An Inclusive Model of Pair Production

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Abstract

The conventional (exclusive) description of pair production requires many functions of many space-time points. An attempt to develope another pair production model which uses several functions of one space-time point only is made. A dynamic system described by the Dirac equation in two-dimensional space-time is considered. The system is described in terms of hydrodynamic variables. Such a description contains a quantum variable κ responsible for quantum effects and sensitive to a collision of wave packets. The effective mass μ of particles depends on κ. Modifying this dependence $\mu(\kappa)$, one obtains another dynamic system (inclusive model). This model describes an effect of pair production, because the mean world lines of the modified model can make zigzag in the time direction. The quantum variable κ describes some force field (κ-field), whose properties are investigated, as well as a mechanism of pair production by means of the κ-field.

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1 Introduction

The quantum field theory (QFT) technique uses an exclusive description. It carries out the "all or nothing" investigation strategy [1], and cannot be effective in the high energy physics, when many pairs are produced. An inclusive mode of description seems to be more effective, because it uses a more appropriate "step by step" investigation strategy. An attempt to develope such an inclusive technique in QFT is presented in this paper.

The respective terms "exclusive" and "inclusive" were suggested by Feynman [2]. An inclusive description of a particle means that the description of the particle includes an indirect description of something else, that is not mentioned explicitly. The inclusive description is used mainly in high energy physics, where too much information is available which cannot be operated on in a detailed way [3-10].

Practically, a usage of an inclusive description means that a state containing N particles ($N \gg 1$) can be described by several functions of *one space-time point*, whereas at the conventional exclusive description such a state is described by several functions of N *space-time arguments*. As the energy of colliding particles increases, diffculties of the conventional exclusive description encrease strongly, because the number of arguments increases rapidly. The inclusive description is not sensitive to such an increase of the energy of colliding particles, because the number of arguments does not depend on the number of particles. It is a very important reason, why a possibility of an inclusive description should be investigated.

A derivation of an inclusive technique of the QFT dynamics from the exclusive one seems to be impossible, because the conventional exclusive technique of the QFT dynamics meets serious difficulties (non-stationarity of a vacuum, ultraviolet and infrared divergences). An independent derivation of an inclusive description seems to be more effective. Unfortunately, the frames of the conventional quantum mechanics are too narrow, and the corresponding quantum principles are too restrictive. It is desirable therefore to look at the problem of inclusive description from a more general viewpoint.

Idea of this more general viewpoint is referred to as the statistical ensemble technique (SET). It is presented in the second section, where the conceptual statement of the pair production problem is discussed. In SET the dynamical and interpretational sides of the statistical ensemble are separated. As a result one can discuss the dynamics and dynamic equations independently of their interpretation, i.e. of the way how to calculate the mean values. This approach gives a possibility to go outside the scope of QM principles. For interpretation of physical quantities it permits to use other considerations, than QM principles. At the same time a separation of the dynamics from the interpretation does not pertmit to use the interpretation for a derivation of dynamic equations.

The leading idea of such a derivation can be formulated as follows. Quantum mechanics (even non-relativistic one) is a kind of relativistic statistical description (RSD) of randomly moving particles. It means that the mean motion of particles can be non-relativistic, but the chaotic motion is always relativistic and should be described by means of relativistic statistical mechanics. The non-relativistic statistical description ($NRSD$) uses the probability density ρ as a main concept. In RSD the 3-scalar ρ is substituted by a 4-vector j^i of the "probability" flux. From the mathematical point of view the 4-vector j^i describes a continuous medium (Madelung fluid). The wave function is merely such a way of the inviscid fluid description which removes the so called convective nonlinearity, connected with the term $(\mathbf{v}\nabla)\mathbf{v}$ in hydrodynamic equations [11]. Description in terms of the wave function leads to linear dynamic equations and to the interpretation of the quantum mechanics (QM) in terms of linear operators and their eigenvalues.

A description of the Dirac equation in terms of the flux j^i leads to appearance of an additional variable κ responsible for quantum effects. It means that, setting $\kappa \equiv 0$ in the action $\mathcal{A}_D$ for the Dirac equation, one obtains the action of a statistical ensemble of classical relativistic particles. Rewriting the action $\mathcal{A}_D$ in the Lagrangian coordinates one obtains an ensemble (a set) of classical relativistic particles moving in a self-consistent force field κ which is determined by the motion of the whole ensemble of particles. A particle has an effective mass $\mu_D = m\cos\kappa$ which depends on κ. All these features permit one to interpret the Dirac equation in terms of classical particles without referring to a wave function and linear operators. The same can be done for other dynamic equations of QM.

Now the QM conception, as anything defined by the principles of QM and connected with equations linear in terms of wave functions and linear operators, is "parameterized" by means of a field κ. Cutting off the κ-field, one removes all quantum effects. Such a parametrization is very convenient for possible modifications of QM. For instance, modifying μ_D as a function of κ, one obtains another quantum mechanics, where the dynamic equation is not linear in terms of wave function, and linear operators cannot be used effectively. Nevertheless, if a corresponding modification of the effective mass μ is small, this conception can be very close to the conventional QM. At the same time it can be treated in terms of world lines, i.e. in classical terms.

We construct an inclusive mode of QM description, choosing such a modification of μ, which enables one to describe pair production and coincides with the conventional QM, if $\mid \kappa \mid \ll 1$. Such a modification of the conventional QM is called an inclusive mode of description (inclusive model). Pair production properties of this model are investigated, because the fact that some fluid model enables to describe pair production is of great interest. The statement of the problem is rather close to papers [12,13], where the pair production is considered to be a result of entropy increase of two colliding drops of "nuclear fluid".

In the third section a connection between different modes of the statistical description is discussed. In the fourth section a "collision" of two non-interacting relativistic particles is described in terms of the exclusive and inclusive descriptions. The problems connected with the model modification are considered in the fifth section. An investigation of the κ-field is performed in the sixth section. The seventh section is devoted to an investigation of the pair production mechanism.

2 Dynamics, Interpretation, Conceptual Statement of the Pair Production Problem

Let us define first what the term "statistical description" means actually. Let $\mathcal{S}$ be a stochastic physical system. It means that experiments with a single system $\mathcal{S}$ are irreproducible. Examples of stochastic systems $\mathcal{S}$ are: Brownian particle in a gas, or an electron moving through a narrow slit. There are no dynamic equations for a single stochastic system.

In order to investigate the stochastic system $\mathcal{S}$, let us consider a set $\mathcal{E}$ (statistical ensemble) of many similar independent stochastic systems $\mathcal{S}$. Experiments with the statistical ensemble $\mathcal{E}$ are reproducible, and $\mathcal{E}$ can be considered as a deterministic dynamic system, associated with the stochastic system $\mathcal{S}$. There exist dynamic equations for $\mathcal{E}$. By definition, an investigation of dynamics of the statistical ensemble $\mathcal{E}$ of stochastic systems $\mathcal{S}$ and its interpretation form a statistical description of the stochastic systems $\mathcal{S}$. In other words, the statistical description is a kind of reduction of a stochastic system $\mathcal{S}$ to a deterministic system $\mathcal{E}$.

There are two main functions of the statistical ensemble:

(1) Dynamics (determined by properties of stochastic systems) describes the time evolution of the statistical ensemble considered as a dynamic system.

(2) Interpretation (a set of rules for calculting mean values of quantities relating to a single stochastic system) describes average properties of stochastic systems, basing on the statistical ensemble properties.

Such a statistical description will be refered to as the statistical ensemble technique (SET). The general scheme (SET-scheme) of statistical description is shown in the top of Fig.1.

The conventional scheme (C-scheme) of the statistical description is shown in the bottom of Fig.1. It distinguishes from SET-scheme by the place which the interpretation takes. The SET-scheme does not determine the way how to obtain the dynamic equations for the statistical ensemble. The C-scheme permits to establish the dynamic equations, using the interpretation as a base. Practically, it is possible only in the case, when the deterministic limit of the stochastic system is known (for quantum stochastic (QS) systems it is known as a classical limit). Then, using the quantization procedure for the (classical) dynamic system (which is a deterministic limit of the QS system), one obtains dynamic equations for the statistical ensemble of such QS systems on the base of QM principles. The quantization procedure includes a substitution of dynamic variables by linear operators, introduction of wave functions, etc.

But the quantization procedure cannot be used in the case, when a classical analog (deterministic limit) of the QS system is not known. For instance, classical analogs of such physical phenomena as half-integer spin and pair production process are not known, and in this case the interpretation does not permit to obtain dynamic equation for the corresponding statistical ensemble.

Two kinds of interpretation (rules for calculation of mean values of dynamic variables functions) are known now. The first (classical) way of interpretation uses the probability density as the main concept of the interpretation. The second one (quantum interpretation) uses the probability amplitude (wave function) as the main concept of the interpretation. Respectively, in the C-scheme there are two kinds of dynamics generated by these interpretations. There is a conceptual boundary between the two dynamics, and most of scientists are sure that any description of physical phenomena can be either classical, or quantum, and there can be nothing other, than the two kinds.

The SET-scheme of a statistical description is more general. Although it does not contain prescriptions for deriving the dynamic equations, but it has not also a conceptual boundary between classical and quantum descriptions. Here the dynamic equations for the statistical ensemble are not restricted by the QM principles, or by some other kind of interpretation. Here the interpretation is only a way of obtaining the average properties of the stochastic system from the statistical ensemble dynamics. If anybody is not sure that the QM principles are always valid, he should use the more general SET-scheme, where the dynamics and interpretation are separated. Here there is no rigid connection between the dynamics and the interpretation, although, of course, they must be agreed.

Our statement of the pair production problem is based on the following proposition. Dynamics of a statistical ensemble $\mathcal{E}$ of independent (and non-interacting) quantum stochastic particles is equivalent to the dynamics of the ensemble $\mathcal{S}'$ of similar deterministic classical particles interacting through some self-consistent field κ. The field κ is responsible for quantum effects in the sense that, if one sets $\kappa \equiv 0$, the ensemble $\mathcal{S}'$ turns to a statistical ensemble $\mathcal{E}_{\rm cl}$ of independent similar deterministic classical particles. If $\kappa \not\equiv 0$, $\mathcal{S}'$ is not a statistical ensemble, because the last is defined as a set of *independent* and , hence, non-interacting particles (systems). $\mathcal{S}'$ is a set (ensemble, but not a statistical ensemble) of interacting deterministic particles. In other words, a stochasticity of the statistical ensemble particles is substituted by an interaction between the deterministic particles of the ensemble $\mathcal{S}'$.

Mathematically it means that by means of a change of variables the action $\mathcal{A}$ for the statistical ensemble $\mathcal{E}$ can reduce to the action $\mathcal{A}'$ of the ensemble $\mathcal{S}'$. This proposition was proved for the dynamic systems $\mathcal{E}_{\rm S}, \mathcal{E}_{\rm KG}, \mathcal{E}_{\rm P}, \mathcal{E}_{\rm D}$, described respectively by the equations of Scrödinger, Klein-Gordon, Pauli and Dirac [14].

In this paper only the dynamic system $\mathcal{S}_{\rm D}$ described by the 2–d Dirac equation will be considered, and an equivalence of two system of dynamic equations described by the actions $\mathcal{A}_{\rm D}[\bar\psi, \psi]$ and $\mathcal{A}'_{\rm D}[x, \kappa]$ will be proved.

$$\mathcal{A}_{\rm D}[\bar\psi, \psi] = \int (-m\bar\psi\psi + \frac{i}{2}\bar\psi\gamma^l\partial_l\psi - \frac{i}{2}\partial_l\bar\psi\gamma^l\psi)d^2x \tag{2.1}$$

where

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\mathcal{A}'_{\rm D}[x, \kappa] = \int \{-\mu(\kappa)\sqrt{1 - (dx/dt)^2} + \frac{1}{2}(\kappa_x + \frac{dx}{dt}\kappa_t)\} dt d\xi \tag{2.2}$$

where

$$x = x(t, \xi), \qquad \kappa = \kappa(t, x), \qquad \mu(\kappa) = m\cos\kappa \tag{2.3}$$

The current is defined by the relations

$$j^0 = \bar\psi\gamma^0\psi = \frac{\partial\xi}{\partial x} = \frac{\partial(\xi, t)}{\partial(x, t)} = (\frac{\partial x}{\partial \xi})^{-1},$$

$$j^1 = \bar\psi\gamma^1\psi = -\frac{\partial\xi}{\partial t} = -\frac{\partial(\xi, x)}{\partial(x, t)} = \frac{dx}{dt}(\frac{\partial x}{\partial \xi})^{-1} = j^0\frac{dx}{dt} \tag{2.4}$$

κ is a dynamic variable, responsible for quantum effects in the sense, that if $\kappa \equiv 0$, the system $\mathcal{S}_D$ turns into a statistical ensemble of free deterministic particles.

As far as the statistical ensemble $\mathcal{E}$ of quantum stochastic (QS) particles reduces to an ensemble $\mathcal{S}'$ of interacting classical particles, it means that $\mathcal{E}$ can be interpreted in terms of deterministic (classical) particles without an usage of QM principles and not in terms of linear operators and wave functions. Thus, interpreting the conventional quantum dynamics, one can use the interpretation other, than ascribed by the QM principles. It is a result of the SET-scheme of the statistical description, where dynamics and interpretation are separated.

It should be mentioned that most of scientists use the C-scheme of the statistical description (many of them use simply QM technique without going into details, whether the wave function describes a statistical ensemble, or a single quantum system). They are sure that there are either quantum description of a phenomenon, or classical one and nothing more. If the system is described in terms of wave functions and operators, this is a quantum system. If a system is described in terms of particle world lines, this is a classical system. Of course, at such an approach the puzzling questions arise. For instance, of such a kind: (1) Is the description in terms of variables x, κ classical, or quantum? (2) What is a strange field κ, responsible for quantum effects? Can this field κ be regarded as so called "hidden variable", considered sometimes as being responsible for quantum effects? (see [15-20] and references therein)

Strictly speaking, from the point of view of SET these questions are put not quite correctly, because the term "quantum" may mean in SET quite different concepts. It may mean a single stochastic system which stochasticity has a quantum nature, or a statistical ensemble of such systems, or a even a system of the QM postulates. Nevertheless we try to answer these questions.

(1) The answer to the first question is as follows. The dynamics is quantum in the sense that it relates to description of stochastic systems with a quantum nature of the stochasticity, but the interpretation is classical in the sense that it uses neither wave function, nor linear operators. There is no contradiction here, because in SET the dynamics and the interpretation are quite different properties of the statistical description.

(2) The response to the second question is as follows. The field κ is a dynamic variable constructed from the wave function components. The κ is not a hidden variable, because usually the term "hidden variables" means a set of variables which carry out more deterministic and causal description, than the QM description does. The hidden variables are supposed to describe the quantum system completely causally. When the information described by the hidden variables is ignored, the quantum description arises. In other words, the conception of "hidden variables" supposes a possibility of more complete and detailed description, than the QM description. But an usage of the field κ does not assume such a possibility, and κ is not a hidden variable in a commonly accepted sense of the word.

Interpretation in terms of world lines (WL-interpretation) is more general, because it does not need fulfilling the QM principles, which seem to be too restrictive for the statement of the pair production problem.

The Dirac equation does not describe a pair production. Pair production problem is stated as follows. Is it possible to modify the form of the function $\mu(\kappa)$, determined by Eq.(2.3), in such a way that the world lines have a shape shown in Fig.2? If it is possible, then, defining the mean 2-flux n_p^i of particles and the mean 2-flux n_a^i of antiparticles by means of relations

$$n_p^i = j^i\theta(j^0)\theta(j^k j_k), \qquad n_a^i = -j^i\theta(-j^0)\theta(j^k j_k) \tag{2.5}$$

$$\theta(x) = \begin{cases} 1 & \text{if } \ x > 0 \\ 0 & \text{if } \ x \le 0 \end{cases} \tag{2.6}$$

one obtains a possibility to describe the pair production effect in terms of functions of one space-time argument.

It is to keep in mind that the definition (2.5) is rather rough. It is valid, provided the flux j^i contains either particles, or antiparticles. If j^i contains both particles and antiparticles, one needs to use higher order fluxes depending on several space-time arguments.

Investigation of a possibility of describing the pair production effect in terms of functions of one space-time argument is a goal of the present paper. A possibility of a comparison with the conventional statement of the pair production problem will be considered in the end of the seventh section, when the technique of the inclusive pair production description becomes somewhat clearer.

3 Modes of statistical description

Let us present briefly a classification of the statistical description modes, which are used for describing a stochastic motion of particles.

1. Non-relativistic statistical description ($NRSD$) is based on the non-relativistic concept of a particle. In the non-relativistic physics the physical object (pointlike particle) is a point in the three-dimensional space. Density ρ of particles (non-relativistic state density) in the vicinity of a point $\mathbf{x}$ is defined by the relation

$$dN = \rho d\mathbf{x}, \tag{3.1}$$

where dN is the number of particles inside the volume $d\mathbf{x}$ containing the point $\mathbf{x}$. ρ is defined as a proportionality factor between $d\mathbf{x}$ and dN. Let $x_k = \{t_k, \mathbf{x}_k\}$ be coordinates of kth particle. Let us use the following notations

$$\mathbf{X}^n = \{\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_n\}, \qquad X^n = \{x_1, x_2, \ldots x_n\},$$
$$d({}^n\mathbf{X}^l) = d\mathbf{x}_{n+1} \wedge d\mathbf{x}_{n+2} \wedge \ldots \wedge d\mathbf{x}_l, \qquad d\mathbf{X}^l = d({}^0\mathbf{X}^l), \tag{3.2}$$
$${}^lV^n = \bigotimes_{k=l+1}^{k=n} V, \qquad V^n = {}^0V^n$$

where $\otimes$ means a direct product.

Let $p_n(\mathbf{X}^n)$, $n = 0, 1, \ldots$ be the probability density of the event that inside V there are n and only n particles, one of them at $\mathbf{x}_1$, another at $\mathbf{x}_2, \ldots$ the nth at $\mathbf{x}_n$. The $p_n(\mathbf{X}^n)$ will be referred to as the probability density of the state $\mathbf{X}^n$, or exclusive probability density. Any state $\mathbf{X}^n$ describes some possible spatial distribution of particles, and the quantity $p_n(\mathbf{X}^n)$ describes probability density of this distribution.

The $p_n(\mathbf{X}^n)$ are non-negative and independent in the sense that the normalization condition takes place:

$$\sum_{n=0}^{\infty} \frac{1}{n!} \int_{V^n} p_n(\mathbf{X}^n) d\mathbf{X}^n = 1. \qquad (3.3)$$

An inclusive mode of description can be achieved by means of quantities $w_n(\mathbf{X}^n)$, $n = 0, 1, \ldots$ determined by relations

$$w_n(\mathbf{X}^n) = \sum_{l=0}^{\infty} \frac{1}{l!} \int_{{}^nV^{n+l}} p_{n+l}(\mathbf{X}^{n+l}) d({}^n\mathbf{X}^{n+l}), \qquad n = 0, 1, \ldots \qquad (3.4)$$

The $w_n(\mathbf{X}^n)$ is the probability density of the event that there are at least n particles, one of them at $\mathbf{x}_1$, another at $\mathbf{x}_2, \ldots$ the nth at $\mathbf{x}_n$. There can be more particles besides. The $w_n(\mathbf{X}^n)$ will be referred to as the probability density of at least the state $\mathbf{X}^n$, or inclusive probability density. In particular, the probability w_0 of at least the state $\mathbf{X}^0$, containing 0 particles is

$$w_0 = 1. \qquad (3.5)$$

This relation follows from Eqs.(3.3), (3.4). Exclusive description and inclusive one are equivalent, because p_n, $n = 0, 1, \ldots$ are expressible in terms of w_n, $n = 0, 1, \ldots$ by means of similar relations

$$p_n(\mathbf{X}^n) = \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \int_{{}^nV^{n+l}} w_{n+l}(\mathbf{X}^{n+l}) d({}^n\mathbf{X}^{n+l}), \qquad n = 0, 1, \ldots \qquad (3.6)$$

2. Relativistic statistical description (RSD) is based on the concept that the physical object is the world line of a particle, i.e. one-dimensional line in the four-dimensional space-time (but not a point in three-dimensional space). Density j^i of world lines (relativistic state density) in the vicinity of a point $x = \{t, \mathbf{x}\}$ is defined by the relation

$$dN = j^i dS_i \qquad (3.7)$$

where dN is a flux of world lines (the number of particles) through the three-dimensional spacelike area dS_i containing the point x. Now the state density is a 4-vector in the space-time, and j^0 can have any sign ($j^0 > 0$ for particles, and $j^0 < 0$ for antiparticles). Thus, in the relativistic case the probability density $p_1(\mathbf{x})$ (3-scalar) is replaced by the 4-vector $j^i(x)$. In other words, the 4-flux is a relativistic analog of the probability density. Correspondingly, $p_n(\mathbf{X}^n)$ is replaced by the n-point tensor $j^{i_1 i_2 \ldots i_n}(X^n)$, which is a vector at the point x_1, a vector at the point $x_2, \ldots$ a vector at the point x_n.

Let us consider a very important particular case, when the stochastic motion is relativistic, but the mean motion is non-relativistic. In this case one can use the non-relativistic approximation of RSD. It means that there exist such a coordinate system, where all time components of the tensors j are much larger, than their spatial components. Then the component $j^{00..0}(X^n)$ is non-negative and can be considered as a probability density of the state X^n. It is convenient to use a notation $j(^0I^n, X^n)$ instead of $j^{i_1 i_2 ... i_n}(X^n)$, where $^lI^n$ is a set of tensor indices

$$^lI^n = \{i_{l+1}, i_{l+2}, ..i_n\}, \qquad I^n \equiv^0 I^n,$$
$$^lO^n =^l I^n = \{i_{l+1}, i_{l+2}, ..i_n\}, \qquad i_k = 0, \qquad k = l+1, l+2, \ldots n, \tag{3.8}$$

index i_k being a tensor index at the point x_k, and $^lO^n$ is a set of indices equal to zero.

The inclusive fluxes $j_{\mathrm{IN}n}(I^n; t, \mathbf{X}^n)$ are related to exclusive fluxes $j_{\mathrm{EX}n}(I^n; t, \mathbf{X}^n)$ by means of

$$j_{\mathrm{IN}n}(I^n; t, \mathbf{X}^n) = \sum_{l=0}^{\infty} \frac{1}{l!} \int\limits_{^nV^{n+l}} j_{\mathrm{EX}n+l}(I^n,^n/!O^{n+l}; t, \mathbf{X}^{n+l}) d(^n\mathbf{X}^{n+l}), \tag{3.9}$$
$$n = 0, 1, \ldots$$
$$j_{\mathrm{EX}n}(I^n; t, \mathbf{X}^n) = \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \int\limits_{^nV^{n+l}} j_{\mathrm{IN}n+l}(I^n,^n O^{n+l}; t, \mathbf{X}^{n+l}) d(^n\mathbf{X}^{n+l}), \tag{3.10}$$
$$n = 0, 1, \ldots$$

The integration is performed over some volume V of the 3-plane t=const in the coordinate system, where all fluxes are non-relativistic. All fluxes j are supposed to be symmetric with respect to simultaneous transposition of any two arguments $i_l, \mathbf{x}_l$ and $i_k, \mathbf{x}_k$.

Both fluxes j_{IN} and j_{EX} can be expressed through the wave function

$$j_n(I^n; t, \mathbf{X}^n) = \sum_{\alpha_1, \alpha_2, ..\alpha_n = 1}^{s} \psi^*(A^n; t, \mathbf{X}^n) \bigotimes_{k=1}^{k=n} \mathcal{L}_k^{i_k}) \psi(A^n; t, \mathbf{X}^n), \qquad n = 1, 2, \ldots$$
$$\mathcal{L}_k = \{\frac{1}{2}, -\frac{i\hbar}{2m}\frac{\partial}{\partial x_k^1}, -\frac{i\hbar}{2m}\frac{\partial}{\partial x_k^2}, -\frac{i\hbar}{2m}\frac{\partial}{\partial x_k^3}\}, \tag{3.11}$$
$$A^n = \{\alpha_1, \alpha_2, \ldots \alpha_n\}, \qquad \alpha_k = 1, 2, \ldots s, \qquad k = 1, 2, \ldots n,$$

where ψ^* is the complex conjugate to ψ, and the Hermitian operator $\mathcal{L}_k$ acts on the argument $\mathbf{x}_k$ of both wave functions ψ and ψ^*. For instance, for $n = 1$ $(s = 2)$ one has

$$j_1^0(\mathbf{x}) = \psi^*(\mathbf{x})\psi(\mathbf{x}), \qquad j_1^\alpha(\mathbf{x}) = -\frac{i\hbar}{2m}[\psi^*(\mathbf{x})\partial_\alpha\psi(\mathbf{x}) - \partial_\alpha\psi^*(\mathbf{x})\psi(\mathbf{x})],$$
$$\alpha = 1, 2, 3 \tag{3.12}$$
$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \qquad \psi^* = (\psi_1^*, \psi_2^*) \tag{3.13}$$

Although the wave function can be introduced for both fluxes, but only the exclusive wave function $\psi_{\rm EX}(\mathbf{X}^n)$ is used conventionally, because dynamic equations for the $\psi_{\rm EX}$ are linear. Such a statistical description in terms of $\psi_{\rm EX}$ is known as the non-relativistic quantum mechanics (QM). Linearity of dynamic equations of QM seems to be so important, that QM is considered conventionally as a special (rather mysterious) mode of statistical description, where a complex probability amplitude ψ is used instead of non-negative probability density.

Actually the QM description is a rather reasonable way of a statistical description which is used in the case, when the mean motion is non-relativistic, but the chaotic motion of particles is essentially relativistic. $NRSD$ cannot be used in this case. The fact that the quantum mechanics can be presented in the hydrodynamical form (in terms of fluxes) was known long ago [21-25]. The fact that the pecularities of QM description can be considered as a result of the relativistical character of chaotic motion is now in a process of realization [26-29].

The pair production effect arises only in the case of relativistic (or ultrarelativistic) mean motion of particles. In this case a usage of a sequential RSD is necessary.

In the present investigation the inclusive mode of RSD is used. Both inclusive flux and inclusive wave function are used. Dynamic equations are nonlinear in terms of the inclusive wave function, and conventional QM interpretation based on linear operators and linear dynamic equations cannot be given. A more general viewpoint of RSD is to be used. In particular, an interpretation of the inclusive RSD is produced on the base of the mean world lines. It means the following. The inclusive flux $j_{\rm IN}$ considered as a system of vector fields determines a system of oriented world lines (or surfaces). These world lines are regarded as mean world lines of particles and antiparticles. All collision processes are interpreted on the base of this system of world lines.

4 Inclusive description of "non-interacting particle scattering"

Let us consider a description of two identical non-interacting relativistic particles. The complete exclusive description can be achieved by means of the second order exclusive flux $j^{ik}_{\rm EX2}(x_1,x_2)$. All other exclusive fluxes vanish. This situation can be described also in terms of the exclusive wave function $\Psi_{\rm EX2}(\alpha_1,x_1;\alpha_2,x_2)$. The wave function satisfy the Dirac equation with respect to each of two arguments x_1,x_2.

The exclusive flux and the exclusive wave function are connected by means of the relations

$$j^{ik}_{\rm EX2}(x_1,x_2)=\Psi^*_{\rm EX2}(\alpha'_1,x_1;\alpha'_2,x_2)(\gamma^0\gamma^i)_{\alpha'_1\alpha_1}(\gamma^0\gamma^k)_{\alpha'_2\alpha_2}\Psi_{\rm EX2}(\alpha_1,x_1;\alpha_2,x_2) \quad (4.1)$$

Here Ψ^* is Hermitian conjugate to Ψ, γ^i, $i=0,1,2,3$ are the Dirac γ-matrices. Indices α numerate columns and rows of γ-matrices. A summation is presumed over repeated Greek indices (1-4).

Inclusive description of this situation is carried out by means of inclusive fluxes $j_{\rm IN0}=1$, $j^i_{\rm IN1}(x)$, $j^{ik}_{\rm IN2}(x_1,x_2)=j^{ik}_{\rm EX2}(x_1,x_2)$. All other inclusive fluxes vanish.

$$j^i_{\rm IN1}(t,\mathbf{x}_1)=\int j^{i0}_{\rm EX2}(t,\mathbf{x}_1,t,\mathbf{x}_2)d\mathbf{x}_2 \tag{4.2}$$

Let the exclusive wave function have the form

$$\Psi_{\rm EX2}(\alpha_1,x_1;\alpha_2,x_2)=\psi_B(\alpha_1,x_1)\psi_T(\alpha_2,x_2)-\psi_T(\alpha_1,x_1)\psi_B(\alpha_2,x_2) \tag{4.3}$$

Here the wave functions $\psi_B(\alpha_1,x_1),\psi_T(\alpha_2,x_2)$ have the form of a wave packets of size L moving to meet each other with the velocities $\mathbf{v}$ and $-\mathbf{v}$, ψ_B, ψ_T both satisfy the Dirac equation, and are orthogonal:

$$\int \psi^*_B(\alpha,t,\mathbf{x})\psi_T(\alpha,t,\mathbf{x})d\mathbf{x}=0 \tag{4.4}$$

Then it follows from Eqs. (4.1)-(4.4) that

$$j^i_{\rm IN1}(t,\mathbf{x})=\sum_{k=B,T}\psi^*_k(\alpha,t,\mathbf{x})(\gamma^0\gamma^i)_{\alpha\alpha'}\psi_k(\alpha',t,\mathbf{x}) \tag{4.5}$$

The first order inclusive wave function is defined by the relation

$$j^i_{\rm IN1}(t,\mathbf{x})=\psi^*_{\rm IN}(\alpha,t,\mathbf{x})(\gamma^0\gamma^i)_{\alpha\alpha'}\psi_{\rm IN}(\alpha',t,\mathbf{x}) \tag{4.6}$$

Let us set

$$\psi_{\rm IN}(\alpha,x)=\psi_B(\alpha,x)+\psi_T(\alpha,x) \tag{4.7}$$

Expressions for the inclusive first order flux $j^i_{\rm IN1}$ determined by Eq.(4.5) and Eqs.(4.6), (4.7) coincide in those space-time regions where the two wave packets do not overlap. In the region of overlapping they differ by the rapidly oscillating term

$$\mathrm{Re}[\psi^*_B(\alpha,t,\mathbf{x})(\gamma^0\gamma^i)_{\alpha\alpha'}\psi_T(\alpha',t,\mathbf{x})].$$

After averaging the expression (4.6), (4.7) coincides with Eq.(4.5).

A typical picture of world lines generated by the j^i_{IN1} is shown in Fig.3. It can be regarded as two fluid drops (beam and target) colliding elastically and recoiling. Interaction between the two drops appears to be very strong. It is a statistical effect known as exchange interaction. It can be treated also as reconnection of world lines. This effect takes place for classical particles and has nothing to do with quantum effects.

The exchange interaction is caused by an identity of the target and beam particles. It leads to the symmetry of $j^{ik}_{\rm IN2}(x_1,x_2)$ with respect to interchange of its arguments. If the particles of the beam and of the target are not identical and do not interact, the flux $j^{ik}_2(x_1,x_2)$ can be presented in the form

$$j^{ik}_2(x_1,x_2)=j^i_B(x_1)j^k_T(x_2) \tag{4.8}$$

The first order fluxes j_B^i and j_T^k generate two systems of almost straight world lines. If those world lines are crossing in some space-time region, then one can treat this as an overlapping of wave packets. In the case of identical particles it is more correct to treat this as a collision of two fluid drops. For non-interacting particles this collision is balanced in such a way that there are neither scattered particles, nor produced pairs.

Let us imagine that the colliding particles interact and the balance of the exchange interaction is violated. In this case the first order inclusive flux $j_{\mathrm{IN}1}$ generates a set of world lines shown in Fig.2. It means that, in general, $j_{\mathrm{IN}1}$ can describe some interesting processes, for instance, one can determine the cross section of antiparticle production

$$\sigma_a =\mid \int\limits_{V_F} j^0 d\mathbf{x} \mid Q^{-1}. \qquad Q = \int\limits_{-\infty}^{\infty} \sqrt{(j_B^i j_{Ti})^2 - j_B^i j_{Bi} j_T^k j_{Tk}}\, dt\, d\mathbf{x} \tag{4.9}$$

where the volume V_F is a result of section of F' and F'' by the plane $t = t_0 \gg L$ (see Fig.2). In the coordinate system of the target where $\mathbf{j}_T = 0$, $\mathbf{j}_B = j_B^0 \mathbf{v}_B$, the invariant integral Q reduces to the conventional form

$$Q = \int\limits_{-\infty}^{\infty} j_T^0 j_B^0 v_B\, dt\, d\mathbf{x} \tag{4.10}$$

Notice that in the case of identical particles the fluxes j_T and j_B inside the collision region can be obtained from the first order inclusive flux $j_{\mathrm{IN}1}$ only as a result of extrapolation. Outside the collision region the flux $j_{\mathrm{IN}1}$ determines two sets of almost straight world lines which define the fluxes j_T and j_B outside the collision region. Extrapolating world lines as straight lines into the collision region, one obtains the fluxes j_T and j_B which can be used for calculation of the expression (4.9) inside the collision region.

Now let us consider the case of two non-relativistic quantum interacting paticles. Let the Hamiltonian has the form

$$\mathcal{H} = -\frac{\hbar^2}{2m}(\nabla_1^2 + \nabla_2^2) + U(\mathbf{x}_1 - \mathbf{x}_2) \tag{4.11}$$

where U is a potential energy of interaction. Then the dynamic equations for the first order inclusive flux $j_{\mathrm{IN}1} = \{\rho, \mathbf{j}\}$ has the form

$$\partial_t \rho + \nabla \mathbf{j} = 0 \tag{4.12}$$

$$\partial_t \mathbf{j} = \mathbf{W} - \frac{1}{m}\int \mid \Psi_{\mathrm{IN}}(\mathbf{x}, \mathbf{x}') \mid^2 \nabla U(\mathbf{x} - \mathbf{x}') d\mathbf{x}' \tag{4.13}$$

where $\Psi_{\mathrm{EX}}(\mathbf{x}, \mathbf{x}') = \Psi_{\mathrm{IN}}(\mathbf{x}, \mathbf{x}')$ is the second order inclusive wave function and $\mathbf{W}$ contains all the terms of the dynamic equation which do not depend on the potential $U(\mathbf{x}-\mathbf{x}')$. Let $\Psi_{\mathrm{IN}}(\mathbf{x}, \mathbf{x}')$ describe two moving wave packets of size L, and $U(\mathbf{x}-\mathbf{x}') \not\equiv$

0 only for $\mid \mathbf{x}-\mathbf{x}' \mid < a \ll$L. Then the last term of Eq.(4.13) vanishes outside the collision region. Outside this region the rhs of Eq.(4.13) is expressed through the first order inclusive flux $j_{\mathrm{IN}1}$. But inside the collision region the rhs of Eq.(4.13) depends on the second order flux, and the system of Eqs.(4.12), (4.13) is not closed. Thus, the last term of Eq.(4.13) is sensitive to collision and responsible for scattering.

To close relativistic dynamic equations for $j_{\mathrm{IN}1}$ and to obtain dynamic equations describing pair production, let us employ the following ideas.

There are quantum effects which are independent of any special interaction and have no classical analog, e.g. a spreading of the wave packet. This effect is caused by the quantum stochasticity, i.e. by a stochastic wandering of a quantum particle in spatial directions. The pair production appears to be a quantum effect of the same kind, but in this case a stochastic wandering in the time direction takes place. Indeed, no special interaction is known that would have a classical analog and be responsible for pair production. The pair production takes place in a collision of any particles, provided the collision energy is high enough.

(1) Thus, we have the following idea. Considering relativistic non-interacting particles, let us look for a variable responsible for quantum effects and make this variable responsible for pair production.

(2) Another idea. Considering collision of two relativistic fluid drops of non-interacting particles, let us try to find a variable sensitive to collision of the two drops and make it responsible for pair production.

It appears that there is such a variable κ which is sensitive to collision and responsible for quantum effects simultaneously.

To show this, suppose that the first order inclusive wave function (4.7) satisfies the Dirac equation. Further for simplicity only the case of two-dimensional space-time will be considered. Index "IN" will be omitted. The following notations are used.

$$\psi=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix},\qquad \psi^*=(\psi_1^*,\psi_2^*),\qquad \bar\psi=\psi^*\gamma^0,$$

$$\gamma^0=\begin{pmatrix}0&1\\1&0\end{pmatrix},\qquad \gamma^1=\begin{pmatrix}0&1\\-1&0\end{pmatrix},\tag{4.14}$$

Here ψ_1 and ψ_2 are complex variables, ψ^* means the Hermitian conjugate.

The action for the Dirac equation has the form

$$\mathcal{A}_{\mathrm{D}}[\bar\psi,\psi]=\int(-m\bar\psi\psi+\frac{i}{2}\bar\psi\gamma^l\partial_l\psi-\frac{i}{2}\partial_l\bar\psi\gamma^l\psi)d^2x\tag{4.15}$$

Here $\hbar=c=1$, and a summation is to be performed over Latin indices (0-1). 2-flux j^l, $l=0,1$ and the momentum density p_i, $i=0,1$ are defined by the relations

$$j^l=\bar\psi\gamma^l\psi,\qquad p_l=-\frac{i}{2\psi^*\psi}(\bar\psi\gamma^0\partial_l\psi-\partial_l\bar\psi\gamma^0\psi),\qquad l=0,1.\tag{4.16}$$

Let us go from two complex variables ψ_1,ψ_2 to real variables j^0,j^1,φ,κ, using the transformation

$$\psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \begin{pmatrix}[(j^0-j^1)/2]^{1/2}\exp[i(\varphi+\kappa/2)]\\ [(j^0+j^1)/2]^{1/2}\exp[i(\varphi-\kappa/2)]\end{pmatrix}, \qquad j^0 = j_0, \qquad j^1 = -j_1$$

$$\psi^* = \left([(j^0-j^1)/2]^{1/2}\exp[-i(\varphi+\kappa/2)], \quad [(j^0+j^1)/2]^{1/2}\exp[-i(\varphi-\kappa/2)]\right). \tag{4.17}$$

Substituting relations (4.17) into the action (4.15), the following expressions can be obtained:

$$\mathcal{A}_{\rm D}[j,\varphi,\kappa] = \int[-\mu(\kappa)\sqrt{j_0^2-j_1^2} - j^i\partial_i\varphi + \frac{1}{2}(j^0\partial_1\kappa + j^1\partial_0\kappa)]d^2x. \tag{4.18}$$

$$\mu(\kappa) = \mu_{\rm D}(\kappa) \equiv \mu_{\rm EX}(\kappa) \equiv m\cos\kappa, \qquad p_k = \partial_k\varphi. \tag{4.19}$$

The action (4.18) provides a description of the Dirac particle ensemble in terms of hydrodynamic variables (first order inclusive flux).

The dynamic system described by the action (4.18) can be regarded as a fluid in the two-dimensional space-time. World lines of the fluid particles are supposed to be tangent to the flux density vector j^i, $i = 0, 1$. $\mu(\kappa)$ is the mass of the fluid particle, κ is some variable describing the internal state of the fluid (something like internal energy density).

Notice that quantum properties of the dynamic system described by the action (4.18) are determined by the variable κ. Indeed, if $\kappa \equiv 0$, the action (4.18) reduces to the form

$$\mathcal{A}_{\rm cl}[j,\varphi] = \int(-m\sqrt{j_0^2-j_1^2} - j^i\partial_i\varphi)d^2x. \tag{4.20}$$

which describes a pure ensemble of free classical relativistic particles in two-dimensional space-time.

Indeed, the relations

$$j^0 = \partial\xi/\partial x^1, \qquad j^1 = -\partial\xi/\partial x^0 \tag{4.21}$$

satisfy the continuity equation identically and can be used for a change of variables. Eliminating j^i from Eq.(4.20) by means of substitution of (4.21), one obtains

$$\mathcal{A}_{\rm cl}[\xi,\varphi] = \int[-m\sqrt{(\partial\xi/\partial x^1)^2 - (\partial\xi/\partial x^0)^2} - \frac{\partial(\varphi,\xi)}{\partial(x^0,x^1)}]d^2{\rm x}. \tag{4.22}$$

The last term can be omitted, because it does not contribute to dynamic equations. Considering x^1 as a function of $x^0 = t$ and ξ, one obtains instead of Eq.(4.22)

$$\mathcal{A}_{\rm cl}[x^1] = -\int m\sqrt{1-(dx^1/dt)^2}dtd\xi \tag{4.23}$$

The action (4.23) describes a statistical ensemble of free independent classical particles characterized by the Lagrangian coordinate ξ. Thus, the variable κ is responsible for quantum properties of the dynamic system (4.18), (4.19).

It follows from Eq. (4.17) that

$$\tan\kappa = i\bar{\psi}\gamma^0\gamma^1\psi/(\bar{\psi}\psi) \tag{4.24}$$

Let us consider a stationary state of the Dirac equation

$$i\gamma^k\partial_k\psi - m\psi = 0 \tag{4.25}$$

It has the form

$$\psi_0(t,x,P) = \exp(-ik_0t)[\exp(-ik_1x)\left({(k_0+k_1)^{1/2} \atop (k_0-k_1)^{1/2}}\right)C+$$

$$\exp(ik_1x)\left({(k_0-k_1)^{1/2} \atop (k_0+k_1)^{1/2}}\right)D] \tag{4.26}$$

$$x^0 = t, \qquad x^1 = x, \qquad C = \mid C \mid \exp(i\varphi_1), \qquad D = \mid D \mid \exp(i\varphi_2)$$

where $P = \{\mid C \mid, \mid D \mid, k_0, k_1, \varphi_1, \varphi_2\}$ is a set of constant parameters, determining the stationary state. All constants are real, k_0 and k_1 satisfy the condition

$$(k_0)^2 - (k_1)^2 = m^2 \tag{4.27}$$

Using relations (4.16), (4.24) and (4.26), one obtains

$$j_0 = 2k_0(\mid C \mid^2 + \mid D \mid^2) + 4m \mid CD \mid \cos(2k_1x + \varphi_{12}),$$

$$j_1 = 2k_1(\mid C \mid^2 - \mid D \mid^2), \qquad \varphi_{12} = \varphi_1 - \varphi_2, \qquad p_0 = -k_0, \tag{4.28}$$

$$\tan\kappa = -\frac{2k_1 \mid CD \mid \sin(2k_1x + \varphi_{12})}{(\mid C \mid^2 + \mid D \mid^2)m + 2 \mid CD \mid k_0 \cos(2k_1x + \varphi_{12})}, \tag{4.29}$$

j_0 is an oscillatory function of x. Eq.(4.29) determines κ as an oscillatory function of x, if

$$\mid CD \mid < m(\mid C \mid^2 + \mid D \mid^2)(2 \mid k_0 \mid)^{-1}, \tag{4.30}$$

and the denominator in Eq. (4.29) does not change its sign.

The κ is a monotone function of x, if

$$\mid CD \mid > m(\mid C \mid^2 + \mid D \mid^2)(2 \mid k_0 \mid)^{-1}, \tag{4.31}$$

and the denominator in Eq. (4.29) can change its sign.

The cases (4.30) and (4.31) will be referred to as an oscillatory mode and a rotational mode respectively. Here an analogy with a pendulum is used. If it oscillates, then the angle determining the state of the pendulum oscillates. If the pendulum rotates around the fixed point, then this angle changes monotonelously.

Now let us consider a solution of the equation (4.25) in the form of two similar wave packets of the size $L \gg m^{-1}$ which pass one through another in the vicinity of the coordinate origin. Then the solution of Eq. (4.25) can be presented in the form

$$\psi(t,x,q_1) = \int\limits_{-\infty}^{\infty} \psi_0(t,x,P)g(q_1 - k_1)dk_1 \tag{4.32}$$

where ψ_0 is determined by Eq.(4.26) with

$$P = \{\mid C \mid, \mid D \mid, k_0, k_1, \varphi_1, \varphi_2\} = \{1, 1, \sqrt{m^2+k^2}, k_1, 0, 0\} \tag{4.33}$$

$$g(k_1) = A(L^2/2\pi)^{1/2} \exp(-L^2 k_1^2/2), \qquad \epsilon = (Lm)^{-1} \ll 1, \qquad A = \text{const.} \tag{4.34}$$

Let us consider the region

$$R_S : \mid t \mid < L\epsilon^{-1/2} = m^{-1}\epsilon^{-3/2}, \qquad \mid x \mid < L\epsilon^{-1/2} = m^{-1}\epsilon^{-3/2} \tag{4.35}$$

Calculation of the integral (4.32) inside this region leads to the following result

$$\psi(t, x, q_1) = \psi_0(t, x, P(q_1, T, X)) + o(\epsilon^2). \tag{4.36}$$

Now parameters $P = \{\mid C \mid, \mid D \mid, k_0, k_1, 0, 0\}$ of the function (4.26) are some functions of $q_1, T = \epsilon t, X = \epsilon$x. $\epsilon = (Lm)^{-1} \ll 1$ is a small parameter, and parameters P are slowly changing functions of x and t. Inside R_S

$$\begin{gathered} C = A \exp[-m^2(X+vT)^2/2 + O(\epsilon^{1/2})], \\ D = A \exp[-m^2(X-vT)^2/2 + O(\epsilon^{1/2})], \\ v = q_1(m^2+q_1^2)^{-1/2}, \qquad \Gamma = (m^2+q_1^2)^{1/2} m^{-1} \end{gathered} \tag{4.37}$$

$$k_0 = \sqrt{m^2+k_1^2} = m\Gamma + O(\epsilon^2), \qquad k_1 = q_1 + O(\epsilon^2), \tag{4.38}$$

where Γ is the Lorentz factor.

Space-time regions R_B and R_T occupied by drops B and T can be described approximately by

$$R_B : (X+vT)^2 < 2m^{-2}, \qquad R_T : (X-vT)^2 < 2m^{-2}, \tag{4.39}$$

It follows from Eqs.(4.29) and (4.37) that

$$\tan\kappa = -\frac{v \sin(2q_1 x)}{\cosh(2m^2 vTX) + \Gamma\cos(2q_1 x)}. \tag{4.40}$$

According to Eqs (4.31), (4.37) the rotational mode takes place inside the region

$$R_C : \quad \mid xt \mid < \frac{L^2}{2v} \log[\Gamma + (\Gamma^2-1)^{1/2}] \tag{4.41}$$

This region coincides approximately with the collision region $R_B \bigcap R_T$. Inside the region R_C $\mid< \partial\kappa/\partial x >\mid\cong\mid 2k_1 \mid$, and $\mid \kappa \mid\cong\mid k_1 L \mid\cong v\Gamma\epsilon^{-1} \gg 1$. As follows from Eq.(4.40), $\mid \tan\kappa \mid\ll 1$ inside $R_B \bigcup R_T \backslash (R_B \bigcap R_T)$. Then $\mid \kappa - n\pi \mid\ll 1$, and one can set $n = 0$ inside the fluid drops before the collision. At the time of the collision $\mid \kappa \mid\cong v\Gamma\epsilon^{-1} \gg 1$. It means that the variable κ is sensitive to a collision of drops.

Thus, several important collaries are obtained.

(1) The variable κ is responsible for quantum effects.

(2) The quantum variable κ is sensitive to the collision of fluid drops (overlapping of wave packets).
(3) A collision of two fluid drops can be considered as a quasi-stationary state with parameters changing slowly in the space-time.
(4) There are different modes of changing the quantum variable κ in the stationary state. These modes can convert one into another adiabatically.
(5) Non-interacting identical particles interact rather strongly via statistical exchange interaction.

5 Modification of the Dirac equation

Particles of the fluid interact in such a balanced way that there is no scattered fluid drops (particles). Internal energy κ and the effective mass $\mu(\kappa)$ change inside the collision region. A violation of the balance of this interaction must lead, in general, to appearing scattered particles.

To get the scattering one has to violate the balance. The most simple way of this violation is a modification of the variable $\mu = \mu(\kappa)$.

This modification must satisfy at least two conditions
(1) The modification is to be very small at $\mid \kappa \mid \ll 1$, to prevent from a change in behaviour of a single drop.
(2) The modification is to be such that j^0 could change its sign and could describe a creation of particle-antiparticle pairs.

Let us note that the definition (4.16) imposes a constraint on the 2-flux j^i, that is

$$j^0 = \bar{\psi}\gamma^0\psi = \psi^*\psi \geq 0. \tag{5.1}$$

The action itself (4.18) does not impose any restriction on the sign of j^0, but as will be shown in sequent the dynamic equations generated by the action (4.18),(4.19) prevent the sign from a change.

To provide a possibility for changing the sign of j^0, we modify the action (4.18), replacing $\mu_{\mathrm{D}}(\kappa) = m\cos\kappa$ by

$$\mu_{\mathrm{MD}}(\kappa) \equiv \mu_{\mathrm{IN}}(\kappa) \equiv m\sqrt{1-\kappa^2} = m\cos\kappa + O(\kappa^4) \tag{5.2}$$

Then the action for the modified Dirac equation takes the form

$$\mathcal{A}_{\mathrm{MD}}[j,\varphi,\kappa] = \int (-m\sqrt{(1-\kappa^2)(j_0^2-j_1^2)} - j^i\partial_i\varphi + \frac{1}{2}(j^0\partial_1\kappa + j^1\partial_0\kappa))d^2\mathrm{x}. \tag{5.3}$$

This action, in general, does not prevent the j^0 from changing its sign, because in the region where $\kappa^2 > 1$ the flux j^i is spacelike ($j^k j_k < 0$), so that in this region j^0 can change its sign.

For the action (4.18) the dynamic equations have the form

$$\frac{\partial j_0}{\partial t} - \frac{\partial j_1}{\partial x} = 0, \qquad j_0 = j^0, \qquad j_1 = -j^1,$$

$$
\begin{aligned}
\frac{\partial j_0}{\partial x}-\frac{\partial j_1}{\partial t}&=-2\mu'(\kappa)\sqrt{j_0^2-j_1^2},\qquad \mu'(\kappa)\equiv\frac{\partial\mu(\kappa)}{\partial\kappa}\\
\frac{\partial\varphi}{\partial t}-\frac{1}{2}\frac{\partial\kappa}{\partial x}&=-\mu(\kappa)j_0(j_0^2-j_1^2)^{-1/2},\\
\frac{\partial\varphi}{\partial x}-\frac{1}{2}\frac{\partial\kappa}{\partial t}&=-\mu(\kappa)j_1(j_0^2-j_1^2)^{-1/2}.
\end{aligned}\tag{5.4}
$$

where $\mu(\kappa)$ is defined either by Eq.(4.19) or by Eq. (5.2).

One can return to description in terms of the wave function ψ, using the transformation reciprocal to Eq.(4.17). This transformation is described by relations (4.16), (4.24) and

$$4i\varphi=\log[\psi_1\psi_2/(\psi_1^*\psi_2^*)]$$

Unfortunately, this reciprocal transformation is not complete, because it is restricted by the relation (5.1), whereas the action (4.18) permits both signs of j^0.

A more adequate representation of the wave function can be reached, if the four-component wave function

$$f=\begin{pmatrix}f_1\\ f_2\\ f_3\\ f_4\end{pmatrix},\qquad \tilde{f}=(f_1,f_2,f_3,f_4)\tag{5.5}$$

is used instead of the four dynamic variables j_0,j_1,φ,κ. The wave function is defined by relations

$$
\begin{aligned}
f&=w\exp[\tau_1(\varphi-\gamma^0\gamma^1\kappa/2)]\Pi_0\chi,\\
\tilde{f}&=\tilde{\chi}\Pi_0\exp[-\tau_1(\varphi-\gamma^0\gamma^1\kappa/2)]w,
\end{aligned}\tag{5.6}
$$

$$w=w_+\Pi_++w_-\Pi_-,\qquad \tilde{\chi}\Pi_0\chi=1,\tag{5.7}$$

χ and $\tilde{\chi}$ being constant column and row. Here tilde ($\tilde{}$) denotes transposition. $\gamma^0,\gamma^1,\tau_0,\tau_1,\Pi_\pm,\Pi_0$ are real constant 4×4 matrices defined by relations

$$
\begin{gathered}
\tilde{\gamma}^0=\gamma^0,\qquad \tilde{\gamma}^1=-\gamma^1,\qquad \tilde{\tau}_0=\tau_0,\qquad \tilde{\tau}_1=-\tau_1,\\
\gamma^i\gamma^k+\gamma^k\gamma^i=2g^{ik}I,\qquad \gamma^i\tau^k=\tau^k\gamma^i,\qquad i,k=0,1\\
\tau_0^2=I,\qquad \tau_1^2=-I,\qquad \tau_0\tau_1+\tau_1\tau_0=0,\\
\Pi_\pm=\frac{1}{2}(I\pm\gamma^0\gamma^1),\qquad \Pi_0=\frac{1}{4}(I+\gamma^0)(I+\tau_0)
\end{gathered}\tag{5.8}
$$

where I is the unit 4×4 matrix,

$$w_+=\sqrt{j^0+j^1},\qquad w_-=\sqrt{j^0-j^1},\tag{5.9}$$

and g^{ik} =diag$\{1,-1\}$ is the metric tensor. All quantities in the right hand side of Eq. (5.6) are real except for the quantities $w_\pm$ which can be either real or imaginary. Then some components of f can be complex. The quantities $f,\tilde{f},\ \bar{f}=\tilde{f}\gamma^0$ will be referred to as a spinor, a transposed spinor and a conjugate spinor respectively.

According to definition (5.8) matrices $\Pi_\pm, \Pi_0$ are projection operators

$$\Pi_\pm^2 = \Pi_\pm, \qquad \Pi_+\Pi_- = \Pi_-\Pi_+ = 0, \qquad \Pi_0\Pi_0 = \Pi_0,$$
$$\Pi_+\gamma^i = \gamma^i\Pi_-, \qquad i = 0, 1, \tag{5.10}$$
$$\Pi_0\tau_0\Pi_0 = \Pi_0, \qquad \Pi_0\gamma^0\Pi_0 = \Pi_0, \qquad \Pi_0\tau_1\Pi_0 = 0, \qquad \Pi_0\gamma^1\Pi_0 = 0.$$

The relations (5.8) can be realized, for instance, as follows

$$\gamma^0 = \begin{pmatrix} 0 & \sigma_0 \\ \sigma_0 & 0 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} 0 & \sigma_0 \\ -\sigma_0 & 0 \end{pmatrix},$$
$$\tau_1 = \begin{pmatrix} \sigma_{13} & 0 \\ 0 & \sigma_{13} \end{pmatrix}, \qquad \tau_0 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix}, \tag{5.11}$$

where σ are 2×2 matrices

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_{13} \equiv \tau = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{5.12}$$

Using relations (5.6)-(5.10), one can show that the reciprocal transformation has the form

$$j^i = \bar{f}\gamma^i f, \qquad i = 0, 1, \qquad p_i = \partial_i\varphi = -\bar{f}\tau_1\partial_i f/(\bar{f}f),$$
$$\tan(2\varphi) = \frac{\bar{f}\tau_1\tau_0 f}{\bar{f}\tau_0 f}, \qquad \tan(\kappa) = -\frac{\bar{f}\nu f}{\bar{f}f}, \qquad \nu = -\tau_1\gamma^0\gamma^1 \tag{5.13}$$

In terms of the wave function f the dynamical equations (5.4) can be written in the form [30]

$$\tau_1\gamma^i\partial_i f + [\mu(\kappa) - \frac{\partial\mu(\kappa)}{\partial\kappa}\nu]e^{-\nu\kappa} f = 0, \tag{5.14}$$
$$\kappa = -\arctan(\bar{f}\nu f/\bar{f}f) + n\pi$$

where n is some integer variable.

One can see that the wave function f describes the fluid state (j^i, p_i, κ) to within a certain integer variable n. In the special case, when

$$\mu(\kappa) = \mu_{\mathrm{EX}}(\kappa) \equiv m\cos\kappa = \mu_{\mathrm{IN}}(\kappa) + O(\kappa^4), \tag{5.15}$$

the equation (5.14) takes the form of a linear equation with constant coefficients

$$\tau_1\gamma^i\partial_i f + mf = 0. \tag{5.16}$$

Its form is the same as that of the two-dimensional Dirac equation. The formal expression (5.13) for j^k, $k = 0, 1$ is also the same as in the Dirac's theory, if the spinor f is treated as the wave function, and γ^i in Eq.(5.8) are treated as γ-matices of the Dirac's theory. However there are some differences. The factor $i = \sqrt{-1}$ of the derivatives in the Dirac equation is replaced by the antisymmetric matrix $-\tau_1$.

In the Dirac's theory a transition to a conjugate spinor is realized by the Hermitian conjugation (transposition + complex conjugation), but in our model it is realized by a transposition only. It would lead to the same result, if the spinor f were real. But according to Eqs.(5.6), (5.9) some components of f can be imaginary (for instance, if $j^0 < j^1$), because in our model the components j^i are arbitrary, whereas in the Dirac's theory $j^0 \geq 0$ always takes place. Thus, at the adequate description of the model the matrix τ_1 cannot be replaced by the imaginary unit. Essentially, τ_1 is the second imaginary unit, which is introduced together with $i = \sqrt{-1}$. Imaginary unit $i = \sqrt{-1}$ arises at rooting in Eq.(5.9).

If matrices γ^i and τ_1 are chosen in the form of Eqs. (5.11), (5.12), all coefficients in Eq. (5.16) are real. Then, if the wave function f of the linear model is real at some time t, it remains real all the time. According to Eq.(5.9), if $j^0 \geq 0$, $j^i j_i \geq 0$, then w_+, w_- and f are real. Reverse, if f is real , then according to the first relation (5.13) $j^0 \geq 0$, $j^i j_i \geq 0$. It means that j^0 cannot change its sign and the pair production does not take place due to the dynamic equations.

A distinction between the action (4.18), (4.19) and the action (5.3) will be treated as follows. The action $\mathcal{A}_{\rm D}$ (4.18), (4.19) describes the first order exclusive flux $j^i_{\rm EX}$ (or exclusive wave function), where there is one and only one particle which may belong to either target or beam. But by no means it can be both a beam particle and a target particle at the same time. For this reason both wave packets pass one through another without interaction, and the dynamic equation (5.16) for the exclusive wave function is linear. The action $\mathcal{A}_{\rm MD}$ (5.3) describes the first order inclusive flux $j_{\rm IN}$. In this case there is at least one particle (and, maybe, anything else). The inclusive flux $j_{\rm IN}$ describes the mean flux of particles. It describes both a beam particle and a target particle at the same time. In this case one has to expect scattered particles and generated pairs. The dynamic equation (5.14) for the inclusive wave function is nonlinear. This nonlinearty is supposed to represent the last term of the equation (4.13), describing inclusively an interaction of particles at the collision.

Further the cases, when $\mu(\kappa)$ is defined by expressions (5.2) and (5.15), will be referred to as a nonlinear (inclusive) model and a linear (exclusive) one correspondingly.

Nothing prevents from regarding the linear model and the nonlinear one as two different description modes (exclusive and inclusive respectively) of the *same physical phenomenon*. Indeed, they are incomplete (only first order fluxes are considered), and a validity of the relations (3.9), (3.10) cannot be tested. Besides, the dynamic equation for the first order exclusive wave function has the form (5.16) even in the case of interacting particles.

The nonlinearity of the dynamic equation for the wave function is incompatible with the QM principles (in particular, with the linear superposition principle, usage of linear operators and their eigenvalues).

In the interpretation of the inclusive model only statistical principles of RSD can be used [30]. It means that the interpretation of the inclusive model is based on a use of conservative quantities: the flux j^i and the energy-momentum tensor T_{ik}. One can draw world lines $\mathcal{L}$ tangent to the flux j^i which describe the mean motion of particles.

The description in terms of the inclusive wave function f is not complete in the nonlinear model. When the effective mass $\mu(\kappa)$ is defined by Eq.(5.2), the fluid state (j^i, p_i, κ) is described by the wave function ambiguously, as follows from the second equation (5.14). If $j^i j_i > 0$, then $\kappa^2 < 1$, and the second equation (5.14) is satisfied only as $n = 0$. If $j^i j_i < 0$, then one cannot determine the integer variable n single-valuedly, and the description in terms of the wave function is incomlete in the nonlinear model.

At the same time the description in terms of the flux is complete for both models. Eliminating variables κ, φ from the last three equations (5.4), one obtains for the linear model

$$\frac{1}{2}\partial_k\partial^k[\arcsin\frac{B_{01}}{2m\sqrt{j^l j_l}}] = \partial_i[m\sqrt{1 - B_{01}^2(4m^2 j^l j_l)^{-1}}\frac{j^i}{\sqrt{j^s j_s}}]$$

$$B_{01} \equiv \partial_1 j_0 - \partial_0 j_1, \qquad \kappa = \arcsin\frac{B_{01}}{2m\sqrt{j^l j_l}} + n\pi, \qquad n = \text{ integer} \tag{5.17}$$

Corresponding equation for the nonlinear model has the form

$$\frac{1}{2}\partial_k\partial^k\frac{B_{01}}{2\sqrt{m^2 j^l j_l + B_{01}^2/4}} = \partial_i\frac{m^2 j^i}{\sqrt{m^2 j^l j_l + B_{01}^2/4}} \tag{5.18}$$

$$\kappa = \frac{B_{01}}{2\sqrt{m^2 j^l j_l + B_{01}^2/4}}, \qquad B_{01} \equiv \partial_1 j_0 - \partial_0 j_1$$

One can see that formally the descriptions in terms of flux are similar for both models. Both equations have the same order. They coincide as $|B_{01}| \ll m(j^i j_l)^{1/2}$. But the definition of κ in terms of B_{01} and j^l is different. It is a single-valued one for the nonlinear model and multivalued for the linear case.

From the viewpoint of RSD the fluxes seem to be more fundamental quantities, than the wave functions which are defined through the fluxes, and the formulation of the pair production problem in terms of fluxes seems to be preferable.

Finally, in the nonlinear model the inclusive wave function f describes at least one particle and "anything else". The integer variable n describes this "anything else", apparently, the number of pairs. In this case it is not correct to treat the spacelike flux j^i as a flux of particles with superlight velocity. Rather it is a total flux of particles with $j^0 > 0$ and antiparticles with $j^0 < 0$. Time components j^0 of particles and antiparticles are compensated partially and the resulting flux becomes spacelike.

6 κ-field

In both models above the quantity κ can be considered as a force field (κ-field) acting on particles. Indeed, let us introduce the mean momentum π_i of particles defined by relations

$$\pi_i = -j_i/K, \qquad i=0,1; \qquad K=(j^k j_k)^{1/2}/\mu(\kappa), \qquad \pi_0^2-\pi_1^2=\mu^2(\kappa). \tag{6.1}$$

Then Eq.(5.4) can reduce to the form

$$\begin{gathered}
(\frac{\partial \pi_0}{\partial t}-\frac{\partial \pi_1}{\partial x})K+(\pi_0\frac{\partial K}{\partial t}-\pi_1\frac{\partial K}{\partial x})=0 \\
(\frac{\partial \pi_0}{\partial x}-\frac{\partial \pi_1}{\partial t})K+(\pi_0\frac{\partial K}{\partial x}-\pi_1\frac{\partial K}{\partial t})=K\frac{d}{d\kappa}[\mu^2(\kappa)] \\
\frac{\partial \varphi}{\partial t}-\frac{1}{2}\frac{\partial \kappa}{\partial x}=\pi_0 \\
\frac{\partial \varphi}{\partial x}-\frac{1}{2}\frac{\partial \kappa}{\partial t}=\pi_1
\end{gathered} \tag{6.2}$$

Equations (6.2) are invariant with respect to a transformation

$$K\to aK, \qquad a=\mathrm{const} \tag{6.3}$$

It means that the quantities φ, π_i, κ do not vanish, in general, in those space-time regions, where $j_i=0$, $K=0$, and the matter is absent. In the empty space-time $j_i=0$, $K=0$, and the first two equations (6.2) are satisfied identically. Two remaining equations reduce to the form

$$\frac{\partial^2\kappa}{\partial t^2}-\frac{\partial^2\kappa}{\partial x^2}=2\{\frac{\partial}{\partial x}[\mu^2(\kappa)+\pi_1^2]^{1/2}-\frac{\partial \pi_1}{\partial t}\} \tag{6.4}$$

where π_1 is an arbitrary quantity. This equation has a lot of nontrivial solutions. At the same time the energy-momentum tensor T_k^i vanishes for those solutions, as one can see from the expressions

$$\begin{aligned}
T_0^0 &= -\pi_0 j^0-\frac{1}{2}(j^0\frac{\partial \kappa}{\partial x}-j^1\frac{\partial \kappa}{\partial t}) = K(\pi_0^2+\pi_0\kappa_x/2+\pi_1\kappa_t/2) \\
T_1^0 &= -\pi_1 j^0-\frac{1}{2}(j^0\frac{\partial \kappa}{\partial t}-j^1\frac{\partial \kappa}{\partial x}) = K(\pi_0\pi_1+\pi_0\kappa_t/2+\pi_1\kappa_x/2) \\
T_1^1 &= -\pi_1 j^1+\frac{1}{2}(j^0\frac{\partial \kappa}{\partial x}-j^1\frac{\partial \kappa}{\partial t}) = -K(\pi_1^2+\pi_0\kappa_x/2+\pi_1\kappa_t/2)
\end{aligned} \tag{6.5}$$

For brevity we shall use the term "dark region" for the space-time (space) region, where $j_i\equiv 0$, and the term "highlighted region" for the space-time region, where $j_i\not\equiv 0$. Inside the dark region the energy-momentum tensor T^{ik} vanishes, and all particles are unobservable. However, it does not mean that this region is empty and can be ignored. Indeed, let us imagine that the system (6.2) is solved for two sets of initial conditions distinguished only inside dark regions of the space. In general, one cannot be sure that the two solutions will distinguish only inside dark regions. The distinction of the two solutions at the initial moment of time inside dark regions can extend in highlighted regions.

Inside highlighted regions T^{ik} does not vanish, the particles have an energy and are observable. The highlight can be described by the Lagrangian coordinate ξ defined by Eq.(4.21). The highlight spread is described by the equation

$$\xi_{tt} - \xi_{xx} = 2\mu'(\kappa)\sqrt{\xi_x^2 - \xi_t^2}$$

It is operated by the variable κ.

The κ-field can be considered as a force field existing independently of the matter and acting on the world lines of particles.

Indeed, applying transformations (4.21) to the action (4.18) and turning to Lagrangian coordinates t, ξ, one obtains instead of Eq.(4.18)

$$\mathcal{A}[x,\kappa] = \int \{-\mu(\kappa)\sqrt{1-(dx/dt)^2} + \frac{1}{2}(\kappa_x + \frac{dx}{dt}\kappa_t)\} dt d\xi \tag{6.6}$$

$$\kappa_x \equiv \frac{\partial(\kappa,t)}{\partial(x,t)}, \qquad \kappa_t \equiv \frac{\partial(\kappa,x)}{\partial(t,x)}$$

where $x = x(t,\xi)$ and $\kappa = \kappa(t,x)$. When the action (6.6) is varied with respect to κ, the κ is considered as a function of arguments $t,\ \xi$: $\quad \kappa = \kappa(t, x(t,\xi))$. However if the κ-field is fixed as a definite function of t, x, the action (6.6) describes dynamics of classical particles in the given classical field κ.

Corresponding Jacobi-Hamilton equation can be obtained either from the action (6.6), or from the last two equations (6.2). In both cases it has the form

$$(\frac{\partial\varphi}{\partial t} - \frac{1}{2}\kappa_x)^2 - (\frac{\partial\varphi}{\partial x} - \frac{1}{2}\kappa_t)^2 = \mu^2(\kappa) \tag{6.7}$$

which does not depend on the highlighting intensity.

Relations (6.6), (6.7) are valid for both models and differ only by the choice of $\mu(\kappa)$.

An action of the κ-field on the matter associates with the action of the electromagnetic field. Indeed, setting in Eq.(6.7)

$$\mu(\kappa) = m = \text{const.} \tag{6.8}$$

$$\kappa_x = \partial_1\kappa = -2eA_0, \qquad \kappa_t = \partial_0\kappa = -2eA_1 \tag{6.9}$$

one obtians the Hamilton-Jacobi equation for a charged particle moving in a given electromagnetic field with the 2-potential $A_i,\ i = 0, 1$.

The resemblance between the electromagnetic field and the κ-field can be displayed also in the following way. In the two-dimensional space-time the ensemble of classical relativistic particles moving in the self-consistent electromagnetic field is described by the action

$$\mathcal{A}[j,\varphi,A] = \int (-m\sqrt{j_0^2 - j_1^2} - j^i\partial_i\varphi - ej^iA_i - \frac{1}{16\pi}F_{ik}F^{ik})d^2\text{x}. \tag{6.10}$$

$$F_{ik} = \partial_i A_k - \partial_k A_i, \tag{6.11}$$

Without loss of generality one can impose the Lorentz gauge constraint on A_i

$$\partial_0 A_0 - \partial_1 A_1 = 0. \tag{6.12}$$

Eq.(6.12) can be solved in the form of (6.9). If we drop from Eq.(6.10) the last term (describing the proper energy of the electromagnetic field) and eliminate A_i by means of (6.9), we arrive at the action (4.18) with $\mu(\kappa)$ defined by Eq.(6.8).

In view of the association between the κ-field and the electromagnetic field the relations (5.2) or (4.19) mean that the proper energy of the field is included directly in the rest mass of the ensemble particles. At the same time the field can escape from the matter without escaping the energy.

According to Eq.(6.9) the κ-field can be treated as a potential of the electromagnetic potential A_i, provided the electromagnetic field A_i has not proper energy and the Maxwell equations are replaced by some other dynamic equations.

The model described by the action (4.18) with the condition (6.8), or by the dynamic equations (6.2), (6.8), can be solved completely, because two first equations in Eq.(6.2) are linear with respect to variables $j_0 = -K\pi_0$, $j_1 = -K\pi_1$. Using null coordinates $\alpha = t + x$, $\beta = t - x$, the general solution of the system (6.2), (6.8) can be presented in the form

$$j_0 = -K\pi_0 = \frac{1}{2}(H_+^2 + H_-^2), \qquad j_1 = -K\pi_1 = \frac{1}{2}(H_+^2 - H_-^2),$$
$$K = \frac{1}{m} H_+ H_- \tag{6.13}$$

$$\varphi = -\frac{m}{2}\left(\frac{R_+}{H_-} + \frac{R_-}{H_+}\right) + A(\alpha) + B(\beta), \qquad \kappa = -m\left(\frac{R_+}{H_-} - \frac{R_-}{H_+}\right) + 2A(\alpha) - 2B(\beta) \tag{6.14}$$

where

$$R_+ \equiv R_+(\alpha) \equiv R_+(t+x), \qquad R_- \equiv R_-(\beta) \equiv R_-(t-x),$$
$$H_+ \equiv H_+(\alpha) \equiv dR_+(\alpha)/d\alpha, \qquad H_- \equiv H_-(\beta) \equiv dR_-(\beta)/d\beta, \tag{6.15}$$

Here R_+, R_-, A, B are arbitrary functions of one variable, while A, B are real and R_+, R_- are either both real, or both imaginary.

The energy-momentum tensor has the form

$$T_0^0 = \frac{m}{4}[2H_+H_- - R_-H_+' - R_+H_-'] - \frac{1}{2}[H_+^2 A'(\alpha) + H_-^2 B'(\beta)]$$
$$T_1^0 = \frac{m}{4}[R_+H_-' - R_-H_+'] - \frac{1}{2}[H_+^2 A'(\alpha) - H_-^2 B'(\beta)] \tag{6.16}$$
$$T_1^1 = \frac{m}{4}[2H_+H_- + R_-H_+' + R_+H_-'] + \frac{1}{2}[H_+^2 A'(\alpha) + H_-^2 B'(\beta)]$$

where prime ($'$) means the derivative $H_+'(\alpha) \equiv dH_+(\alpha)/d\alpha$. The wave function (5.5), (5.6) takes the form

$$f = \begin{pmatrix} H_+ \cos[-mR_+/2H_- + A(\alpha)] \\ H_+ \sin[-mR_+/2H_- + A(\alpha)] \\ H_- \cos[-mR_-/2H_+ + B(\beta)] \\ H_- \sin[-mR_-/2H_+ + B(\beta)] \end{pmatrix} \qquad (6.17)$$

Thus the dynamic system (4.18) can be considered as an ensemble of relativistic classical particles moving in the self-consistent force κ-field. The κ-field has no specific proper energy, but a presence of the κ-field changes the total energy of particles.

7 The pair production mechanism

To understand the pair production process, let us consider a motion of particles in a given κ-field. Following the world lines of particles we are interested mainly in existence of branch points which can be responsible for pair production effect. Incidentally the turning point should be distinguished from the branch point. At both points the particle velocity dx/dt changes its sign, but at the turning point $dx/dt = 0$, and x is a single-valued function of t, whereas at the branch point $dt/dx = 0$, and x is a multivalued function of t. At the branch point the world line of a particle describes either the pair production, or annihilation.

Let us consider the most simple case, when in the vicinity of the branch point the κ depends only on one variable y. There are three different cases: (I) y is the time coordinate $y = t$, (II) y is a spatial coordinate $y = x$, (III) y is the null coordinate $y = t$+x. All other cases can reduce to these three ones by means of a proper Lorentz transformation.

If κ depends only on y, then a complete integral of Eq. (6.7) can be easily found so that dx/dt and the time component p_0 can be obtained as functions of the coordinate y and some parameter. In the three cases the following expressions are found.

I. $y = t, \quad \kappa = \kappa(t), \quad p_1$ is a parameter,

$$\frac{dx}{dt} = \frac{p_1 - \kappa_t(t)/2}{\sqrt{(p_1 - \kappa_t(t)/2)^2 + \mu^2(\kappa)}}, \qquad p_0 = \sqrt{(p_1 - \kappa_t(t)/2)^2 + \mu^2(\kappa)} \qquad (7.1)$$

II. $y = x, \quad \kappa = \kappa(x), \quad p_0$ is a parameter,

$$\frac{dx}{dt} = \frac{\sqrt{(p_0 - \kappa_x(x)/2)^2 - \mu^2(\kappa)}}{p_0 - \kappa_x(x)/2}, \qquad p_0 = \text{ const} \qquad (7.2)$$

III. $y = t + x, \quad \kappa = \kappa(y), \quad q$ is a parameter,

$$\frac{dx}{dt} = \frac{4q^2 - \mu^2}{4q^2 + \mu^2}, \qquad p_0 = q + \frac{\mu^2}{4q} + \kappa_y/2. \qquad (7.3)$$

In the case I the branch point [of the function $x = x(t)$] is determined by the relation:

$$(p_1 - \kappa_t(t)/2)^2 = -\mu^2(\kappa) \tag{7.4}$$

If $\mu^2(\kappa) > 0$, the denominator in Eq. (7.1) does not change sign, and the branch point does not exist. The parameter p_1 can be treated as the spatial component of the canonical momentum of the particle which is conserved in this case. In the vicinity of the branch point the world line is directed along the lines of constant κ.

In the case II the branch point [of the function $x = x(t)$] is determined by the relation

$$p_0 - \kappa_y(y)/2 = 0, \qquad \mu^2(\kappa) < 0 \tag{7.5}$$

where y=x. If $\mu^2(\kappa) > 0$ the branch point is located in the space-time region inaccesible for the world line. The parameter p_0 is the time component of the canonical momentum of the particle which is conserved in this case. In the vicinity of the branch point the world line is directed across the lines of constant κ.

In the case III the branch point is determined by the same equation (7.5) with $y = t + x$ and p_0 defined by the last equation (7.3).

If instead of Eq.(7.5) the condition

$$p_0 - \kappa_y(y)/2 = 0, \qquad \mu^2(\kappa) = 0, \qquad y = x, \text{ or } \quad y = t + x, \tag{7.6}$$

takes place, a special investigation is necessary. It shows that, if κ is an analytical function of x, then there are no branch points satisfying the condition (7.6).

Let the constraint (7.4) be supplemented with a condition

$$\mu^2(\kappa) = 0. \tag{7.7}$$

Then for the linear model ($\mu^2 = m^2 \cos^2 \kappa$) there are no branch points satisfying Eqs.(7.4), (7.7). In the case of nonlinear model ($\mu^2 = m^2(1 - \kappa^2)$) such a branch point $t = t_b$ does exist, if

$$p_1 = 0, \qquad \kappa(t) = 1 - 2m^2(t - t_b)^2 + \dots \tag{7.8}$$

Thus there are no branch points at all in the linear model, where $\mu^2(\kappa) = m^2 \cos^2 \kappa \geq 0$, i.e. in this model the pair production is impossible.

In all cases the branch point is located in the space-time region where $\mu^2(\kappa) \leq 0$. Near the branch point $\mid dx/dt \mid \gg 1$, and the world line is spacelike.

In the case II it follows from Eq.(7.5) that the pair production is connected with a spatial gradient of κ-field. If the parameter p_0 is large enough ($\mid p_0 \mid \gg m$), and the spatial gradient of the κ-field is small ($\mid \kappa_x \mid \leq m$), then the pair production does not take place.

In the case I the pair production is connected mainly with the value of κ, (but not with the value of the time gradient of κ), although $\kappa_t(t)$ influences a location of the branch point. In a slowly varying κ-field, $\mid \kappa_t^2 + \kappa_x^2 \mid \leq m^2$, the branch points are possible in the case I provided the parameter p_1 is not extremely small. In this case the particles of the pair produced are pushed out into the space-time region where $\mu^2(\kappa) > 0$ (see Fig.4). In a slowly varying κ-field the main pair production mechanism is determined by the branch points corresponding to Eq.(7.4).

The particle's energy is defined by the relation

$$E = \frac{1}{N_0}\int\limits_V T^{00}dx, \qquad N_0 = \int\limits_V j^0 dx \tag{7.9}$$

where V is a small volume around the particle, and T^{00} is the energy density defined by Eqs.(6.5), (6.1)

$$T^{00} = \frac{j^0 j^0}{K} - \frac{1}{2}(j^0\frac{\partial\kappa}{\partial x} - j^1\frac{\partial\kappa}{\partial t}) \tag{7.10}$$

$$K = j^0[1-(dx/dt)^2]^{1/2}\mu^{-1}(\kappa) \tag{7.11}$$

In general, the energy defined by Eq.(7.9) does not coincide with the time component p_0 of the canonical momentum which is defined as a quantity conjugate canonically to the time t. In the absence of pair production the two quantities do coincide [31]. If the κ-field changes slowly, the first term in Eq.(7.10) dominates and the sign of T^{00} (and energy E) coincides with the sign of K.

Combining Eq.(7.11) with Eqs. (7.1)-(7.3), one obtains in these three cases

$$\text{I.} \qquad K = \frac{j^0}{\sqrt{(p_1 - \kappa_t(t)/2)^2 + \mu^2(\kappa)}} \tag{7.12}$$

$$\text{II.} \qquad K = j_0/(p_0 - \kappa_x/2) \tag{7.13}$$

$$\text{III.} \qquad K = j_0/(p_0 - \kappa_y/2), \qquad y = t + \mathrm{x}. \tag{7.14}$$

According to Eqs. (7.4), (7.5) both the numerator and the denominator in Eqs.(7.12)-(7.14) change their signs only at the branch point. It means that the sign of K does not change along any world line, so that the first term in Eq.(7.10) has the same sign along the whole world line. As far as the first term dominates at $t = \pm\infty$, where $\kappa \to 0$, the sign of the particle energy is the same at $t = -\infty$ and at $t = +\infty$, although it can change inside the collision region. If a world line describes a particle and an antiparticle as well, the signs of their energies are the same at $t = +\infty$, even though p_0 has different signs for particle and antiparticle.

An existence of branch points is not sufficient for an actual pair production, since it is necessary also to bring an energy to the branch points. Only in this case the dark world lines will be highlighted. The projections $\epsilon_{||}$, $\epsilon_\perp$ of the energy flux in the direction of the world line and in the transverse direction are defined by Eq.(6.5) as follows:

$$\epsilon_{||} = \mid\mu\mid^{-1}\pi^i T_i^0 = K\mid\mu\mid(\pi_0 + \kappa_x/2) = m\sqrt{\mid j_0^2 - j_1^2\mid}(\pi_0 + \kappa_x/2)$$

$$\epsilon_\perp = \mid\mu\mid^{-1}(\pi^1 T_1^0 + \pi^0 T_0^0) = K\mid\mu\mid\kappa_t/2 = m\sqrt{\mid j_0^2 - j_1^2\mid}\kappa_t/2 \tag{7.15}$$

It means that the energy flows practically along the world lines in those space-time regions where the κ-field changes slowly $(\kappa_t^2 + \kappa_x^2)^{1/2} \ll m$.

Inside the collision region, where gradients of the κ-field can be large, the energy flows across world lines and is brought into the region, where $\mu^2(\kappa) < 1$ and there are branch points. In this case the transversal energy flux highlights dark regions of the space-time providing the pair production with energy. Unfortunately, this process is rather subtle and needs a complete solution of the collision problem.

Let us try to compare our statement of the pair production problem with the conventional one. The conventional statement of this problem is based on the QM principles and is considered conventionally in the frame of the quantum field theory (QFT) as a part of the collision problem. Considering a conceptual statement of the problem we follow the clear presentation given by Henley and Thirring [32].

According to QM principles, introducing concepts of particles and antiparticles, one needs to define corresponding operators. One determines operators of flux densities of particles $n_p^i[\varphi_{\mathbf{x}}]$ and antiparticles $n_a^i[\varphi_{\mathbf{x}}]$ as some functionals of operator-valued dynamic variables $\varphi_{\mathbf{x}}$ (the label $\mathbf{x}$ means a point of some 3-plane t=const). They are to be determined in such a way that the current

$$j^i[\varphi_{\mathbf{x}}] = n_p^i[\varphi_{\mathbf{x}}] - n_a^i[\varphi_{\mathbf{x}}] \tag{7.16}$$

Solving dynamic equations, one determines the dynamic variables $\varphi_{\mathbf{x}}$ as functions of the time t. Then, averaging over two vector states $|\Psi\rangle_b$ and $|\Psi\rangle_{bt}$, one can determine the mean values of different powers of operators $n_p^i[\varphi_{\mathbf{x}}]$ and $n_a^i[\varphi_{\mathbf{x}}]$. The state vector $|\Psi\rangle_b$ describes the situation, when the beam goes past the target, $|\Psi\rangle_{bt}$ describes the case, when the beam collides with the target. Comparing the two sets of mean values labelled by "bt" and "b", one can conclude about cross sections.

The number of particles (antiparticles) in a certain volume V is determined by the operator

$$N_V = \int\limits_V n^0 d^3\mathbf{x} \tag{7.17}$$

The operators $n_p^i[\varphi_{\mathbf{x}}]$ of the flux densities of particles (antiparticles) are to satisfy the following conditions:
(1) Under the Lorentz transformations $n_p^i[\varphi_{\mathbf{x}}]$ is to be transformed as a 4-vector.
(2) Expression $n_p^i[\varphi_{\mathbf{x}}]$ is Lorentz covariant, i.e. it is expressed through dynamic variables in the same form in all Lorentz coordinate systems, provided $\varphi_{\mathbf{x}}$ are transformed according to some representation of the Lorentz group, i.e. $\varphi_{\mathbf{x}}$ are scalars, vectors, spinors, etc.
(3) Eigenvalues of the operator N_V are only natural numbers for any volume V.

In the non-relativistic collision theory only the last condition takes place. Two first conditons are unessential. Vice versa, in the relativistic QFT the first two condition are essential. In the relativistic QFT the last condition is not fulfilled even in the case of free fields, when there is neither scattering, nor pair production. It is fulfilled approximately, provided V is large enough ($V^{1/3} \gg \hbar/mc$). This can be treated either in the sense that the particle is not pointlike, or in the sense that the QM principles are incompatible with the relativity principles. Both alternatives seem to be unfavourable.

Let us note that in the general case the division (7.16) of the current into particles and antiparticles is rather complicated problem which needs a solution of dynamic equations. Indeed, to test a fulfilment of the second condition, one needs to recalculate the dynamic variables $\varphi_{\mathbf{x}}$ from the 3-plane t=const to another 3-plane t'=const in other coordinate system. Such a recalculation needs a usage of dynamic equations.

The relativistic QFT cannot satisfy the third condition and gets around this obstacle as follows. Let us consider the number of particles only inside rather large volumes $dV_{\mathbf{k}}$. Any volume $dV_{\mathbf{k}}$ is located around the point $\mathbf{x} = \hbar T\mathbf{k}/m$, where $\hbar\mathbf{k}$ is the particle momentum, and T is a period of time, which is large enough. Then the volume $dV_{\mathbf{k}} = (\hbar T/m)^3 d^3\mathbf{k} \gg (\hbar/m)^3$, provided $T^3 d^3\mathbf{k} \gg 1$. If $T \to \infty$, the last condition is fulfilled, the eigenvalues of the operator $n(\mathbf{k})d^3\mathbf{k}$ of the particle number in the momentum space

$$N_{dV_{\mathbf{k}}} = n(\mathbf{k})d^3\mathbf{k} = \int\limits_{dV_{\mathbf{k}}} n_p^0[\varphi_{\mathbf{x}}]d^3\mathbf{x} \qquad (7.18)$$

are natural numbers, and all $n(\mathbf{k})$ commute between themselves. It means that the operators $n(\mathbf{k})$ can be presented in the form

$$n(\mathbf{k}) = a^*(\mathbf{k})a(\mathbf{k}), \qquad [a(\mathbf{k}), a^*(\mathbf{k}')]_{\pm} = \delta(\mathbf{k} - \mathbf{k}') \qquad (7.19)$$

where $a(\mathbf{k})$ and $a^*(\mathbf{k})$ are so called annihilation and creation operators. The collision is supposed to take place in the coordinate origin, and T is the time after collision ($T \to \infty$). QFT studies results of the collision only at $T \to \infty$, using S-matrix technique. In the case of free particles, when there are neither scattering, nor pair production, the operators $a(\mathbf{k}), a^*(\mathbf{k})$ are Fourier components of the dynamic variables $\varphi_{\mathbf{x}}$, and can be considered as dynamic variables.

In the general case, when there is scattering and pair production, a connection between $a(\mathbf{k}), a^*(\mathbf{k})$ and dynamic variables $\varphi_{\mathbf{x}}$ is not clear and, generally, depends on the solution of dynamic equations. As far as for solution of dynamic equations one uses a perturbation theory, the defintion of the number of particles arises as a result of perturbation theory (or diagram technique). But practically, in all cases, one considers Fourier components of the dynamic variables $\varphi_{\mathbf{x}}$ as the creation and annihilation operators of particles (antiparticles) $a^*(\mathbf{k}), a(\mathbf{k})$.

Let us note that a solution of dynamic equations and a choice of the operators $n^i[\varphi_{\mathbf{x}}]$ in such a way, that to satisfy the three constraints, are different problems. These problems are difficult, but different. Solving them, there is no necessity for uniting them into one more complicated problem, as the contemporary QFT does. A solution of a problem by parts is more simpler as a rule. Let us note that, using operators $n(\mathbf{k})$ instead of $n^i[\varphi_{\mathbf{x}}]$, it is rather difficult to distinguish between the states $|\Psi\rangle_b$ and $|\Psi\rangle_{bt}$, and one uses either cut off of the interaction, or so called $|\text{in}\rangle$ and $|\text{out}\rangle$ states.

Finally, the conventional QFT description is a kind of exclusive description [1]. Recalculation of the QFT statement of the pair production problem to the inclusive description and comparison with our statement of the problem is very difficult, because for the conventional statement of the problem the concept of the particle number is not defined directly (but only via solution of the dynamic equations). At the same time it seems impossible to reduce our statement of the problem to the conventional exclusive form of description, because our inclusive description is incomplete (it uses only functions of one space-time argument). Besides, it is hardly worth to consider the up-to-date version of QFT as a standard for the statement of the pair production problem. But even such a statement of the problem is better, than nothing.

Of course, it should be well to compare results of our investigation with the experimental data. Unfortunately, now we are only starting a long way for constructing the inclusive model of pair production. We succeded only to show that the pair production can be described conceptually in terms of functions of one space-time argument. For comparison to the experimental data it is necessary:
1) A generalization of results to the case of 4–d space-time.
2) Investigation of stationary states of the modified Dirac equations.
3) Development of a technique for solution of non-linear equation describing inclusively the collision problem.

Unfortunately, such a program cannot be carried out in a frame of one paper.

8 Concluding remarks

The results obtained can be devided into two sets. The first one is a corollary of transforming the Dirac equation to hydrodynamic variables $j_0, j_1, \varphi, \kappa$. These results are formal and can be summarized as follows.
(1) The variable κ describes some force field which is responsible for quantum effects and sensitive to a collision of two wave packets.
(2) With $\kappa \equiv 0$ the Dyrac dynamic system degenerates into a statistical ensemble of free classical particles which can be treated without use of quantum mechanical principles at all.
(3) The statistical interpretation given at $\kappa \equiv 0$ can be applied in the general case. It removes constraints imposed by QM principles (dynamic equations linear in wave functions, linear operators, interpretation in terms of linear operators).
(4) The field κ has no proper energy, but the effective rest mass of particles μ depends on κ. The κ-field influences world lines of particles. This action resembles that of the electromagnetic field.

It is necessary to stress that all these results are formal mathematical corolaries of the transformation of the Dirac equation to hydrodynamic variables. Their derivation does not require any additional hypothesis. Nevertheless many of these results are very difficult for perception psychologically, because it requires a replacement of the quantum axiomatics by a statistical description. (A similar transition from the thermodynamic axiomatics to the statistical physics was also rather difficult psychologically, as the history of the Waterston's paper described by Jammer shows [33]).

Another set of results is connected with a generalization of the Dirac equation [the substitution for the Dirac effective mass $\mu_{\mathrm{D}} = m\cos\kappa$ another effective mass $\mu_{\mathrm{MD}} = m(1-\kappa^2)^{1/2}$]. Such a substitution changes properties of the κ-field. This change is very slight within the limits of one wave packet, when $\kappa^2 \ll 1$ (that includes the problems connected with the atom structure). But this change is very essential, if $\kappa^2 \geq 1$ and the pair production is possible. The second set of results based on this substitution can be summarized as follows.
(1) A possibility of description of many particles by means of several functions of one point (inclusive description).
(2) Investigation of details of the pair production mechanism.

The κ-field discovery bring about problems which cannot be solved in frame of one paper. Let us list only the most evident of them:

(1) A generalization of results to the case of four-dimensional space-time and inclusion of electromagnetic and other fields.

(2) The nature of the κ-field in view of quantum effects.

(3) A calculation of high energy collision process in terms of inclusive description and a comparison with experimental data.

(4) A statistical interpretation of the inclusive description without a usage of too restrictive QM principles.

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References

1. Rylov Yu.A., *J. Math. Phys.* **35**, 3922, (1994).
2. Feynman R.P. Phys. Rev. Lett. **23**, 1415, (1969).
3. Logunov A.A., Mestvirishvili M.A., Nguen Van Hiew. *Phys. Lett.* **25**, 611, (1967).
4. Feinberg E.L. *Phys. Rept. C.* **5**, 237, (1972).
5. Grishin V.G. *Inclusive Processes at Hadron Interactions at High Energies.* Moscow, Atomizdat, 1982. (in Russian).
6. Grishin V.G. *Quarks and Hadrons in Interactions of High Energy Particles.* Moscow, Energoatomizdat, 1988. (in Russian).
7. Taylor R.E. *Rev. Mod. Phys.* **63,** 573, (1991).
8. Kendall H.W. *Rev. Mod. Phys.* **63**, 597, (1991).
9. Friedman J.I. *Rev. Mod. Phys.* **63**, 615, (1991).
10. Day D.B., McCarthy J.S., Meziani Z.B., Minchart R., Sealock R., Thornton S.T.,Jourdan J., Sick I., Filippone B.W., McKelown R.D., Milner R.G., Potterveld D.H., Szalata Z. *Phys. Rev.C.* **48**, 1849, (1993).
11. Rylov Yu.A., *J. Math. Phys.* **30**, 2516, (1989).
12. Fermi E., *Progr. Theor. Phys.* **5**, 570, (1950).
13. Landau L.D., *Izvestiya Akademii Nauk USSR.* **17**, 5, (1953).
14. Rylov Yu.A., *Advances Appl. Clifford Algebras* **5**, 1, (1995).
15. Bohm D. *Phys.Rev.* **85**, 166, 180, (1952)
16. de la Pena-Auerbach L. and Garcia-Colin Leopoldo L., *J.Math. Phys.* **9**, 916, (1968)
17. de la Pena-Auerbach L. and Cetto A.M., *Found. Phys.* **5**, 355, (1975)
18. Kaliski S., *Proc. Vibr. Phys.* **3**, (1970).
19. Belinfante F.J., *A Survey of Hidden-Variables Thories*, Pergamon, Oxford, 1973.
20.Holland P., *The quantum theory of motion.* Cambridge University Press, 1993.
21. Madelung E., *Z. Phys.* **40**, 322, (1926).
22. Takabayasi T., *Progr. Theor. Phys.* **9**, 187, (1953).
23. Janossy L., Ziegler M., *Acta Phys. Hung.* **16**, 37, (1963).
24. Janossy L., Ziegler M., *Acta Phys. Hung.* **16**, 345, (1964).
25. Janossy L., Ziegler-Naray M., *Acta Phys. Hung.* **20**, 233, (1966).
26. Rylov Yu.A. *Ann. Phys. (Leipzig)* **27**, 1 (1971).
27. Rylov Yu.A. *Int. J. Theor. Phys.* **8**, 65, (1973).
28. Rylov Yu.A. *Int. J. Theor. Phys.* **8**, 123, (1973).
29. Rylov Yu.A. *Int. J. Theor. Phys.* **19**, 645, (1980).
30. Rylov Yu.A. *Phys. Essays.* **4**, 300, (1991) .
31. Rylov Yu.A., *Theor. and Math. Phys.* (USA) **5**, 333, (1970), (translated from Russsian)
32. Henley E.M. and Thirring W. *Elementary quantum field theory.* New York, McGraw-Hill, 1962.
33. Jammer M., *The conceptual development of quantum mechanics.* McGraw-Hill book Company, 1967. Sec.1.2.