Neutrino world chain in framework of skeleton conception of particle dynamics.

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Abstract

One considers a free neutral particle whose world chain is a spacelike helix with timelike axis. Such world chain appears to be possible in some discrete space-time geometry. Radius of the helix may be macroscopic. This fact agrees with the results of the OPERA experiment, where superluminal velocity of a neutrino has been discovered. The helical world chain can be approximated by a world tube of macroscopic radius. Discovery of the neutrino superluminal velocity is an end effect of the thick world tube, but not the mean superluminal velocity of neutrino. The discrete space-time geometry has no definite metric dimension. Mathematical technique of linear vector space (technique of differential geometry) cannot be used in the discrete space-time geometry. Coordinateless description of the discrete space-time geometry is used.

Key words: discrete geometry; metric dimension and coordinate dimension; geometry without metric dimension; superluminal velocity; tachyons;

1 Introduction

Investigation of dynamic system $\mathcal{S}_{\rm D}$, described by the Dirac equation, has shown, that the classical approximation of $\mathcal{S}_{\rm D}$ is a classical dynamic system $\mathcal{S}_{\rm Dcl}$, which has ten degrees of freedom [1, 2, 3, 4, 5]. The dynamic system $\mathcal{S}_{\rm Dcl}$ describes a free particle, whose world line is a spacelike helix with timelike axis. The particle moves along the world line with superluminal velocity. At first sight, a free particle cannot move with superluminal velocity along a helix. Such a viewpoint is conditioned by the fact, that we do not deal with discrete space-time geometry and do not know its properties.

It appears, that there exist such space-time geometries, where a free particle may move along a spacelike helical world line [6]. This helical world line forms a world tube, describing the mean particle motion. Such a particle has no electric charge. It may be interpreted as a neutrino. In the paper [6] one considered a model of such a space-time geometry, where a free particle may have a helical world line. Here we hope to investigate more real space-time geometry, were parameters of the neutrino world tube are close to results of observation.

Experiment [7] shows, that neutrinos pass the distance 730km faster, than the light signal. The time of the lead is about 60.7 ± 6.9ns. It corresponds to difference $(v-c)\,c^{-1} \approx 3 \times 10^{-5}$. This experiment is interpreted usually as a discovery of superluminal speed of the neutrino mean motion, and generates problems, connected with the relativity principles. Such interpretation is based on the supposition, that the mean neutrino motion is described by one-dimensional straight world line. However, if the mean neutrino motion is described by a world tube, the OPERA experiment is explained freely, as an effect, conditioned by the neutrino world tube thickness. Estimation of the world tube radius from the OPERA experiment gives a macroscopic size of the tube radius: $R > 2.5$km. Such a result looks rather unexpected, because one believes that all parameters of the elementary particles motion are microscopic.

In this paper we investigate the neutrino world tube in the case of a more realistic space-time geometry. As a result one obtains the neutrino motion structure. It appears, that the space-time geometry may be such one that the radius of the neutrino tube appears to be macroscopic. Space-time geometry in microcosm is either discrete, or partly discrete. The simplest example of a discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ is described by the world function

$$\sigma_{\mathrm{d}}(P,Q) = \sigma_{\mathrm{M}}(P,Q) + \frac{\lambda_0^2}{2}\mathrm{sgn}\left(\sigma_{\mathrm{M}}(P,Q)\right), \quad \forall P,Q \in \Omega \tag{1.1}$$

where σ_{M} is the world function of the Minkowski geometry $\mathcal{G}_{\mathrm{M}}$, and Ω is the set of points (events) where the space-time geometry of Minkowski is given. The geometry (1.1) is discrete in the sense that the distance $\rho_{\mathrm{d}}(P,Q) = \sqrt{2\sigma_{\mathrm{d}}(P,Q)}$ between the points P and Q satisfies the restriction

$$|\rho_{\mathrm{d}}(P,Q)| \notin (0,\lambda_0), \quad \forall P,Q \in \Omega \tag{1.2}$$

Here λ_0 is the characteristic length of the discrete geometry. The inequality means that in the discrete geometry $\mathcal{G}_{\mathrm{d}}$ there are no close points. The constraint (1.2) may be described also in terms of the relative density of points

$$n_{\mathrm{d}} \equiv \frac{d\sigma_{\mathrm{M}}}{d\sigma_{\mathrm{d}}} = \begin{cases} 1 & \text{if} \quad |\sigma_{\mathrm{d}}| \geq \frac{\lambda_0^2}{2} \\ 0 & \text{if} \quad |\sigma_{\mathrm{d}}| < \frac{\lambda_0^2}{2} \end{cases} \tag{1.3}$$

One can compare relative densities of points in $\mathcal{G}_{\mathrm{d}}$ and in $\mathcal{G}_{\mathrm{M}}$, because they given on the same set of points. Here the points density in the interval $(0,\lambda_0)$ of values

$|\rho_{\rm d}|$ vanishes, whereas in other intervals of values $|\rho_{\rm d}|$ the point density is the same as in the Minkowski geometry.

The granular space-time geometry $\mathcal{G}_{\rm g}$, described by the world function

$$\sigma_{\rm g}=\sigma_{\rm M}+\frac{\lambda_0^2}{2}\left\{\begin{array}{lll}\operatorname{sgn}(\sigma_{\rm M}) & \text{if} & |\sigma_{\rm M}|>\sigma_0>0\\ \frac{\sigma_{\rm M}}{\sigma_0} & \text{if} & |\sigma_{\rm M}|<\sigma_0\end{array}\right. \tag{1.4}$$

is discrete only partly. Here $\sigma_0>0$ is a constant of the order of λ_0^2. The relative density $n_{\rm g}$ of points in $\mathcal{G}_{\rm g}$ has the form

$$n_{\rm g}=\frac{d\sigma_{\rm M}}{d\sigma_{\rm g}}=\left\{\begin{array}{lll}1 & \text{if} & |\sigma_{\rm g}|\geq\frac{\lambda_0^2}{2}\\ \frac{\sigma_0}{\sigma_0+\lambda_0^2/2} & \text{if} & |\sigma_{\rm g}|<\frac{\lambda_0^2}{2}\end{array}\right. \tag{1.5}$$

If $\sigma_0\rightarrow0$, the relative density $n_{\rm g}$ of points in $\mathcal{G}_{\rm g}$ tends to the relative density of points in the discrete space-time geometry $\mathcal{G}_{\rm d}$. In general, the space-time geometry $\mathcal{G}_{\rm g}$ may be considered as a partly discrete geometry, because in the interval $(0,\lambda_0)$ of the values $|\rho|$ the relative point density is less, than in the geometry of Minkowski. We shall refer to space-time geometry (1.4) as the granular space-time geometry $\mathcal{G}_{\rm g}$. The granular geometry $\mathcal{G}_{\rm g}$ has the properties of the discrete geometry. It is a kind of discrete geometry, and sometimes we shall use the name discrete geometry for the granular geometry.

Granular geometry and discrete geometry are special cases of the physical geometry, which is defined as a geometry, described in terms and only in terms of the world function, given on some point set Ω

$$\sigma:\quad\Omega\times\Omega\rightarrow\mathbb{R},\quad\sigma(P,Q)=\sigma(Q,P),\quad\sigma(P,P)=0,\quad\forall P,Q\in\Omega \tag{1.6}$$

The metric geometry is a special case of the physical geometry, equipped by the triangle axiom. The distant geometry [8, 9] is free of the triangle axiom, but it uses the restriction $\sigma(P,Q)\geq0,\quad\forall P,Q\in\Omega$.

The discrete geometry (and other physical geometries) is constructed as a generalization of the proper Euclidean geometry. To carry out such a generalization, the proper Euclidean geometry $\mathcal{G}_{\rm E}$ has to be presented in the σ-representation [10], where all concepts, quantities, and geometrical objects of the proper Euclidean geometry are expressed via the world function $\sigma_{\rm E}$ of the proper Euclidean geometry. The reason of such a demand lies in the fact that usually all concepts of the Euclidean geometry (dimension, manifold, smooth line, differential relations) relate to the differential (continuous) geometry, and there are no such concepts in the discrete geometry. The only concept which is common for the discrete geometry and for the proper Euclidean geometry is the world function σ (or the distance function $\rho=\sqrt{2\sigma}$).

Being presented in terms of the world function $\sigma_{\rm E}$, the proper Euclidean geometry $\mathcal{G}_{\rm E}$ contains two kinds of relations: (1) general geometric relations, which contains only world function $\sigma_{\rm E}$, and (2) special relations of the geometry $\mathcal{G}_{\rm E}$, which are constraints, imposed on the world function $\sigma_{\rm E}$.

Let us adduce some general geometric definitions:

Vector $\mathbf{PQ}$ is an ordered set $\{P, Q\}$ of two points P, Q (but not an element of the linear vector space as usually). Scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) \qquad (1.7)$$

The length $|\mathbf{PQ}|$ of the vector $\mathbf{PQ}$ is defined by the relation

$$|\mathbf{PQ}|^2 = (\mathbf{PQ}.\mathbf{PQ}) = 2\sigma(P, Q) \qquad (1.8)$$

n vectors $\mathbf{P}_0\mathbf{P}_1, \mathbf{P}_0\mathbf{P}_2, ...\mathbf{P}_0\mathbf{P}_n$ are linear dependent, if and only if the Gram determinant

$$F_n(\mathcal{P}_n) = \det ||(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)||, \quad i, k = 1, 2, ...n, \quad \mathcal{P}_n \equiv \{P_0, P_2, ...P_n\} \qquad (1.9)$$

vanishes

$$F_n(\mathcal{P}_n) = 0 \qquad (1.10)$$

Two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent (equal) $(\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1)$, if the vectors are in parallel

$$(\mathbf{P}_0\mathbf{P}_1 \upuparrows \mathbf{Q}_0\mathbf{Q}_1): \quad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \qquad (1.11)$$

and their lengths are equal

$$\sigma(P_0, P_1) = \sigma(Q_0, Q_1) \qquad (1.12)$$

According to (1.11), (1.12) the equivalence definition has the form

$$\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1: \quad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1|^2 \wedge |\mathbf{P}_0\mathbf{P}_1|^2 = |\mathbf{Q}_0\mathbf{Q}_1|^2 \qquad (1.13)$$

All general geometric relations (1.7) - (1.13) are obtained as properties of the linear vector space. However, they do not contain any reference to the linear vector space. They are written in terms of the world function σ_E of the proper Euclidean geometry, and they may be used in any physical geometry even in the case, when one cannot introduce linear vector space in this geometry. To use the relation (1.7) - (1.13) in a discrete geometry, it is sufficient to use the world function σ_d of the discrete geometry $\mathcal{G}_\mathrm{d}$ in them.

Formally general geometric relations (1.7) - (1.13) realize some processing of information, contained in the world function. Such a processing is to be universal, i.e. it is uniform for all generalized geometries. This method of processing is known for the proper Euclidean geometry $\mathcal{G}_\mathrm{E}$. It may applied for construction of general geometric relations for other generalized geometries. In the case, when one can introduce linear vector space, such a processing admits one to construct the particle dynamics in the space-time geometry, equipped by the linear vector space. As far as the general geometric relations (1.7) - (1.13) are universal in the sense that they do not refer to the linear vector space, they may be used for construction of the particle dynamics in those space-time geometries, where introduction of the linear vector space is impossible.

The special relations of the proper Euclidean geometry have the form [11]:

I. Definition of the metric dimension:

$$\exists \mathcal{P}_n \equiv \{P_0, P_1, ...P_n\} \subset \Omega, \qquad F_n(\mathcal{P}_n) \neq 0, \qquad F_k\left(\Omega^{k+1}\right) = 0, \qquad k > n \quad (1.14)$$

where $F_n(\mathcal{P}_n)$ is the n-th order Gram's determinant (1.9). Vectors $\mathbf{P}_0\mathbf{P}_i$, $i = 1, 2, ...n$ are basic vectors of the rectilinear coordinate system K_n with the origin at the point P_0. The covariant coordinates of the point P in the coordinate system K_n are defined by the relation

$$x_i(P) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}), \qquad i = 1, 2, ...n \qquad (1.15)$$

The metric tensors $g_{ik}(\mathcal{P}_n)$ and $g^{ik}(\mathcal{P}_n)$, $i, k = 1, 2, ...n$ in K_n are defined by the relations

$$\sum_{k=1}^{k=n} g^{ik}(\mathcal{P}_n)\, g_{lk}(\mathcal{P}_n) = \delta_l^i, \qquad g_{il}(\mathcal{P}_n) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_l), \qquad i, l = 1, 2, ...n \quad (1.16)$$

II. Linear structure of the Euclidean space:

$$\sigma_{\mathrm{E}}(P, Q) = \frac{1}{2}\sum_{i,k=1}^{i,k=n} g^{ik}(\mathcal{P}_n)\,(x_i(P) - x_i(Q))\,(x_k(P) - x_k(Q)), \qquad \forall P, Q \in \Omega \qquad (1.17)$$

where coordinates $x_i(P)$, $x_i(Q)$, $i = 1, 2, ...n$ of the points P and Q are covariant coordinates of the vectors $\mathbf{P}_0\mathbf{P}$, $\mathbf{P}_0\mathbf{Q}$ respectively in the coordinate system K.

III: The metric tensor matrix $g_{lk}(\mathcal{P}^n)$ has only positive eigenvalues g_k

$$g_k > 0, \qquad k = 1, 2, ..., n \qquad (1.18)$$

IV. The continuity condition: the system of equations

$$(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}) = y_i \in \mathbb{R}, \qquad i = 1, 2, ...n \qquad (1.19)$$

considered to be equations for determination of the point P as a function of coordinates $y = \{y_i\}$, $i = 1, 2, ...n$ has always one and only one solution. Conditions I – IV contain a reference to the dimension n of the Euclidean space, which is defined by the relations (1.14).

Special relations of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ may be not valid for other physical geometries. In some cases these relations may used partly. For instance, the metric dimension may be defined locally. Instead of constraint (1.14) one uses the condition

$$\forall P_0 \in \Omega, \quad \exists \mathcal{P}_n \equiv \{P_0, P_1, ...P_n\} \subset \Omega, \quad F_n(\mathcal{P}_n) \neq 0, \quad F_k(\mathcal{P}_k) = 0, \quad k > n \qquad (1.20)$$

where all skeletons $\mathcal{P}_n$ contain only infinitely close points. The conditions (1.20) determine the metric dimension for locally flat (Riemannian) geometry.

All relations I – IV are written in terms of the world function. They are constraints on the form of the world function of the proper Euclidean geometry.

The proper Euclidean geometry looks in the σ-representation quite different, than in conventional representation on the basis of the linear vector space. For instance, such a quantity as dimension has two different meanings in the σ-representation. On one hand, the metrical dimension $n_{\rm m}$ is the maximal number of linear independent vectors, which is determined by the relations (1.14). On the other hand, the coordinate dimension $n_{\rm c}$, is a number of coordinates, which is used at the description of the point set Ω. In the proper Euclidean geometry $\mathcal{G}_{\rm E}$ the coordinate dimension $n_{\rm c} = n_{\rm m}$, and this fact is a corollary of special (not general geometric) relations (1.14), (1.15)

In general, the coordinate labelling of points of Ω has no relation to the geometry. In the proper Euclidean geometry the two dimensions coincide, because the coordinate dimension $n_{\rm c}$ is determined by the special conditions (1.14), (1.15), which are characteristic for the proper Euclidean geometry. In the geometry $\mathcal{G}_{\rm d}$ the number $n_{\rm m}$ of linear independent vectors is more, than the number of coordinates $n_{\rm c}$. For instance, for six points $\mathcal{P}_5 = \{P_0, P_1...P_5\}$ and five vectors

$$\begin{aligned}
\mathbf{P}_0\mathbf{P}_1 &= \{l,0,0,0\}\,, \quad \mathbf{P}_0\mathbf{P}_2 = \{0,l,0,0,0\}\,, \quad \mathbf{P}_0\mathbf{P}_3 = \{0,0,l,0\}\,, \\
\mathbf{P}_0\mathbf{P}_4 &= \{0,0,0,l\}\,, \quad \mathbf{P}_0\mathbf{P}_5 = \{a,0,0,0\}
\end{aligned}$$

the Gram determinant $F_5(\mathcal{P}_5)$ does not vanish in the geometry $\mathcal{G}_{\rm d}$ with the world function (1.1). One obtains for the case $d = \lambda_0^2/2 \ll a^2, l^2$

$$F_4(\mathcal{P}_4) = \begin{vmatrix} l^2+d & 0 & 0 & 0 \\ 0 & -l^2-d & -2d & -2d \\ 0 & -2d & -l^2-d & -2d \\ 0 & -2d & -2d & -l^2-d \end{vmatrix} = -l^8 - 4l^6 d + O\left(d^2\right) \qquad (1.21)$$

$$\begin{aligned}
F_5(\mathcal{P}_5) &= \begin{vmatrix} l^2+d & 0 & 0 & 0 & al+\frac{3}{2}d \\ 0 & -l^2-d & -2d & -2d & d \\ 0 & -2d & -l^2-d & -2d & d \\ 0 & -2d & -2d & -l^2-d & d \\ al+\frac{3}{2}d & d & d & d & a^2+d \end{vmatrix} \\
&= d\left(-a^2l^6 + 3al^7 - l^8\right) + O\left(d^2\right) \qquad (1.22)
\end{aligned}$$

It means that, in general, the metric dimension $n_{\rm m} \geq 5$ in $\mathcal{G}_{\rm d}$. In $\mathcal{G}_{\rm d}$ the metric dimension $n_{\rm m}$ cannot coincide with the coordinate dimension $n_{\rm c}$. It means essentially that one cannot introduce a finite number of linear independent basic vectors and expand space-time vectors over these basic vectors. It is very unexpected, because the conventional construction of a differential geometry (for instance, the Riemannian one) starts, giving n-dimensional manifold with a coordinate system on it. Of course, one assumes, that the number of linear independent basic vectors at any point is equal to $n = n_{\rm m} = n_{\rm c}$. Only in this case one can expand vectors over basic vectors and use operations, defined in the linear vector space. In the case of a discrete space-time geometry, where $n_{\rm m} \neq n_{\rm c}$, the linear vector space cannot be introduced, although the coordinate system can be introduced, and the coordinate dimension $n_{\rm c} = 4$ as in the space-time geometry of Minkowski. Four coordinates $x = \{x^0, x^1, x^2, x^3\}$, $x^k \in \mathbb{R}$ are defined as usually.

Note, that the conditions (1.14), defining metric dimension $n_{\rm m}$ contain a lot of constraints, and all they are special conditions of the proper Euclidean geometry. It means that there is a lot of physical geometries, where $n_{\rm m} \neq n_{\rm c}$, and one cannot introduce a linear vector space there. In the limit $d \rightarrow 0$, $F_5(\mathcal{P}^5) = 0$ in (1.22), and $\mathcal{G}_{\rm d}$ transforms to $\mathcal{G}_{\rm M}$. In this case the metric dimension $n_{\rm m} = 4$ coincides with the coordinate dimension $n_{\rm c} = 4$. It means that one may use approximately the space-time geometry $\mathcal{G}_{\rm M}$ in the case, when typical lengths l of vectors is much greater, than the elementary length λ_0. In this case one may set approximately $\lambda_0 = 0$, and suppose that $n_{\rm m} = n_{\rm c}$.

The set of the Gram determinants values $F_n(\mathcal{P}_n)$, $n = 2, 3, ...$ may be such, that one cannot introduce the metric dimension $n_{\rm m}$. Apparently, the discrete space-time geometries are geometries without a definite metric dimension. Such "dimensionless" geometries look especially exotic. Contemporary researchers deal only with space-time geometries of definite dimension. They can hardly conceive properties of "dimensionless" space-time geometries. On the other hand, the classical particle dynamics does not work in microcosm, described by the geometry of Minkowski. As far as the discrete ("dimensionless") space-time geometries are not known for most researchers, they use quantum dynamics, which imitates the discrete geometry properties. This imitation is arbitrary and desultory. Besides, this imitation is not complete. There are such properties of real particle dynamics, which cannot be imitated by quantum dynamics in the space-time of Minkowski.

We see that coincidence of metric dimension $n_{\rm m}$ with the coordinate dimension $n_{\rm c}$ and a construction of a smooth manifold with the dimension $n = n_{\rm m} = n_{\rm c}$ is a special property of the proper Euclidean geometry $\mathcal{G}_{\rm E}$, which is not a general geometric property. The conventional method of the differential geometry construction starts from the definition of a smooth manifold with fixed dimension. Such a method is not a general method of the generalized geometries construction, because it uses special properties of $\mathcal{G}_{\rm E}$, which, generally speaking, are not characteristic for all generalized geometries. In general, a use of the coordinate description for the generalized geometries construction is a use of special properties of the proper Euclidean geometry $\mathcal{G}_{\rm E}$ for such a construction. Such an approach cannot be a general method of the generalized geometries construction. Using special properties of $\mathcal{G}_{\rm E}$, one obtains only a part of possible generalized geometries. In particular, a use of the coordinate description does not admit to construct geometries with indefinite metric dimension and with intransitive equality relation. However, the coordinate labelling of points of Ω has nothing to do with a construction of a manifold. The coordinate labelling of points may be used always, and it has no relation to a construction of generalized geometries. The coordinate labelling becomes to deal with the generalized geometry construction, when one imposes the condition $n_{\mathrm{c}} = n_{\mathrm{m}}$.

The relation $n_{\mathrm{c}} = n_{\mathrm{m}}$ is a special property of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$, and it may be wrong for many physical geometries, because physical geometries may have no definite metric dimension. Using the relation $n_{\mathrm{c}} = n_{\mathrm{m}}$ at the construction of a generalized geometry, one may meet such a situation, when the real space-time geometries appear beyond the scope of consideration.

2 Particle dynamics in the discrete space-time geometry

In the discrete space-time geometry there are no smooth world lines and no differential relations. The state of a particle cannot be described by its position and momentum, because the momentum is defined as a derivative along a smooth world line. In this case one uses the skeleton conception of particle dynamics [12]. According to this conception the particle state is described by its skeleton $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$. The skeleton is a discrete analog of the frame, attached rigidly to a physical body. Tracing the skeleton motion one traces the physical body motion. Dynamics of an elementary particle, having initial skeleton $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$, is described by the world chain

$$\mathcal{C}_{\mathcal{P}_n} = \bigcup_{k=0}^{\infty} \mathcal{P}_n^{(k)}, \qquad \mathcal{P}_n^{(s)} = \left\{P_0^{(s)}, P_1^{(s)}, ...P_n^{(s)}\right\}, \qquad \mathcal{P}_n^{(0)} = \mathcal{P}_n, \tag{2.1}$$

$$P_0^{(s+1)} = P_1^{(s)} \qquad s = 0, 1, 2, ... \tag{2.2}$$

Connection between adjacent links (skeletons) of the chain is realized by the relation (2.2). Direction of the skeleton evolution in the space-time is described by the leading vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)} = \mathbf{P}_0^{(s)}\mathbf{P}_0^{(s+1)}$. If the motion of the elementary particle is free, the adjacent links $\mathcal{P}_{(s)}^n$ and $\mathcal{P}_{(s+1)}^n$ are equivalent in the sense that

$$\mathcal{P}_n^{(s)} \mathrm{eqv} \mathcal{P}_n^{(s+1)}: \qquad \mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)} \mathrm{eqv} \mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}, \qquad i, k = 0, 1, ...n, \qquad s = ...0, 1, 2, ... \tag{2.3}$$

Relations (2.1) - (2.3) realizes coordinateless description of the free elementary particle motion. In the simplest case, when the space-time is the space-time of Minkowski, and the skeleton consists of two points P_0, P_1 with timelike leading vector $\mathbf{P}_0\mathbf{P}_1$, the coordinateless description by means of relations (2.1) - (2.3) coincides with the conventional description. The conventional classical dynamics is well defined only in the Riemannian space-time. The coordinateless dynamic description (2.1) - (2.3) of elementary particles is a generalization of the conventional classical dynamics onto the case of arbitrary space-time geometry.

We investigate now, whether a world chain with a spacelike leading vector may form a helix with timelike axis. If it is possible, then we try to investigate, under which world function such a situation is possible. We consider the world function $\sigma_{\rm g}$ of the form

$$\sigma_{\rm g}=\sigma_{\rm M}+\frac{\lambda_0^2}{2}f\left(\frac{\sigma_{\rm M}}{\sigma_0}\right),\qquad f(x)=\left\{\begin{array}{ccc}\mathrm{sgn}(x) & \text{if} & |x|>1\\ Cx+\varepsilon g(x) & \text{if} & |x|\leq 1\end{array}\right.,\qquad \sigma_0=\text{const}>0 \tag{2.4}$$

$$g(x)=-g(-x),\quad 0\leq\varepsilon\ll 1 \tag{2.5}$$

where C is a constant, which is determined from the relation

$$C+\varepsilon g(1)=1$$

The function $f\left(\frac{\sigma_{\rm M}}{\sigma_0}\right)$ should be determined from the condition that the world chain with spacelike leading vectors $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ forms a helix with timelike axis. The shape of the chain is determined by leading vectors.

To estimate the form of $\sigma_{\rm g}$ as a function of $\sigma_{\rm M}$ at $\sigma_{\rm M}<\sigma_0$, it is useful to consider the world chain, consisting only of spacelike leading vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_2\mathbf{P}_3$,...Other vectors of the skeleton will be considered later, when one needs to reduce the chain wobbling.

The world chain wobbling depends on difference between the number $N_{\rm dyn}$ of dynamic equations and the number $N_{\rm v}$ of dynamic variables. In the case of simplest skeleton with the number of points $n+1=2$, $N_{\rm dyn}=2$ and $N_{\rm v}=4$ the vectors of the chain are determined ambiguous, and world chain wobbles. In the case of timelike vectors the wobbling amplitude is of the order of λ_0, and one obtains quantum effects [13]. In the case of spacelike vectors the wobbling amplitude is infinite, as a result one cannot trace the world chain of spacelike vectors. It does not mean that the tachyon does not exist. It means simply that tachyon is unobservable.

If the number of the skeleton points $n+1=3$, $N_{\rm dyn}=6$ and $N_{\rm v}=8$. Although the difference $N_{\rm v}-N_{\rm dyn}=2$ is the same, the situation may be changed, because some skeleton vectors are timelike, and the wobbling amplitude may be reduced. As a result the particle (neutrino) with three-point skeleton and with the spacelike leading vector may appear to be observable. This case should be investigated.

Finally, if the number of the skeleton points $n+1=4$, $N_{\rm dyn}=N_{\rm v}=12$, one should expect that the wobbling will be absent.

The chain describes the free particle motion, and its links satisfy the equations (2.3) We suppose that the chain is a helix with timelike axis in the space-time. Let the points $...P_0,P_1,...$ have the coordinates

$$P_k=\left\{kl_0,R\cos(k\varphi),R\sin(k\varphi),0\right\},\qquad k=...0,1,2,... \tag{2.6}$$

All points (2.6) lie on a helix with timelike axis. The quantities R,l_0,φ are parameters of the chain. We suppose, that the radius R of the helix has macroscopic size. We investigate, if it is possible such a space-time geometry (2.4), that the world chain, consisting of connected vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_2\mathbf{P}_3$,... form a helix with macroscopic radius R, although its parameters l_0, $l_1=2R\sin\frac{\varphi}{2}$ are small in the sense, that

$$\left|l_0\right|,\left|l_1\right|<\sqrt{2\sigma_0},\quad l_1=2R\sin\frac{\varphi}{2} \tag{2.7}$$

To obtain connection between parameters l_0, l_1, φ, it is sufficient to solve equations, connecting adjacent leading vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, which have the form

$$\left(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2\right)_{\mathrm{g}}=\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{g}}^2 \tag{2.8}$$

$$\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{g}}^2=\left|\mathbf{P}_1\mathbf{P}_2\right|_{\mathrm{g}}^2 \tag{2.9}$$

Here index "g" means that the quantities are calculated in the space-time geometry $\mathcal{G}_{\mathrm{g}}$, whose world function σ_{g} is chosen in the form (2.4) where g is some function $g(x)=-g(-x)$, $x\in(-1,1)$ and $\varepsilon\ll1$.

We are to verify that two adjacent vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ satisfy the relations (2.8), (2.9), if

$$P_0=\{0,0,0,0\},\quad P_1=\{l_0,l_1,0,0\},\quad P_2=\{2l_0,l_1\cos\varphi,l_1\sin\varphi,0\} \tag{2.10}$$

and $l_0^2<l_1^2$. The points (2.10) correspond to three points of the helix (2.6). It is sufficient to verify, that the points (2.10) satisfy equations (2.8), (2.9), because in this case all other pairs of adjacent points (2.6) will satisfy equations of the form (2.8), (2.9).

It is important to keep in mind that the vectors

$$\mathbf{P}_0\mathbf{P}_1=\{l_0,l_1,0,0\},\quad \mathbf{P}_1\mathbf{P}_2=\{l_0,l_1(\cos\varphi-1),l_1\sin\varphi,0\} \tag{2.11}$$

are not unique solution of the equations (2.8), (2.9). There is a lot of other solutions, which lead to unpredictable wobbling of the world chain (2.6) [12]. Amplitude of this wobbling is infinite. The world chain of a pointlike particle, described by two-point skeleton $\mathcal{P}_2=\{P_0,P_1\}$ with spacelike vector $\mathbf{P}_0\mathbf{P}_1$, is unobservable, because it is impossible to trace such a world chain. One cannot trace the world chain, because the spatial distance between points P_s and P_{s+1} may be infinite in any coordinate system. It means that the statement of the relativity theory on impossibility of the tachyons existence is strongly overstated. Tachyons may exist, but they are unobservable.

However, the tachyon with the skeleton $\mathcal{P}_3=\{P_0,P_1,Q_1\}$ and spacelike leading vector $\mathbf{P}_0\mathbf{P}_1$ may be observable, because wobbling of its world chain is finite, and one may trace this world chain.

Considering equations (2.8), (2.9), we write them in the Minkowski space-time, setting

$$\sigma_{\mathrm{g}}(P_0,P_1)=\sigma_{\mathrm{M}}(P_0,P_1)+d(P_0,P_1),\quad d(P_0,P_1)\equiv\frac{\lambda_0^2}{2}f\left(\frac{\sigma_{\mathrm{M}}(P_0,P_1)}{\sigma_0}\right) \tag{2.12}$$

Then equations (2.8), (2.9) take the form

$$\left(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2\right)_{\mathrm{M}}+w(P_0,P_1,P_1,P_2)=\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{M}}^2+2d(P_0,P_1) \tag{2.13}$$

$$|\mathbf{P}_0\mathbf{P}_1|_{\mathrm{M}}^2 = |\mathbf{P}_1\mathbf{P}_2|_{\mathrm{M}}^2 \tag{2.14}$$

where

$$w(P_0, P_1, P_3, P_4) = d(P_0, P_4) + d(P_1, P_3) - d(P_0, P_3) - d(P_1, P_4) \tag{2.15}$$

Dynamic equations (2.13), (2.14) may be treated as a description of the particle motion in the space-time geometry of Minkowski under influence of force fields w and d . In other words, we pass from description in $\mathcal{G}_{\mathrm{g}}$ to description in the Minkowski space-time geometry $\mathcal{G}_{\mathrm{M}}$, introducing additional force fields, generated by the geometry $\mathcal{G}_{\mathrm{g}}$. Such a passage admits one to use conventional mathematical technique of the Minkowski geometry.

Further we shall use the scalar product only in the space-time of Minkowski. For brevity index "M" will be omitted. We present points (2.10) in the form

$$P_0 = \{0,0,0,0\}, \qquad P_1 = l, \qquad P_2 = l + q + \alpha \tag{2.16}$$

$$\mathbf{P}_0\mathbf{P}_1 = l, \quad \mathbf{P}_1\mathbf{P}_2 = q + \alpha, \quad \mathbf{P}_0\mathbf{P}_2 = l + q + \alpha \tag{2.17}$$

Here

$$l = \{l_0, l_1, 0, 0\}, \quad q = \{l_0, l_1\cos\varphi, l_1\sin\varphi, 0\}, \quad \alpha = \{\alpha_0, \alpha_1, \alpha_2, \alpha_3\} = \{\alpha_0, \boldsymbol{\alpha}\} \tag{2.18}$$

Vector α describes wobbling of the point P_2 near the "helical" position of the point $P_2 = l + q$.

To determine the form of the world function , we set $\alpha = 0$ in (2.16), (2.17). For $|\mathbf{P}_0\mathbf{P}_1|^2$, $|\mathbf{P}_1\mathbf{P}_2|^2$, $|\mathbf{P}_0\mathbf{P}_2|^2$ and w in (2.13) one obtains dynamic equations

$$|\mathbf{P}_0\mathbf{P}_1|_{\mathrm{M}}^2 = |\mathbf{P}_1\mathbf{P}_2|_{\mathrm{M}}^2 = 2\sigma_{\mathrm{M}}(P_0, P_1) = l_0^2 - l_1^2 \equiv l^2, \quad l_0^2 < l_1^2 < 0 \tag{2.19}$$

$$|\mathbf{P}_0\mathbf{P}_2|_{\mathrm{M}}^2 = 4l^2 + 4l_1^2\sin^2\frac{\varphi}{2}, \quad l^2 < 0, \quad l_0^2, l_1^2 < \sigma_0 \tag{2.20}$$

$$w(P_0, P_1, P_1, P_2) = \frac{\lambda_0^2}{2}\left(f\left(\frac{2l_1^2\sin^2\frac{\varphi}{2} + 2\left(l_0^2 - l_1^2\right)}{\sigma_0}\right) - 2f\left(\frac{l_0^2 - l_1^2}{2\sigma_0}\right)\right) \tag{2.21}$$

Setting

$$l^2 = l_0^2 - l_1^2 = -2\nu\sigma_0, \qquad \nu > 0 \tag{2.22}$$

$$a = \frac{2l_1^2}{\sigma_0}\sin^2\frac{\varphi}{2}, \qquad \varkappa = \frac{\sigma_0}{\lambda_0^2} \tag{2.23}$$

dynamic equation (2.8) may be written in the form

$$a\varkappa + f(a - 4\nu) = -4f(\nu) \tag{2.24}$$

Here the function f is an antisymmetric function, defined by the relation (2.4). Dynamic equation (2.6) transforms to the identity.

After a use of (2.4) equation (2.24) turns into

$$a(\varkappa + 1) - \varepsilon g(4\nu - a) + 4\varepsilon g(\nu) = 0 \tag{2.25}$$

As far as $a = O(\varepsilon)$, then

$$a(\varkappa + 1) - \varepsilon (g(4\nu) - ag'(4\nu) - 4g(\nu)) = O(\varepsilon^2) \tag{2.26}$$

$$a = \frac{\varepsilon (g(4\nu) - 4g(\nu))}{\varkappa + 1 - \varepsilon g'(4\nu)} = \frac{\varepsilon (g(4\nu) - 4g(\nu))}{\varkappa + 1} + O(\varepsilon^2) \tag{2.27}$$

It follows from (2.27), that a may be a small quantity, if $\varepsilon \ll 1$. According to (2.23) a must be positive. It is possible, if

$$g(4\nu) > 4g(\nu), \quad \nu > 0, \quad 0 < \varepsilon \ll 1 \tag{2.28}$$

According to (2.7) and (2.23) one obtains

$$R = \frac{l_1}{2\sin\frac{\varphi}{2}} = \frac{l_1^2}{\sqrt{2a\sigma_0}} = \frac{l_1}{\sqrt{\varepsilon}} \frac{\frac{l_1}{\sqrt{2\sigma_0}}\sqrt{1 + \frac{\sigma_0}{\lambda_0^2}}}{\sqrt{(g(4\nu) - 4g(\nu))}} \tag{2.29}$$

It means that the radius R of helix may be macroscopic, if ε is small enough.

The result obtained

$$\mathbf{P}_1\mathbf{P}_2 = q, \quad \mathbf{P}_0\mathbf{P}_2 = l + q \tag{2.30}$$

corresponds to position of the point P_2 on the helix (2.6). However, there are another solutions of equations (2.8), (2.9), where the point P_2 is described by relations (2.16) and vectors (2.17)

$$\mathbf{P}_1\mathbf{P}_2 = q + \alpha, \quad \mathbf{P}_0\mathbf{P}_2 = l + q + \alpha \tag{2.31}$$

Here vector α describes wobbling of the point P_2. It satisfies the dynamic equations

$$l^2 = (q + \alpha)^2 \tag{2.32}$$

$$(l.q + \alpha) + w(P_0, P_1, P_1, P_2) = l^2 + 2d\left(\frac{l^2}{2}\right) \tag{2.33}$$

which are reduced to the form

$$\alpha^2 + 2(q.\alpha) = 0 \tag{2.34}$$

$$2l_1^2 \sin^2\frac{\varphi}{2} + (l.\alpha) + \frac{\lambda_0^2}{2} f\left(\frac{2l^2 + 2l_1^2\sin^2\frac{\varphi}{2} + (l.\alpha)}{\sigma_0}\right) - 2\lambda_0^2 f\left(\frac{l^2}{2\sigma_0}\right) = 0 \tag{2.35}$$

Supposing that $(l.\alpha) = l_0\alpha_0 - \mathbf{l}\boldsymbol{\alpha}$ is a small quantity and expanding (2.35) over $(l.\alpha)$, one obtains from (2.35)

$$(l.\alpha) + \varepsilon \frac{\lambda_0^2}{2} g'\left(\frac{2l^2 + 2l_1^2\sin^2\frac{\varphi}{2}}{\sigma_0}\right)\frac{(l.\alpha)}{\sigma_0} = 0 \tag{2.36}$$

or

$$(l.\alpha) = l_0\alpha_0 - l_1\alpha_1 = 0, \quad \alpha_0 = \frac{l_1\alpha_1}{l_0} \tag{2.37}$$

Substituting α_0 from (2.37) in (2.34), one obtains

$$2\left(l_1 - l_1\cos\varphi\right)\alpha_1 - 2l_1\sin\varphi\alpha_2 + \left(\frac{l_1\alpha_1}{l_0}\right)^2 - \alpha_1^2 - \alpha_2^2 - \alpha_3^2 = 0 \qquad (2.38)$$

Taking into account that φ is small and setting for simplicity $\varphi = 0$, one obtains for spatial components of vector α

$$\left(\left(\frac{l_1}{l_0}\right)^2 - 1\right)\alpha_1^2 - \alpha_2^2 - \alpha_3^2 = 0 \qquad (2.39)$$

As far as $l_1^2 > l_0^2$, the first term in (2.39) is positive, and components of 3-vector $\boldsymbol{\alpha}$ may be infinitely large. It means that the wobbling amplitude is infinite. Thus, the helical world chain (2.6) with the two-point skeleton $\mathcal{P}_1^{(s)} = \left\{P_0^{(s)}, P_1^{(s)}\right\}$ is unstable with respect to metric wobbling.

3 Helical world chain with three-point skeleton

Suppression of the wobbling of the world chain, consisting of spacelike vectors, can be achieved, if we consider the world chain with composed links, whose skeleton consists of three points $\{P_k, P_{k+1}, Q_{k+1}\}$, $k = ...1.2, ...$ Let $\mathbf{P}_k\mathbf{P}_{k+1}$ be a spacelike vector, whereas the vector $\mathbf{P}_k\mathbf{Q}_{k+1}$ be a timelike vector in $\mathcal{G}_{\mathrm{M}}$. To investigate the effect of stabilization, it is sufficient to consider the points P_0, P_1, P_2, Q_1, Q_2, having coordinates

$$\begin{aligned} P_0 &= \{0\}, \qquad P_1 = \{l\}, \qquad P_2 = \{l+q+\alpha\}, \\ Q_1 &= \{s\}, \qquad Q_2 = \{s+q+\beta\}, \end{aligned} \qquad (3.1)$$

Corresponding vectors have the form

$$\mathbf{P}_0\mathbf{P}_1 = l, \qquad \mathbf{P}_1\mathbf{P}_2 = q+\alpha, \qquad \mathbf{P}_0\mathbf{P}_2 = l+q+\alpha, \qquad (3.2)$$

$$\mathbf{P}_0\mathbf{Q}_1 = s, \qquad \mathbf{P}_1\mathbf{Q}_2 = s+q-l+\beta, \qquad \mathbf{P}_0\mathbf{Q}_2 = s+q+\beta, \qquad (3.3)$$

$$\mathbf{P}_1\mathbf{Q}_1 = s-l, \qquad \mathbf{P}_2\mathbf{Q}_2 = s-l+\gamma, \qquad \mathbf{Q}_1\mathbf{Q}_2 = q+\beta, \qquad (3.4)$$

$$\mathbf{Q}_1\mathbf{P}_2 = l+q-s+\alpha, \qquad \gamma = \beta-\alpha \qquad (3.5)$$

$$l = \{l_0, l_1, 0, 0\} \qquad q = \{l_0, l_1\cos\varphi, l_1\sin\varphi, 0\}, \qquad s = \{s_0, 0, 0, 0\} \qquad (3.6)$$

Vector $\mathbf{P}_0\mathbf{Q}_1 = s$ is directed along the axis of the helix. Vectors α, β, $\gamma = \beta - \alpha$ are vectors describing wobbling, connected with points P_2 and Q_2. On needs to write six dynamic equations corresponding to equalities $\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_0\mathbf{Q}_1\mathrm{eqv}\mathbf{P}_1\mathbf{Q}_2$, and $\mathbf{P}_1\mathbf{Q}_1\mathrm{eqv}\mathbf{P}_2\mathbf{Q}_2$. Two equations, corresponding to $\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{P}_1\mathbf{P}_2$, have been written and investigated in the previous section: equations (2.8), (2.9)

One obtains for the case $\mathbf{P}_0\mathbf{Q}_1\mathrm{eqv}\mathbf{P}_1\mathbf{Q}_2$

$$s^2 = (s+q-l+\beta)^2 \qquad (3.7)$$

$$s^2 + (\beta.s) + w(P_0, Q_1, P_1, Q_2) = s^2 + 2d\left(\frac{s^2}{2}\right) \tag{3.8}$$

where according to (2.15) and (3.2) - (3.5)

$$-w(P_0, Q_1, P_1, Q_2) = d(P_0, Q_2) + d(Q_1, P_1) - d(P_0, P_1) - d(Q_1, Q_2)$$

$$= \frac{\lambda_0^2}{2}\left(f\left(\frac{(s+q+\beta)^2}{2\sigma_0}\right) + f\left(\frac{(s-l)^2}{2\sigma_0}\right) - f\left(\frac{l^2}{2\sigma_0}\right) - f\left(\frac{(q+\beta)^2}{2\sigma_0}\right)\right) \tag{3.9}$$

As far as $(s.q - l) = 0$, these equations are transformed to the form

$$2(s.\beta) + \beta^2 = 0 \tag{3.10}$$

$$(\beta.s) + \frac{\lambda_0^2}{2}\left(\begin{array}{c} f\left(\frac{(s+q+\beta)^2}{2\sigma_0}\right) + f\left(\frac{(s-l)^2}{2\sigma_0}\right) - f\left(\frac{l^2}{2\sigma_0}\right) \\ -f\left(\frac{(q+\beta)^2}{2\sigma_0}\right) - 2f\left(\frac{s^2}{2\sigma_0}\right) \end{array}\right) = 0 \tag{3.11}$$

The necessary condition of the fact, that $\beta = 0$, has the form

$$f\left(\frac{(s+q)^2}{2\sigma_0}\right) + f\left(\frac{(s-l)^2}{2\sigma_0}\right) - fd\left(\frac{l^2}{2\sigma_0}\right) - fd\left(\frac{s^2}{2\sigma_0}\right) = 0 \tag{3.12}$$

Substituting f from (2.4) in (3.12), equation (3.12) takes the form

$$\varepsilon g\left(\frac{(s_0+l_0)^2 - l_1^2}{2\sigma_0}\right) + \varepsilon g\left(\frac{(s_0-l_0)^2 - l_1^2}{2\sigma_0}\right) - 2\varepsilon g\left(\frac{l_0^2 - l_1^2}{2\sigma_0}\right) - 2\varepsilon g\left(\frac{s_0^2}{2\sigma_0}\right) = 0 \tag{3.13}$$

It determines the connection between parameters l_0, l_1, s_0 of the helical world chain. This connection depends on the form of function g. One should verify that there exist such functions g, for which equation (3.13) admits solutions $l_0^2, l_1^2, s_0^2 < \sigma_0$ and $l_0^2 < l_1^2$. Let us show that such a function g does exist. In particular, if

$$g(x) = x^3 \tag{3.14}$$

equation (3.13) takes the form

$$\left((s_0+l_0)^2 - l_1^2\right)^3 + \left((s_0-l_0)^2 - l_1^2\right)^3 - 2\left(l_0^2 - l_1^2\right)^3 - 2\left(s_0^2\right)^3 = 0 \tag{3.15}$$

After simplification (3.15) is reduced to equation

$$-6s_0^2\left(l_1^2 - 5l_0^2\right)\left(l_0^2 - l_1^2 + s_0^2\right) = 0 \tag{3.16}$$

which admits solutions satisfying the relation

$$l_0^2, l_1^2, s_0^2 < \sigma_0 \quad \text{and } l_0^2 = \frac{l_1^2}{5} < l_1^2 \tag{3.17}$$

Let us now solve equations (3.10), (3.11) with respect to β, supposing that parameters l_0^2, l_1^2, s_0^2 satisfy relations (3.16), (3.17). Let us suppose that β is a small quantity and expand equation (3.11) over β. Keeping in mind (3.13), one obtains

$$(\beta.s) + \frac{\lambda_0^2}{2}\varepsilon g'\left(\frac{(s+q)^2}{2\sigma_0}\right)\left(\frac{(2(s+q).\beta)+\beta^2}{2\sigma_0}\right) - \frac{\lambda_0^2}{2}\varepsilon g'\left(\frac{q^2}{2\sigma_0}\right)\frac{2(q.\beta)+\beta^2}{2\sigma_0} = 0 \tag{3.18}$$

Taking into account (3.10), one obtains from (3.18)

$$(\beta.s)\left(1+\frac{\lambda_0^2}{2\sigma_0}\varepsilon g'\left(\frac{(s+q)^2}{2\sigma_0}\right)\right) = 0 \tag{3.19}$$

It means that

$$(\beta.s) = \beta_0 s_0 = 0, \quad \beta_0 = 0 \tag{3.20}$$

Then it follows from (3.20) and (3.10) that

$$\beta^2 = \beta_0^2 - \boldsymbol{\beta}^2 = 0, \quad \beta_k = 0, \quad k = 0,1,2,3 \tag{3.21}$$

Thus, the vector $\mathbf{P}_0\mathbf{Q}_1$ directed along the helix axis does not wobbles at all. In other words the wobbling of point $\mathbf{Q}_2$ is absent.

We obtain from the condition $\mathbf{P}_1\mathbf{Q}_1\mathrm{eqv}\mathbf{P}_2\mathbf{Q}_2$

$$(s-l)^2 = (s-l+\gamma)^2 \tag{3.22}$$

$$(s-l.s-l+\gamma) + w(P_1,Q_1,P_2,Q_2) = (s-l)^2 + 2d\left(\frac{(s-l)^2}{2}\right) \tag{3.23}$$

where according to (2.15) and (3.2) - (3.5)

$$w(P_1,Q_1,P_2,Q_2) = d(\sigma_{\mathrm{M}}(P_1,Q_2)) + d(\sigma_{\mathrm{M}}(Q_1,P_2)) - d(\sigma_{\mathrm{M}}(P_1,P_2)) - d(\sigma_{\mathrm{M}}(Q_1,Q_2))$$

$$= \frac{\lambda_0^2}{2}f\left(\frac{(s+q-l+\beta)^2}{2\sigma_0}\right) + \frac{\lambda_0^2}{2}f\left(\frac{(l+q-s+\alpha)^2}{2\sigma_0}\right) - \frac{\lambda_0^2}{2}f\left(\frac{l^2}{2\sigma_0}\right) - \frac{\lambda_0^2}{2}f\left(\frac{(q+\beta)^2}{2\sigma_0}\right) \tag{3.24}$$

Equations (3.22) and (3.23) take the form

$$\gamma^2 + 2((s-l).\gamma) = 0, \quad \gamma = \beta - \alpha \tag{3.25}$$

$$\begin{aligned}((s-l).\gamma) &+ \frac{\lambda_0^2}{2}f\left(\frac{(s+q-l+\beta)^2}{2\sigma_0}\right) + \frac{\lambda_0^2}{2}f\left(\frac{(l+q-s+\alpha)^2}{2\sigma_0}\right) \\ -\frac{\lambda_0^2}{2}f\left(\frac{l^2}{2\sigma_0}\right) &- \frac{\lambda_0^2}{2}f\left(\frac{(q+\beta)^2}{2\sigma_0}\right) - \lambda_0^2 f\left(\frac{(s-l)^2}{2\sigma_0}\right) = 0\end{aligned} \tag{3.26}$$

In the case $\alpha = \beta = \gamma = 0$ equation (3.26) turns to the equation

$$\varepsilon \left(g \left(\frac{(s+q-l)^2}{2\sigma_0} \right) + g \left(\frac{(l+q-s)^2}{2\sigma_0} \right) - 2g \left(\frac{l^2}{2\sigma_0} \right) - 2g \left(\frac{(s-l)^2}{2\sigma_0} \right) \right) = 0 \tag{3.27}$$

One should show that there exist such a function g that the system of two equations (3.13) and (3.27) has a solution for parameters l_0, l_1, s_0 of the helix (2.6). For equation (3.13) such a solution has been obtained for the function g of the form (3.14). For this form of the function g equation (3.27) takes the form

$$\begin{aligned} &\left(s_0^2 - 2l_1^2 (1-\cos\varphi)\right)^3 + \left((s_0 - 2l_0)^2 - 2l_1^2 (1+\cos\varphi)\right)^3 \\ &-2\left(l_0^2 - l_1^2\right)^3 - 2\left((s_0 - l_0)^2 - l_1^2\right)^3 = 0 \end{aligned} \tag{3.28}$$

After simplification this equation takes the form

$$6\left(2l_0^2 - 2l_0 s_0 - 2l_1^2 + s_0^2\right) \left(\begin{array}{c} 5l_0^4 - 10l_0^3 s_0 - 8l_0^2 l_1^2 \cos\varphi - 2l_0^2 l_1^2 + 5l_0^2 s_0^2 \\ +8l_0 l_1^2 s_0 \cos\varphi + 2l_0 l_1^2 s_0 + 4l_1^4 \cos^2\varphi + l_1^4 - l_1^2 s_0^2 \end{array} \right) = 0 \tag{3.29}$$

Comparing (3.29) with (3.16)

$$-6s_0^2 \left(l_1^2 - 5l_0^2\right)\left(l_0^2 - l_1^2 + s_0^2\right) = 0$$

we see that the system of the two equations has the solution

$$(s_0 - l_0)^2 = 2l_1^2 - l_0^2, \quad s_0 = l_0 \pm \sqrt{2l_1^2 - l_0^2} \tag{3.30}$$

$$l_1 = \pm\sqrt{5} l_0, \quad s_0 = l_0 \pm 3l_0 = l_0 \left\{ \begin{array}{c} 4 \\ -2 \end{array} \right. \tag{3.31}$$

If the ratio l_0^2/σ_0 is small enough, the lengths of all vectors (3.2) -(3.5) are less, than $\sqrt{2\sigma_0}$, and application of that part of the world function (2.4), where $|\sigma_{\rm M}| < \sigma_0$, is justified. As a result we obtain the rather rigid connection between the helical world chain parameters. One should expect that solution exist for other forms of the functions g.

4 Stabilization of helical world chain with three-point skeleton

To obtain additional constraints, imposed on the wobbling vector α, we return to equations (2.34), (3.25), and (3.26). Let us assume that conditions (3.27) and (3.13) are fulfilled due to a proper choice of function g. Then keeping in mind that according to (3.20) $\beta = 0$. Equations (2.34), (3.25), and (3.26) are written in the form

$$\alpha^2 + 2(q.\alpha) = 0 \tag{4.1}$$

$$\alpha^2 - 2\left(\left(s-l\right).\alpha\right) = 0 \tag{4.2}$$

$$-\left(\left(s-l\right).\alpha\right) + \frac{\lambda_0^2}{2} f\left(\frac{\left(s+q-l\right)^2}{2\sigma_0}\right) + \frac{\lambda_0^2}{2} f\left(\frac{\left(l+q-s+\alpha\right)^2}{2\sigma_0}\right)$$
$$-\frac{\lambda_0^2}{2} f\left(\frac{l^2}{2\sigma_0}\right) - \frac{\lambda_0^2}{2} f\left(\frac{q^2}{2\sigma_0}\right) - \lambda_0^2 f\left(\frac{\left(s-l\right)^2}{2\sigma_0}\right) = 0 \tag{4.3}$$

Supposing that α is a small quantity, we expand equation (4.3) over powers of α. Taking into account (2.4), and (3.27), one obtains from (4.3)

$$-\left(\left(s-l\right).\alpha\right) + \frac{\lambda_0^2}{2}\varepsilon g'\left(\frac{\left(l+q-s\right)^2}{2\sigma_0}\right) \frac{2\left(l+q-s.\alpha\right)+\alpha^2}{2\sigma_0} = 0 \tag{4.4}$$

Substituting α^2 from (4.1) in (4.4), one obtains

$$\left(l-s.\alpha\right)\left(1+\frac{\lambda_0^2}{2\sigma_0}\varepsilon g'\left(\frac{\left(l+q-s\right)^2}{2\sigma_0}\right)\right) = 0 \tag{4.5}$$

It follows from (4.5) that

$$\left(l-s.\alpha\right) = 0 \tag{4.6}$$

From (4.2) and (4.6) one obtains that

$$\alpha^2 = 0 \tag{4.7}$$

If vector $s-l$ is timelike $\left(s-l\right)^2 > 0$, it follows from (4.6) and (4.7) that vector $\alpha = 0$. In the partial case, when the function g is determined by (3.14) we have according to (3.31) that $s_0 = 4l_0$, $l_1^2 = 5l_0^2$, and hence

$$\left(s-l\right)^2 = \left(s_0-l_0\right)^2 - l_1^2 = 4l_0^2 > 0 \tag{4.8}$$

In this case

$$\alpha_k = 0, \quad k = 0,1,2,3 \tag{4.9}$$

and wobbling of the helical world line (2.6) is absent.

In the general case one obtains from (4.1) and (4.7) that

$$\alpha_0 l_0 - \alpha_1 l_1 \cos\varphi - \alpha_2 l_1 \sin\varphi = 0$$
$$\alpha_0 = -\frac{l_1\left(\alpha_1\cos\varphi + \alpha_2\sin\varphi\right)}{l_0} \tag{4.10}$$

Substituting (4.10) in (4.6) and (4.7), one obtains

$$-\left(s_0-l_0\right)\frac{l_1\left(\alpha_1\cos\varphi+\alpha_2\sin\varphi\right)}{l_0} - l_1\alpha_1 = 0 \tag{4.11}$$

$$\left(\frac{l_1\left(\alpha_1\cos\varphi+\alpha_2\sin\varphi\right)}{l_0}\right)^2-\alpha_1^2-\alpha_2^2-\alpha_3^2=0 \tag{4.12}$$

If the radius R of the helix is macroscopic, the angle φ is a small quantity. Then one may set $\varphi=0$. It follows from (4.11) (4.10) that $\alpha_0=\alpha_1=0$, and one obtains from (4.12)

$$\alpha_2^2+\alpha_3^2=0$$

As a result one obtains (4.9).

Thus, the helical world chain (2.6) is stable for large radius R, provided it is described by the three point skeleton with the vector $\mathbf{P}_0\mathbf{Q}_1$, directed along the timelike helix axis. The coordinate system K, where the vector $\mathbf{P}_0\mathbf{Q}_1$ is directed along the time axis of the coordinate system K may be considered as a coordinate system, where the particle (neutrino) is at rest. Such a statement is based on the fact that the mean 4-momentum of the particle is directed along the timelike vector $\mathbf{P}_0\mathbf{Q}_1$, which is the temporal basic vector of the coordinate system K. In reality the particle rotates in this coordinate system K with the superluminal velocity. Speaking about the coordinate system K, we mean the coordinate system in the geometry $\mathcal{G}_{\mathrm{M}}$ of Minkowski, which is associated with the discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$. This coordinate system K is used simply for labelling of points of the space-time, which is not a manifold.

5 Simulation of the OPERA experiment

The principal space-time scheme of the OPERA experiment is shown in the figure. Two vertical lines are world lines of radiator and of detector. Neutrino and photon are radiated simultaneously at the time moment $t=0$ at the origin of the coordinate system. The photon is detected at the time T_{L}. The neutrino world line is replaced by a world tube. The surface of the world tube is formed by helical world line of neutrino. The neutrino may be detected practically at any point of the tube surface. In the figure the projection of the tube on two-dimensional section of the space-time is shown. World lines of the anterior and back fronts of the tube are presented by inclined lines. The neutrino may be detected at any point between these fronts. The time of detecting neutrino lies in interval $(t_{\min},t_{\max})$. Time of passage of the neutrino world tube through the radiator position is $2t_{\mathrm{in}}$, $t_{\max}-t_{\min}=2t_{\mathrm{in}}$. Let the distance between the radiator and detector be L, and the neutrino velocity be V. The radius of the neutrino world tube is R. The distance between the fronts in the motionless coordinate system reduces to $2R\sqrt{1-c^{-2}V^2}$

We obtain

$$t_{\mathrm{in}}=\frac{R\sqrt{1-\beta^2}}{V},\qquad \beta=\frac{V}{c} \tag{5.1}$$

$$T_{\mathrm{L}}=\frac{L}{c},\qquad t_{\min}=\frac{L}{V}-t_{\mathrm{in}}=\frac{L}{V}-\frac{R}{V}\sqrt{1-\beta^2} \tag{5.2}$$

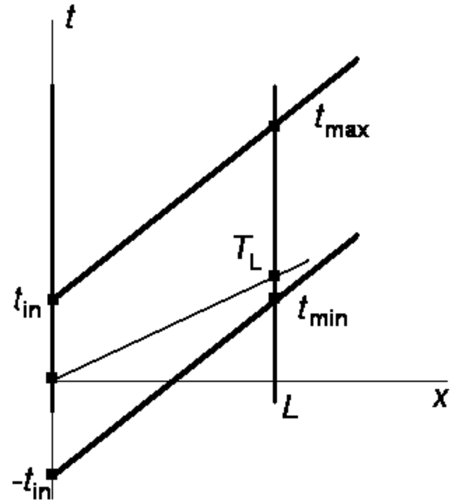

Figure 1: Space-time scheme of neutrino propagation

$$T_{\rm L}-t_{\rm min}=\frac{L}{c}-\left(\frac{L}{V}-\frac{R}{V}\sqrt{1-\beta^2}\right)=\frac{L}{V}\left(-1+\beta+\frac{R\sqrt{1-\beta^2}}{L}\right) \tag{5.3}$$

or

$$T_{\rm L}-t_{\rm min}=\frac{L}{V}(1-\beta)\left(\frac{R}{L}\sqrt{\frac{1+\beta}{1-\beta}}-1\right) \tag{5.4}$$

As far as $\varepsilon=1-\beta\ll 1$, the retardation of the photon detection with respect to the neutrino detection may be written in the form

$$\Delta t=T_{\rm L}-t_{\rm min}=\frac{\sqrt{\varepsilon}}{c}\left(R\sqrt{2}-L\sqrt{\varepsilon}\right) \tag{5.5}$$

In the OPERA experiment the quantities Δt and L are known. We try to estimate the minimally possible radius of the neutrino world tube. We have

$$R>\frac{L\sqrt{\varepsilon}}{\sqrt{2}},\quad \Delta t<\frac{\sqrt{\varepsilon}}{c}R\sqrt{2},\quad R>\frac{c\Delta t}{\sqrt{2\varepsilon}}$$

The radius R of the neutrino world tube may be minimal, if

$$\frac{L\sqrt{\varepsilon}}{\sqrt{2}}=\frac{c\Delta t}{\sqrt{2\varepsilon}},\quad \varepsilon=\frac{c\Delta t}{L},\quad R>\frac{L\sqrt{\varepsilon}}{\sqrt{2}}=2^{-1/2}\sqrt{Lc\Delta t} \tag{5.6}$$

According to the results of the OPERA experiment [7]

$$L\simeq 7.3\times 10^7{\rm cm},\quad \Delta t\simeq 6\times 10^{-8}{\rm s} \tag{5.7}$$

Estimation of the neutrino world tube radius has the form

$$R>2^{-1/4}\sqrt{Lc\Delta t}=2.5\times 10^5{\rm cm}\simeq 2.5{\rm km} \tag{5.8}$$

One obtains from (5.6)

$$\varepsilon = \sqrt{2}\frac{c\Delta t}{L} \approx 3.5 \times 10^{-5} \tag{5.9}$$

The neutrino Lorentz factor γ

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{2\varepsilon}} =\simeq 1.2 \times 10^2 \tag{5.10}$$

According to (5.5) the retardation of the photon with respect to neutrino is not proportional to the distance L. On the contrary, at fixed parameters of neutrino an increase of L leads to a reduction of the retardation Δt. It means, that the OPERA effect does not evidence the superluminal mean speed of neutrino. This fact may be tested by other experiment with other value of the distance L between the radiator and detector. This is conditioned by the fact that the retardation of the photon with respect to neutrino depends essentially on the phase of the helical world chain at the moment of the neutrino detection. As one can see on the figure the moment $t_{\rm d}$ of the neutrino detection lies in the interval $(t_{\rm min}, t_{\rm max})$.

6 Discussion

The lengths of vectors (3.2) - (3.5), which enter in dynamic equations are shorter, than $\sqrt{2\sigma_0}$, where the world function $\sigma_{\rm g}$ differs from the $\sigma_{\rm M}$ only by a gage factor and by a small addition

$$\sigma_{\rm g} = \left(1 + \frac{\lambda_0^2}{2\sigma_0}C\right)\sigma_{\rm M} + \varepsilon\frac{\lambda_0^2}{2}g\left(\frac{\sigma_{\rm M}}{\sigma_0}\right), \quad |\sigma_{\rm M}| < \sigma_0, \quad \varepsilon \ll 1 \tag{6.1}$$

As a result the spacelike world chain (2.6) differs slightly from the straight line. Stabilization of the world chain is achieved due to two timelike vectors $\mathbf{P}_0\mathbf{Q}_1$ and $\mathbf{P}_1\mathbf{Q}_1$ in the skeleton $\mathcal{P}_3 = \{P_0, P_1, Q_1\}$ of the world chain. Quantum wobbling of these timelike vectors is absent because of small term $\varepsilon\lambda_0^2 g/2$ for $|\sigma_{\rm g}| < \sigma_0$ added to $\sigma_{\rm M}$ (instead of $\lambda_0^2 \text{sgn}(\sigma_{\rm M})/2$ for $|\sigma_{\rm g}| > \sigma_0$, where the quantum wobbling takes place).

In the conventional approach to the space-time geometry the spacelike world lines (with superluminal velocities) are absent, because the tachyons have not been discovered experimentally. This experimental fact is considered as a principle of the relativity theory. However, it is too strong generalization of the experimental fact. In reality, tachyons may exist, but they cannot be traced because of infinite wobbling amplitude. Tachyons have not electric charge, and they cannot be discovered by means of the electromagnetic interaction. However, the gravitational interaction of the tachyon gas may be observed. Maybe, the tachyon gas contributes to so called dark matter, which has been discovered around galaxies. Due to great tachyons mobility they can form powerful "tachyionspheres" around galaxies, changing essentially their gravitational fields. The radius of the tachyon sphere is greater, than the radius of a galaxy. As a result the rotation velocities of stars do not decrease, if their distance from the galaxy center increases. Such a situation seems to be rather believable. Of course, one should investigate gravitational properties of the tachyon gas for certain conclusions.

One should stress that a use of the space-time geometry with indefinite metric dimension n_{m} is not a hypothesis. It is a corollary of the fact, that one does not take into account the numerous restrictions (1.14), which describe special properties of the proper Euclidean geometry. Considering these properties as general geometric ones, one restricts strongly the set of possible space-time geometries. Thus, we do not use fitting (invention of new hypotheses). We simply ignore unjustified supposition on definite metric dimension n_{m} of the space-time geometry. At the same time we may label space-time points by coordinates, and the number n_{c} of coordinates (coordinate dimension) is a definite natural number. The coordinate dimension n_{c} is an attribute of the geometry description. It has nothing to do with the metric dimension n_{m} which is an attribute of the Euclidean geometry itself. Coincidence $n_{\mathrm{m}} = n_{\mathrm{c}}$ is a corollary of unjustified restrictions, taken from the proper Euclidean geometry.

The difference between n_{m} and n_{c} is perceived hardly, because in the conventional presentation of differential (Riemannian) geometry one starts from consideration of a smooth manifold, where $n_{\mathrm{m}} = n_{\mathrm{c}}$. One considers the dimension $n = n_{\mathrm{m}} = n_{\mathrm{c}}$ as a natural number, and one does not think on existence of restrictions (1.14), which appear only in the σ-representation of the proper Euclidean geometry. In order one may introduce the dimension $n = n_{\mathrm{m}}$, the numerous restrictions (1.14) have to be fulfilled. These restrictions are special properties of the proper Euclidean geometry. Unfortunately, one does not take into account this fact and believes that there exist only geometries with a definite metric dimension. One believes that there are no geometries with indefinite metric dimension $n = n_{\mathrm{m}}$. If the world function of a generalized geometry is a such one, that several of numerous constraints (1.14) are not fulfilled, one cannot introduce the metric dimension n_{m}, But a coordinate system with the coordinate dimension $n = n_{\mathrm{c}}$ can be introduced always independently of the restrictions (1.14). The coordinate system depends on the point set Ω, where the geometry is given, but not on the world function of the generalized geometry.

Unfortunately, one does not differ usually between the metric dimension n_{m} and the coordinate dimension n_{c}. One suppose usually, that giving coordinate system and its dimension n_{c}, one determines also n_{m}. Application of the discrete (or granular) space-time geometry is conceptual, because in this case the spacelike world chains (superluminal velocities) are admissible. Simultaneously one explains, why tachyons cannot be discovered.

7 Concluding remarks

Describing real space-time geometry in microcosm, one should take into account all possible kinds of geometries. It is an incorrect use only those geometries, where metric dimension $n_{\rm m}$ coincides with the coordinate dimension $n_{\rm c}$. The fact that we do not know properties of "dimensionless" space-time geometries cannot be a reason for disregard of them. Such a disregard leads to the supposition on impossibility of superluminal velocities, based on absence of the tachyons observation. The lack of tachyon observation means only that tracing of tachyons is impossible, but it does not mean, that tachyons do not exist. Tachyons may be discovered in the form of neutrino, which is a bound tachyon whose mean velocity is less, than the speed of the light.

The main problem of the microcosm physics lies in the fact, that we do not know discrete (dimensionless) geometries and do not possess coordinateless technique of these geometries. Progress in the development of discrete geometries is connected with the progress in the development of the world function technique. First it appeared as a method of description of the Riemannian geometry [14, 15]. Description of a geometry in terms of scalar world function is more effective, than description in terms of metric tensor g_{ik}, because description in terms of a scalar function admits one to use a coordinateless description. Monistic conception of geometry, when the Euclidean geometry is described in terms of one scalar quantity admits one to discover hidden connections between different basic concepts of geometry (manifold, dimension , coordinate system, etc.). Some connections appeared to be specific Euclidean connections, which prevent from construction of more general geometries and from coordinateless geometry description.

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