibility of antigravitation, which could resist to collapsing of a dust cloud, it would be very important for construction of correct cosmological models.

Note, that the approach to gravitation from the dynamic viewpoint and that from the geometric viewpoint are different. From the geometric viewpoint an influence of gravitation is maximal at the point $\mathbf{x} = 0$. Variation of the metric tensor component $g_{00} = c^2(1 - V(\mathbf{x}, \mathbf{x})$ is maximal at the point $\mathbf{x} = 0$, as it follows from (1.3). However, from the dynamic viewpoint an influence of gravitation is minimal at the point $\mathbf{x} = 0$, because the gravitational force (1.4) vanishes at this point. Such a difference is connected with the fact, that the dynamitic description is local (differential), whereas the geometrical description is non-local (integral). It is a reason, why it is difficult to distinguish between the gravitation and antigravitation at the geometric description.

Already in the first (Newtonian) approximation the world function σ_1, determined by relations (4.8), (4.9), appears to be non-Riemannian (integral difference), although the metric tensor coincides in this approximation with the metric tensor of the Schwarzchild solution in the case of very small $\varepsilon = r_g/R = 2GM/(c^2R)$ (differential coincidence).

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