Explanation of OPERA effect in skeleton conception of the elementary particle dynamics

Yuri A.Rylov

Institute for Problems in Mechanics, Russian Academy of Sciences,
101-1, Vernadskii Ave., Moscow, 119526, Russia.
e-mail: rylov@ipmnet.ru
Web site: *http : //rsfq1.physics.sunysb.edu/˜rylov/yrylov.htm*
or mirror Web site:
http : //gasdyn − ipm.ipmnet.ru/˜rylov/yrylov.htm

Abstract

The OPERA experiment does not measure the neutrino velocity. It is an end effect conditioned by tubular character of the neutrino world chain (line). This world chain is a helix with timelike axis. On average this world chain looks as a timelike straight tube. Such a structure of the world chain is conditioned by a discrete space-time geometry, where elementary length is connected with the quantum constant. In the discrete space-time geometry there are no smooth world lines. The state of a particle is described by its skeleton (several space-time points connected rigidly). The particle evolution is described by world chains, which wobble. Statistical description of this wobbling is equivalent to quantum description. Pointlike tachyons are possible, but they are nonobservable. Composite tachyons (with many-point skeleton) are observable in the form of fermions.

Key words: geometric paradigm; helical world line; superluminal velocity

1 Introduction

Experiment [1] shows, that neutrinos pass the distance 730km faster, than the light signal. The time of the lead is about 60.7 ± 6.9ns. It corresponds to difference $(v - c)\, c^{-1} \approx 3 \times 10^{-5}$. This experiment is interpreted as a discovery of superluminal speed of the neutrino mean motion, and generates problems, connected with the relativity principles. Such interpretation is based on the supposition, that the mean neutrino motion is described by one-dimensional straight world line. If the mean neutrino motion is described by a world line tube, the OPERA experiment is explained freely, as an effect, conditioned by the neutrino world tube thickness.

The world tube for neutrino description is obtained in the skeleton conception of the elementary particle dynamics [2]. In this conception the world line of any fermion is a helix with timelike axis. The helix may be spacelike or timelike. In any case such a world line is perceived on the average as a thick world line (world tube), describing a pointlike particle, which can be detected at any point of the world tube surface.

In general, the quantum mechanics is a kind of a theory of the continuous medium. Madelung [3], L. de Broglie [4], and Bohm [5] have shown, that the Schrödinger equation can be reduced to a nonrotational flow of some quantum fluid. Besides, it appears, that the reverse statement is true. The wave function can be considered as a way of description of any ideal fluid [6]. The statistical ensemble of stochastic particles looks as an ideal continuous medium (fluid). It is natural, that it can be described by a wave function.

However, equations of continuous medium describe only regular component of the continuous medium. Physical phenomena conditioned by the irregular (chaotic) component of the gas motion cannot be described by the gas dynamics equations. For instance, the Brownian motion of dust particles in a gas cannot be described by the gas dynamics equations. The OPERA effect is a physical phenomenon which is connected with the helical shape of the neutrino world line (rotational motion of neutrino). This rotation forms an irregular component of the neutrino motion. The rotation is connected with γ-matrices in the Dirac equation. This rotation is characteristic for any fermion, i.e. a particle described by the Dirac equation. This rotation known as zitterbewegung is described by dynamic variables, which appear after a change of variables. This change of variables excludes γ-matrices from dynamic equations.

However, it is very difficult to explain a very rapid rotation of a free particle. If this particle is one of two rigidly connected particle, then the rotation can be explained. However, it is difficult to understand the nature of this rigid coupling. This helical shape of the neutrino world line can be explained in the framework of the skeleton conception of the particle dynamics [2].

The skeleton conception of elementary particle dynamics is based on a discrete space-time geometry. In the discrete space-time geometry there is an elementary length λ_0, which is a new characteristic of the space-time geometry. The elementary length λ_0 is connected with the quantum constant $\hbar$ ($\lambda_0^2 = \hbar/bc$, where b and c are universal constants), and all quantum effects may be explained as geometrical effects of the discrete space-time geometry. There is no necessity of using quantum principles in the discrete space-time geometry.

The conventional conception of the elementary particle dynamics is based on the continuous space-time geometry. As far as continuous space-time geometry together with classical conception of dynamics cannot describe correctly physical phenomena in microcosm, the classical dynamics was replaced by the quantum dynamics.

The discrete space-time geometry is based on the metric approach to geometry [7, 8]. Metric approach admits one to consider physical geometries, which are described completely by their world function. The set of physical geometries is more powerful, than the set of differential geometries. The physical geometries are nonaxiomatizable, and their mathematical formalism distinguishes strongly from that of differential geometries. A discrete geometry is a special case of a physical geometry. Description of physical geometries was developed in papers [9] - [16]. Relation of mathematicians to physical geometries reminds relations of mathematicians to geometry of Lobachevski - Bolyai in the beginning of XIX century. As a result a very simple mathematical formalism of the discrete geometry is slightly known now.

In this paper we shall explain the OPERA experiment as an effect of a "thick world line". We shall show, how the helical shape of the world line can be deduced from the Dirac equation. Thereafter we shall describe briefly concepts of discrete space-time geometry and skeleton conception of elementary particle dynamics, based on the discrete space-time geometry.

2 Simulation of the OPERA experiment

The principal space-time scheme of the OPERA experiment is shown in the figure. Two vertical lines are world lines of radiator and of detector. Neutrino and photon are radiated simultaneously at the time moment $t = 0$ at the origin of the coordinate system. The photon is detected at the time $T_{\rm L}$. The neutrino world line is replaced by a world tube. The surface of the world tube is formed by helical world line of neutrino. The neutrino may be detected practically at any point of the tube surface. In the figure the projection of the tube on two-dimensional section of the space-time is shown. World lines of the anterior and back fronts of the tube are presented by inclined lines. The neutrino may be detected at any point between these fronts. The time of detecting neutrino lies in interval $(t_{\rm min}, t_{\rm max})$. Time of passage of the neutrino world tube through the radiator position is $2t_{\rm in}$, $t_{\rm max} - t_{\rm min} = 2t_{\rm in}$. Let the distance between the radiator and detector be L, and the neutrino velocity be V. The radius of the neutrino world tube is R. The distance between the fronts in the motionless coordinate system reduces to $2R\sqrt{1-c^{-2}V^2}$

We obtain

$$t_{\rm in} = \frac{R\sqrt{1-\beta^2}}{V}, \qquad \beta = \frac{V}{c} \tag{2.1}$$

$$T_{\rm L} = \frac{L}{c}, \qquad t_{\rm min} = \frac{L}{V} - t_{\rm in} = \frac{L}{V} - \frac{R}{V}\sqrt{1-\beta^2} \tag{2.2}$$

$$T_{\rm L} - t_{\rm min} = \frac{L}{c} - \left(\frac{L}{V} - \frac{R}{V}\sqrt{1-\beta^2}\right) = \frac{L}{V}\left(-1+\beta+\frac{R\sqrt{1-\beta^2}}{L}\right) \tag{2.3}$$

or

$$T_{\rm L} - t_{\rm min} = \frac{L}{V}(1-\beta)\left(\frac{R}{L}\sqrt{\frac{1+\beta}{1-\beta}} - 1\right) \tag{2.4}$$

As far as $\varepsilon = 1 - \beta \ll 1$, the retardation of the photon detection with respect to the neutrino detection may be written in the form

$$\Delta t = T_{\rm L} - t_{\rm min} = \frac{\sqrt{\varepsilon}}{c}\left(R\sqrt{2} - L\sqrt{\varepsilon}\right) \tag{2.5}$$

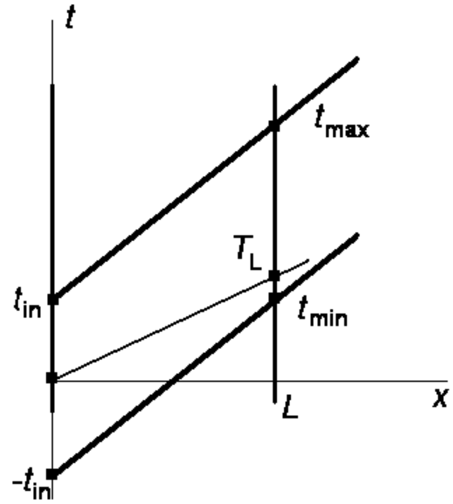

Figure 1: Space-time scheme of neutrino propagation

In the OPERA experiment the quantities Δt and L are known. We try to estimate the minimally possible radius of the neutrino world tube. We have

$$R > \frac{L\sqrt{\varepsilon}}{\sqrt{2}}, \quad \Delta t < \frac{\sqrt{\varepsilon}}{c} R\sqrt{2}, \quad R > \frac{c\Delta t}{\sqrt{2\varepsilon}}$$

The radius R of the neutrino world tube may be minimal, if

$$\frac{L\sqrt{\varepsilon}}{\sqrt{2}} = \frac{c\Delta t}{\sqrt{2\varepsilon}}, \quad \varepsilon = \frac{c\Delta t}{L}, \quad R > \frac{L\sqrt{\varepsilon}}{\sqrt{2}} = 2^{-1/2}\sqrt{Lc\Delta t} \tag{2.6}$$

According to the results of the OPERA experiment [1]

$$L \simeq 7.3 \times 10^7 \text{cm}, \quad \Delta t \simeq 6 \times 10^{-8} \text{s} \tag{2.7}$$

Estimation of the neutrino world tube radius has the form

$$R > 2^{-1/4}\sqrt{Lc\Delta t} = 2.5 \times 10^5 \text{cm} \simeq 2.5 \text{km} \tag{2.8}$$

One obtains from (2.6)

$$\varepsilon = \sqrt{2}\frac{c\Delta t}{L} \approx 3.5 \times 10^{-5} \tag{2.9}$$

The neutrino Lorentz factor γ

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{2\varepsilon}} =\simeq 1.2 \times 10^2 \tag{2.10}$$

According to (2.5) the retardation of the photon with respect to neutrino is not proportional to the distance L. On the contrary, at fixed parameters of neutrino an increase of L leads to a reduction of the retardation Δt. It means, that the OPERA effect does not evidence the superluminal mean speed of neutrino. This fact may be tested by other experiment with other value of the distance L.

Description of the neutrino world line by means of a tube is demonstrative. However, it is rather rough, since it does not take into account the rotation phase of the helical world line.

Let instead of the tube one considers a helical world line. In the coordinate system K', where neutrino is at rest it is described by the relations

$$x'^0 = c\tau, \quad x' = R\sin(\Omega\tau), \quad y' = R\cos(\Omega\tau), \quad z' = 0 \tag{2.11}$$

In the coordinate system K one obtains

$$x^0 = \gamma(c\tau + \beta R\sin(\Omega\tau)), \quad x = \gamma(R\sin(\Omega\tau) + \beta c\tau), \quad y = R\cos(\Omega\tau), \quad z = 0 \tag{2.12}$$

Here τ is a parameter along the neutrino world line. At the value τ_n of the parameter τ the neutrino is detected. One has

$$ct_\mathrm{n} = \gamma(c\tau_\mathrm{n} + \beta R\sin(\Omega\tau_\mathrm{n})), \quad x_\mathrm{n} = L = \gamma(R\sin(\Omega\tau_\mathrm{n}) + \beta c\tau_\mathrm{n}) \tag{2.13}$$

One obtains from (2.13)

$$T_\mathrm{ph} - t_\mathrm{n} = \frac{L}{c} - \gamma\left(\tau_\mathrm{n} + \frac{R\beta}{c}\sin(\Omega\tau_\mathrm{n})\right) = \gamma(1-\beta)\left(\frac{R}{c}\sin(\Omega\tau_\mathrm{n}) - \tau_\mathrm{n}\right) \tag{2.14}$$

Introducing the phase $\Phi = \Omega\tau_\mathrm{n}$, one obtains from (2.14)

$$T_\mathrm{ph} - t_\mathrm{n} = \frac{1}{\Omega}\sqrt{\frac{(1-\beta)}{1+\beta}}\left(\frac{\Omega R}{\beta c}\sin\Phi - \Phi\right) \tag{2.15}$$

The first term in (2.15) is positive and maximal, when $\Phi = \left(2n + \frac{1}{2}\right)\pi$. As far as $\Phi > 0$, expression (2.15) is positive, if

$$\frac{\Omega R}{\beta c} \approx \frac{\Omega R}{c} > \frac{\pi}{2} \tag{2.16}$$

As far as ΩR is neutrino velocity in the coordinate system K', relation (2.16) means that the photon retardation is possible only in the case, when the neutrino velocity is superluminal.

Eliminating τ_n from relation (2.14) by means of the second relation (2.13), one obtains

$$T_\mathrm{ph} - t_\mathrm{n} = (1-\beta)\left(\frac{R}{c}\gamma\sin(\Omega\tau_\mathrm{n})\left(1 + \frac{1}{\beta}\right) - \frac{L}{\beta c}\right) \tag{2.17}$$

For $t_\mathrm{n} = t_\mathrm{min}$ relation (2.17) coincides with relation (2.5)

Thus, the photon retardation is possible only in the case, when the neutrino rotational velocity is superluminal $\frac{\Omega R}{c} > 1$.

3 Investigation of the classical approximation of Dirac equation

The idea of helical world line appeared as a result of the Dirac equation investigation. [17] - [21].

The Dirac equation

$$i\hbar\gamma^l\partial_l\psi - m\psi = 0 \tag{3.1}$$

describes the mean motion of a Dirac particle (fermion) of the mass m, which may be very small. The units is chosen in such a way, that the speed of the light $c = 1$. The Dirac equation describes only the mean motion of the Dirac particle. The same situation we have in the theory of continuous medium, when the gas dynamic equations describe only the mean motion of gas molecules, ignoring chaotic component of their motion. Such quantities as angular momentum (spin) and magnetic moment are classical quantities, although they contain the quantum constant $\hbar$. In the conventional quantum description an existence of spin and of magnetic momentum is postulated by a reference to the Dirac equation, because these quantities are attributes of the Dirac equation. To obtain any additional information on the Dirac particle, one needs to simplify the description in terms of Dirac equation. The Dirac equation is a system of partial differential equations, which describe a dynamical system with infinite number of the freedom degrees.

To investigate the structure of the Dirac equation, one needs to reduce the equation (3.1) to a system of ordinary differential equations, which describe a statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{dcl}}\right]$ of classical dynamic systems $\mathcal{S}_{\mathrm{dcl}}$, having a finite number of the freedom degrees. Such a reduction is produced by means of the procedure of dynamic disquantization, when all derivatives ∂_k in (3.1) are projected onto the direction of the current $j^l = \bar{\psi}\gamma^l\psi$

$$\partial_k \rightarrow \partial_{k\|} = \frac{j_k j^l}{j^s j_s}\partial_l, \qquad j^l = \bar{\psi}\gamma^l\psi \tag{3.2}$$

The dynamic disquantization is a relativistic dynamical procedure, which does not refer to quantum principles. The name of this procedure is conditioned by the following circumstance. Application of the dynamic disquantization (3.2) to the Shrödinger equation leads to dynamic equation for statistical ensemble of free classical nonrelativistic particles. After application of the procedure (3.2) to the Shrödinger equation the quantum constant disappears from dynamic equations. The procedure (3.2) may be applied to any system of partial differential equations. After application of the dynamic disquantization the derivatives with respect to all independent variables disappear except of one of them. These independent variables turn into parameters labelling dependent dynamic variables. Only derivatives remain in the direction of 4-vector $j^k = \bar{\psi}\gamma^k\psi$. Choosing this independent variable as a temporal variable, one obtains a system of ordinary differential equations, which depends on parameters.

Applying the dynamic disquantization (3.2) to the action for the Dirac equation, one obtains after a proper change of variables (details see in [19])

$$\mathcal{A}_{\mathrm{Dqu}}[x,\boldsymbol{\xi}]=\int\mathcal{A}_{\mathrm{Dcl}}[x,\boldsymbol{\xi}]d\boldsymbol{\tau},\qquad \mathbf{d}\boldsymbol{\tau}=d\tau_1 d\tau_2 d\tau_3 \tag{3.3}$$

where $\mathcal{A}_{\mathrm{Dcl}}[x,\boldsymbol{\xi}]$ is the action for the classical dynamic system $\mathcal{S}_{\mathrm{Dcl}}$, having 10 degrees of freedom

$$\mathcal{S}_{\mathrm{Dcl}}:\qquad \mathcal{A}_{\mathrm{Dcl}}[x,\boldsymbol{\xi}]=\int\left\{-\kappa_0 m\sqrt{\dot{x}^i\dot{x}_i}+\hbar\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{2(1+\boldsymbol{\xi}\mathbf{z})}+\hbar\frac{(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}}{2\sqrt{\dot{x}^s\dot{x}_s}(\sqrt{\dot{x}^s\dot{x}_s}+\dot{x}^0)}\right\}d\tau_0 \tag{3.4}$$

Here variables $x=\{x^0,\mathbf{x}\}=\{x^0,x^1,x^2,x^3\}$, $\dot{x}^i\equiv\frac{dx^i}{d\tau_0}$ describe six translational degrees of freedom, whereas variables $\boldsymbol{\xi}=\{\xi_1,\xi_2,\xi_3\}$ describe some internal rotational degrees of freedom. The variables x are described relativistically, whereas the variables $\boldsymbol{\xi}$ are described non-relativistically, although the dynamic disquantization (3.2) is a relativistic procedure [20]. It means that the initial Dirac equation (3.1) is relativistic only with respect to external (translational) degrees of freedom. It is non-relativistic with respect to internal degrees of freedom, describing zitterbewegung [20]. We shall see, that the internal degrees of freedom describe a rapid rotation, and such a rotation cannot be described relativistically.

Variables $\boldsymbol{\xi}$, responsible for the particle rotation, appear instead of γ-matirces, as a result of their exclusion at a change of dynamic variables, which looks rather complicated.

The classical Dirac particle $\mathcal{S}_{\mathrm{Dcl}}$ is not a pointlike particle, because it has 10 degrees of freedom, whereas the pointlike particle has only 6 degrees of freedom. The classical Dirac particle $\mathcal{S}_{\mathrm{Dcl}}$ may be considered as two coupled particles (rotator), where one describes a motion of only one of the two particles. It is quite obscure, what interaction connects the two particles of the rotator and why the rotational degrees of freedom are described nonrelativistically [20]. It seems, that the rigid connection between two rotating particles can be explained by a restricted divisibility of the space-time geometry, what is characteristic for a discrete space-time geometry.

In the Dirac equation the 4-momentum and 4-velocity have different directions, in general. In the coordinate system, where the canonical momentum four-vector P_k has the form

$$P_k=\{p_0,\mathbf{p}\}=\left\{-\frac{\kappa_0 m}{\gamma},0,0,0\right\} \tag{3.5}$$

solution of dynamic equations, generated by the action (3.4) leads to the result

$$\frac{d\mathbf{x}}{dt}=\left\{\frac{\sqrt{\gamma^2-1}}{\gamma}\cos(\Omega t),-\frac{\sqrt{\gamma^2-1}}{\gamma}\sin(\Omega t),0\right\},\qquad \Omega=\frac{2m}{\hbar\gamma^2} \tag{3.6}$$

$$\mathbf{x}=\left\{\frac{\hbar\gamma\sqrt{\gamma^2-1}}{2m}\sin\left(\frac{2m}{\hbar\gamma^2}t\right),\frac{\hbar\gamma\sqrt{\gamma^2-1}}{2m}\cos\left(\frac{2m}{\hbar\gamma^2}t\right),0\right\} \tag{3.7}$$

Here $\kappa_0 = \pm 1$, γ is an arbitrary constant of integration (Lorentz factor) and m is the constant (mass), taken from the Dirac equation (3.1). The units are chosen in such a way, that the light speed $c = 1$. Details of calculation see in [19]. The maximal velocity of rotation is arbitrary, but it is less than the speed of the light $c = 1$. It means, that the world line of a classical Dirac particle is a helix.

In general, variables $\boldsymbol{\xi}$ in the action are described nonrelativistically. The relations (3.6), (3.7) are obtained after correction of the action (3.4), when all terms of the action were made relativistic. We cannot be sure, that the world line (3.6) (3.7) is timelike with necessity because of nonrelativistic character of two last terms in the action (3.1). We are to consider the problem in the framework of the skeleton conception [2].

4 Discrete space-time geometry

Even if the neutrino world line is a spacelike helix, on the average its world line may be considered as a timelike world tube, which describes the mean neutrino motion. Spacelike helix is impossible in the space-time geometry of Minkowski. However it may be obtained in the framework of the skeleton conception of the elementary particle dynamics [22].

The skeleton conception is not an ad hoc conception, created for explanation of the OPERA experiment. The skeleton conception of the elementary particle dynamics has been created several years ago on the basis of the discrete space-time geometry, which has many unexpected properties. The mathematical formalism of the discrete geometry distinguishes strongly from the formalism of the Minkowski geometry, which is a continuous (differential) geometry.

The distance function $\rho_{\rm d}$ of a discrete geometry $\mathcal{G}_{\rm d}$ satisfies the condition

$$|\rho_{\rm d}(P,Q)| \notin (0,\lambda_0), \qquad \forall P,Q \in \Omega \tag{4.1}$$

where Ω is the point set, where the geometry $\mathcal{G}_{\rm d}$ is given. In means that in the geometry $\mathcal{G}_{\rm d}$ there are no distances, which are shorter, than the elementary length λ_0. The distance $\rho_{\rm d}(P,Q) = 0$ is admissible. This condition takes place, if $P = Q$.

Conventionally one considers a geometry on a lattice as a discrete geometry. The geometry on a lattice satisfies the condition (4.1). However, the condition (4.1) is a restriction on the values of the distance function $\rho_{\rm d}$, but not on values of its argument (points of Ω). As a result the geometry on a lattice is a caricature on the discrete geometry, because it cannot be uniform and isotropic.

The discrete geometry is given on the same point set, where the geometry of Minkowski is given. It is obtained by means of such a deformation of the distance function form, that it becomes to satisfy the relation (4.1). At the description of indefinite geometries (such as geometry of Minkowski) it is more convenient to use the world function $\sigma_{\rm d} = \frac{1}{2}\rho_{\rm d}^2$, because it is always real. In particular, the world function $\sigma_{\rm d}$ of the simplest discrete space-time geometry $\mathcal{G}_{\rm d}$ looks as follows

$$\sigma_{\rm d}(P,Q) = \sigma_{\rm M}(P,Q) + \frac{\lambda_0^2}{2}\mathrm{sgn}(\sigma_{\rm M}(P,Q)), \quad \forall P,Q \in \Omega \tag{4.2}$$

where $\sigma_{\mathrm{M}}(P,Q)$ is the world function of the geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$.

The geometry $\mathcal{G}_{\mathrm{d}}$ is uniform and isotropic as well as the geometry of Minkowski. At the large scale one may set $\lambda_0 = 0$ and $\mathcal{G}_{\mathrm{d}}$ reduces to $\mathcal{G}_{\mathrm{M}}$. The discrete geometry $\mathcal{G}_{\mathrm{d}}$ contains the additional parameter λ_0. If the squared elementary length distinguishes from the quantum constant by universal factor, the classical mechanics of statistical ensembles of indeterministic particles describes quantum effects in the discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ without referring to quantum principles [16].

Mathematical formalism of a discrete geometry differs from that of a differential geometry, because in the discrete geometry there are no infinitesimal distances and one cannot differentiate and use differential relations. Besides, in the discrete space-time geometry there are no smooth world lines, and one is forced to use world chains instead of world lines. In the simplest case of a pointlike particle its world chain is a broken line, consisting of straight links of length μ. This new parameter μ of the world chain is connected with the particle mass m by the universal factor b

$$m = b\mu \tag{4.3}$$

The relation (4.3) geometrizes the particle mass. As a result a pointlike particle is described completely by geometric parameters.

In the discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ the particle state cannot be described by the position x^k and 4-momentum p_k, because p_k is defined by a differentiation of a smooth world line, which does not exists in $\mathcal{G}_{\mathrm{d}}$. As a result the pointlike particle state is described by the particle skeleton $\mathcal{P}_1 = \{P_0, P_1\}$. The points P_0 and P_1 describe ends of a link of the world chain. Dynamic equations are difference equations, which describe evolution of the particle skeleton.

The discrete geometry $\mathcal{G}_{\mathrm{d}}$ is obtained as a generalization of the proper Euclidean geometry. Such a generalization is possible, only if both geometries are described in terms of quantities, which are well defined in both geometries. Conventional description of the proper Euclidean geometry is produced on the basis of the linear vector space. Quantities of the linear vector space and the linear vector space in itself are not determined in a discrete geometry. The only quantity, which is described in the discrete geometry is the distance function ρ (or world function $\sigma = \frac{1}{2}\rho^2$). To generalize the proper Euclidean geometry on the case of a discrete geometry, one needs to produce a logical reloading in the description of the proper Euclidean geometry [23]. It means, that the proper Euclidean geometry is to be described in terms of the Euclidean world function and only in its terms. Such a presentation of the proper Euclidean geometry is called σ-presentation of the proper Euclidean geometry.

In the σ-representation of the proper Euclidean geometry there are relations of two sorts: (1) general geometric relations in terms of the Euclidean world function σ_{E}, which are valid for all physical geometries (i.e. for geometries which are described completely in terms of the world function), (2) special geometric relations, which describes special properties of the Euclidean world function. The general geometric relations are mainly relations, which describe properties of the linear vector space. Replacing the Euclidean world function σ_{E} in general geometric relations by the world function $\sigma_{\rm d}$ of the discrete geometry $\mathcal{G}_{\rm d}$, one obtains general geometric relations of the discrete geometry.

For instance, equivalence $(\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1)$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is described by two relations

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \tag{4.4}$$

$$|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1| \tag{4.5}$$

where the scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ of vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is determined via the world function by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_1, Q_1) - \sigma(P_0, Q_0) \tag{4.6}$$

$$|\mathbf{P}_0\mathbf{P}_1| = \sqrt{2\sigma(P_0, P_1)} \tag{4.7}$$

Equation (4.4) describes parallelism of vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$, whereas equation (4.5) describes equality of their lengths. Equations (4.4) - (4.7) are general geometric relations, which are valid in all discrete (and physical) geometries.

5 Skeleton conception of elementary particle dynamics

There are composite particles, which are described by a skeleton $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$, consisting of $n+1$ space-time points. This skeleton is a discrete analog of a coordinate system, attached rigidly to a physical body. Tracing the motion of the skeleton, one may trace the motion of the physical body. The particle skeleton describes a state of the particle. Evolution of skeleton is described by a world chain of connected skeletons

$$\mathcal{C} = \bigcup_s \mathcal{P}_n^{(s)}, \qquad \mathcal{P}_n^{(s)} = \left\{P_0^{(s)}, P_1^{(s)}, ...P_n^{(s)}\right\} \tag{5.1}$$

The adjacent skeletons are connected in the sense, that

$$P_1^{(s)} = P_0^{(s+1)}, \qquad s = ...0, 1, .. \tag{5.2}$$

The vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)} = \mathbf{P}_0^{(s)}\mathbf{P}_0^{(s+1)}$ is the leading vector, which determines a shape of the world chain. All skeletons are similar

$$\mu_{ik} = \left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right| = \text{const}, \qquad i, k = 0, 1, ...n, \qquad s = ...0, 1, .. \tag{5.3}$$

$n(n+1)/2$ quantities μ_{ik} are invariant characteristics of the composite particle. In the simplest case, when $n = 1$, there is one invariant - geometrical mass μ which is connected with the usual mass m be means of the relation (4.3). In the case of a composite particle the meaning of invariant parameters μ_{ik} is not yet known.

In the case of a free particle motion the orientation of vectors $\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}$ $i,k = 0,1,...n$, is conserved. It means

$$\left(\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}.\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}\right) = \left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|\cdot\left|\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}\right| = \mu_{ik}^2, \quad i,k = 0,1,...n, \qquad s = ...0,1,.. \tag{5.4}$$

Equations (5.3), (5.4) may be written in the form

$$\mathcal{P}_n^{(s)}\text{eqv}\mathcal{P}_n^{(s+1)}, \qquad s = ...0,1,... \tag{5.5}$$

where equivalence of two skeletons $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$ and $\mathcal{P}'_n = \{P'_0, P'_1, ...P'_n\}$ means

$$(\mathcal{P}_n\text{eqv}\mathcal{P}'_n): \qquad \mathbf{P}_i\mathbf{P}_k\text{eqv}\mathbf{P}'_i\mathbf{P}'_k, \qquad i,k = 0,1,,...n \tag{5.6}$$

Dynamic equations (5.2), (5.5) are difference dynamic equations written in a coordinateless form. The number of dynamic equations is equal to $n(n+1)$, whereas the number of dynamic variables, which are liable to determination, is equal to nD, where D is the number of coordinates, describing position of a point in the space-time.

If $n(n+1) < nD$, sth link position determines the position of $(s+1)$th link by the nonunique manner, in general. If $n(n+1) > nD$, the dynamic equations may have solutions only for several skeletons, discriminating other ones. Most skeletons appear to be forbidden, in general. It leads to existence of composite particles with discrete values of their invariant parameters μ_{ik}.

Thus, the skeleton conception of the elementary particle dynamics (interrelation between the number of dynamic equations and the number of dynamic variables) distinguishes strongly from the classical conception of the elementary particle dynamics, where the number of dynamic equations coincides with the number of dynamic variables. Coincidence of numbers of dynamic equations with the number of dynamic variables means, that a description of the particle motion is deterministic, and dynamic equations may be obtained from the variational principle. Even if the particle motion is indeterministic (random), a description of this indeterministic motion can be made deterministic. In this case one considers a statistical ensemble of indeterministic particles. Dynamic equation for the statistical ensemble appear to be deterministic. They can be obtained from a variational principle. Dynamic equations for the statistical ensemble exist, although dynamic equations for a single particle do not exist [24]. In other words, although a motion of a single particle is indeterministic, a mean motion of this particle can be deterministic. The mean motion of a particle can be described in the framework of the classical conception of the particle dynamics.

In the framework of the skeleton conception one can describe indeterministic particle motion directly, and one can obtain a more detailed information on the indeterministic particle. For instance, in the framework of the classical conception of the particle dynamics such classical quantities as spin and magnetic moment of a fermion are postulated by means the statement that the particle is described by the Dirac equation, keeping in mind that the spin and the magnetic moment are attributes of the Dirac equation. In the framework of the skeleton conception the spin and the magnetic moment are corollaries of the helical shape of the particle world line. It is a more detailed information on the fermion motion, which appears to be essential in the explanation of the OPERA experiment results.

6 Manifestation of pointlike particle indeterminism

A pointlike particle is described by the skeleton $\mathcal{P}_1$, consisting of two points P_0, P_1. In this case $n = 1$ and the number of dynamic equations is equal to $n(n+1) = 2$. The number of dynamic variables $nD = 4$. It is larger, than the number of dynamic equations. Whether or not the particle motion is deterministic, this depends on the number of solutions of two equations

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2) = |\mathbf{P}_0\mathbf{P}_1|^2, \quad |\mathbf{P}_0\mathbf{P}_1| = |\mathbf{P}_1\mathbf{P}_2| \tag{6.1}$$

In terms of the world function they are written in the form

$$\sigma_{\mathrm{d}}(P_0, P_2) = 4\sigma_{\mathrm{d}}(P_0, P_1), \quad \sigma_{\mathrm{d}}(P_1, P_2) = \sigma_{\mathrm{d}}(P_0, P_1) \tag{6.2}$$

We shall solve these equations in the case of timelike vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, setting

$$P_0 = \{0,0,0,0\}, \quad P_1 = \{l,0,0,0\}, \quad P_2 = \{ct, x, y, z\} \tag{6.3}$$

The discrete space-time is considered, and the world function σ_{d} is defined by the relation (4.2). Solution of these equations has the form

$$ct = 2l + \frac{3}{2}\frac{\lambda_0^2}{l}, \quad x^2 + y^2 + z^2 = r^2 = 3\lambda_0^2 + \frac{9}{4}\frac{\lambda_0^4}{l^2} \tag{6.4}$$

As a result the point P_2 has coordinates

$$P_2 = \left\{2l + \frac{3}{2}\frac{\lambda_0^2}{l}, r\sin\theta\cos\varphi, r\sin\theta\sin\varphi, r\cos\theta\right\}, \quad r = \lambda_0\sqrt{3 + \frac{9}{4}\frac{\lambda_0^2}{l^2}} \tag{6.5}$$

where θ and φ are arbitrary quantities. Thus, spatial coordinates of the point P_2 are determined to within $\sqrt{3}\lambda_0$. In the limit $\lambda_0 \to 0$ the point P_2 is determined uniquely. Two solutions

$$P_2' = \left\{2l + \frac{3}{2}\frac{\lambda_0^2}{l}, 0, 0, r\right\}, \quad P_2'' = \left\{2l + \frac{3}{2}\frac{\lambda_0^2}{l}, 0, 0, -r\right\}$$

are divided by spatial distance $i\,|\mathbf{P}_2'\mathbf{P}_2''| = 2r \approx 2\sqrt{3}\lambda_0$ $(\lambda_0 \ll l)$. It is a maximal distance between two solutions $\mathbf{P}_2'$ and $\mathbf{P}_2''$.

Let us consider the same problem for spacelike vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, when

$$P_0 = \{0,0,0,0\}, \quad P_1 = \{0,l,0,0\}, \quad P_2 = \{ct, x, y, z\} \tag{6.6}$$

We have the same equations (6.2), but now we have another solution

$$x=2l+\frac{3\lambda_0^2}{2l},\quad c^2t^2-y^2-z^2=r^2=3\lambda_0^2+\frac{9}{4}\frac{\lambda_0^4}{l^2} \tag{6.7}$$

The point P_2 has coordinates

$$P_2=\left\{\sqrt{a_2^2+a_3^2+r^2},2l+\frac{3\lambda_0^2}{2l},a_2,a_3\right\},\quad r^2=3\lambda_0^2\left(1+\frac{3\lambda_0^2}{4l^2}\right) \tag{6.8}$$

where a_2 and a_3 are arbitrary quantities. The difference between two solutions P_2' and P_2''

$$P_2'=\left\{\sqrt{a_2^2+a_3^2+r^2},2l+\frac{3\lambda_0^2}{2l},a_2,a_3\right\},\quad P_2''=\left\{\sqrt{b_2^2+b_3^2+r^2},2l+\frac{3\lambda_0^2}{2l},b_2,b_3\right\}$$

may be infinitely large

$$|\mathbf{P}_2'\mathbf{P}_2''|=\sqrt{2a_2b_2+2a_3b_3-2\sqrt{r^2+a_2^2+a_3^2}\sqrt{r^2+b_2^2+b_3^2}+2r^2}$$

This difference remains very large, even if $\lambda_0\rightarrow 0$.

A particle with spacelike world chain is known as tachyon. Tachyons have not been discovered experimentally. From viewpoint of the skeleton conception it is natural, because links of the tachyon world chain are placed in the space-time chaotically, and one cannot identify different points of the taxion world chain as belonging to the same world chain. As result one cannot trace the tachyon world chain. On the contrary, the links of a timelike world chain wobble, but the spatial amplitude of such a wobbling is of the order of the elementary length λ_0, and one can trace the world chain of usual particle [16].

In the conventional conception of particle motion one supposes, that particles with the spacelike world line are impossible because of the relativity principle, which asserts, that a motion of real bodies with the superluminal velocity is impossible. In the skeleton conception the spacelike world chains are admissible, but one cannot trace the spacelike world chain of a pointlike particle (tachyon). However, if the spacelike world chain forms a helix with the timelike axis, one can trace such a world chain, because it looks as a timelike world tube.

However, are the helical world chains of free particles possible? They are impossible in the discrete space-time geometry with the world function (4.2). However, they appear to be possible, if the world function has the form

$$\sigma_{\rm g}=\sigma_{\rm M}+\frac{\lambda_0^2}{2}\begin{cases}\operatorname{sgn}(\sigma_{\rm M}) & \text{if}\quad |\sigma_{\rm M}|>\sigma_0>0\\ f\left(\frac{\sigma_{\rm M}}{\sigma_0}\right) & \text{if}\quad |\sigma_{\rm M}|<\sigma_0\end{cases} \tag{6.9}$$

where function f satisfies the relations

$$f(x)=-f(x),\quad f(1)=1,\quad |f(x)|<|x|,\quad x\in(-1,1) \tag{6.10}$$

and σ_0 =const. The space-time geometry with the world function (6.9) is discrete only partly. We shall refer to this geometry as granular space-time geometry. The world function (6.9) differs from the world function (4.2) in the small region near the value $\sigma_{\rm g} = 0$, where $|\sigma_{\rm M}| < \sigma_0$.

The form of the function f is not known now, but this form is to be universal, because it describes the space-time geometry. In the paper [22] the world function $\sigma_{\rm d}$ has been chosen in the form

$$\sigma_{\rm d} = \sigma_{\rm M} + \lambda_0'^2 \begin{cases} \operatorname{sgn}(\sigma_{\rm M}) & \text{if} \quad |\sigma_{\rm M}| > \sigma_0 > 0 \\ \left(\frac{\sigma_{\rm M}}{\sigma_0}\right)^3 & \text{if} \quad |\sigma_{\rm M}| < \sigma_0 \end{cases} \tag{6.11}$$

i.e. instead of the elementary length λ_0 the quantity $\lambda_0' = \lambda_0/\sqrt{2}$ has been used. The world function (6.11) did not pretend to description of the real space-time geometry. One investigated only a possibility of a helical world chain. The idea of the helical world chain and association of such a world chain with a fermion (Dirac particle) appeared from investigation of the Dirac equation.

Investigation in [22] confirmed the idea of helical world chains. The helical world chain may be spacelike or timelike. But now we are interested only in spacelike world chains.

The infinite wobbling amplitude of spacelike world chains of pointlike tachyons may destroy the helical shape of the spacelike world chain. To stabilize the helical shape, the number of points in the skeleton should be increased. It may be made by different manners. In [22] the skeleton, consisting of three points has been considered. It appears to be sufficient to reduce the wobbling amplitude and to fix the helical shape of the spacelike world chain. It appeared that the helical shape of the world chain imposes some restrictions on the parameters of the particle skeleton. Investigations in [22] were produced in the 4-dimensional space-time, which is available for description of neutral fermions. For description of charged fermions one needs to use the 5-dimensional discrete space-time of Kaluza-Klein.

7 Concluding remarks

OPERA experiments were carried out for determination of neutrino oscillations. Discovery of "superluminal velocity" of neutrino was an unexpected result. The skeleton conception of particle dynamics is not an ad hoc conception, created for explanation of the OPERA experiment results. The skeleton conception of particle dynamics is a natural corollary of the geometric paradigm, where the space-time geometry is discrete, and quantum principles are not used. In the geometric paradigm the spacelike word chains and the tachyons are not forbidden. Pointlike tachyons cannot be detected only because of infinite amplitude of their world chains wobbling.

The results of the OPERA experiments cannot be explained in the framework of the quantum paradigm, where space-time geometry is continuous and quantum principles are used. As a result the OPERA experiment appears to be a crucial experiment, which evidences in virtue of the geometrical paradigm.

The skeleton conception of the particle dynamics admits one to obtain a more detailed information on the particle structure, than it is possible in the framework of classical conception of the particle dynamics. In particular, in the skeleton conception one constructs the neutrino spin model, whereas in the classical conception of the particle dynamics the spin of neutrino is simply postulated.

In the skeleton model there exists a natural discrimination mechanism, choosing possible skeletons of composite particles. This mechanism is responsible for discrete values of the elementary particle parameters. In the conventional conception such a discrimination mechanism can be only man-made.

References

[1] The OPERA Collaboration: T. Adam, N. Agafonova et al. Measurement of the neutrino velocity with the OPERA detector in the CNGS beam. *e-print 1109.4897.*

[2] Yu. A. Rylov Discrete space-time geometry and skeleton conception of particle dynamics. *e-print 1110.3399v1*

[3] E. Madelung, *Z.Phys.* **40,** 322, (1926).

[4] L. de Broglie, *Comptes Rendus* **183**, 447, (1926).

[5] D. Bohm, *Phys.Rev.* **85** (1952) 166, 180.

[6] Yu.A.Rylov, Spin and wave function as attributes of ideal fluid. (*Journ. Math. Phys.* **40**, *pp. 256 - 278, (1999).*

[7] K. Menger, Untersuchen über allgemeine Metrik, *Mathematische Annalen,* **100**, 75-113, (1928).

[8] L.M. Blumenthal, *Theory and Applications of Distance Geometry*, Oxford, Clarendon Press, 1953

[9] Yu.A. Rylov, Geometry without topology *e-print math.MG/0002161.*

[10] Yu.A.Rylov, Geometry without topology as a new conception of geometry. *Int. Jour. Mat. & Mat. Sci.* **30**, *iss. 12, 733-760, (2002)*, (see also *e-print math.MG/0103002*).

[11] Yu.A.Rylov, Deformation principle as foundation of physical geometry and its application to space-time geometry *e-print physics/0411103*)

[12] Rylov Yu. A. Multivariance as a crucial property of microcosm *Concepts of Physics* **6**, iss.1, 89 -117, (2009), see also *e-print 0806.1716*

[13] Yu. A. Rylov, Tubular geometry construction as a reason for new revision of the space-time conception. in *What is Geometry?* polimetrica Publisher, Italy, pp.201-235 http://www.polimetrica.com/polimetrica/406/

[14] Yu. A. Rylov, Different conceptions of Euclidean geometry. *e-print 0709.2755*

[15] Yu. A. Rylov, Non-Euclidean method of the generalized geometry construction and its application to space-time geometry. in *Pure and Applied Differential geometry pp.238-246.* eds. Franki Dillen and Ignace Van de Woestyne. Shaker Verlag, Aachen, 2007. (see also *e-print math/0702552.*)

[16] Yu.A. Rylov, Non-Riemannian model of the space-time responsible for quantum effects. *Journ. Math. Phys.* **32(8)**, *2092-2098, (1991).*

[17] Yu.A.Rylov, Dirac equation in terms of hydrodynamic variables. *Advances in Applied Clifford Algebras*, **5**, *pp 1-40, (1995).* (see also *e-print* 1101.5868)

[18] Yu.A. Rylov, Dynamic disquantization of Dirac equation. *e-print 0104060..*

[19] Yu. A. Rylov, Is the Dirac particle composite? *e-print /physics /0410045*

[20] Yu. A. Rylov, Is the Dirac particle completely relativistic? *e-print /physics /0412032.*

[21] Yu. A. Rylov, Dynamical methods of investigation in application to the Dirac particle. *e-print /physics/0507084.*

[22] Yu. A.Rylov, Geometrical dynamics: spin as a result of rotation with superluminal speed. *e-print 0801.1913.*

[23] Yu. A.Rylov, Different conceptions of Euclidean geometry and their application to the space-time geometry. *e-print 0709.2755v4.*

[24] Yu. A. Rylov, Uniform formalism for description of dynamic, quantum and stochastic systems. *e-print 0603237v6.*