Non-Riemannian geometry as a reason of the third modification of the space-time geometry.

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Abstract

The third modification of the space-time geometry is considered. (The first modification is the spacial relativity, the second one is the general relativity.) After the third modification of the space-time geometry the motion of free particles become to be primordially stochastic (multivariant). This circumstance forces one to construct a multivariant dynamics. The multivariant dynamics is the classical dynamics in the non-Riemannian space-time geometry. The multivariant dynamics explains quantum effects without a reference to the quantum principles. Elimination of quantum principles admits one to solve the main problem of relativistic quantum theory: unification of the principles of relativity with the nonrelativistic quantum principles.

1 Introduction

In the 20th century dynamics was developed mainly by means of modification of the space-time model. Two essential modifications of the space-time geometry were produced by Albert Einstein. The first modification solved the problem of motion with large velocities. It is known as the special relativity. The first modification led to Minkowski space-time as a result of replacement of two invariants of the event space by one invariant.

The second modification is known as the general relativity. This modification led to the Riemannian space-time geometry, which is a result of influence of the matter distribution on the space-time curvature.

The third modification has not yet a short name. This modification led to non-Riemannian space-time geometry (T-geometry). This modification of the space-time geometry admitted one to describe quantum phenomena in the framework of the classical dynamics (the quantum principles are not used). In the non-Riemannian space-time geometry the motion of free particles is multivariant (stochastic), and one needs a special multivariant dynamics, which must be compatible with the space-time geometry. Note that the space-time geometry in itself is single-variant (deterministic), whereas the motion of particles in the single-variant space-time is multivariant (stochastic). This is a corollary of the multivariance of the parallelism concept.

In the Minkowski space-time geometry the deterministic motion is natural in the sense, that there are no special reasons for the deterministic motion. The natural stochastic motion in the Minkowski geometry is impossible. The stochastic motion is artificial in the sense, that there must be some special reason for the stochasticity.

In the non-Riemannian space-time the motion of any particle is stochastic and there is no special reason for the stochasticity. In this sense the stochastic motion is natural. We shall use a special term "multivariant motion" for the natural stochastic motion. In some special case (the particle of large mass) the multivariant motion degenerates into the single-variant (deterministic) motion, which is a special case of the multivariant motion.

The conception, generated by the third modification of the space-time geometry, describes the quantum effects without a use of quantum principles. It has the following properties.

1. It is a model conception, whereas the conventional quantum mechanics is an axiomatic one.

2. It uses dynamical methods, which are not constrained by quantum principles. The dynamical methods admit one to obtain results, which cannot be obtained in the framework of conventional quantum mechanics: for instance, incompatibility of the Copenhagen interpretation with the formalism of quantum mechanics [1, 2], existence of internal degrees of freedom of the Dirac particle, which are described nonrelativistically [3, 4, 5, 6], and the nature of the pair production mechanism [7].

3. The fact, that the conception is *more fundamental (primary)*, than the conventional quantum description, is of most importance.

We can see this in the figure 1, where one presents some fundamental conception, based on some primary (fundamental) propositions, which are shown in the bottom. In the top we see experimental data, which may be explained by means of corollaries of the primary propositions. It is possible such a situation: the experimental data may be explained by means of a set of corollaries, placed near the experimental data, without a direct reference to primary propositions of the fundamental theory. In this case the list of these propositions may be considered as primary propositions of some theory (a curtailed theory). This curtailed theory may be considered to be a self-sufficient theory, which does not need references to the fundamental theory and does not use these references. The curtailed theory contains more primary propositions, than the fundamental conception does, because it contains corollaries of the primary propositions of the fundamental theory, obtained in application to nonrelativistic phenomena.

If we do not know primary principles of the fundamental theory, we cannot separate, what in the primary propositions of the curtailed theory is conditioned by the primary propositions of the fundamental theory and what is conditioned by the nonrelativistic character of the described phenomena. In this case one may perceive the curtailed theory as a fundamental theory with primary propositions other, than those of the fundamental theory. The curtailed theory is axiomatic as a rule. Its application to explanation of experimental data is easier and simpler, than the application of the fundamental theory, because some corollaries of the fundamental theory are contained in the curtailed theory in a ready form.

From the practical viewpoint the applications of the curtailed theory is simpler, than the application of the fundamental theory. The curtailed theory looks as a simpler theory, which is more convenient for explanation of experimental data, than the fundamental theory. Besides, as a rule, the curtailed theory is obtained earlier, than the fundamental theory, because its primary propositions are nearer to experimental data and it is simpler, than the fundamental theory. For instance, the axiomatic thermodynamics (curtailed theory) had been constructed earlier, than the statistical physics (kinetic theory), which is a fundamental theory with respect to thermodynamics. As a rule, a transition from a known curtailed theory to the corresponding unknown fundamental theory is difficult for perception of researchers.

The nonrelativistic quantum theory is a curtailed theory, and there is a fundamental theory for the nonrelativistic quantum theory. Unfortunately, at the present stage of the science development, most researchers consider the nonrelativistic quantum mechanics as a fundamental theory. The primary propositions contain quantum principles, which are nonrelativistic. As a result the problem of the relativistic quantum theory construction is formulated as a join of nonrelativistic quantum principles with the principles of relativity. Such a statement of problem is inconsistent.

The true statement of the problem is formulated as follows. One needs to separate the nonrelativistic character of described phenomena from the primary propositions of the theory. It means that one needs to construct a fundamental theory, whose primary propositions are insensitive to the character (relativistic or nonrelativistic) of the described phenomena.

2 Why do we need T-geometry?

In the thirties of the 20th century, one had discovered that free microparticles move stochastically. Motion of free particles depends only on the space-time geometry. To explain the stochasticity of the particle motion, the space-time geometry was to possess the following properties. Motion of a free particle in such a space-time geometry is primordially stochastic (multivariant). Intensity of this stochasticity (multivariance) is to depend on the particle mass. Such a geometry was not known till the nineties of the 20th century.

In T-geometry the property of parallelism is intransitive, i.e. if we have the relations $\mathbf{a} \parallel \mathbf{b}$ and $\mathbf{b} \parallel \mathbf{c}$ for vectors $\mathbf{a,b,c}$, then, in general, vector $\mathbf{a}$ is not in parallel with vector $\mathbf{c}$. In reality, in T-geometry there are many vectors $\mathbf{b}_1, \mathbf{b}_2, ...$, which are in parallel with vector $\mathbf{a}$, but they are not in parallel between themselves. The intransitivity of parallelism is connected with the multivariance of the parallelism, which means, that there are many vectors (directions) $\mathbf{b}_1, \mathbf{b}_2, ...$, which are in parallel with the vector (direction) $\mathbf{a}$, but are not in parallel between themselves.

The world lines of free particles are shown in Figure 2. In the first case the world line is shown in the Minkowski space-time. In the second case the multivariant world line is shown in the space-time with intransitive parallelism. In the first case we have the conventional single-variant dynamics of the special relativity. In the second case one succeedes to construct the corresponding multivariant dynamics (or a statistical description). Such a dynamics is obtained, when the single particle (world line) is replaced by a statistical ensemble of particles (world lines).

3 Construction of T-geometry

Any generalized geometry is obtained as a result of a deformation of the proper Euclidean geometry. The proper Euclidean geometry is formalized, i.e. any statement $\mathcal{S}$ and any geometrical object $\mathcal{O}$ is represented in terms of the Euclidean world function σ_{E} in the form $\mathcal{S}(\sigma_{\mathrm{E}})$ and $\mathcal{O}(\sigma_{\mathrm{E}})$ respectively. (There is a theorem, which states that it is always possible [9]). *The set of all* $\mathcal{S}(\sigma_{\mathrm{E}})$ *and* $\mathcal{O}(\sigma_{\mathrm{E}})$ *forms the proper Euclidean geometry.* The world function [8] is defined by the relation $\sigma(P,Q) = \frac{1}{2}\rho^2(P,Q)$, where $\rho(P,Q)$ is a distance between the points P and Q. It has the properties

$$\sigma(P,P) = 0, \qquad \sigma(P,Q) = \sigma(Q,P), \qquad \forall P,Q \in \Omega \tag{3.1}$$

To obtain some generalized geometry $\mathcal{G}$, described by the world function σ, it is sufficient to replace σ_{E} by σ in all expressions $\mathcal{S}(\sigma_{\mathrm{E}})$ and $\mathcal{O}(\sigma_{\mathrm{E}})$. Then *the set of all* $\mathcal{S}(\sigma)$ *and* $\mathcal{O}(\sigma)$ *forms the generalized geometry* $\mathcal{G}$, described by the world function σ.

The representation of the proper Euclidean geometry in the formalized (σ-immanent) form does not contain any theorems. All theorems are replaced by definitions.

We shall explain this in the example of the cosine theorem, which states

$$\begin{aligned} |\mathbf{BC}|^2 &= |\mathbf{AB}|^2 + |\mathbf{AC}|^2 - 2(\mathbf{AB}.\mathbf{AC}) \\ &= |\mathbf{AB}|^2 + |\mathbf{AC}|^2 - 2|\mathbf{AB}||\mathbf{AC}|\cos\alpha \end{aligned} \tag{3.2}$$

where the points A, B, C are vertices of a triangle, $|\mathbf{BC}|$, $|\mathbf{AB}|$, $|\mathbf{AC}|$ are lengths of the triangle sides and α is the angle $\angle BAC$. The relation (3.2) is the cosine theorem which is proved on the basis of the axioms of the proper Euclidean geometry.

Using expression of the length of the triangle side **AB** via the world function σ

$$|\mathbf{AB}| = \sqrt{2\sigma(A,B)} \tag{3.3}$$

we may rewrite the relation (3.2) for $|\mathbf{BC}|^2$ in the form

$$(\mathbf{AB}.\mathbf{AC}) = \sigma(A,B) + \sigma(A,C) - \sigma(B,C) \tag{3.4}$$

This relation is a definition of the scalar product $(\mathbf{AB}.\mathbf{AC})$ of two vectors **AB** and **AC**, having the common origin A. Thus, the theorem is replaced by the definition of a new concept (the scalar product), which appears now not to be connected directly with the concept of the linear space.

Another example the Pythagorean theorem for the rectangular triangle ABC with the right angle $\angle BAC$. It is written in the form

$$|\mathbf{BC}|^2 = |\mathbf{AB}|^2 + |\mathbf{AC}|^2$$

In the formalized form (in T-geometry) we have a definition of the right angle$\angle BAC$ instead of the Pythagorean theorem. In terms of the world function this definition has the form. *The angle $\angle BAC$ is right, if the relation*

$$\sigma(A,B) + \sigma(A,C) - \sigma(B,C) = 0 \tag{3.5}$$

takes place.

Thus, the cosine theorem turns into the definition of the scalar product, whereas the Pythagorean theorem turns into the definition of the right angle. In a like way all theorems of the Euclidean geometry turn into definitions.

Thus, we see that theorems of the proper Euclidean geometry are replaced by definitions of T-geometry. The situation is very unusual and strange for mathematicians, who cannot imagine any geometry without theorems, because formulation and proof of theorems is the main work of geometers. Many of geometers cannot accept the geometry without theorems, i.e. the formalized form of the Euclidean geometry.

In reality, the Euclidean geometry is taken into account in the process of the Euclidean geometry formalization. The conventional method of the generalized geometry construction repeats all the Euclidean constructions at other original axioms. The alternative method, based on the deformation principle, does not need a repetition of all Euclidean constructions at the obtaining of the generalized geometry. All Euclid's results are contained in the formalized (σ-immanent) form of the Euclidean geometry.

4 Parallelism of remote vectors. Multivariance of parallelism

Scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ of two remote vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{Q}_0\mathbf{Q}_1$ is defined by the σ-immanent relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma(P_0,Q_1) + \sigma(P_1,Q_0) - \sigma(P_0,Q_0) - \sigma(P_1,Q_1) \tag{4.1}$$

Two vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{Q}_0\mathbf{Q}_1$ are linear dependent (collinear $\mathbf{P}_0\mathbf{P}_1||\mathbf{Q}_0\mathbf{Q}_1$), if the Gram determinant

$$\mathbf{P}_0\mathbf{P}_1||\mathbf{Q}_0\mathbf{Q}_1: \qquad \begin{vmatrix} (\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_0\mathbf{P}_1) & (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) \\ (\mathbf{Q}_0\mathbf{Q}_1.\mathbf{P}_0\mathbf{P}_1) & (\mathbf{Q}_0\mathbf{Q}_1.\mathbf{Q}_0\mathbf{Q}_1) \end{vmatrix} = 0 \tag{4.2}$$

or

$$\mathbf{P}_0\mathbf{P}_1||\mathbf{Q}_0\mathbf{Q}_1: \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)^2 = |\mathbf{P}_0\mathbf{P}_1|^2\,|\mathbf{Q}_0\mathbf{Q}_1|^2 \tag{4.3}$$

Here we see the definition of the linear dependence of two vectors, which does not refer to the linear space. Again the theorem on necessary and sufficient condition of linear dependence turns into definition of the linear dependence.

Two vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{Q}_0\mathbf{Q}_1$ are in parallel $\mathbf{P}_0\mathbf{P}_1 \uparrow\uparrow \mathbf{Q}_0\mathbf{Q}_1$, if

$$\mathbf{P}_0\mathbf{P}_1 \uparrow\uparrow \mathbf{Q}_0\mathbf{Q}_1: \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \tag{4.4}$$

The set of such points R, that the vector $\mathbf{Q}_0\mathbf{R}$ is collinear with the vector $\mathbf{P}_0\mathbf{P}_1$, forms the straight (tube) $\mathcal{T}_{Q_0;P_0P_1}$, passing through the point Q_0 collinear to the vector $\mathbf{P}_0\mathbf{P}_1$.

$$\mathcal{T}_{Q_0;P_0P_1} = (R|\mathbf{P}_0\mathbf{P}_1||\mathbf{Q}_0\mathbf{R}) \tag{4.5}$$

If the straight passes in the four-dimensional space, the straight is, in general, three-dimensional surface (multivariant straight). In the Minkowski space-time the straight $\mathcal{T}_{Q_0;P_0P_1}$ is one-dimensional (single-variant), if the vector $\mathbf{P}_0\mathbf{P}_1$ is timelike ($\sigma(P_0, P_1) > 0$). The straight $\mathcal{T}_{Q_0;P_0P_1}$ in the Minkowski space-time is three-dimensional (multivariant), if the vector $\mathbf{P}_0\mathbf{P}_1$ is spacelike ($\sigma(P_0, P_1) < 0$). It is a reason, why one cannot discover taxyons, when one searches them as one-dimensional spacelike lines.

If the space-time is deformed in such a way, that the world function σ_d has the form

$$\sigma_\mathrm{d} = \sigma_\mathrm{M} + d(\sigma_\mathrm{M}), \qquad d(\sigma_\mathrm{M}) = \begin{cases} \frac{\hbar}{2bc}, & \sigma_\mathrm{M} > \sigma_0 \\ 0, & \sigma_\mathrm{M} < 0 \end{cases}, \tag{4.6}$$

the straight $\mathcal{T}_{Q_0;P_0P_1}$ is always a three-dimensional surface (multivariant straight). Here σ_M is the world function of the Minkowski space-time, the quantities $\hbar$, b, c are constants.

Segment $\mathcal{T}_{[P_0P_1]}$ of timelike straight $\mathcal{T}_{P_0;P_0P_1}$ between the basic points P_0, P_1 may be presented in the form

$$\mathcal{T}_{[P_0P_1]} = \left(R|\sqrt{2\sigma(P_0, R)} + \sqrt{2\sigma(P_1, R)} - \sqrt{2\sigma(P_0, P_1)} = 0\right) \tag{4.7}$$

A chain of such segments form the particle world line (tube)

$$\mathcal{T}_\mathrm{br} = \bigcup_i \mathcal{T}_{[P_iP_{i+1}]} \tag{4.8}$$

The world line describe a free particle, if the vectors $\mathbf{P}_i\mathbf{P}_{i+1}$, $i = 0, \pm1, \pm2, ...$ are in parallel

$$\mathbf{P}_i\mathbf{P}_{i+1} \uparrow\uparrow \mathbf{P}_{i+1}\mathbf{P}_{i+2} \qquad i = 0, \pm1, \pm2, ... \tag{4.9}$$

and $|\mathbf{P}_i\mathbf{P}_{i+1}| = \mu, \qquad ni = 0, \pm 1, \pm 2, .., \mu$ is a geometrical mass of the particle. In the case of Minkowski space-time and timelike vector $\mathbf{P}_0\mathbf{P}_1$, one obtains the one-dimensional (single-variant) straight line, passing through the points P_0, P_1.

In the case of the distorted space-time, described by the world function σ_d, the world line of a free particle has the shape of a multivariant broken tube. To describe such world tubes one needs a multivariant dynamics.

5 Multivariant dynamics

Sir Isaac Newton had constructed his deterministic (single-variant) dynamics for the Newtonian conception of space-time. The single-variant dynamics is used for description of relativistic particles. However, for description of multivariant world lines, one needs a multivariant dynamics. Multivariant dynamics is used for description of particle motion in the Newtonian space-time, or in the Minkowski space-time, when the initial conditions are not known exactly. In this case one uses the concept of the statistical ensemble.

We display in the example of free nonrelativistic particles, how the statistical ensemble is introduced without a reference to the probability theory, (i.e. only dynamically). The action $\mathcal{A}_{\mathcal{S}_\mathrm{d}}$ for the free deterministic particle $\mathcal{S}_\mathrm{d}$ has the form

$$\mathcal{A}_{\mathcal{S}_\mathrm{d}}[\mathbf{x}] = \int \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 dt \tag{5.1}$$

where $\mathbf{x} = \mathbf{x}(t)$.

For the pure statistical ensemble $\mathcal{A}_{\mathcal{E}[\mathcal{S}_\mathrm{d}]}$ of free deterministic particles we obtain the action

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_\mathrm{d}]}[\mathbf{x}] = \int \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 dt d\boldsymbol{\xi} \tag{5.2}$$

where $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$ is a 3-vector function of independent variables $t, \boldsymbol{\xi} = \{\xi_1, \xi_2, ...\xi_n\}$. The variables (Lagrangian coordinates) $\boldsymbol{\xi}$ label particles $\mathcal{S}_\mathrm{d}$ of the statistical ensemble $\mathcal{E}[\mathcal{S}_\mathrm{d}]$. The statistical ensemble $\mathcal{E}[\mathcal{S}_\mathrm{d}]$ is a dynamic system of hydrodynamic type. Note that the number n of variables $\xi_1, \xi_2, ...\xi_n$ may be chosen arbitrary, but it is useful to choose them in such a way, that the relations $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$ may be resolved in the form $\boldsymbol{\xi} = \boldsymbol{\xi}(t, \mathbf{x})$. In this case we set $n = 3$. The statistical ensemble $\mathcal{E}[\mathcal{S}_\mathrm{d}]$ realizes a multivariant description in the sense, that different values of $\boldsymbol{\xi}$ describe different world lines, determined by different initial conditions.

If the particle $\mathcal{S} = \mathcal{S}_\mathrm{st}$ is stochastic, dynamic equations for the statistical ensemble $\mathcal{E}[\mathcal{S}_\mathrm{st}]$ exist, whereas there are no dynamic equations for the single stochastic particle $\mathcal{S}_\mathrm{st}$.

As a rule the statistical ensemble $\mathcal{E}[\mathcal{S}]$ is considered as a derivative object. The basic object is a single dynamic system. To construct the multivariant dynamics, we shall consider the statistical ensemble $\mathcal{E}[\mathcal{S}]$ as a basic object, whereas the single particle is considered as a partial case of the statistical ensemble with δ-like initial data.

In the case of deterministic dynamic system $\mathcal{S}_{\rm d}$ the dynamic equations, generated by the action for the single particle $\mathcal{S}_{\rm d}$, and those, generated by the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm d}\right]$, are similar

$$\frac{d^2\mathbf{x}}{dt^2}=0 \tag{5.3}$$

The objects $\mathcal{S}_{\rm d}$ and $\mathcal{E}\left[\mathcal{S}_{\rm d}\right]$ distinguish in the relation, that $\mathcal{S}_{\rm d}$ is described by one vector function $\mathbf{x}=\mathbf{x}\left(t\right)$, whereas $\mathcal{E}\left[\mathcal{S}_{\rm d}\right]$ is described by many different functions $\mathbf{x}=\mathbf{x}\left(t,\boldsymbol{\xi}\right)$. In this case it is of no importance, which of two objects: $\mathcal{S}$ or $\mathcal{E}\left[\mathcal{S}\right]$ is basic.

But for the stochastic particles the choice of basic object is important. If the statistical ensemble $\mathcal{E}\left[\mathcal{S}\right]$ is a basic object, the dynamic equations exist always for it. The fact, that there are no dynamic equations for a single stochastic particle is of no importance, because the single particle is not a basic object, and the dynamics is a dynamics of basic objects (statistical ensembles).

Thus, *choosing the statistical ensemble as a basic object of dynamics, we may construct a multivariant dynamics.*

The statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$ of free *stochastic* particles $\mathcal{S}_{\rm st}$ is a dynamic system, described by the action

$$\mathcal{A}_{\mathcal{E}\left[\mathcal{S}_{\rm st}\right]}\left[\mathbf{x},\mathbf{u}_{\rm df}\right]=\int\left\{\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2+\frac{m}{2}\mathbf{u}_{\rm df}^2-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}_{\rm df}\right\}dtd\boldsymbol{\xi} \tag{5.4}$$

where $\mathbf{u}_{\rm df}=\mathbf{u}_{\rm df}\left(t,\mathbf{x}\right)$ is a diffusion velocity, describing the mean value of the stochastic component of velocity, whereas $\frac{d\mathbf{x}}{dt}\left(t,\boldsymbol{\xi}\right)$ describes the regular component of the particle velocity, and $\mathbf{x}=\mathbf{x}\left(t,\boldsymbol{\xi}\right)$ is the 3-vector function of independent variables $t,\boldsymbol{\xi}=\left\{\xi_{1,}\xi_2,\xi_3\right\}$. The variables $\boldsymbol{\xi}$ label particles $\mathcal{S}_{\rm st}$, substituting the statistical ensemble. The operator

$$\boldsymbol{\nabla}=\left\{\frac{\partial}{\partial x^1},\frac{\partial}{\partial x^2},\frac{\partial}{\partial x^2}\right\} \tag{5.5}$$

is defined in the coordinate space of $\mathbf{x}$. Note that the transition from the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm d}\right]$ to the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$ is purely dynamic. The concept of probability is not used. The character of stochasticity is determined by the form of two last terms in the action (5.4) for $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$. For instance, if we replace $\boldsymbol{\nabla}\mathbf{v}_{\rm df}$ by some function $f\left(\boldsymbol{\nabla}\mathbf{v}_{\rm df}\right)$, we obtain another type of stochasticity, which does not coincide with the quantum stochasticity.

The action for the single stochastic particle is obtained from the action (5.4) for $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$ by omitting integration over $\boldsymbol{\xi}$. We obtain the action

$$\mathcal{A}_{\mathcal{S}_{\rm st}}\left[\mathbf{x},\mathbf{u}_{\rm df}\right]=\int\left\{\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2+\frac{m}{2}\mathbf{u}_{\rm df}^2-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}_{\rm df}\right\}dt \tag{5.6}$$

where $\mathbf{x}=\mathbf{x}\left(t\right),\quad \mathbf{u}_{\rm df}=\mathbf{u}_{\rm df}\left(t,\mathbf{x}\right)$. However, this action has only a symbolic sense, as far as the operator $\boldsymbol{\nabla}$ is defined in some vicinity of the point $\mathbf{x}$, but not at the point $\mathbf{x}$ itself. It means, that this action does not determine dynamic equations for the single particle $\mathcal{S}_{\mathrm{st}}$, and the *particle appears to be stochastic*, although dynamic equations exist for the statistical ensemble of such particles. They are determined by the action (5.4) for $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$. Thus, the particles described by the action for $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ are stochastic, because there are no dynamic equations for a single particle. In the case, when the quantum constant $\hbar=0$, the actions (5.4) for $\mathcal{S}_{\mathrm{st}}$ and (5.2) for $\mathcal{S}_{\mathrm{d}}$ coincide, because in this case it follows from dynamic equation, that $\mathbf{u}_{\mathrm{df}}=0$.

Variation of action for $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ with respect to variable $\mathbf{u}_{\mathrm{df}}$ leads to the equation

$$\mathbf{u}_{\mathrm{df}}=-\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho, \tag{5.7}$$

where the particle density ρ is defined by the relation

$$\rho=\left[\frac{\partial\left(x^1,x^2,x^3\right)}{\partial\left(\xi_1,\xi_2,\xi_3\right)}\right]^{-1}=\frac{\partial\left(\xi_1,\xi_2,\xi_3\right)}{\partial\left(x^1,x^2,x^3\right)} \tag{5.8}$$

Eliminating $\mathbf{u}_{\mathrm{df}}$ from the dynamic equation for $\mathbf{x}$, we obtain dynamic equations of the hydrodynamic type.

$$m\frac{d^2\mathbf{x}}{dt^2}=-\boldsymbol{\nabla}U\left(\rho,\boldsymbol{\nabla}\rho\right) \tag{5.9}$$

$$U\left(\rho,\boldsymbol{\nabla}\rho\right)=\frac{\hbar^2}{8m}\left(\frac{\left(\boldsymbol{\nabla}\rho\right)^2}{\rho^2}-2\frac{\boldsymbol{\nabla}^2\rho}{\rho}\right) \tag{5.10}$$

By means of the proper change of variables these equations can be reduced to the Schrödinger equation [10].

However, there is a serious mathematical problem here. The fact is that the hydrodynamic equations are to be integrated, in order they can be described in terms of the wave function. The fact, that the Schrödinger equation can be written in the hydrodynamic form, is well known [11]. However, the inverse transition from the hydrodynamic equations to the description in terms of wave function was not known until the end of the 20th century [10], and the necessity of integration of hydrodynamic equations was a reason of this fact.

Derivation of the Schrödinger equation as a partial case of dynamic equations, describing the statistical ensemble of random particles, shows that the *wave function is simply a method of description of hydrodynamic equations*, *but not a specific quantum object*, whose properties are determined by the quantum principles. At such an interpretation of the wave function the *quantum principles appear to be superfluous*, because they are necessary only for explanation, what is the wave function and how it is connected with the particle properties. All remaining information is contained in the dynamic equations. It appears that the *quantum particle is a kind of stochastic particle*, and all its exhibitions can be interpreted easily in terms of multivariant dynamics (in terms of the statistical ensemble of stochastic particles).

The idea of that, the quantum particle is simply a stochastic particle, is quite natural. It was known many years ago. However, the mathematical form of this idea could not be realized for a long time because of the two problems considered above (incorrect conception on the statistical ensemble of relativistic particles and necessity of integration of the hydrodynamic equations).

One can show, that quantum systems are a special sort of dynamic systems, which could be obtained from the statistical ensemble of classical dynamic systems by means of a change of parameters P of the dynamic system by its effective value P_{eff}. In particular, the free uncharged particle is described by an unique parameter: its mass m.

Statistical ensemble of free classical relativistic particles is described by the action

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\text{d}}]}[x] = -\int mc\sqrt{g_{ik}\dot{x}^i\dot{x}^k}d\tau d\boldsymbol{\xi}, \qquad \dot{x}^k \equiv \frac{dx^k}{d\tau} \tag{5.11}$$

where $x^k = x^k(\tau, \boldsymbol{\xi})$. To obtain the quantum description, we are to consider the statistical ensemble $\mathcal{E}[\mathcal{S}_{\text{st}}]$ of free stochastic relativistic particles $\mathcal{S}_{\text{st}}$, which is the dynamic system described by the action

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\text{st}}]}[x, u] = -\int m_{\text{eff}}c\sqrt{g_{ik}\dot{x}^i\dot{x}^k}d\tau d\boldsymbol{\xi}, \qquad \dot{x}^k \equiv \frac{dx^k}{d\tau} \tag{5.12}$$

where $x^k = x^k(\tau, \boldsymbol{\xi})$, $u^k = u^k(x)$, $k = 0, 1, 2, 3$. Here the effective mass m_{eff} is obtained from the mass m of the deterministic (classical) particle by means of the change

$$m^2 \to m_{\text{eff}}^2 = m^2\left(1 + g_{ik}\frac{u^iu^k}{c^2} + \frac{\hbar}{mc^2}\partial_k u^k\right) \tag{5.13}$$

where $u^k = u^k(x)$ the mean value of the 4-velocity stochastic component. Using the change of variables

$$\kappa^k = \frac{m}{\hbar}u^k, \tag{5.14}$$

it is convenient to introduce the 4-velocity $\kappa = \{\kappa^0, \boldsymbol{\kappa}\}$ with $\boldsymbol{\kappa}$, having dimensionality of the length. The action takes the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\text{st}}]}[x, \kappa] = -\int mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k}d\tau d\boldsymbol{\xi}, \tag{5.15}$$

$$K = \sqrt{1 + \lambda^2\left(g_{ik}\kappa^i\kappa^k + \partial_k\kappa^k\right)} \tag{5.16}$$

where $\lambda = \frac{\hbar}{mc}$ is the Compton wave length of the particle and the metric tensor g_{ik} =diag$\{c^2, -1, -1, -1\}$. In the nonrelativistic approximation this action turns into the action

$$\mathcal{A}_{\mathcal{S}_{\text{st}}}[\mathbf{x}, \mathbf{u}] = \int\left\{-mc^2 + \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right\}dtd\boldsymbol{\xi} \tag{5.17}$$

which coincides with the action (5.4) and generates the Schrödinger equation for the irrotational flow of the fluid, described by this action. In the relativistic case we obtain the Klein-Gordon equation.

Thus, we see that the proper modification of the space-time geometry generates the multivariant dynamics, which describes quantum phenomena *without any additional suppositions.* It means, that we obtain a *fundamental theory*, which replaces the curtailed theory (conventional nonrelativistic quantum mechanics). Having a fundamental theory, we may hope to construct the relativistic quantum theory without any additional hypotheses.

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