Mechanism of pair production in classical dynamics

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Abstract

It is shown that description of pair production is possible in terms of classical dynamics of stochastic particles. Any particle generates the κ-field, which can change the particle mass. Interaction of a relativistic particle beam with a returning beam reflected from the potential barrier generates such a κ-field, that a tachyon region arises around the particle. World lines of other particles can change the direction in time inside the tachyon region. As a result a pair of particle - antiparticle can arise in the tachyon region.

Key words: classical dynamics of stochastic particles; κ-field; pair production; tachyon region .

1 Introduction

It is used to think that pair production is possible only in the framework of quantum mechanics. However, mechanism of pair production is not known. The force field, producing pair production is not known also. It is known only, that the pair particle-antiparticle is produced as a jump. Particle and antiparticle are produced together, because the particle and the antiparticle are two different states of the world line, which is a real physical object in the relativistic particle dynamics. We shall use the special term "emlon" for the world line, considered as the basic object of dynamics. It is a reading of abbreviation ML of the Russian term "world line". Particle and antiparticle are two different states of emlon. This terminology differs from the conventional terminology, where particle and antiparticle are basic objects of dynamics, whereas the world line is not an object of dynamics. It is a history of the particle motion.

In the beginning of the twentieth century one discovered micro particles (electrons, protons etc), which move indeterministically. One tried to describe these stochastic particles in terms of nonrelativistic statistical description [1, 2]. However, this statistical description failed. As a result the conception of particle dynamics has been changed. The classical particle dynamics has been replaced by the quantum particle dynamics. The change of the conception meant a change of the mathematical formalism of the particle dynamics.

However, a failure of the nonrelativistic statistical description of the stochastic particle dynamics was connected with the fact, that nonrelativistic quantum particles were in reality relativistic particles. They have nonrelativistic regular component of velocity, but the stochastic component of velocity is relativistic. For description of relativistic stochastic particles one should use a relativistic statistical description. Relativistic statistical description and nonrelativistic one differ in the definition of the particle state.

The nonrelativistic state (n-state) of a particle is described as a point in the phase space, whereas the relativistic state (r-state) of a particle is the world line in the space-time [3, 4, 5]. The density of states is defined differently for n-states and r-states.

The density $\rho(x, \mathbf{p})$ of n-states is defined by the relation

$$dN = \rho d\Omega$$

where dN is the number of states in the volume $d\Omega$ of the phase space. The quantity ρ is nonnegative. It may serve for introduction of the probability density and for the statistical description in terms of probability.

The density $j^k(x)$ of r-states is defined by the relation

$$dN = j^k dS_k$$

where dN is the flux of world lines through the 3-dimensional area dS_k. Four-vector j^k cannot serve for introduction of the probability density and for the statistical description in terms of probability.

Statistical description of relativistic particles is produced in terms of a statistical ensemble. Statistical ensemble $\mathcal{E}[\mathcal{S}]$ of particles $\mathcal{S}$ is a set of N $(N \to \infty)$ independent identical particles $\mathcal{S}$. In conventional conception of particle dynamics the deterministic particle $\mathcal{S}_{\rm d}$ is a basic object of dynamics. The statistical ensemble $\mathcal{E}[\mathcal{S}_{\rm d}]$ of deterministic particle $\mathcal{S}_{\rm d}$ is a derivative object. If one has dynamic equations for $\mathcal{S}_{\rm d}$, one obtains dynamic equations for $\mathcal{E}[\mathcal{S}_{\rm d}]$, because all dynamic systems $\mathcal{S}_{\rm d}$ in $\mathcal{E}[\mathcal{S}_{\rm d}]$ are independent. Dynamic equations for $\mathcal{E}[\mathcal{S}_{\rm d}]$ are the same as for $\mathcal{S}_{\rm d}$, but the number of the freedom degrees of $\mathcal{E}[\mathcal{S}_{\rm d}]$ is larger, than the number of the freedom degrees of $\mathcal{S}_{\rm d}$. If n is the number of degrees of $\mathcal{S}_{\rm d}$, then nN is the number of the freedom degrees of $\mathcal{E}[\mathcal{S}_{\rm d}]$, where N is the number of $\mathcal{S}_{\rm d}$ in $\mathcal{E}[\mathcal{S}_{\rm d}]$.

Idea of the relativistic statistical description looks as follows. The statistical ensemble $\mathcal{E}[\mathcal{S}]$ of particles $\mathcal{S}$ is considered as a basic object of particle dynamics. It means that $\mathcal{E}[\mathcal{S}]$ is a dynamical system, and the dynamic equations are written directly for $\mathcal{E}[\mathcal{S}]$. Dynamic equations for $\mathcal{E}[\mathcal{S}]$ have the form of dynamic equation for the continuous medium. If the particle $\mathcal{S}$ is a deterministic particle $\mathcal{S}_{\rm d}$, then dynamic equations for $\mathcal{S}$ can be obtained from dynamic equations for $\mathcal{E}[\mathcal{S}_{\mathrm{d}}]$. Dynamic equations for $\mathcal{S}_{\mathrm{d}}$ coincide with dynamic equations for $\mathcal{E}[\mathcal{S}_{\mathrm{d}}]$, provided they are written in the Lagrangian representation.

However, if the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ consists of stochastic particles $\mathcal{S}_{\mathrm{st}}$, one cannot obtain dynamic equations for $\mathcal{S}_{\mathrm{st}}$ from dynamic equations for $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$, because dynamic equations for $\mathcal{S}_{\mathrm{st}}$ do not exist. In this case the dynamic equations for $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ describe some mean motion of $\mathcal{S}_{\mathrm{st}}$ (regular component of the stochastic particle $\mathcal{S}_{\mathrm{st}}$ motion). A simple example of such a situation is given by the gas dynamic equations, which describe a mean motion of the gas molecules. Their exact motion is stochastic. It cannot be described exactly. Thus, a change of the basic object of the particle dynamics changes the conception of particle dynamics. The procedure of the basic object replacement (logical reloading [6]) changes mathematical formalism of the particle dynamics: dynamics of a single particle transforms to the dynamics of continuous media. But in both cases the particle dynamics remains to be a classical dynamics.

Note that the quantum mechanics is essentially dynamics of continuous media, but this conception is restricted by a set of constraints (quantum principles, linearity of dynamic equations, etc.), which are absent in the dynamics, based on the statistical ensemble as a basic object of dynamics.

The quantum mechanics may be founded as a classical dynamics of stochastic particles [6, 7, 8]. Classical dynamics of stochastic particles appears, when the main object of dynamics is changed. The main object of dynamics becomes the statistical ensemble $\mathcal{E}[\mathcal{S}]$ of particles S (instead of a single particle $\mathcal{S}$). The statistical ensemble $\mathcal{E}[\mathcal{S}]$ is a dynamic system independently of whether or not particles S are deterministic particles. If the number N of particles $N\rightarrow\infty$, the statistical ensemble $\mathcal{E}[\mathcal{S}]$ turns to fluidlike deterministic dynamic system. The statistical ensemble $\mathcal{E}[\mathcal{S}]$ is a deterministic system, even if the systems $\mathcal{S}$ constituting the statistical ensemble are stochastic. Thus, being a continuous medium, the statistical ensemble can be described in terms of the wave function, because the wave function is a way of description of ideal continuous medium [9]. If the internal energy of the statistical ensemble has the form $E=\frac{m\mathbf{v}_{\mathrm{dif}}^{2}}{2}\rho=\frac{\hbar^{2}(\nabla\rho)^{2}}{8m\rho}$, where ρ is the ensemble fluid density and $\mathbf{v}_{\mathrm{dif}}=\frac{\hbar}{2m}\boldsymbol{\nabla}\log\rho$ is the mean velocity of a particle, the dynamic equations for this fluid in terms of the wave function coincide with the Schrödinger equation (in the case of nonrotational flow).

In the case of the relativistic stochastic particle the action for the statistical ensemble of emlons has the form

$$\mathcal{E}[\mathcal{S}]:\quad \mathcal{A}[x,\kappa]=\int_{V_{\xi}}\left(-Kmc\sqrt{g_{lk}\dot{x}^{l}\dot{x}^{k}}-\frac{e}{c}A_{l}\dot{x}^{l}\right)d^{4}\xi,\quad \dot{x}^{i}=\frac{\partial x^{i}}{\partial\xi_{0}} \tag{1.1}$$

$$K=\sqrt{1+\lambda^{2}\left(\kappa_{l}\kappa^{l}+\partial_{l}\kappa^{l}\right)},\quad \lambda=\frac{\hbar}{mc},\quad \partial_{l}\equiv\frac{\partial}{\partial x^{l}} \tag{1.2}$$

Here $\xi=\{\xi_{0},\xi_{1},\xi_{2},\xi_{3}\}$ are independent variables, and $x=x(\xi)$, $\kappa=\kappa(x)$ $x=\{x^{0}(\xi),x^{1}(\xi),x^{2}(\xi),x^{3}(\xi)\}$, $\kappa=\{\kappa^{0}(x),\kappa^{1}(x),\kappa^{2}(x),\kappa^{3}(x)\}$ are dependent variables. The quantity A_l is the 4-potential of the electromagnetic field. Dynamic equations are obtained as a result of varying with respect to x and κ. After a proper change of variables the dynamic equations are reduced to [6]

$$\begin{aligned}&\left(-i\hbar\partial_k+\frac{e}{c}A_k\right)\left(-i\hbar\partial^k+\frac{e}{c}A^k\right)\psi-m^2c^2\psi\\ =\ &\frac{\hbar^2}{2\rho}\partial_l\left(\partial^l s_\alpha\rho\right)s_\alpha\psi-\frac{\hbar^2}{4}\partial_l\partial^l s_\alpha s_\alpha\psi+\frac{\hbar^2}{2\rho}\partial_l\left(\partial^l s_\alpha\rho\right)\sigma_\alpha\psi\end{aligned}\tag{1.3}$$

where ρ and 3-vector $\mathbf{s}=\{s_1,s_2,s_3,\}$ are defined by the relations

$$\rho=\psi^*\psi,\qquad s_\alpha=\frac{\psi^*\sigma_\alpha\psi}{\rho},\qquad \alpha=1,2,3\tag{1.4}$$

$$\psi=\binom{\psi_1}{\psi_2},\qquad \psi^*=\left(\psi_1^*,\psi_2^*\right),\tag{1.5}$$

Here σ_α, $\alpha=1,2,3$ are the Pauli matrices.

In the special case, when the flow is nonrotational and the wave function ψ is one-component $\psi=\binom{\psi_1}{\psi_1}$, s_α =const, $\partial_l s_\alpha=0$, and equation (1.3) turns to the Klein-Gordon equation

$$\left(-i\hbar\partial_k+\frac{e}{c}A_k\right)\left(-i\hbar\partial^k+\frac{e}{c}A^k\right)\psi-m^2c^2\psi=0\tag{1.6}$$

Thus, the action (1.1), (1.2) describes the mean motion of a relativistic quantum particle (relativistic classical stochastic particle). In this special case the wave function ψ can be presented in the form

$$\psi=\sqrt{\rho_0}\exp\left(\kappa+i\varphi\right),\quad \rho_0=\text{const}\tag{1.7}$$

where κ is a potential of the κ-field κ_l

$$\kappa_l=\partial_\lambda\kappa\tag{1.8}$$

Existence of potential κ follows from dynamic equations, derived by variation of the action (1.1) with respect to κ^l. The variable φ is a potential of the momentum p_l

$$p_l=\frac{\partial}{\partial\dot{x}^l}\left(-Kmc\sqrt{g_{lk}\dot{x}^l\dot{x}^k}-\frac{e}{c}A_l\dot{x}^l\right)=-Kmc\frac{g_{ls}\dot{x}^s}{\sqrt{g_{ik}\dot{x}^i\dot{x}^k}}-\frac{e}{c}A_l\tag{1.9}$$

considered as a function of coordinates x.

The classical force field κ_l, $l=0,1,2,3$ is an internal field of any particle. It changes the particle mass m, replacing the usual particle mass m by an effective mass M

$$M=\sqrt{m^2+\frac{\hbar^2}{c^2}\left(\kappa_l\kappa^l+\partial_l\kappa^l\right)},\tag{1.10}$$

Using relation (1.8), this relation can be presented in the form

$$M=Km,\quad K=\sqrt{1+\lambda^2e^{-\kappa}\partial_l\partial^l e^\kappa},\quad \lambda^2=\frac{\hbar^2}{m^2c^2}\tag{1.11}$$

The κ-field κ_l is responsible for quantum effects. The same κ-field is responsible for pair production, because the κ-field can return the world line in the time direction. In this sense one may say, that pair production is possible only in the framework of quantum theory.

From formal viewpoint the action (1.1), (1.2) describes a charged fluid, whose particles interact via self-consistent force field κ. Dynamics of this fluid is classical. However, if this fluid is described in terms of the wave function and the κ-field is included into the wave function, one obtains a quantum description, which does not contain the κ-field explicitly.

Connection between the Schrödinger equation and the fluid dynamics is known long ago [10]. However, this connection was one-way in the sense, that one could obtain the equation for the fluid from the Schrödinger equation, whereas one could not obtain the Schrödinger equation from the equation for a fluid. Now, when it is known that the wave function is a way of the fluid description [9], one can deduce the Schrödinger equation as equation of the fluid dynamics.

Representation of quantum mechanics as a special case of the fluid dynamics (or as a classical dynamics of stochastic relativistic particles (CDSRP)) is useful in the relation, that it is a more general conception, which does not need a use of quantum principles. There are no necessity to unite the quantum principles with the principles of relativity theory. It is sufficient to use relativistic lagrangian for description of the fluid. Besides, in the framework of CDSRP one describes the pair production phenomenon.

Being an axiomatic conception, the conventional quantum theory describes only part of the quantum phenomena properties. This part concerns mainly nonrelativistic quantum phenomena. Relativistic quantum phenomena appear outside the scope of quantum principles. Theorists speak about necessity of uniting of the quantum principles and the relativity principles. The quantum field theory must solve this problem. Unfortunately, the quantum field theory cannot solve properly the problem of pair production, which is a special problem of the relativistic quantum theory. Mechanism of pair production is unclear in the quantum field theory. Some theorists believe that in the framework of second quantization some nonlinear terms added to the Klein-Gordon equation could explain the pair production phenomenon. Unfortunately, this approach is inconsistent, because it uses identification of energy with Hamiltonian, that is inconsistent in the case, when pair production is possible [11].

In reality in the case of return in time of the world line the energy E_{p} of a particle and energy E_{a} of antiparticle are positive always, whereas the temporal component $p_0 = H$ of the canonical momentum has different sign for particle and antiparticle [12]. At the second quantization one identifies energy E with Hamiltonian $H = p_0$. Such identification is inconsistent. It leads to nonstationarity of the vacuum state, which evidences on inconsistency of the approach.

What is profit from the classical dynamics of stochastic (quantum) particles? Conventional quantum mechanics is a axiomatic conception of quantum phenomena, whereas the classical dynamics of stochastic particles is a model conception of quantum phenomena. In application to theory of atomic phenomena both conception give the same result. However, in application to the elementary particle theory the results are different. The axiomatic conception leads to empirical approach, where any elementary particle is considered as a pointlike object, which is provided by some set of quantum numbers. An arrangement of a particle is not considered. The model conception leads to the structural approach, where an elementary particle is a complicated object, having an inner structure. In particular, the κ-field is an element of this structure.

Taking into account the classical force field κ_l, the quantum mechanics can be considered as the classical mechanics of stochastic particles. It is reasonable to think that the pair production phenomenon can be described in the framework of classical dynamics. Some sides of this problem were considered in the papers [13, 14]. In this paper we shall consider (1) mechanism of pair production in the external κ-field and (2) generation of the κ-field at a collision of relativistic particles.

2 Pair production in the κ-field

Let us consider motion of a particle in a given κ-field, which is presented by a factor K in the action (1.1). The action, describing world line of the particle, has the form

$$\mathcal{A}[x] = -\int c\sqrt{M^2 g_{lk}\dot{x}^l\dot{x}^k}d\tau, \quad \dot{x}^k \equiv \frac{dx^k}{d\tau} \tag{2.1}$$

where $x = x(\tau)$ and the effective mass $M = Km = M(x)$ is a given function of x. Metric tensor has the form $g_{ik} = \mathrm{diag}\,(c^2, -1, -1, -1)$. World line can be placed only in the region, where expression under radical $M^2 g_{lk}\dot{x}^l\dot{x}^k > 0$. The particle momentum has the form

$$p_i = -Mc\frac{g_{ik}\dot{x}^k}{\sqrt{g_{lk}\dot{x}^l\dot{x}^k}} \qquad i = 0, 1, 2, 3 \tag{2.2}$$

$$\mathbf{p} = M\frac{\frac{d\mathbf{x}}{dt}}{\sqrt{1 - c^{-2}\left(\frac{d\mathbf{x}}{dt}\right)^2}}, \qquad p_0 = -\frac{Mc^2}{\sqrt{1 - c^{-2}\left(\frac{d\mathbf{x}}{dt}\right)^2}} \tag{2.3}$$

The Hamilton-Jacobi equation has the form

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \left(\frac{\partial S}{\partial \mathbf{x}}\right)^2 = K^2 m^2 c^2 = M^2 c^2 \tag{2.4}$$

We consider the simplest case, when $K = K(t)$. Then the full integral of the Hamilton-Jacobi equation (2.4) has the form

$$S - S_0 = \int mc^2\sqrt{K^2(t) + \frac{\mathbf{p}^2}{m^2c^2}}dt + p_\beta x^\beta, \quad \mathbf{p} = \{p_1, p_2, p_3\} = \mathrm{const} \tag{2.5}$$

World line of the particle has the form

$$\frac{\partial S}{\partial p_\beta} = x_0^\beta, \quad x_0^\beta = \text{const}, \ \beta = 1, 2, 3 \tag{2.6}$$

Substituting (2.5) in (2.6) and setting $p_2 = p_3 = 0$, one obtains

$$\int \frac{p_1}{m\sqrt{K^2(t) + \frac{p_1^2}{m^2c^2}}} dt + x^1 = x_0^1 \tag{2.7}$$

$$x^2 = x_0^2, \quad x^3 = x_0^3 \tag{2.8}$$

Integral in (2.7) is produced only over the region, where expression under radical is positive. For instance, let K have the form

$$K^2 = \begin{cases} 1 + \frac{t}{t_0} & \text{if} \quad t < 0 \\ 1 & \text{if} \quad t > 0 \end{cases}$$

$$S - S_0 = \begin{cases} S_1(t) & \text{if} \quad t < 0 \\ S_2(t) & \text{if} \quad t > 0 \end{cases} \tag{2.9}$$

Equation (2.7) has the form

$$\frac{\partial S_1}{\partial p_1} = \int_{-t_0\gamma^2}^{t} \frac{p_1}{m\sqrt{\frac{t}{t_0} + \gamma^2}} dt + x^1 = x_0^1, \quad t < t_0, \tag{2.10}$$

$$\frac{\partial S_2}{\partial p_1} = \pm \int_0^t \frac{p_1}{m\gamma} dt + x^1 = y_0, \quad t > t_0 \tag{2.11}$$

where γ is the Lorentz-factor

$$\gamma = \sqrt{\frac{m^2c^2 + p_1^2}{m^2c^2}} = \frac{E}{mc^2} \geq 1 \tag{2.12}$$

Here $E = \sqrt{m^2c^4 + p_1^2c^2}$ is the particle energy. Setting $x^1 = x$, calculation gives for (2.10) and (2.11)

$$2t_0 \frac{p_1}{m} \sqrt{\frac{t}{t_0} + \gamma^2} = -(x - x_0), \quad t < t_0 \tag{2.13}$$

$$\pm \frac{p_1}{m\gamma} t = -(x - y_0), \quad t > t_0 \tag{2.14}$$

where x_0 and y_0 are arbitrary constants, which are connected by the condition of the world line continuity. One obtains two branches of the solution

$$x_- = \begin{cases} x_0 - 2t_0 \frac{p_1}{m} \sqrt{\frac{t}{t_0} + \gamma^2} & \text{if} \quad t < 0 \\ x_0 - 2t_0 \frac{p_1 \gamma}{m} - \frac{p_1}{m\gamma} t & \text{if} \quad t > 0 \end{cases} \tag{2.15}$$

$$x_+ = \begin{cases} x_0 + 2t_0\frac{p_1}{m}\sqrt{\frac{t}{t_0}+\gamma^2} & \text{if} \quad t<0 \\ x_0 + 2t_0\frac{p_1\gamma}{m} + \frac{p_1}{m\gamma}t & \text{if} \quad t>0 \end{cases} \tag{2.16}$$

The two branches coincide at the point with coordinates $t = -t_0\gamma^2$, $x = x_0$. At this point

$$K^2 - 1 + \gamma^2 = 0 \tag{2.17}$$

As a result two branches form a continuous world line, which reflects from the tachyon region, where $K^2 \leq 0$. Only external κ-field of a world line generates the pair production. Internal κ-field generates only a wobbling of the world line and quantum effects connected with this wobbling. The world line penetrates in the tachyon region, where $K^2 \leq 0$. At $-t_0 < t$ the particle velocity

$$\left|\frac{dx_\pm}{dt}\right| = \left|\frac{p_1}{m}\frac{1}{\sqrt{\frac{t}{t_0}+\gamma^2}}\right| < c, \quad \text{if } t > -t_0 \tag{2.18}$$

However $\left|\frac{dx}{dt}\right| > c$, if $-t_0\gamma^2 < t < -t_0$, and $K^2 < 0$. In this region the world line is spacelike. We shall refer to the space-time region, where $K^2 < 0$ and world line is spacelike, as the tachyon region. The world line is reflected from the tachyon region, although it may penetrate in the tachyon region.

Energy of the produced pair is taken from the energy of the κ-field. This statement follows from the expression for the energy-momentum tensor T^{ik} for a system of N identical particles [6]

$$\begin{aligned} T^{ik} \quad &= \quad \sum_{A=1}^{A=N} \frac{\hbar^2}{c} \frac{\sqrt{\dot{x}^s_{(A)}\dot{x}_{(A)s}}\left(\kappa^i\left(x_A\right)\kappa^k\left(x_A\right) + \frac{1}{2}\frac{\partial}{\partial x_{(A)i}}\kappa^k\left(x_A\right) + \frac{1}{2}\frac{\partial}{\partial x_{(A)k}}\kappa^i\left(x_A\right)\right)}{M_{(A)}\left(x_{(A)}\right)} \\ &\quad + \sum_{A=1}^{A=N} M_{(A)}\left(x_{(A)}\right) c \frac{\dot{x}^i_{(A)}\dot{x}^k_{(A)}}{\sqrt{\dot{x}^s_{(A)}\dot{x}_{(A)s}}} \end{aligned} \tag{2.19}$$

Here $x_{(A)}$ are coordinates of the Ath particle and $M_{(A)} = K_{(A)}m$ is its effective mass. The second term in (2.19) describes energy-momentum of particles, whereas the first term describes the energy-momentum of the κ-field. It follows from (2.19) that the κ-field energy is maximal at the points, where the effective mass $M_{(A)}$ is minimal, i.e. in vicinity of the boundary of the tachyon region, whereas the energy of particles at these points is minimal.

3 Generation of κ-field at the particle collisions

We have shown that the external κ-field can produce pair production. One needs also to show that the κ-field can be generated at the collision of relativistic particles. One should expect from experimental data that the pair production takes place at the head-on collision of two charged relativistic particles. We consider the case, when the relativistic particle reflects from the wall generated by an electromagnetic potential.

Let the wave function of two particle beams has the form

$$\psi = \psi_1 + \psi_2 \tag{3.1}$$

$$\psi_1 = A \int e^{-\frac{\Delta^2 (k-p)^2}{\hbar^2}} \exp\left(\frac{ik}{\hbar}x - \frac{ik_0}{\hbar}t\right) dk \tag{3.2}$$

$$\psi_2 = A \int e^{-\frac{\Delta^2 (k+p)^2}{\hbar^2}} \exp\left(\frac{ik}{\hbar}x - \frac{ik_0}{\hbar}t\right) dk \tag{3.3}$$

Here Δ is a characteristic size of the wave packets. The speed of the light is taken $c = 1$ We consider the ultrarelativistic case, when $k_0 \simeq |\mathbf{k}|$. In this case calculation of ψ_1 and ψ_2 gives

$$\psi_1 = \frac{A\hbar}{\sqrt{\pi}\Delta} \exp\left(\frac{ip}{2\hbar}x - \frac{ip}{2\hbar}t\right) \exp\left(-\frac{1}{4\Delta^2}(x-t)^2\right) \tag{3.4}$$

$$\psi_2 = \frac{A\hbar}{\sqrt{\pi}\Delta} \exp\left(-\frac{ip}{2\hbar}x - \frac{ip}{2\hbar}t\right) \exp\left(-\frac{1}{4\Delta^2}(x+t)^2\right) \tag{3.5}$$

Both wave packets have the effective width Δ in the vicinity of the collision point $t = 0, x = 0$.

Then the wave function (3.1) has the form

$$\psi_1 + \psi_2 = \frac{A\hbar}{\sqrt{\pi}\Delta} \exp\left(-\frac{ip}{2\hbar}t\right) \exp\left(-\frac{1}{4\Delta^2}\left(x^2+t^2\right)\right) \left(\begin{array}{c} \exp\left(\frac{ip}{2\hbar}x + \frac{xt}{2\Delta^2}\right) \\ +\exp\left(-\frac{ip}{2\hbar}x - \frac{xt}{2\Delta^2}\right) \end{array} \right) \tag{3.6}$$

$$\rho = |\psi_1 + \psi_2|^2 = 4\left(\frac{A\hbar}{\sqrt{\pi}\Delta}\right)^2 \exp\left(-\frac{1}{2\Delta^2}\left(x^2+t^2\right)\right) \cos\left(\frac{px}{2\hbar} - \frac{ixt}{2\Delta^2}\right) \cos\left(\frac{px}{2\hbar} + \frac{ixt}{2\Delta^2}\right) \tag{3.7}$$

$$\begin{aligned} \sqrt{\rho} &= 2\left(\frac{A\hbar}{\sqrt{\pi}\Delta}\right) \exp\left(-\frac{1}{4\Delta^2}\left(x^2+t^2\right)\right) \frac{1}{\sqrt{2}} \left(\cosh\frac{xt}{\Delta^2} + \cos\frac{px}{\hbar}\right)^{1/2} \\ &= 2\left(\frac{A\hbar}{\sqrt{\pi}\Delta}\right) \exp\left(-\frac{1}{4\Delta^2}\left(x^2+t^2\right)\right) \frac{1}{\sqrt{2}} \sqrt{\cosh\frac{xt}{\Delta^2} + \cos\frac{px}{\hbar}} \end{aligned} \tag{3.8}$$

One obtains for K^2

$$K^2 = 1 + \frac{\hbar^2}{m^2}\left(\partial_l \log\sqrt{\rho}\,\partial^l \log\sqrt{\rho} + \partial_i \partial^i \log\sqrt{\rho}\right) \tag{3.9}$$

$$\partial_0 \log\sqrt{\rho} = -\frac{t}{2\Delta^2} + \frac{\frac{x}{\Delta^2}\sinh\frac{xt}{\Delta^2}}{2\left(\cosh\frac{xt}{\Delta^2} + \cos\frac{px}{\hbar}\right)} \tag{3.10}$$

$$\partial_x \log\sqrt{\rho} = -\frac{x}{2\Delta^2} + \frac{\frac{t}{\Delta^2}\sinh\frac{xt}{\Delta^2} - \frac{p}{\hbar}\sin\frac{px}{\hbar}}{2\left(\cosh\frac{xt}{\Delta^2} + \cos\frac{px}{\hbar}\right)} \tag{3.11}$$

$$\partial_0\partial_0 \log\sqrt{\rho} = -\frac{1}{2\Delta^2} - \frac{\left(\frac{x}{\Delta^2}\sinh\frac{xt}{\Delta^2}\right)^2}{2\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)^2} + \frac{\left(\frac{x}{\Delta^2}\right)^2\sinh\frac{xt}{\Delta^2}\cosh\frac{xt}{\Delta^2}}{2\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)} \tag{3.12}$$

$$\begin{aligned}\partial_x\partial_x \log\sqrt{\rho} &= -\frac{1}{2\Delta^2} - \frac{\left(\frac{t}{\Delta^2}\sinh\frac{xt}{\Delta^2}-\frac{p}{\hbar}\sin\frac{px}{\hbar}\right)^2}{2\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)^2} \\ &\quad + \frac{\left(\frac{t}{\Delta^2}\right)^2\sinh\frac{xt}{\Delta^2}\cosh\frac{xt}{\Delta^2}-\left(\frac{p}{\hbar}\right)^2\sin\frac{px}{\hbar}\cos\frac{px}{\hbar}}{2\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)}\end{aligned} \tag{3.13}$$

$$\begin{aligned}\partial_l\partial^l \log\sqrt{\rho} &= -\frac{\left(\frac{x}{\Delta^2}\sinh\frac{xt}{\Delta^2}\right)^2-\left(\frac{t}{\Delta^2}\sinh\frac{xt}{\Delta^2}-\frac{p}{\hbar}\sin\frac{px}{\hbar}\right)^2}{2\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)^2} \\ &\quad + \frac{\left(\frac{x}{\Delta^2}\right)^2\sinh\frac{2xt}{\Delta^2}-\left(\frac{t}{\Delta^2}\right)^2\sinh\frac{2xt}{\Delta^2}+\left(\frac{p}{\hbar}\right)^2\sin\frac{2px}{\hbar}}{4\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)}\end{aligned}$$

$$\begin{aligned}\partial_l\partial^l \log\rho &= -\frac{\left(\frac{x}{\Delta^2}\sinh\frac{xt}{\Delta^2}\right)^2-\left(\frac{t}{\Delta^2}\sinh\frac{xt}{\Delta^2}-\frac{p}{\hbar}\sin\frac{px}{\hbar}\right)^2}{2\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)^2} \\ &\quad + \frac{\frac{x^2-t^2}{\Delta^4}\sinh\frac{2xt}{\Delta^2}+\left(\frac{p}{\hbar}\right)^2\sin\frac{2px}{\hbar}}{4\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)}\end{aligned} \tag{3.14}$$

$$\left(\partial_0\log\sqrt{\rho}\right)^2 = \frac{t^2}{4\Delta^4} - \frac{t}{2\Delta^2}\frac{\frac{x}{\Delta^2}\sinh\frac{xt}{\Delta^2}}{\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)} + \frac{\frac{x^2}{\Delta^4}\sinh^2\frac{xt}{\Delta^2}}{4\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)^2} \tag{3.15}$$

$$\begin{aligned}\left(\partial_x\log\sqrt{\rho}\right)^2 &= \frac{x^2}{4\Delta^4} - \frac{x}{2\Delta^2}\frac{\frac{t}{\Delta^2}\sinh\frac{xt}{\Delta^2}-\frac{p}{\hbar}\sin\frac{px}{\hbar}}{\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)} \\ &\quad + \frac{\left(\frac{t}{\Delta^2}\sinh\frac{xt}{\Delta^2}-\frac{p}{\hbar}\sin\frac{px}{\hbar}\right)^2}{4\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)^2}\end{aligned} \tag{3.16}$$

$$\begin{aligned}\partial_l\log\sqrt{\rho}\,\partial^l\log\sqrt{\rho} &= \frac{t^2-x^2}{4\Delta^4} - \frac{\frac{x}{2\Delta^2}\frac{p}{\hbar}\sin\frac{px}{\hbar}}{\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)} \\ &\quad + \frac{\frac{x^2}{\Delta^4}\sinh^2\frac{xt}{\Delta^2}-\left(\frac{t}{\Delta^2}\sinh\frac{xt}{\Delta^2}-\frac{p}{\hbar}\sin\frac{px}{\hbar}\right)^2}{4\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)^2}\end{aligned} \tag{3.17}$$

$$\begin{aligned}\partial_l\partial^l \log\sqrt{\rho} &= -\frac{\left(\frac{x}{\Delta^2}\sinh\frac{xt}{\Delta^2}\right)^2-\left(\frac{t}{\Delta^2}\sinh\frac{xt}{\Delta^2}-\frac{p}{\hbar}\sin\frac{px}{\hbar}\right)^2}{2\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)^2} \\ &\quad + \frac{\frac{x^2-t^2}{\Delta^4}\sinh\frac{2xt}{\Delta^2}+\left(\frac{p}{\hbar}\right)^2\sin\frac{2px}{\hbar}}{4\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)}\end{aligned} \tag{3.18}$$

$$K^2 = 1 + \frac{\hbar^2}{m^2}\left(\begin{array}{c} \frac{t^2-x^2}{4\Delta^4} - \frac{\frac{x}{2\Delta^2}\frac{p}{\hbar}\sin\frac{px}{\hbar}}{\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)} + \frac{\frac{x^2}{\Delta^4}\sinh^2\frac{xt}{\Delta^2}-\left(\frac{t}{\Delta^2}\sinh\frac{xt}{\Delta^2}-\frac{p}{\hbar}\sin\frac{px}{\hbar}\right)^2}{4\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)^2} \\ -\frac{\left(\frac{x}{\Delta^2}\sinh\frac{xt}{\Delta^2}\right)^2-\left(\frac{t}{\Delta^2}\sinh\frac{xt}{\Delta^2}-\frac{p}{\hbar}\sin\frac{px}{\hbar}\right)^2}{2\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)^2} + \frac{\frac{x^2-t^2}{\Delta^4}\sinh\frac{2xt}{\Delta^2}+\left(\frac{p}{\hbar}\right)^2\sin\frac{2px}{\hbar}}{4\left(\cosh\frac{xt}{\Delta^2}+\cos\frac{px}{\hbar}\right)} \end{array}\right)$$

Let consider the region, where two beams overlapped, i.e. where $x, t \ll \Delta$. one obtains

$$K^2 = 1 + \frac{\hbar^2}{m^2}\left(\frac{-\left(-\frac{p}{\hbar}\sin\frac{px}{\hbar}\right)^2}{4\left(1+\cos\frac{px}{\hbar}\right)^2} - \frac{-\left(-\frac{p}{\hbar}\sin\frac{px}{\hbar}\right)^2}{2\left(1+\cos\frac{px}{\hbar}\right)^2} + \frac{+\left(\frac{p}{\hbar}\right)^2\sin\frac{2px}{\hbar}}{4\left(1+\cos\frac{px}{\hbar}\right)}\right)$$

$$K^2 = 1 + \frac{\hbar^2}{m^2}\left(\frac{\left(\frac{p}{\hbar}\sin\frac{px}{\hbar}\right)^2}{4\left(1+\cos\frac{px}{\hbar}\right)^2} + \frac{+\left(\frac{p}{\hbar}\right)^2\sin\frac{2px}{\hbar}}{4\left(1+\cos\frac{px}{\hbar}\right)}\right) \tag{3.19}$$

or

$$K^2 = 1 + \frac{\left(\frac{p}{m}\right)^2\left(\sin^2\frac{px}{\hbar}+\sin\frac{2px}{\hbar}\left(1+\cos\frac{px}{\hbar}\right)\right)}{4\left(1+\cos\frac{px}{\hbar}\right)^2} \tag{3.20}$$

The second term of (3.20) is negative, when $-\pi/2 < 2px/\hbar < 0$. As far as $p/m \gg 1$, $K^2 < 0$ and $|K^2| \gg 1$. It means that in the region of overlapping of two beams a tachyon region arises. Such a region is necessary for explanation of the pair production. It means that pair production can be explained in the framework of the classical dynamics of relativistic stochastic particles. However, one failed to explain it in the framework of quantum field theory.

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