Necessity of the general relativity revision and free motion of particles in non-Riemannian space-time geometry

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Abstract

It is shown, that a free motion of microparticles (elementary particles) in the gravitational field is multivariant (stochastic). This multivariance is conditioned by multivariant physical space-time geometry. The physical geometry is described completely by a world function. The Riemannian geometries form a small part of possible physical geometries. The contemporary theory of gravitation ignores existence of physical geometries. It supposes, that any space-time geometry is a Riemannian geometry. It is a mistake. As a result the contemporary theory of gravitation needs a revision. Besides, the Riemannian geometry is inconsistent, and conclusions of the gravitational theory, based on inconsistent geometry may be invalid. Free motion of macroparticles (planets), consisting of many connected microparticles, is deterministic, because connection of microparticles inside the macroparticle averages stochastic motion of single microparticles.

Key words: non-Riemannian geometry; stochastic motion of microparticles

1 Introduction

The physical geometry describes the space-time in terms of a finite space-time distance ρ (or in terms of world function $\sigma = \frac{1}{2}\rho^2$). This description in terms of distance ρ and only in terms of ρ is a complete description. The Riemannian geometry tries to describe the space-time in terms of infinitesimal space-time distance $d\rho$, which is determined by the metric tensor g_{ik}, given at any point of the space-time.

$$d\rho^2 = g_{ik}(x)\, dx^i dx^k \tag{1.1}$$

The distance $\rho = \rho(x, x')$ is a function of two space-time points x and x'. The distance ρ as a function of two points contains much more information, than ten functions g_{ik}, which are functions of one point. To obtain the finite distance ρ from the infinitesimal distance $d\rho$, a set of additional conditions (and additional information) is to be fulfilled. In particular, the obtained *finite distance ρ is to be a single-valued function of any two points of the space-time.* Construction of the space-time geometry by methods of the Riemannian geometry construction leads, in general, to a many-valued finite distance ρ. This fact is a nonsense. Besides, the Riemannian geometry in itself appeared to be inconsistent [1, 2]. However, in the twentieth century nobody paid attention to this fact, because there were no alternative to the Riemannian geometry.

When this alternative appeared [3], the problem of the general relativity revision arose. Let us stress, that it was just the problem of revision, but not a problem of a new gravitational theory construction, because the main idea of the general relativity (geometrization of physics) remains changeless.

The problem of the general relativity revision contains two essential points

1. A use of more effective and general mathematical method: the physical geometry, described completely by the finite space-time distance and only by it.

2. Description of the relativity theory in terms, which are adequate to this theory.

In fact, when the relativity theory came in the stead of the nonrelativistic physics, some terms and concepts of the nonrelativistic physics remained in the relativity theory. These concepts do not prevent one to solve concrete physical problems. However, they prevent from development of the relativity theory. In particular, nonrelativistic concept of two events nearness prevents from the general relativity generalization [4].

The present paper is written in the framework of the physics geometrization. The program of the physics geometrization is a generalization of the relativity theory on the case of non–Riemannian space-time geometry. The general relativity (theory of gravitation) is created at the supposition, that the space-time geometry is a Riemannian geometry. In the twentieth century the Riemannian geometry was considered as the most general kind of geometry, which is available for the space-time description. However, it appears, that there exist non-Riemannian geometries, which are more general in the sense, that the set of Riemannian geometries is only a negligible part of the set of non-Riemannian (physical) geometries. Physical geometries are described completely by a finite space-time interval, but not by infinitesimal space-time interval as Riemannian geometries. Physical geometry is a constructive (nonaxiomatizable) geometry, which cannot be deduced from axiomatics.

It has been shown [4], that a heavy sphere creates a deformation of the space-time in such a way, that the space-time geometry ceases to be a Riemannian geometry. The fact is that, the metric tensor, given in the whole space-time, determines the space-time geometry only under condition that the space-time geometry is Riemannian. In the twentieth century the Riemannian geometry was considered as a most general space-time geometry. The condition, that the space-time geometry is a Riemannian geometry, seemed to be very reasonable.

The Riemannian geometry is a mathematical geometry in the sense, that it is a logical construction, and all propositions of mathematical geometry are deduced from a system of axioms. The main property of a mathematical geometry is the fact, that any mathematical geometry is a logical construction. It is a secondary circumstance, whether or not the mathematical geometry describes mutual disposition of geometrical objects. There are such mathematical geometries (for instance, symplectic geometry), which do not describe a disposition of geometrical objects.

On the contrary, any physical geometry is defined as a science on mutual disposition of geometrical objects in the space or in the space-time. It is a secondary circumstance, whether or not the physical geometry is a logical construction. The physical geometry is described completely by means of a distance $\rho(P,Q)$ between any two points P and Q of the space, or of the space-time. In this sense the physical geometry is a distance geometry [5]. Physical geometry distinguishes from the distance (metric) geometry in the sense, that the distance geometry is not completely a metric geometry. For instance, in the distance geometry the concept of a curve is formulated not only in terms of a distance, whereas in physical geometry all geometrical concepts are formulated in terms of a distance ρ and only in terms of distance. In fact the physical geometry is formulated in terms of the world function $\sigma(P,Q) = \frac{1}{2}\rho^2(P,Q)$ [6]. A use of world function is more effective from technical viewpoint. (For instance, the scalar product $(\mathbf{PQ}.\mathbf{RS})$ of vectors $\mathbf{PQ}$ and $\mathbf{RS}$ is a linear function of world functions of points P,Q,R,S, whereas in terms of the distance ρ the scalar product is a more complicated expression.

In reality there exist nonaxiomaitzable space-time geometries, which cannot be deduced from a system of axioms. Riemannian geometries form a small subset of all possible physical space-time geometries, which are nonaxiomatizable, in general. Different nonaxiomatizable space-time geometries may have the same metric tensor. As a result metric tensor does not determine the space-time geometry uniquely.

Besides, the Riemannian geometry is inconsistent, in general. The Riemannian geometry is considered as a kind of a mathematical geometry, i.e. it can be deduced from a system of axioms. These axioms are inconsistent at some points. As a result the Riemannian have at least three well known defects. First, a parallel transport of a vector from the point P_0 to the point P_1 depends on the path of the transport. It means that there is no absolute parallelism in the Riemannian geometry. Second, from physical viewpoint the main characteristic of the space-time geometry is a distance $\rho(P,Q)$ between two space-time points P and Q. This distance must be single-valued. However, in the Riemannian geometry the distance $\rho(P,Q)$ is defined as the length of the geodesic, connecting points P and Q. In the Riemannian geometry there are such points P and Q , which my be connected by several geodesics of different length. It leads to multiformity of the distance $\rho(P,Q)$.

Third, the Euclidean geometry is a partial case of a Riemannian geometry. Let us construct Euclidean geometry on a two-dimensional plane $\mathcal{P}_2$ by means of the method of the Riemannian geometry construction, i.e. we define the distance $\rho(P,Q)$ as a length of a geodesic (the shortest line, connecting points P and Q). As far as in the case of Euclidean geometry the geodesics are straight lines, one obtains in the Cartesian coordinate system

$$\rho(P,Q) = \rho(\mathbf{x},\mathbf{x}') = \sqrt{(\mathbf{x}-\mathbf{x}')^2} \tag{1.2}$$

Let us cut a hole in the two-dimensional plane $\mathcal{P}_2$.The geodesics, passing through the hole, become to be impossible, and the shortest lines pass around the hole. Their lengths change, and the formula (1.2) ceases to be valid for some points. As a result the plane $\mathcal{P}_2$ with a hole cannot be embedded isometrically into the same plane $\mathcal{P}_2$ without the hole.

This result is a paradox, because, if experimentally one cuts a hole in a flat piece of tin-plate, the obtained piece with a hole can be embedded isometrically in the original piece of tin-plate. Mathematicians know this paradoxical result, which means, that the conventional method of the Riemannian geometry construction is inconsistent. However, they have no alternative to the conventional method of the Riemannian geometry construction. As a result they prefer to consider Riemannian geometries on convex sets of points.

Of course, such a restriction by the convex point sets does not solve the problem of the Riemannian geometry inconsistency. This problem may be solved only by a change of the geometry construction method.

Deduction of the geometrical propositions from a system of basic axioms is a very laborious process. One needs to prove numerous theorems. Besides, one should be sure that the basic axioms are compatible between themselves. A test of this compatibility is a very laborious process. For any new space-time geometry one needs to repeat this test of the geometry consistency.

However, the main problem of the mathematical geometries construction is a doubt, that any space-time geometry may be deduced from a finite system of basic axioms. Indeed, any geometry is a continual set of geometrical propositions. It follows from no quarter, that a continual set of geometrical propositions can be deduced from a finite number of basic propositions by means of the formal logic. It is true, that Euclid succeeded to deduce all propositions of the Euclidean geometry from several axioms. However, it does not mean that such a deduction is possible for other geometries. In reality such a deduction is impossible for most of physical geometries, i.e. for geometries, which can be used for the space-time description [1, 2].

For construction of physical geometries one should use the proper Euclidean geometry itself (but not the method of its construction). The proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ is a mathematical geometry and a physical geometry at the same time. It means, that the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ can be deduced from a system of basic axioms, and all its propositions can be expressed in terms of the world function σ_{E} of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. Replacing σ_{E} in all propositions of the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ by the world function σ of other physical geometry $\mathcal{G}$, one obtains all propositions of the physical geometry $\mathcal{G}$.

The procedure of the world function replacement is a change of distances between the points of the space (or space-time). Such a change is a deformation of the

Euclidean geometry. This method of a physical geometry construction is called the deformation principle [3]. The deformation principle is very simple. It does not need a proof of numerous theorems and a test of the geometry consistency. (All theorems has been proved at the construction of the proper Euclidean geometry). A use of the deformation principle for the physical geometry construction does not need an application of the formal logic. The deformation principle admits one to construct nonaxiomatizable geometries. Most of physical geometries are nonaxiomaitzable, and they cannot be constructed by the conventional method (deduction of a geometry from axioms). In particular, the Riemannian geometry (σ-Riemannian one), constructed by means of the deformation principle, has not defects, which are characteristic for the Riemannian geometry, constructed by the conventional method. There is a fern-parallelism in the σ-Riemannian geometry. The world function is single-valued. Cutting a hole in the space-time, one does not change the space-time geometry in the remaining part of the space-time.

Contemporary theory of gravitation as well as the contemporary cosmology are based on the supposition, that the space-time geometry is a Riemannian geometry. However, generalization of the general relativity on the case of a physical space-time geometry shows that the deformation of the space-time geometry of Minkowski generates a non-Riemannian space-time geometry [4]. The obtained generalization admits one to obtain the world function of the space-time geometry directly [4]. In particular, the space-time geometry, generated by a heavy sphere is non-Riemannian.

The world function of a Riemannian geometry satisfies the equation

$$\sigma_{,i}\left(x,x'\right)g^{ik}\left(x\right)\sigma_{,k}\left(x,x'\right)=2\sigma\left(x,x'\right),\qquad \sigma_{,i}\left(x,x'\right)\equiv\frac{\partial}{\partial x^{i}}\sigma\left(x,x'\right)\tag{1.3}$$

The first approximation of the world function of the space-time, generated by the heavy sphere of mass M has the form

$$\sigma\left(t_{1},\mathbf{x}_{1};t_{2},\mathbf{x}_{2}\right)=\frac{1}{2}\left(c^{2}\left(1-\frac{4GM}{c^{2}\left|\mathbf{x}_{2}+\mathbf{x}_{1}\right|}\right)\left(t_{2}-t_{1}\right)^{2}-\left(\mathbf{x}_{2}-\mathbf{x}_{1}\right)^{2}\right)\tag{1.4}$$

where G is the gravitational constant. The world function (1.4) does not satisfy equation (1.3). It describes non-Riemannian space-time geometry, although the metric tensor coincides with the metric tensor of Newtonian approximation

$$g_{00}\left(\mathbf{x}\right)=c^{2}-2\frac{Gm}{\left|\mathbf{x}\right|},\qquad g_{0\alpha}=g_{\alpha\beta}=0,\qquad \alpha,\beta=1,2,3\tag{1.5}$$

Note, that in Riemannian geometry, constructed for the metric tensor (1.5), the world function is many-valued, whereas the function of non-Riemannian geometry is single-valued [4], as it follows from (1.4). This fact tells in behalf of non-Riemannian geometry.

Note, that a use of physical (non-Riemannian) geometry is not a hypothesis, which should be tested by experiments. It is a logical necessity, because the Riemannian geometry is incosistent. It means that contemporary theory of gravitation and cosmology need a revision. We do not state, that such a revision will lead to a change of our cosmological conceptions. However, such a change may take place. For instance, the concept of dark matter, made on the basis of unsatisfactory theory of gravitation may appear to be invalid.

In this paper we try to obtain the law of free particle motion in non-Riemannian space-time geometry. This law has been formulated for microparticles, moving in the arbitrary space-time geometry [7]. This law has been presented in invariant form (in terms of the world function). Now we present it in terms of differential equations (in the conventional coordinate form).

2 Motion of a free microparticle in physical space-time geometry

The state of a simple microparticle is described by its skeleton $\mathcal{P}^1$, which consists of two points P_0, P_1. These two points form a vector $\mathbf{P}_0\mathbf{P}_1 = \{P_0, P_1\}$, which describes the energy-momentum of the microparticle. The length $|\mathbf{P}_0\mathbf{P}_1|$ of vector $\mathbf{P}_0\mathbf{P}_1$ is the geometrical mass μ of the microparticle

$$\mu = |\mathbf{P}_0\mathbf{P}_1| = \sqrt{2\sigma\left(P_0, P_1\right)} \tag{2.1}$$

where σ is the world function of space-time geometry. The geometrical mass μ is connected with the usual mass m of the microparticle by means of relation

$$m = b\mu \tag{2.2}$$

where b is some universal constant. There are complex microparticles, whose skeleton $\mathcal{P}^n$ consists of $n+1$, $n = 1, 2, ..$ space-time points.

The world function of the space-time of Minkowski has the form

$$\sigma_{\mathrm{M}}\left(P, P'\right) = \sigma_{\mathrm{M}}\left(x, x'\right) = \frac{1}{2}g_{ik}\left(x^i - x'^i\right)\left(x^k - x'^k\right) \tag{2.3}$$

$$g_{ik} = \mathrm{diag}\left\{c^2, -1, -1, -1\right\} \tag{2.4}$$

Evolution of the microparticle in the space-time is described by a world chain $\mathcal{T}_{\mathrm{br}}$ of connected links

$$\mathcal{T}_{\mathrm{br}} = \bigcup_s \mathcal{P}_s^1 = \bigcup_s \mathcal{T}_{[s,s+1]} \tag{2.5}$$

where any link $\mathcal{T}_{[s,s+1]}$ is a set of points R, defined by the relation

$$\mathcal{T}_{[s,s+1]} = \left\{R | \sqrt{2\sigma\left(P_s, R\right)} + \sqrt{2\sigma\left(R, P_{s+1}\right)} = \sqrt{2\sigma\left(P_s, P_{s+1}\right)}\right\} \tag{2.6}$$

The links $\mathbf{P}_s\mathbf{P}_{s+1}$ of the world chain have the same length. According to (2.1) it means that they have the same mass. If the particle motion is free, the adjacent vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$ are in parallel. It means that

$$\left(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2}\right) = \left|\mathbf{P}_s\mathbf{P}_{s+1}\right| \cdot \left|\mathbf{P}_{s+1}\mathbf{P}_{s+2}\right|, \qquad s = 0, \pm 1, \pm 2, ... \tag{2.7}$$

where $(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2})$ is the scalar product of vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$.

The scalar product $(\mathbf{PQ}.\mathbf{RS})$ of two vectors $\mathbf{PQ}$ and $\mathbf{RS}$ in physical geometry is defined by the relation

$$(\mathbf{PQ}.\mathbf{RS}) = \sigma(P,S) + \sigma(Q,R) - \sigma(P,R) - \sigma(Q,S) \tag{2.8}$$

where P, Q, R, S are the points, which determine the vectors $\mathbf{PQ}$ and $\mathbf{RS}$. In the proper Euclidean geometry the definition of the scalar product (2.8) is equivalent to the conventional definition of the scalar product in the linear vector space. The definition (2.8) via world function does not refer to the linear vector space, and it may be used in the case of such a physical geometry, where one cannot introduce a linear vector space.

Equivalence (parallelism and equality of lengths) of two vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$ is written in the form of two equations

$$(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2}) \equiv \sigma(P_s, P_{s+2}) - \sigma(P_s, P_{s+1}) - \sigma(P_{s+1}, P_{s+2}) = 2\sigma(P_s, P_{s+1}) \tag{2.9}$$

$$\sigma(P_s, P_{s+1}) = \sigma(P_{s+1}, P_{s+2}), \qquad s = 0, \pm 1, \pm 2, ... \tag{2.10}$$

By means of relation (2.10) the equation (2.9) can be reduced to the form

$$\sigma(P_s, P_{s+2}) = 4\sigma(P_s, P_{s+1}), \qquad s = 0, \pm 1, \pm 2, ... \tag{2.11}$$

Two equations (2.10), (2.11) describe the world chain of a free microparticle. In the space-time of Minkowski the timelike links (2.6) of this chain are segments of the straight line. These segments became infinitesimal, if the lengths $|\mathbf{P}_s\mathbf{P}_{s+1}|$ of links tend to zero, and world chain transforms into a world line in space-time of Minkowski. At such a limit the geometrical length $\mu \to 0$, and the relation (2.2) cannot be used for geometrization of the finite mass m of the particle. In this case the particle mass m becomes some external characteristic of a particle, which is not connected with the space-time geometry directly.

In the case of arbitrary space-time geometry the link (2.6) is not a segment of one-dimensional straight, in general. Indeed, according to definition (2.6) the link $\mathcal{T}_{[P_sP_{s+1}]}$ is a three-dimensional surface in the 4-dimensional space-time. In the case of the space-time of Minkowski and timelike vector $\mathbf{P}_s\mathbf{P}_{s+1}$ the surface $\mathcal{T}_{[P_sP_{s+1}]}$ degenerates into a segment of one-dimensional straight line. The same degeneration takes place in the case of the Riemannian space-time geometry.

This degeneration is connected with original suppositions on the space-time geometry and on geometry, in general. One supposes, that the geometry is single-variant and infinitely divisible. In reality the space-time geometry is multivariant and restrictedly divisible.

Let us explain concept of multivariance. Two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent (equal) $(\mathbf{P}_0\mathbf{P}_1 \mathrm{eqv} \mathbf{Q}_0\mathbf{Q}_1)$, if they are in parallel and their length are equal.

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \tag{2.12}$$

$$|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1| \tag{2.13}$$

In terms of the world function the relations (2.12), (2.13) are written in the form

$$\sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_0) = 2\sigma(P_0, P_1) \tag{2.14}$$

$$\sigma(P_0, P_1) = \sigma(Q_0, Q_1) \tag{2.15}$$

The definition of equivalence of two vectors does not refer to a coordinate system, or to dimension of the space-time. If vector $\mathbf{P}_0\mathbf{P}_1$ is given, and we are going to determine vector $\mathbf{Q}_0\mathbf{Q}_1$ at the point Q_0, which is equivalent to vector $\mathbf{P}_0\mathbf{P}_1$, we should solve the system of two equations (2.14), (2.15) with respect to the point Q_1 at given points Q_0, P_0, P_1.

In the proper Euclidean space and in the space of Minkowski for timelike vector $\mathbf{P}_0\mathbf{P}_1$ ($\sigma(P_0, P_1) > 0$) one obtains one and only one solution Q_1 of the two equations (2.14), (2.15). This fact is formulated as follows. The geometry of Minkowski is single-variant with respect to any point Q_0 and with respect to any timelike vector $\mathbf{P}_0\mathbf{P}_1$.

However, the same geometry of Minkowski is multivariant with respect to any point Q_0 and any spacelike vector $\mathbf{P}_0\mathbf{P}_1$ ($\sigma(P_0, P_1) < 0$). It means, that in the case of given point Q_0 and given spacelike vector $\mathbf{P}_0\mathbf{P}_1$ the system of two equations (2.14), (2.15) has many solutions for the point Q_1.

All axiomatizable geometries, constructed on the basis of the linear vector space appear to be single-variant, because the construction of the linear vector space does not admit multivariance. The fact is that, in a multivariant geometry the equivalence relation is intransitive, whereas axioms of the linear vector space demands a transitive equivalence relation. The axiomatizable geometries cannot be multivariant, because they are constructed on the basis of the linear vector space. Thus, multivariance and intransitivity of the equivalence relation are incompatible with axiomatizability of a geometry.

In the nonaxiomatizable space-time geometry the links $\mathcal{T}_{[P_s P_{s+1}]}$ of the world chain (2.5) are surfaces. If the space-time geometry is close to the geometry of Minkowski, the links $\mathcal{T}_{[P_s P_{s+1}]}$ have the shape of narrow tubes. If the space-time geometry tends to the geometry of Minkowski, these tubes degenerate into segments of one-dimensional straight line. However, such a degeneration takes place only, if vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ are timelike.

In the case of spacelike vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ any link $\mathcal{T}_{[P_s P_{s+1}]}$ has the shape of infinite 3-dimensional hyperplanes, which are tangent to the light cone and contain the points of one-dimensional segment, restricted by points P_s, P_{s+1}. Such a shape of spacelike links $\mathcal{T}_{[P_s P_{s+1}]}$ is a reason, why there are only timelike world lines. Spacelike world lines have not been discovered. At the conventional approach to space-time geometry absence of spacelike world lines is simply postulated.

It is worth to note, that the multivariant space-time geometries exist indeed. In such a space-time geometry world chains appear to be stochastic, and one needs to use statistical description to obtain a deterministic description and to make some prediction on possible evolution of a particle.

For instance, the world function

$$\sigma_{\mathrm{d}} = \sigma_{\mathrm{M}} + d \cdot \mathrm{sign}\sigma_{\mathrm{M}}, \qquad d = \frac{\hbar}{2bc} = \mathrm{const} \tag{2.16}$$

where σ_{M} is the world function (2.3) of the Minkowski geometry, describes the space-time geometry, which is multivariant with respect to timelike vectors. Here $\hbar$ is the quantum constant and b is the universal constant, defined by the relation (2.2). World chains are stochastic in this space-time geometry. Statistical description of stochastic timelike world chains is equivalent to the quantum description in terms of the Schrödinger equation [8]. The quantum constant $\hbar$ appears in the description from the expression (2.16) for the world function, whereas the universal constant b disappears, because the statistical description is sensitive to the length μ of links of the world chain. Replacement of μ by its expression $\mu = m/b$, which follows form (2.2), leads to disappearance of b and appearance of m instead of μ.

It should note, that the geometry (2.16) is uniform, isotropic and discrete. At conventional approach to a geometry as a logical construction, an isotropic geometry cannot be discrete. Such a viewpoint takes place, because the conventional approach connects any discreteness with properties of the manifold. According to this approach a discrete geometry cannot be given on continual manifold. In reality, a discreteness is determined by properties of the world function. On the same manifold of Minkowski one can define both a continual geometry and a discrete one. The space-time geometry (2.16) is discrete, because in this geometry there are no vectors $\mathbf{PQ}$ of the length $|\mathbf{PQ}|$, satisfying the condition

$$0 < |\mathbf{PQ}|^2 < d^2 \tag{2.17}$$

where d is the constant, defined in (2.16). Of course, such a space-time should be qualified as a discrete. See details in [9].

3 General properties of dynamic equations

Dynamic equations (2.10), (2.11) describe evolution of microparticle state, which is described by the vector $\mathbf{P}_s\mathbf{P}_{s+1}$. The dynamic equations are finite difference equations (not differential). They are sensitive to the length of a step (link of the chain). They are written in the form, which is insensitive to a choice of a coordinate system and to the dimension of the space-time. All information on dynamic equations is concentrated in the space-time geometry and in the length of links of the world chain. We are going to rewrite equations (2.10), (2.11) in the form of differential equations, tending the length of links to zero. We are interested in the form of dynamic equations in the case of non-Riemannian space-time geometry. In particular, we are interested in the form of dynamic equations in the case, when the space-time geometry is deformed by a presence of a heavy sphere.

Before to write dynamic equations (2.10), (2.11) in the form of differential equations, we rewrite them in terms of distance $\rho = \sqrt{2\sigma}$. We obtain

$$\rho(P_s, P_{s+1}) = \rho(P_{s+1}, P_{s+2}), \qquad s = 0, \pm 1, \pm 2, ... \tag{3.1}$$

$$\rho(P_s, P_{s+2}) = 2\rho(P_s, P_{s+1}), \qquad s = 0, \pm 1, \pm 2, .. \tag{3.2}$$

In the case of the proper Euclidean space all points P_s, $s = 0, \pm 1, \pm 2, ...$ lie on one straight line. In the space-time geometry of Minkowski all points P_s, $s = 0, \pm 1, \pm 2, ...$ lie on one timelike straight, provided $\rho^2(P_s, P_{s+1}) = \rho^2(P_0, P_1) > 0$.

The equations (3.1), (3.2), as well equations (2.10), (2.11) realize the procedure of the straight line construction by means of only compasses. One starts from the segment P_0P_1 of length ρ_0 and draws a sphere $\mathcal{S}_{P_{1,\rho_0}}$ of radius ρ_0 with the center at the point P_1. Besides, one draws a sphere $\mathcal{S}_{P_0,2\rho_0}$ of radius $2\rho_0$ with the center at the point P_0. The spheres $\mathcal{S}_{P_1,\rho_0}$ and $\mathcal{S}_{P_0,2\rho_0}$ have the only common point P_2. It is the point P_2, which is defined as a common point of spheres $\mathcal{S}_{P_1,\rho_0}$ and $\mathcal{S}_{P_0,2\rho_0}$, which are tangent to each other at this point. The segment P_1P_2 has the length ρ_0. One draws spheres $\mathcal{S}_{P_2,\rho_0}$ and $\mathcal{S}_{P_1,2\rho_0}$, which has the only common point P_3 and so on. All points , constructed by this method, lie on the same straight line. This procedure is sensitive to an error of the radius ρ_0 in the sense, that the error $\delta\rho_0 = \alpha\rho_0$, $0 < \alpha \ll 1$ of the radius generates the error $\delta\rho$ of the point P_2 position, which is larger, than $\delta\rho_0$ ($\delta\rho = \sqrt{\alpha}\rho_0 \gg \delta\rho_0$).

One can construct points $P_0, P_1, P_2, ...$ which lie on the same straight in the proper Euclidean space, if one uses the fact, that adjacent vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$ are in parallel. However, in this case one uses usually parallelism, defined in the linear vector space. This definition of parallelism refers to a coordinate system and to the dimension of the Euclidean space. This fact prevents from a use of dynamic equations in the case of arbitrary space-time geometry, where one cannot introduce a linear vector space. Dynamic equation (3.2) has been obtained from the equation (2.9), which describes parallelism of vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$. However, in this case one uses the definition of parallelism in the form (2.7), which refers to the world function only. As a result the form of dynamic equations (2.10), (2.11) appears to be valid in the case of arbitrary physical space-time geometry.

Let us consider dynamic equations (2.10), (2.11) in the space-time, whose geometry is close to that of Minkowski. For simplicity we consider the case of the particle at rest, where coordinates of the points P_0, P_1 are

$$P_0 = \{-\mu, \mathbf{0}\}, \qquad P_1 = \{0, \mathbf{0}\},, \qquad P_2 = \{t, \mathbf{x}\} \tag{3.3}$$

Equations of "spheres" $\mathcal{S}_{P_0,2\mu}$ and $\mathcal{S}_{P_1,\mu}$ are

$$\mathcal{S}_{P_1,\mu} : t^2 - \mathbf{x}^2 + \alpha_1(t, \mathbf{x})\mu^2 = \mu^2, \qquad |\alpha_1(t, \mathbf{x})| \ll 1 \tag{3.4}$$

$$\mathcal{S}_{P_0,2\mu} : (t + \mu)^2 - \mathbf{x}^2 + \alpha_2(t, \mathbf{x})\mu^2 = (2\mu)^2, \qquad |\alpha_2(t, \mathbf{x})| \ll 1 \tag{3.5}$$

where the small quantities $\alpha_1(t, \mathbf{x})\mu^2$ and $\alpha_2(t, \mathbf{x})\mu^2$ take into account a small deflection of the space-time geometry from the geometry of Minkowski. In fact the "spheres" $\mathcal{S}_{P_0,2\mu}$ and $\mathcal{S}_{P_1,\mu}$ are deformed spheres, i.e. the surfaces, which are close to spheres.

The time coordinate t of the intersection point is defined by the relation

$$(t+\mu)^2 - t^2 = 3\mu^2 - (\alpha_2 - \alpha_1)\,\mu^2 \tag{3.6}$$

or

$$t = \mu\left(1 - \frac{\alpha_2\,(t,\mathbf{x}) - \alpha_1\,(t,\mathbf{x})}{2}\right) \tag{3.7}$$

In the space-time of Minkowski, where $\alpha_1 = \alpha_2 = 0$, one obtains $t = \mu,\ \mathbf{x} = \mathbf{0}$. It means that three points P_0, P_1, P_2 lie on the same timelike straight line.

One obtains from (3.7) and (3.4), that the spatial coordinates $\mathbf{x}$ of the intersection point are placed on the two-dimensional surface

$$\mathbf{x}^2 = \left(-\alpha_2 + \frac{(\alpha_2 - \alpha_1)^2}{4}\right)\mu^2 \tag{3.8}$$

If

$$\alpha_2 < 0 \quad \text{and} \quad |\alpha_2|\,,|\alpha_1| \ll 1 \tag{3.9}$$

there are many intersection points, placed in the small spatial region with radius of the order $\sqrt{|\alpha_2|}\mu$. In this case the world chain of the particle is multivariant (stochastic).

In the case, when

$$\alpha_2 > \frac{(\alpha_2 - \alpha_1)^2}{4} \tag{3.10}$$

there are no intersection points between the spheres $\mathcal{S}_{P_0,2\mu}$ and $\mathcal{S}_{P_1,\mu}$, defined by the relations (3.5), (3.4). It means, that the world chain with geometrical mass μ (length of the chain link) cannot exist. Such a region of the space-time is a "dead region" for microparticles of the geometrical mass μ.

The most interesting case is realized, when rhs of (3.8) vanishes. In this case the links of the world chain are determined uniquely. The world chain appears to be deterministic. Deterministic (single-variant) world chain appears in the space-time geometry of Minkowski and in the Riemannian space-time geometry.

The result on stochasticity of world chains, which are obtained in this section, are valid only for a simple microparticle (elementary particle), which is described by the skeleton $\mathcal{P}^1 = \{P_0, P_1\}$, consisting of two points. Macroparticles (metorites, planets, stars) consist of many microparticles, connected between themselves by some force fields. Microparticles cannot move independently and stochastically, because they are connected between themselves. As a result the motion of all microparticles inside the macroparticle is not free. It is described by a deterministic (single-valued) world chain. Any such a deterministic world chain is determined as a mean chain, which is a result of averaging over the surface (3.8). A result of this averaging of the surface (3.8) leads to the point $P_2 = \{t, \mathbf{x}\}$, which together with the given point $P_1 = \{0, \mathbf{0}\}$ determines uniquely the next link (P_1, P_2).

Thus, to obtain a deterministic world chain of a macroparticle, one needs to produce an averaging of stochastic world chains of microparticles, which is equivalent to a statistical description. The obtained mean world chains are described by dynamic equations in finite difference, which depend on the geometrical mass μ of microparticles, constituting the macroparticle. To obtain differential dynamic equations, one needs to go to the limit $\mu \to 0$ in finite difference equations of dynamics. As a result one obtains differential dynamic equations, describing free motion of macroparticles in the given physical space-time.

Multivariant physical space-time geometries are not considered in the contemporary theoretical physics. One believes, that the space-time geometry may not be nonaxiomatizable (multivariant), because it is not known, how to construct such geometries (the deformation principle is either unknown, or is not accepted). One does not admit, that a free motion of microparticles may be stochastic, and statistical averaging of these stochastic world chains is not considered. One believes, that the mean world lines of free macroparticles are geodesic lines of the space-time geometry. Restoring the space-time geometry on the basis of these geodesic lines one obtains the Riemannian geometry, which is determined by its geodesic lines and lengths of their segments. One ignores the fact, that there is an intermediate link between the world lines of free macroparticles and the space-time geometry. This intermediate link has a form of statistical averaging. As a result different physical geometries of space-time, having similar mean world lines of free macroparticles, are substituted by one (Riemannian) geometry.

Such an approach is admissible, when one considers motion of free macroparticles in a fixed space-time geometry. However, this approach appear to be wrong, when a generation of the space-time geometry by the matter distribution is considered. For instance, the world function of the space-time geometry, generated by a heavy sphere gives in the first approximation [4]

$$\sigma_{(1)}\left(t_1,\mathbf{x}_1;t_2,\mathbf{x}_2\right)=\frac{1}{2}\left(c^2\left(1-V_1\left(\mathbf{x}_1,\mathbf{x}_2\right)\right)\left(t_2-t_1\right)^2-\left(\mathbf{x}_2-\mathbf{x}_1\right)^2\right) \tag{3.11}$$

where

$$V_1\left(\mathbf{x}_1,\mathbf{x}_2\right)=\frac{2GM}{c^2\sqrt{\left|\mathbf{x}\right|^2}},\qquad \mathbf{x}=\frac{\mathbf{x}_1+\mathbf{x}_2}{2},\qquad \rho_0=\frac{3M}{4\pi R^3}$$

In the second approximation we obtain [4]

$$\sigma_{(2)}\left(t_1,\mathbf{x}_1;t_2,\mathbf{x}_2\right)=\frac{1}{2}\left(c^2\left(t_2-t_1\right)^2-\left(\mathbf{x}_2-\mathbf{x}_1\right)^2\right)+\delta\sigma_2\left(t_1,\mathbf{x}_1;t_2,\mathbf{x}_2\right) \tag{3.12}$$

where

$$\delta\sigma_2\left(t_1,\mathbf{x}_1;t_2,\mathbf{x}_2\right)=-\frac{1}{2}V_2\left(\mathbf{x}_1,\mathbf{x}_2\right)c^2\left(t_2-t_1\right)^2+B_2\left(\mathbf{x}_1,\mathbf{x}_2\right)c\left(t_2-t_1\right) \tag{3.13}$$

$$V_2(\mathbf{x}_1,\mathbf{x}_2) = V_1(\mathbf{x}_1,\mathbf{x}_2) + \frac{3GM}{2\pi R^3c^2}\int_V \frac{\rho_0(\boldsymbol{\xi})\,V_1(\boldsymbol{\xi},\boldsymbol{\xi})}{\sqrt{(\mathbf{x}-\boldsymbol{\xi})^2}}d\boldsymbol{\xi} \tag{3.14}$$

$$+\frac{3GM}{4\pi R^3c^2}\int_V \frac{\rho_0(\boldsymbol{\xi})\left(-V_1(\boldsymbol{\xi},\mathbf{x})+V_1(\mathbf{x},\mathbf{x})-V_1(\boldsymbol{\xi},\mathbf{x}_2)-V_1(\boldsymbol{\xi},\mathbf{x}_1)\right)}{\sqrt{(\mathbf{x}-\boldsymbol{\xi})^2}}d\boldsymbol{\xi}$$

$$B_2(\mathbf{x}_1,\mathbf{x}_2) = -\frac{3GM}{2\pi R^3c^2}\int_V \rho_0(\boldsymbol{\xi})\left(V_1(\boldsymbol{\xi},\mathbf{x}_2)-V_1(\boldsymbol{\xi},\mathbf{x}_1)\right)d\boldsymbol{\xi} \tag{3.15}$$

and the heavy sphere of the mass M and of radius R is placed at the origin of the coordinate system.

Comparison of the world function (3.11) of the first approximation, with the world function (3.12) of the second approximation (3.12) - (3.15) shows that the iteration process is to converge rapidly, if the gravitational potential is slight, i.e.

$$\frac{2GM}{c^2|\mathbf{x}|} \ll 1 \tag{3.16}$$

The well known Schwarzchild solution for gravitational field of a heavy sphere has the form

$$ds^2 = \left(1-\frac{2GM}{c^2r}\right)c^2dt^2 - \left(1-\frac{2GM}{c^2r}\right)^{-1}dr^2 - r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right) \tag{3.17}$$

Gravitational potential $V(r) = \frac{2GM}{c^2r}$ of (3.17) coincides with the gravitational potential of the first approximation (3.11)

$$V_1(\mathbf{x}_1,\mathbf{x}_2) = \frac{4GM}{c^2|\mathbf{x}_1+\mathbf{x}_2|} \tag{3.18}$$

However, it distinguishes from the gravitational potential (3.14) of the second approximation.

Comparison of expressions (3.11) - (3.14) with exact Schwarzchild solution (3.17) shows the following differences.

1. Geometry (3.11) - (3.14) is not a Riemannian geometry. Corresponding world function is obtained directly, whereas in the case of the Schwarzchild solution (3.17) the space-time geometry is *supposed to be Riemannian.* Under this supposition it is obtained by means of the metric tensor, which is obtained as a solution of the gravitational equations.

2. Potential (3.14) in the metric component g_{00} depends on the particle mass M linearly in the Schwarzchild solution, whereas this dependence is not linear in the case of geometry (3.12) - (3.15).

Both equations (3.17) and (3.12) - (3.15) cannot be true. Although the world function (3.12) - (3.14) is only a second approximation, (but not an exact solution), it is closer to the truth, than the Schwarzchild solution (3.17), because the Schwarzchild solution is based on a use of inconsistent Riemannian geometry.

In the light of hesitations in consistency of the Riemannian geometry the conclusion on existence of the dark matter and other astrophysical conclusions, based on the contemporary (Riemannian) theory of gravitation may appear to be a little too previous.

4 Dynamic equations for free particle in the space-time of Minkowski

At first, we consider application of suggested method to the case of space-time geometry of Minkowski. This method transforms dynamic equations, written in terms of finite differences to differential equations of dynamics. Although the obtained result is trivial, it is interesting in the sense, that it forbids an existence of spacelike world lines.

We consider two connected links of the world chain, defined by the points P_0, P_1, P_2,having coordinates

$$P_0 = \{y - dy_1\}, \qquad P_1 = \{y\}, \qquad P_2 = \{y + dy_2\} \tag{4.1}$$

where

$$y = \{t, \mathbf{y}\}, \qquad dy_1 = \{dt_1, d\mathbf{y}_1\}, \qquad dy_2 = \{dt_2, d\mathbf{y}_2\} \tag{4.2}$$

are coordinates in some inertial coordinate system, where the world function has the form (2.3). Dynamic equations (2.10), (2.11) have the form

$$\sigma_{\mathrm{M}}(y, y - dy_1) = \sigma_{\mathrm{M}}(y, y + dy_2) \tag{4.3}$$

$$4\sigma_{\mathrm{M}}(y, y - dy_1) = \sigma_{\mathrm{M}}(y - dy_1, y + dy_2) \tag{4.4}$$

In the developed form one obtains

$$\frac{1}{2}c^2(dt_1)^2 - \frac{1}{2}(d\mathbf{y}_1)^2 = \frac{1}{2}c^2(dt_2)^2 - \frac{1}{2}(d\mathbf{y}_2)^2 \tag{4.5}$$

$$2c^2(dt_1)^2 - 2(d\mathbf{y}_1)^2 = \frac{1}{2}c^2(dt_1 + dt_2)^2 - \frac{1}{2}(d\mathbf{y}_1 + d\mathbf{y}_2)^2 \tag{4.6}$$

We introduce designations

$$\mathbf{v}_1 = \frac{d\mathbf{y}_1}{dt_1}, \qquad \mathbf{v}_2 = \frac{d\mathbf{y}_2}{dt_2}, \qquad \boldsymbol{\beta}_1 = \frac{\mathbf{v}_1}{c}, \qquad \boldsymbol{\beta}_2 = \frac{\mathbf{v}_2}{c} \tag{4.7}$$

$$\boldsymbol{\beta}_1 = \boldsymbol{\beta} - \frac{1}{2}\dot{\boldsymbol{\beta}}dt, \qquad \boldsymbol{\beta}_1 = \boldsymbol{\beta} + \frac{1}{2}\dot{\boldsymbol{\beta}}dt, \qquad \dot{\boldsymbol{\beta}} \equiv \frac{d\boldsymbol{\beta}}{dt}, \qquad dt = \frac{dt_1 + dt_2}{2} \tag{4.8}$$

where

$$\mathbf{v} = c\boldsymbol{\beta} \qquad \dot{\mathbf{v}} = c\dot{\boldsymbol{\beta}} \tag{4.9}$$

are the mean velocity and the mean acceleration of the particle on the interval (P_0, P_2).

One rewrites equations (4.5), (4.6) in the form

$$1-\boldsymbol{\beta}_1^2 = \frac{dt_2^2}{dt_1^2}-\boldsymbol{\beta}_2^2\frac{dt_2^2}{dt_1^2} \tag{4.10}$$

$$4-4\boldsymbol{\beta}_1^2 = \left(1+\frac{dt_2}{dt_1}\right)^2-\left(\boldsymbol{\beta}_1+\boldsymbol{\beta}_2\frac{dt_2}{dt_1}\right)^2 \tag{4.11}$$

One obtains from equation (4.10) to within $(dt)^2$

$$\frac{dt_2^2}{dt_1^2}=\frac{1-\boldsymbol{\beta}_1^2}{1-\boldsymbol{\beta}_2^2}=\frac{1-\left(\boldsymbol{\beta}-\frac{1}{2}\dot{\boldsymbol{\beta}}dt\right)^2}{1-\left(\boldsymbol{\beta}+\frac{1}{2}\dot{\boldsymbol{\beta}}dt\right)^2}=1+2\frac{\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt}{1-\boldsymbol{\beta}^2}+\frac{2\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt\right)^2}{\left(1-\boldsymbol{\beta}^2\right)^2}=1+\alpha \tag{4.12}$$

where

$$\alpha=2\frac{\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt}{1-\boldsymbol{\beta}^2}+2\frac{\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt\right)^2}{\left(1-\boldsymbol{\beta}^2\right)^2}+O\left(dt^3\right) \tag{4.13}$$

Then

$$\frac{dt_2}{dt_1}=1+\frac{1}{2}\alpha-\frac{1}{8}\alpha^2 \tag{4.14}$$

Substituting (4.8) and (4.14) in (4.11), one obtains

$$4\left(1-\left(\boldsymbol{\beta}-\frac{1}{2}\dot{\boldsymbol{\beta}}dt\right)^2\right)=\left(2+\frac{\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt}{1-\boldsymbol{\beta}^2}+\frac{1}{2}\frac{\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt\right)^2}{\left(1-\boldsymbol{\beta}^2\right)^2}\right)^2$$
$$-\left(2\boldsymbol{\beta}+\left(\boldsymbol{\beta}+\frac{1}{2}\dot{\boldsymbol{\beta}}dt\right)\left(1+\frac{\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt}{1-\boldsymbol{\beta}^2}+\frac{1}{2}\frac{\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt\right)^2}{\left(1-\boldsymbol{\beta}^2\right)^2}\right)\right)^2 \tag{4.15}$$

After simplification one obtains

$$-\dot{\boldsymbol{\beta}}^2dt^2=+3\frac{\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt\right)^2}{\left(1-\boldsymbol{\beta}^2\right)^2}-3\boldsymbol{\beta}^2\left(\frac{\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt}{1-\boldsymbol{\beta}^2}\right)^2-2\frac{\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt\right)^2}{1-\boldsymbol{\beta}^2} \tag{4.16}$$

Note, that terms, which are proportional dt disappear from the equation (4.16). After simplification the equation (4.16) takes the form

$$\dot{\boldsymbol{\beta}}^2dt^2+\frac{\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt\right)^2}{1-\boldsymbol{\beta}^2}=0 \tag{4.17}$$

Let us introduce designation

$$\boldsymbol{\beta}\dot{\boldsymbol{\beta}}=\sqrt{\boldsymbol{\beta}^2\dot{\boldsymbol{\beta}}^2}\cos\phi \tag{4.18}$$

where ϕ is the angle between vectors $\boldsymbol{\beta}$ and $\dot{\boldsymbol{\beta}}$. The equation (4.18) takes the form

$$\dot{\boldsymbol{\beta}}^2 \left(1 + \frac{\boldsymbol{\beta}^2 \cos^2\phi}{1-\boldsymbol{\beta}^2}\right) = 0 \tag{4.19}$$

If the links of the world chain are timelike, then $\boldsymbol{\beta}^2 = \mathbf{v}^2/c^2 < 1$, the expression in brackets of (4.19) is positive, and the equation (4.19) can be satisfied only in the case, when

$$\dot{\boldsymbol{\beta}} = \frac{1}{c}\frac{d\mathbf{v}}{dt} = 0, \qquad \mathbf{v} = \mathbf{v}(t) = \text{const} \tag{4.20}$$

It is the expected unique solution. It should stress, that the unique solutions obtained only in the case of timelike world chain. In the case of spacelike world chain $\dot{\boldsymbol{\beta}}^2 > 1$, and there is such an angle ϕ between vectors $\boldsymbol{\beta}$ and $\dot{\boldsymbol{\beta}}$, that length $\left|\dot{\boldsymbol{\beta}}\right|$ of the vector $\dot{\boldsymbol{\beta}}$ is arbitrary. This angle is defined by the formula

$$\cos^2\phi = \frac{\boldsymbol{\beta}^2 - 1}{\boldsymbol{\beta}^2} < 1 \tag{4.21}$$

There is no unique solution in the case of spacelike world chain.

Note, that for obtaining of differential equations from dynamic equations in finite difference, we use the following representation of finite intervals $dy_1 = \{dt_1, d\mathbf{y}_1\}$, $dy_2 = \{dt_2, d\mathbf{y}_2\}$

$$d\mathbf{y}_1 = c\boldsymbol{\beta}dt - \frac{c}{2}\dot{\boldsymbol{\beta}}(dt)^2, \qquad d\mathbf{y}_2 = c\boldsymbol{\beta}dt + \frac{c}{2}\dot{\boldsymbol{\beta}}(dt)^2$$

$$dt = \frac{dt_1 + dt_2}{2}$$

5 Dynamic equations for motion of free particle in the gravitational field of heavy sphere

We consider the same equations (4.3), (4.4), but now in the space-time geometry with the world function (3.11)

$$\sigma(t, \mathbf{y}; t', \mathbf{y}') = \frac{1}{2}\left(c^2 - \frac{2GM}{\sqrt{\mathbf{x}^2}}\right)(t-t')^2 - \frac{1}{2}(\mathbf{y}-\mathbf{y}')^2 \tag{5.1}$$

where

$$\mathbf{x} = \frac{\mathbf{y}+\mathbf{y}'}{2} \tag{5.2}$$

We shall use designations (4.1), (4.2) and (4.7), (4.8). Besides, we use the designations

$$V = V(\mathbf{y}) = \frac{GM}{\sqrt{(\mathbf{y})^2}}, \qquad U = \frac{V}{c^2} \tag{5.3}$$

The equations (4.3), (4.4) with the world function (5.1) take the form

$$\frac{1}{2}\left(1-2U\left(\mathbf{y}-\frac{d\mathbf{y}_1}{2}\right)\right)-\frac{1}{2}\boldsymbol{\beta}_1^2=\frac{1}{2}\left(1-2U\left(\mathbf{y}+\frac{d\mathbf{y}_2}{2}\right)\right)\frac{dt_2^2}{dt_1^2}-\frac{1}{2}\boldsymbol{\beta}_2^2\frac{dt_2^2}{dt_1^2} \tag{5.4}$$

$$\begin{aligned}2\left(1-2U\left(\mathbf{y}-\frac{d\mathbf{y}_1}{2}\right)\right)-2\boldsymbol{\beta}_1^2 &= \frac{1}{2}\left(1-2U\left(\mathbf{y}+\frac{d\mathbf{y}_2-d\mathbf{y}_1}{2}\right)\right)\left(1+\frac{dt_2}{dt_1}\right)^2\\ &-\frac{1}{2}\left(\boldsymbol{\beta}_1+\boldsymbol{\beta}_2\frac{dt_2}{dt_1}\right)^2\end{aligned} \tag{5.5}$$

It follows from (5.4), that

$$\frac{dt_2^2}{dt_1^2}=\frac{1-2U\left(\mathbf{y}-\frac{d\mathbf{y}_1}{2}\right)-\boldsymbol{\beta}_1^2}{1-2U\left(\mathbf{y}+\frac{d\mathbf{y}_2}{2}\right)-\boldsymbol{\beta}_2^2}=1+\alpha+O\left(dt^3\right) \tag{5.6}$$

where α is an infinitesimal quantity. We shall consider dt as a principal infinitesimal quantity, and all infinitesimal quantities $d\mathbf{y}_1, d\mathbf{y}_2$, α will be expressed via dt and $(dt)^2$. The higher powers of dt will be neglected.

One obtains from (4.7), (4.8) and (5.6)

$$\frac{dt_2}{dt_1}=1+\frac{\alpha}{2}-\frac{\alpha^2}{8},\qquad \frac{dt_1}{dt_2}=1-\frac{\alpha}{2}+\frac{3\alpha^2}{8} \tag{5.7}$$

$$\frac{dt}{dt_1}=\frac{1}{2}+\frac{1}{2}\frac{dt_2}{dt_1}=1+\frac{\alpha}{4}-\frac{\alpha^2}{16} \tag{5.8}$$

$$\frac{dt}{dt_2}=\frac{1}{2}+\frac{1}{2}\frac{dt_1}{dt_2}=1-\frac{\alpha}{4}+\frac{3}{16}\alpha^2 \tag{5.9}$$

$$\frac{dt_1}{dt}=1-\frac{\alpha}{4}+\frac{\alpha^2}{8},\qquad \frac{dt_2}{dt}=1+\frac{\alpha}{4}-\frac{\alpha^2}{8} \tag{5.10}$$

$$dt_1=\left(1-\frac{\alpha}{4}+\frac{\alpha^2}{8}\right)dt,\qquad dt_2=\left(1+\frac{\alpha}{4}-\frac{\alpha^2}{8}\right)dt \tag{5.11}$$

$$d\mathbf{y}_1=c\boldsymbol{\beta}_1dt_1=c\boldsymbol{\beta}_1\left(1-\frac{\alpha}{4}\right)dt=c\left(\boldsymbol{\beta}-\frac{1}{2}\dot{\boldsymbol{\beta}}dt\right)\left(1-\frac{\alpha}{4}\right)dt \tag{5.12}$$

$$d\mathbf{y}_2=c\boldsymbol{\beta}_2dt_2=c\boldsymbol{\beta}_2\left(1+\frac{\alpha}{4}\right)dt=c\left(\boldsymbol{\beta}+\frac{1}{2}\dot{\boldsymbol{\beta}}dt\right)\left(1+\frac{\alpha}{4}\right)dt \tag{5.13}$$

Besides, the following decompositions are useful

$$U\left(\mathbf{y}-\frac{d\mathbf{y}_1}{2}\right)=U\left(\mathbf{y}\right)+\delta_1U,\qquad U\left(\mathbf{y}+\frac{d\mathbf{y}_2}{2}\right)=U\left(\mathbf{y}\right)+\delta_2U \tag{5.14}$$

$$U\left(\mathbf{y}+\frac{d\mathbf{y}_2-d\mathbf{y}_1}{2}\right)=U\left(\mathbf{y}\right)+\delta_{2-1}U \tag{5.15}$$

where

$$\delta_1 U = -\frac{d\mathbf{y}_1}{2}\nabla U + \frac{1}{8}dy_1^\alpha dy_1^\beta U_{,\alpha\beta}, \qquad U_{,\alpha\beta} = \frac{\partial^2 U(\mathbf{y})}{\partial y^\alpha \partial y^\beta} \tag{5.16}$$

$$\delta_2 U = \frac{d\mathbf{y}_2}{2}\nabla U + \frac{1}{8}dy_2^\alpha dy_2^\beta U_{,\alpha\beta} \tag{5.17}$$

$$\delta_{2-1} U = \frac{d\mathbf{y}_2 - d\mathbf{y}_1}{2}\nabla U + \frac{1}{8}\left(dy_2^\alpha - dy_1^\alpha\right)\left(dy_2^\beta - dy_1^\beta\right)U_{,\alpha\beta} \tag{5.18}$$

Using (5.12), (5.13), one obtains for (5.16) - (5.18)

$$\delta_1 U = -\frac{1}{2}c\boldsymbol{\beta}\nabla U dt + \frac{1}{8}c\boldsymbol{\beta}\nabla U \alpha dt + \frac{1}{4}c\dot{\boldsymbol{\beta}}\nabla U (dt)^2 + \frac{c^2}{8}\beta^\alpha\beta^\beta U_{,\alpha\beta}(dt)^2 \tag{5.19}$$

$$\delta_2 U = \frac{1}{2}c\boldsymbol{\beta}\nabla U dt + \frac{1}{8}c\boldsymbol{\beta}\nabla U \alpha dt + \frac{1}{4}c\dot{\boldsymbol{\beta}}\nabla U (dt)^2 + \frac{c^2}{8}\beta^\alpha\beta^\beta U_{,\alpha\beta}(dt)^2 \tag{5.20}$$

$$\delta_{2-1} U = \frac{1}{2}\left(\frac{1}{2}c\boldsymbol{\beta}\alpha dt + c\dot{\boldsymbol{\beta}}(dt)^2\right)\nabla U + O\left(dt^4\right) \tag{5.21}$$

Using (5.6) and (5.14) (5.15), one obtains for the infinitesimal quantity α

$$\begin{aligned}\alpha &= \frac{2\delta_2 U - 2\delta_1 U + \boldsymbol{\beta}_2^2 - \boldsymbol{\beta}_1^2}{1 - 2U - \boldsymbol{\beta}_2^2 - 2\delta_2 U} = \frac{2\delta_2 U - 2\delta_1 U + 2\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt}{1 - 2U - \boldsymbol{\beta}^2} \\ &+ \frac{\left(2\delta_2 U - 2\delta_1 U + 2\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt\right)\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}dt + 2\delta_2 U\right)}{\left(1 - 2U - \boldsymbol{\beta}^2\right)^2} + O\left(dt^3\right)\end{aligned} \tag{5.22}$$

Let us substitute expansions (5.12) - (5.22) in the dynamic equation (5.5)/ We obtain after simplifications

$$\frac{1}{2}\dot{\boldsymbol{\beta}}^2(dt)^2 - c\dot{\boldsymbol{\beta}}\nabla U (dt)^2 + \frac{1}{2}\frac{\left(c\boldsymbol{\beta}\nabla U + \boldsymbol{\beta}\dot{\boldsymbol{\beta}}\right)^2}{1 - 2U - \boldsymbol{\beta}^2}(dt)^2 + \frac{c^2}{2}\beta^\alpha\beta^\beta U_{,\alpha\beta}(dt)^2 = 0 \tag{5.23}$$

Note, that the terms of the order of dt disappear.

In terms of variables $\mathbf{v}, \dot{\mathbf{v}}, V$, defined by relations (4.9), (5.3) the relation (5.23) has the form

$$\frac{1}{2}\dot{\mathbf{v}}^2 - \dot{\mathbf{v}}\nabla V + \frac{1}{2}\frac{\left(\mathbf{v}\nabla V + \mathbf{v}\dot{\mathbf{v}}\right)^2}{c^2\left(1 - 2c^{-2}V - c^{-2}\mathbf{v}^2\right)} + \frac{1}{2c^2}v^\alpha v^\beta V_{,\alpha\beta} = 0 \tag{5.24}$$

One obtains in the nonrelativistic approximation

$$\frac{1}{2}\dot{\mathbf{v}}^2 - \dot{\mathbf{v}}\nabla V = 0 \tag{5.25}$$

It is evident, that one cannot determine three components of vector $\dot{\mathbf{v}}$ from one equation (5.25). One can determine only mean value $\langle\dot{\mathbf{v}}\rangle$ of vector $\dot{\mathbf{v}}$, choosing some principle of averaging.

Let us represent $\mathbf{v}$ in the form

$$\dot{\mathbf{v}} = \dot{\mathbf{v}}_{\parallel} + \dot{\mathbf{v}}_{\perp}, \qquad \dot{\mathbf{v}}_{\parallel} = \boldsymbol{\nabla} V \frac{(\dot{\mathbf{v}}\boldsymbol{\nabla} V)}{|\boldsymbol{\nabla} V|^2}, \qquad \dot{\mathbf{v}}_{\perp} = \dot{\mathbf{v}} - \boldsymbol{\nabla} V \frac{(\dot{\mathbf{v}}\boldsymbol{\nabla} V)}{|\boldsymbol{\nabla} V|^2} \tag{5.26}$$

where $\mathbf{v}_{\parallel}$ and $\mathbf{v}_{\perp}$ are components of $\mathbf{v}$, which are parallel to $\boldsymbol{\nabla} V$ and perpendicular to $\boldsymbol{\nabla} V$ correspondingly. Let us suppose, that the mean value $\langle \dot{\mathbf{v}} \rangle$ of vector $\dot{\mathbf{v}}$ is directed along the vector $\boldsymbol{\nabla} V$. In this case $\langle \dot{\mathbf{v}}_{\perp} \rangle = 0$, although $\langle \dot{\mathbf{v}}_{\perp}^2 \rangle > 0$, in general. One obtains from (5.25)

$$\dot{v}_{\parallel}^2 - 2\dot{v}_{\parallel} |\boldsymbol{\nabla} V| + \left\langle \dot{\mathbf{v}}_{\perp}^2 \right\rangle = 0 \tag{5.27}$$

or

$$\dot{v}_{\parallel} = |\boldsymbol{\nabla} V| \pm \sqrt{|\boldsymbol{\nabla} V|^2 - \langle \dot{\mathbf{v}}_{\perp}^2 \rangle} \tag{5.28}$$

It follows from (5.28), that

$$0 < \left\langle \dot{\mathbf{v}}_{\perp}^2 \right\rangle \leq |\boldsymbol{\nabla} V|^2, \qquad 0 < \dot{v}_{\parallel} < 2 |\boldsymbol{\nabla} V| \tag{5.29}$$

At any admissible value $\langle \dot{\mathbf{v}}_{\perp}^2 \rangle$ the quantity $\dot{v}_{\parallel}$ vibrates around its mean value $\langle \dot{v}_{\parallel} \rangle = |\boldsymbol{\nabla} V|$. Taking into account that $\langle \dot{\mathbf{v}}_{\perp} \rangle = 0$, one obtains, that

$$\langle \dot{\mathbf{v}} \rangle = \boldsymbol{\nabla} V = \boldsymbol{\nabla} \frac{GM}{r} \tag{5.30}$$

Any macroparticle moves in the gravitational field with the acceleration (5.30). Note, that for obtaining of this result only supposition $\langle \dot{\mathbf{v}}_{\perp} \rangle = 0$ is important. Variation of $\langle \dot{\mathbf{v}}_{\perp}^2 \rangle$ do not change the direction of the acceleration $\langle \dot{\mathbf{v}} \rangle$ .According to (5.28) this variation can change only $|\dot{v}_{\parallel}|$, which can be compensated by a proper change of the gravitational constant G.

In the nonrelativistic case the acceleration $\dot{\mathbf{v}}$ of a macroparticle in the gravitational field is determined be the mean value $\langle \dot{\mathbf{v}} \rangle = \boldsymbol{\nabla} V$. This result agrees with the Newtonian theory of gravitation.

In the general case we obtain instead of (5.27)

$$\begin{aligned} & \dot{v}_{\parallel}^2 - 2\dot{v}_{\parallel} |\boldsymbol{\nabla} V| + \left\langle \dot{\mathbf{v}}_{\perp}^2 \right\rangle \\ = & -\frac{(\mathbf{v}\boldsymbol{\nabla} V)^2 + 2(\mathbf{v}\boldsymbol{\nabla} V)\left(\mathbf{v}\left\langle \dot{\mathbf{v}}_{\parallel}\right\rangle\right) + \left\langle \left(v_{\parallel}\dot{v}_{\parallel} + \mathbf{v}_{\perp}\dot{\mathbf{v}}_{\perp}\right)^2 \right\rangle}{c^2 - 2V - \mathbf{v}^2} - \frac{1}{c^2} v^\alpha v^\beta V_{,\alpha\beta} \end{aligned} \tag{5.31}$$

This result distinguishes from the conventional result of the general relativity, because it depends on the second derivatives $V_{,\alpha\beta}$ of the gravitational potential. Equation (5.31) can be written in the form of quadratic equation with respect to $\dot{v}_{\parallel}$

$$\begin{aligned} & \dot{v}_{\parallel}^2 \left(1 + \frac{v_{\parallel}^2}{c^2 - 2V - \mathbf{v}^2}\right) - 2\dot{v}_{\parallel} \left(|\boldsymbol{\nabla} V| - \frac{(\mathbf{v}\boldsymbol{\nabla} V)\, v_{\parallel}}{c^2 - 2V - \mathbf{v}^2}\right) + \left\langle \dot{\mathbf{v}}_{\perp}^2 \right\rangle \\ = & -\frac{(\mathbf{v}\boldsymbol{\nabla} V)^2 + \left\langle (\mathbf{v}_{\perp}\dot{\mathbf{v}}_{\perp})^2 \right\rangle}{c^2 - 2V - \mathbf{v}^2} - \frac{1}{c^2} v^\alpha v^\beta V_{,\alpha\beta} \end{aligned} \tag{5.32}$$

6 Concluding remarks

Our consideration of free particle motion led to the conclusion, that the nonrelativistic motion of a free microparticle is multivariant (stochastic) already in the gravitational field of a heavy sphere. It is multivariant for other gravitational fields also, because the gravitational field has been considered in the general form of gravitational potential. Free motion of a macroparticle appears to be single-variant (deterministic), because stochastic behavior of different microparticles inside the macroparticle is independent. Averaging over many microparticles, one obtains a deterministic free motion. In the nonrelativistic case this average motion of a macroparticle coincides with predictions of the Newtonian theory of gravitation, which have been tested experimentally. In the relativistic case there are differences between predictions of the gravitational theory, based on physical space-time geometry, and predictions of the contemporary gravitational theory (general relativity), based on the Riemannian space-time geometry.

The Riemannian space-time geometry is an approximate theory of the space-time, because it is based on the *false supposition, that the Riemannian geometry is the most general geometry, which could be used for the space-time description.* Besides, the Riemannian geometry is constructed as a mathematical geometry, i.e. the geometry is considered as a logical construction, but not as a science on mutual disposition of geometrical objects. In particular, the space-time distance (world function) appears to be many-valued even in the gravitational field of a heavy sphere. It is inadmissible, if the space-time geometry is a physical geometry, i.e. a science on mutual disposition of geometrical objects. The world function is to be single-valued, as the main characteristic of the geometry.

The contemporary theory of gravitation admits one to determine only metric tensor, which determines the world function *only under supposition on the Riemannian space-time geometry.* The new conception (generalization of the general relativity on the case of a physical space-time geometry) admits one to determine the world function directly, and this world function appears to describe a non-Riemannian geometry even in the case of the gravitational field of a heavy sphere. Generalization of the general relativity appeared to be possible, only when we refuse a use of nonrelativistic concepts in the relativity theory. In particular, the nonrelativistic concept of nearness of two events has been replaced by the relativistic one [4].

Interrelation between the multivariant physical space-time geometry and conventional Riemannian geometry may be described as follows. The Riemannian geometry is a single-variant approximation of the physical geometry, generated by our poor knowledge of a geometry. From viewpoint of physical geometry this approximation looks as follows. Multivariant world chains of microparticles are replaced by averaged world chains, which are interpreted as exact world lines of particles. The space-time geometry is constructed on the basis of these "exact" world lines in such a way, that these world lines are geodesics of the geometry. Such an approximation is possible at large scales for description of the particle motion, although one cannot be sure, that this approximation is effective at description of the matter distribution influence on the space-time geometry, because the Riemannian geometry is inconsistent. At small scales, when the multivariance of the microparticle motion can be observed directly (for instance, diffraction of electrons on small hole), one uses the quantum description, which takes into account some properties of the multivariant motion.

The contemporary theory of gravitation, as well interpretation of astronomical observations, based on this theory, needs a revision. In particular, one needs to revise such concepts of the contemporary theory as "black hole" and "dark matter". At the present stage of investigations, one cannot state, that these concepts describe fictitious objects. However, these concepts becomes disputable, as far as they are introduced on the basis of a doubtful theory of gravitation. In any case a revision of the contemporary gravitational theory is necessary.

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