Physics geometrization in microcosm: discrete space-time and relativity theory

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Abstract

The presented paper is a review of papers on the microcosm physics geometrization in the last twenty years. These papers develop a new direction of the microcosm physics. It is so-called geometric paradigm, which is alternative to the quantum paradigm, which is conventionally used now. The hypothesis on discreteness of the space-time geometry appears to be more fundamental, than the hypothesis on quantum nature of microcosm. Discrete space-time geometry admits one to describe quantum effects as pure geometric effects. Mathematical technique of the microcosm physics geometrization (geometric paradigm) is based on the physical geometry, which is described completely by the world function. Equations, describing motion of particles in the microcosm, are algebraic (not differential) equations. They are written in a coordinateless form in terms of world function. The geometric paradigm appeared as a result of overcoming of inconsistency of the conventional elementary particle theory. In the suggested skeleton conception the state of an elementary particle is described by its skeleton (several space-time points). The skeleton contains all information on the particle properties (mass, charge, spin, etc.). The skeleton conception is a monistic construction, where elementary particle motion is described in terms of skeleton and world function and only in these terms. The skeleton conception can be constructed only on the basis of the physical geometry. Unfortunately, most mathematicians do not accept the physical geometries, because these geometries are nonaxiomatizable. It is a repetition of the case, when mathematicians did not accept the non-Euclidean geometries of Lobachevsky-Bolyai. As a result this review is a review of papers of one author. This situation has some positive sides, because it appears to be possible a consideration not only of papers, but also of motive for writing some papers.

1 Introduction

The conventional paradigm of the microcosm physics development may be classified as quantum paradigm. The quantum paradigm is based on hypothesis of continuous space-time geometry equipped by quantum principles of particle motion. There is an alternative geometric paradigm based on hypothesis on a discrete space-time. There is no necessity to use quantum principles in the geometric paradigm, because all quantum effects can be explained by existence of elementary length of the discrete geometry. The elementary length appears to be proportional to the quantum constant $\hbar$.

The hypothesis on discreteness of the space-time geometry looks more reasonable and natural, than the hypothesis on mysterious quantum nature of microcosm. One of reasons, why the geometric paradigm is not used in the contemporary physics is the circumstance, that the discrete geometry has not been developed properly. One believed that the discrete geometry is a geometry on a lattice. Any lattice point set cannot be uniform and isotropic, and such a point set is not adequate for the space-time.

In reality, the discrete space-time geometry can be defined on the same point set, where the space-time geometry of Minkowski is given. In other words, a discrete geometry may be uniform and isotropic. This unexpected circumstance admits one to use a discrete geometry as a space-time geometry. *The discrete geometry G_{d} is such a geometry, where there are no close points.* Mathematically it means

$$|\varrho(P,Q)| \notin (0,\lambda_0), \qquad \forall P,Q \in \Omega \tag{1.1}$$

Here Ω is the point set, where the geometry is given, and $\varrho(P,Q)$ is a distance between the points P, Q. The quantity λ_0 is the elementary length of the discrete geometry $\mathcal{G}_{\mathrm{d}}$. The geometry on a lattice can satisfy the property (1.1), but such a geometry cannot be uniform and isotropic. The discrete space-time geometry has a set of new unexpected properties, which were unknown in the twentieth century. This fact was one of reasons, why the physics geometrization in microcosm has not been developed in the twentieth century.

This paper is a short review of the physics geometrization development in the last two decades. The physics geometrization began in the end of the nineteenth century. Different stages of the physics geometrization are: (1) connection of the conservation laws with the properties of the space-time geometry (uniformity and isotropy), (2) the special relativity theory,(3) the general relativity theory, (4) the Kaluza-Klein space-time geometry. Most physicists do not believe in the physics geometrization in microcosm. They believe in the quantum nature of physical phenomena in microcosm, and they do not know properties of a discrete geometry, which admits one to explain quantum phenomena as geometrical effects. It is a reason, why practically nobody deal with the physics geometrization now. By necessity this review of papers on the physics geometrization in microcosm is a review of papers of one author.

It should note that we distinguish between a conception and a theory. A conception does not coincide with a theory. For instance, the skeleton conception of elementary particles distinguishes from a theory of elementary particles. A conception investigates connections between concepts of a theory. For instance, the skeleton conception of elementary particles investigates the structure of a possible theory of elementary particles. It investigates, why an elementary particle is described by its skeleton (several space-time points), which contains all information on the elementary particle. The skeleton conceptions explains, why dynamic equations are coordinateless algebraic equations and why the dynamic equations a written in terms of the world functions. However, the skeleton conception does not answer the question, which skeleton corresponds to a concrete elementary particle and what is the world function of the real space-time. In other words, the skeleton conception deals with physical principles, but not with concrete elementary particles. The conception cannot be experimentally tested. However, if the world function of the real space-time geometry has been determined and correspondence between a concrete elementary particle and its skeleton has been established, the skeleton conception turns to the elementary particle theory. The theory of elementary particle (but not a conception) can be tested experimentally.

In other words, it is useless to speak on experimental test of the skeleton conception, because it deals only with physical principles. Discussing properties of a conception, one should discuss only properties of the concept and logical connection between them, but not to what extent they agree with experimental data.

We consider in the review the following problems

1. Conceptual defects of the quantum paradigm, which manifest themselves, in particular, in incorrect use of the relativity principles at a description of indeterministic particles.

2. Explanation of quantum effects as a statistical description of the indeterministic particle motion.

3. Discrete geometry as a special case of a physical geometry and properties of physical geometries.

4. Elementary particle dynamics in physical space-time geometry and skeleton conception of particle dynamics.

Idea of the physics geometrization is based on the following circumstance. Description of the particle motion contains two essential elements: the space-time geometry and the dynamic laws. The two categories are connected. One can investigate the two categories only together, and the boundary between the laws of geometry and the laws of dynamics is not fixed rigidly. One can shift this boundary. For instance, one can choose a very simple space-time geometry, then the laws of dynamics appear to be rather complicated. One may try to use a complicated space-time geometry, which is chosen in such a way, that the dynamic laws be very simple.

For instance, maybe, there exists such a space-time geometry, where the elementary particles move freely. Interaction between particles is realized via the space-time geometry. The Kaluza-Klein geometry is an example of such a space-time geometry, where the electromagnetic field is a property of the space-time geometry. If one uses the space-time geometry of Minkowski (instead of the Kaluza-Klein geometry), the electromagnetic interaction of particles is explained as a result of interaction with the electromagnetic field,

The space-time of Minkowski is uniform and isotropic, and one can easily write the conservation laws of energy-momentum and of angular momentum in the Minkowski space-time. One cannot write the conservation laws in the Kaluza-Klein space-time with electromagnetic field, because this space-time is not uniform and isotropic, in general. Such a difference is conditioned by the circumstance, that in the space-time of Minkowski the electromagnetic field is a substantive essence, whereas in the Kaluza-Klein space-time the electromagnetic field is only a property of the space-time geometry.

What point of view is true? We believe, that one should use both approaches. In the geometrical approach the number of essences is less (in the limit of a complete geometrization there is only one essence), and it is easier to establish physical (and geometrical) principles responsible for description of different sides of a physical phenomenon. On the other hand, when the physical principles and connection between different sides of a physical phenomenon have been established, one may consider the different sides of a physical phenomenon as different essences. Such an approach admits one to describe concrete physical phenomena easier and more convenient, considering them as a result of interaction of different essences.

Developing the physics geometrization, one tries to work with physical principles, assuming that the good old classical principles are true. We stand aback from introducing new physical principles on the basis of consideration of single physical phenomena. We believe that classical physical principles are valid, although they are applied sometimes incorrectly. We have succeeded to discover several mistakes in application of classical principles of physics. Some of mistakes were connected with our imperfect knowledge of geometry and, in particular, with imperfect knowledge of a discrete geometry.

At the complete geometrization of physics the space-time geometry is chosen in such a way, that all particles move freely. The force fields and their interaction with particles appear only in the case, when the space-time geometry is chosen incorrectly. In this case, when the chosen space-time geometry differs from the true geometry, the deviation of geometries generates appearance of force fields. The complete geometrization of physics is known for classical (gravitational an electromagnetic) interactions . However, it is not yet known in microcosm. The reason of this circumstance lies mainly in the fact, that our knowledge of geometry is imperfect. The complete geometrization of physics is possible only at a perfect description of the space-time geometry.

A geometry as a science on disposition of geometrical objects in space or in the event space (space-time) is described completely by the distance $\rho(P,Q)$ between any two points P and Q, or by the world function $\sigma = \frac{1}{2}\rho^2$. The geometry, which is described completely by the world function will be referred to as a physical geometry. After complete physics geometrization the particle dynamics turns to a monistic conception, which is described completely in terms of one quantity (world function). Any conception, which contains several basic concepts (quantities), needs an agreement between all concepts, used in the conception. Achievement of such an agreement is a very difficult problem. One can see this in the example of a geometry. The physical geometry is a monistic conception, because it is described by means of only world function. One uses a few concepts (manifold, coordinate system, metric tensor) in the conventional description of Riemannian geometries, and a Riemannian geometry appears to be a less general conception, than a physical geometry.

Albert Einstein dreamed on creation of a united field theory. Such a theory was to be a monistic conception, and this circumstance was the most attractive feature of such a theory. However, a monistic theory on the basis of a geometry seems to be more attractive, than a monistic theory based on a united field, because the main object of a physical geometry (world function) is a simpler object, than a force field of the united field theory.

Problems of the physical geometrization appeared, when physicists began to investigate physical phenomena in microcosm. We cannot know exactly the microcosm space-time geometry. It is rather natural, that the space-time geometry in microcosm may appear to be discrete. Contemporary researchers consider a discrete geometry as a geometry on a lattice point set. In particular, there is a special section in the ArXiv publications, entitled High Energy Physics - Lattice. A lattice point set cannot be uniform and isotropic. In accordance with this circumstance the discrete space-time geometry (geometry on a lattice) is considered to be not uniform and isotropic.

In reality a discrete space-time geometry is not a geometry on a lattice. The discrete space-time geometry may be given on a continual point set. In particular, it can be given on the same manifold, where the geometry of Minkowski is given. It is connected with the fact, that the geometry discreteness is a property of the geometry, but not a property of the point set, where the geometry is given. A discrete geometry satisfies the condition (1.1).

Geometry on a lattice satisfies the condition (1.1), but such a geometry is a special kind of a discrete geometry, which cannot be uniform and isotropic.

Let σ_{M} be the world function of the geometry Minkowski $\mathcal{G}_{\mathrm{M}}$

$$\sigma_{\mathrm{M}}\left(x,x'\right) = \frac{1}{2}g_{ik}\left(x^i - x'^i\right)\left(x^k - x'^k\right), \qquad \sigma_{\mathrm{M}}\left(x,x'\right) = \frac{1}{2}\rho_{\mathrm{M}}^2\left(x,x'\right) \tag{1.2}$$

where $\rho_{\mathrm{M}}\left(x,x'\right)$ is the distance (interval) between the points with inertial coordinates $x = \{x^0, x^1, x^2, x^3\}$ and $x' = \{x'^0, x'^1, x'^2, x'^3\}$. The world function σ_{d}

$$\sigma_{\mathrm{d}} = \sigma_{\mathrm{M}} + \frac{\lambda_0^2}{2}\mathrm{sgn}\left(\sigma_{\mathrm{M}}\right), \qquad \mathrm{sgn}\left(x\right) = \begin{cases} 1 & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x = 0 \\ -1 & \text{if} \quad x < 0 \end{cases} \tag{1.3}$$

describes a discrete geometry $\mathcal{G}_{\mathrm{d}}$, which satisfies the restriction (1.1), although the geometry $\mathcal{G}_{\mathrm{d}}$ is given on the same point set Ω_{M}, where the geometry of Minkowski is given. The geometry $\mathcal{G}_{\mathrm{d}}$ appears to be uniform and isotropic.

However, one cannot use coordinates for description of the geometry . It does not that one cannot introduce coordinates. Substituting (1.2) in (1.3), one obtains representation of the world function σ_{d} in terms of coordinates. However, the points, which have close coordinates, are not close in the sense that the distance between them is greater, than λ_0

$$\sqrt{2\sigma_{\mathrm{d}}\left(x,x'\right)} \geq \lambda_0, \qquad 0<\left|x-x'\right|^2<\varepsilon \tag{1.4}$$

It means that coordinate lines and differentiation along them have no relation to the discrete geometry $\mathcal{G}_{\mathrm{d}}$, given on the manifold of Minkowski. It does not mean, that the discrete geometry $\mathcal{G}_{\mathrm{d}}$ does not exist. It means only, that capacities of the coordinate description method are restricted, and one needs to use the coordinateless method of description, which are used at description of physical geometries [1, 2, 3]

Besides, the discrete geometry $\mathcal{G}_{\mathrm{d}}$ appears to be multivariant and nonaxiomatizable [4]. Such properties of a geometry can be obtained only at a use of the coordinateless description method. In the discrete space-time a particle cannot be described by a world line, because any world line is a set of connected infinitesimal segments of a straight line. However, in the discrete geometry $\mathcal{G}_{\mathrm{d}}$ there are no segments, whose length is shorter, than the elementary length λ_0. It means, that instead of world line one has a world chain

$$\mathcal{C}=\bigcup_s \mathbf{P}_s\mathbf{P}_{s+1} \tag{1.5}$$

consisting of geometrical vectors $\mathbf{P}_s\mathbf{P}_{s+1}=\{P_s,P_{s+1}\}$, $s=...-1,0,1,...$of finite length μ. The geometrical vector (g-vector) is an ordered set $\mathbf{PQ}=\{P,Q\}$ of two points P and Q. The first point P is the origin of the vector, whereas the second point Q is the end of the g-vector. Such a definition of the vector is used in physics. However, mathematicians prefer another definition. They define a vector as an element of a linear vector space.

Remark. We used the special term "geometrical vector", because conventionally the term " vector" means some many-component quantity (components of the vector in some coordinate system). In general, a vector is defined in the contemporary geometry as an element of the linear vector space. In this case the vector can be decomposed over basic vectors of a coordinate system and represented as a set of the vector coordinates. Such a definition is convenient, when one speaks about vector field, having several components. In the proper Euclidean geometry the concept of a geometrical vector coincides with the conventional concept of a vector as an element of the linear vector space. In the Euclidean geometry the g-vector can be decomposed over basic vectors. It can be represented as a set of coordinates. However, in the discrete geometry, described by the world function (1.3), a geometric vector cannot be represented as a sum of its projections onto basic vectors, because in the discrete geometry (1.3) one cannot introduce a linear vector space even locally. However, the definition of a vector as a set of two points does not contain a reference to a coordinate system and to special properties of the Euclidean geometry (such as linear vector space). The definition of the geometrical vector is more general, and according to the logic rules the term "vector" should be used with respect to the geometric vector. Another term, for instance, "linear vector" should be used for the vector, defined as an element of the linear vector space.

The discrete geometry $\mathcal{G}_{\mathrm{d}}$ is obtained from the geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$ by means of a deformation of the geometry of Minkowski, when the world function σ_{M} is replaced by the world function σ_{d} [5]. World chains in the discrete space-time geometry appear to be stochastic. Let the elementary length λ_0 have the form

$$\lambda_0^2 = \frac{\hbar}{bc} \tag{1.6}$$

where $\hbar$ is the quantum constant c is the speed of the light, and b is the universal constant, connecting the geometric mass μ (length of the chain link) with the particle mass m by means of

$$m = b\mu \tag{1.7}$$

Then statistical description of the stochastic world chains leads to the Schrödinger equation [6]. As a result the quantum effects can be described as geometrical effects of the discrete space-time geometry. Quantum principles cease to be prime physical principles. They become secondary principles, which should not be applied always and everywhere. In particular, there is no necessity of the gravitational field quantization.

In the discrete space-time geometry the relativity theory appears to be incomplete. The fact is that, the transition from the nonrelativistic physics to the relativistic one is followed only by a modification of dynamic equations, describing the particle motion. Description of the particle state remains the same as in the nonrelativistic physics. The particle state is described as a point in the phase space of coordinates and momenta. The particle momentum p_k is defined as a tangent vector to the particle world line $x^k = x^k(\tau)$, $k = 0, 1, 2, 3$.

$$p_k(\tau) = \frac{m g_{kl} u^l(\tau)}{\sqrt{g_{js} u^j(\tau) u^s(\tau)}}, \qquad u^l(\tau) = \lim_{d\tau \to 0} \frac{x^l(\tau + d\tau) - x^l(\tau)}{d\tau} \tag{1.8}$$

where τ is a parameter along the world line. In the discrete space-time geometry there are no world lines, and the limit (1.8) does not exist. This limit does not exist also in the case, when the particle is indeterministic and its world line (if it exists) is random (stochastic). In the physics of usual scale, when characteristic lengths much more, than the elementary length λ_0, restricting the link length of the world chain. In this case it is admissible to use the limit (1.8) as a good approximation. However, in the microcosm physics such an approximation appears to be unsatisfactory, because characteristic lengths of physical phenomena appear of the order of the elementary length λ_0. As a result the concepts of the elementary particle theory, based on the particle state as point of the phase space appear to be incomplete.

Consequent relativistic description of particles in microcosm does not use the phase space and its points. Instead, one uses a skeleton conception of the elementary particle description, where a particle is described by its skeleton $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$, which consists of $n+1$ rigidly connected points $P_0, P_1, ...P_n$. In the case of pointlike particle its skeleton consists of two points P_0, P_1, which define the particle momentum vector. In the given case all characteristics of the particle (mass, charge, momentum) are defined geometrically by the two points P_0, P_1. In the case of a more complicated particle, described by the skeleton $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$, there are $n(n+1)/2$ invariants $|\mathbf{P}_k\mathbf{P}_i|$, $i, k = 0, 1, ...n$, describing geometrically all characteristics of the particle. The question about nature of connection between the points of the skeleton does not arise, because the discrete space-time geometry may have a restricted divisibility. Such a question is conditioned by the hypothesis on continuous space-time geometry.

In the beginning of the twentieth century it was natural to think, that the quantum particles are simply indeterministic (stochastic) particles, something like Brownian particles. There were attempts to obtain quantum mechanics as a statistical description of stochastically moving particles [7, 8]. However, these attempts failed, because a *probabilistic conception of the statistical description* was used.

Statistical description is used in physics for description of indeterministic particles (or systems), when there are no dynamic equations, or initial conditions are indefinite. One considers statistical ensemble of indeterministic particles, i.e. many independent similar particles. It appears, that there are dynamic equations for the statistical ensemble $\mathcal{E}$ of indeterministic particles, although there are no dynamic equations for a single indeterministic particle, which is a constituent of this statistical ensemble $\mathcal{E}$. Consideration of the statistical ensemble as a dynamic system is *the dynamic conception of the statistical description* (DCSD). It is a primordial conception of statistical description. A use of DCSD is founded on independence of constituents of the statistical ensemble. Random components of motion are compensated due to their independence, whereas regular components of motion are accumulated. As a result the statistical ensemble, considered as a dynamic system, describes a mean motion of an indeterministic particle.

In the nonrelativistic physics one uses the probabilistic conception of the statistical description (PCSD). PCSD is used successfully, for instance, for description of Brownian motion. In PCSD one traces the motion of a point in the phase space. The point represents the state of indeterministic particle, and motion of the point in the phase space is described by the transition probability. Attempts of obtaining the quantum mechanics as a result of statistical description in the framework PCSD failed [7, 8], whereas the statistical description in the framework of DCSD appeared to be successful [9, 10, 11]. This fact is explained by a use of the dynamic conception of statistical description (DCSD), which does not use a concept of the phase space.

In the relativistic case the ensemble state is described by a 4-vecotor $j^k(x)$, which described the density of world lines in the vicinity of the point x. The ensemble state does not contain a reference to the phase space. In the nonrelativistic case the ensemble state is described by a 3-scalar $\rho(\mathbf{x}, \mathbf{p})$, which describes the particle density in vicinity of the point $(\mathbf{x}, \mathbf{p})$ of the phase space. PCSD is based on a use of the nonnegative quantity ρ, which is used as a probability density of the particle position at the point $(\mathbf{x}, \mathbf{p})$ of the phase space.

Nonrelativistic quantum mechanics is a relativistic construction in reality, because the stochastic component of the quantum particle motion may be relativistic. At such a situation and one has to use the dynamic conception of statistical description (DCSD), which does not use the nonrelativistic concept of the phase space. Besides, one may not use the limit (1.8) in the definition for the particle momentum of stochastic world lines, which can have no tangent vectors.

Indeed, in terms of DCSD one succeeded to obtain the quantum mechanics as a statistical description of stochastically moving particles [9, 10, 11]. This use of dynamic conception of statistical description was not a stage of the physics geometrization. DCSD was simply an overcoming of the incompleteness of the relativity theory, when relativistic dynamic equations are combined with non-relativistic concept of the particle state. However, the explanation of quantum mechanics effects as a result of statistical description of stochastic particle motion arose the *question on the nature of stochasticity of such a stochastic particle motion.*

Primarily the particle motion stochasticity was interpreted as a result of interaction with an ether. However, further the idea has been appeared, that the space-time geometry in itself may play the role of the ether. In other words, the space-time geometry is to determine the free particle motion. If the free particle motion is stochastic, the space-time geometry cannot be geometry of Minkowski, because in the space-time geometry of Minkowski a free particle motion is deterministic. The real space-time geometry is to be uniform and isotropic, but it is to distinguish from the geometry of Minkowski. It is to be multivariant. It means that at the point Q_0 there are many vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$, $\mathbf{Q}_0\mathbf{Q}_1''$,..., which are equivalent to the vector $\mathbf{P}_0\mathbf{P}_1$ at the point P_0. But vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$,$\mathbf{Q}_0\mathbf{Q}_1''$,...are not equivalent between themselves. It means that the equivalence relation is intransitive. Such a geometry cannot be axiomatizable, because *in any axiomatizable geometry the equivalence relation is to be transitive.* Nonaxiomatizable geometries were not known in seventieth of the twentieth century. The discrete geometry (1.3) was not known also, because in that time the discrete geometry was perceived as a geometry on a lattice point set.

Idea of the physical geometry as a geometry described completely by the world function appeared only in ninetieth of the twentieth century [12]. The close idea of the distance (metrical) geometry appeared earlier [13, 14]. But such a geometry cannot be used as a space-time geometry.

One uses the discrete geometry (1.3) to explain the stochasticity of free particle motion [6]. However, this geometry was used as a simplest multivariant generalization of the geometry of Minkowski, but not as a discrete space-time geometry. The fact, that the space-time geometry (1.3) is discrete, has been remarked several years later. It is rather natural, that starting from idea of a discrete space-time geometry, one comes to a geometry on a lattice, because one cannot obtain the geometry (1.3), if concepts of physical geometry are unknown.

Application of a physical geometry for description of the space-time has serious consequences for microcosm physics. It appears, that quantum principles are not primary principles of nature. The relativity theory appeared to be not completed. One needs to revise concept of the particle state. The mathematical technique of description of the microcosm physical phenomena changed essentially. Dynamic equations become finite difference equations instead of differential equations. Description of particle motion and that of gravitational field becomes coordinateless, and it was a progress in the particle motion description.

Transition from the conventional description in terms of differential equations to coordinateless description in terms of the world function appears rather unexpected. It is connected with degenerative character of the proper Euclidean geometry with respect to physical geometry. It means that some geometrical concepts and some geometrical objects, which are different in a physical geometry appear coinciding in the Euclidean geometry. For instance, the geometrical vector $\mathbf{P}_0\mathbf{P}_1$ defined as the ordered set of two points P_0 and P_1 is a vector in a physical geometry and in the Euclidean geometry. Projections p_l of vector $\mathbf{P}_0\mathbf{P}_1$ on basic coordinate vectors $\mathbf{Q}_0\mathbf{O}_l$, $l = 1, 2, ..n$ are defined by the relation

$$p_l = (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_l), \qquad l = 1, 2, ...3 \tag{1.9}$$

Here $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_l)$ is the scalar product of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_l$, which is defined in terms of the world function by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_l) = \sigma(P_0, Q_l) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_l) \tag{1.10}$$

The expression of the scalar product (1.10) via the world function is the same in a physical geometry and in the proper Euclidean one. In the physical geometry the relation (1.10) is a definition of the scalar product, whereas in the Euclidean geometry the relation (1.10) is obtained as a corollary of the cosine theorem, but in both cases the expression (1.10) is true. The scalar product has conventional linear properties in the Euclidean geometry, but these properties are absent, in general, in the physical geometry. As a result components p_l of the geometrical vector $\mathbf{P}_0\mathbf{P}_1$ do not determine the vector $\mathbf{P}_0\mathbf{P}_1$ in the physical geometry, although they determine the vector $\mathbf{P}_0\mathbf{P}_1$ in the proper Euclidean geometry. It means, that the vector $\mathbf{P}_0\mathbf{P}_1$ and its components p_l, $l = 1, 2, ..n$ mean the same quantity in the Euclidean geometry, whereas they are, in general, different quantities in a physical geometry.

In a like way the expression for a circular cylinder $Cyl_{P_0P_1Q}$, determined by points P_0, P_1 ($P_0 \neq P_1$) on the cylinder axis and by the point Q on cylinder surface, is a set of points R, satisfying the relation

$$Cyl_{P_0P_1Q} = \{R | S_{P_0P_1R} = S_{P_0P_1Q}\} \tag{1.11}$$

where $S_{P_0P_1Q}$ is the area of the triangle, determined by vertices P_0, P_1, Q. The area $S_{P_0P_1Q}$ is calculated by means of the Heron formula via distances between the points P_0, P_1, Q. Let the point $P_3 \in \mathcal{T}_{[P_0P_1]}$, where $\mathcal{T}_{[P_0P_1]}$ is a segment of a straight line between the points P_0, P_1. This segment is defined as a set of points R by the relation

$$\mathcal{T}_{[P_0P_1]} = \left\{ R | \sqrt{2\sigma(P_0, R)} + \sqrt{2\sigma(P_1, R)} = \sqrt{2\sigma(P_0, P_1)} \right\} \qquad (1.12)$$

Then in the proper Euclidean geometry $Cyl_{P_0P_1Q} = Cyl_{P_0P_3Q} = Cyl_{P_1P_3Q}$. However, in a physical geometry, in general, $Cyl_{P_0P_1Q} \neq Cyl_{P_0P_3Q} \neq Cyl_{P_1P_3Q}$. In other words, many different cylinders $Cyl_{P_0P_1Q}, \quad P_0, P_1 \in \mathcal{T}_{[S_1S_2]}$ of a physical geometry degenerate in the proper Euclidean geometry into one cylinder, defined by its axis $\mathcal{T}_{[S_1S_2]}$ and by the point Q on the surface of the cylinder. This fact takes place, because the segment of the straight line (1.12) is one-dimensional in the case of the proper Euclidean geometry, but it is, in general, a many-dimensional surface in the case of a physical geometry.

One-dimensionality of $\mathcal{T}_{[S_1S_2]}$ in the Euclidean geometry is formulated in terms of the world function as follows. Any section $S\left(\mathcal{T}_{[S_1S_2]}, Q\right)$ of the segment $\mathcal{T}_{[S_1S_2]}$ at the point $Q \in \mathcal{T}_{[S_1S_2]}$ consists of one point Q. Section $S\left(\mathcal{T}_{[S_1S_2]}, Q\right)$ is defined as a set of points R

$$S\left(\mathcal{T}_{[S_1S_2]}, Q\right) = \{R | \sigma(S_1, R) = \sigma(S_1, Q) \wedge \sigma(S_2, R) = \sigma(S_2, Q)\} \qquad (1.13)$$

In the proper Euclidean geometry $S\left(\mathcal{T}_{[S_1S_2]}, Q\right) = \{Q\}$, $\forall Q \in \mathcal{T}_{[S_1S_2]}$, whereas in the case of a physical geometry this equality does not take place, in general.

Thus, the physical geometry degenerates, in general, at a transition from a physical geometry to the proper Euclidean geometry. Different geometrical objects and concepts may coincide. On the contrary at transition from the proper Euclidean geometry to a physical geometry some geometrical objects split into different geometrical objects. Transition from a general case to a special one, followed by a degeneration, is perceived easily, whereas a transition from a special case to a general one, followed by a splitting of geometrical objects and of geometrical concepts, is perceived hard.

2 Relativistic invariance

The relativistic invariance is presented usually as an invariance of dynamic equation with respect to the Poincare group of inertial coordinate transformations. Nonrelativistic dynamic equations are considered to be invariant with respect to Galilean group of inertial coordinates transformation. Is it possible to formulate difference between relativistic physics and nonrelativistic one in invariant terms, i.e. without a reference to coordinate system and the laws of their transformation? Yes, it is possible.

In the relativistic physics the space-time geometry is described by means of one structure σ, which is known as the squared space-time interval, or the world function. In the nonrelativistic physics the event space (space-time) is described by two invariant geometrical structures. Such a two-structure description is not a space-time geometry, because the space-time geometry is described by one structure σ.

If there exist another space-time structure, such a construction should be referred to as a fortified geometry, i.e. a geometry with additional geometric structure. This additional structure is the time structure $T(P,Q)$ which is a difference of absolute times between the points P and Q. One can construct another geometrical structure $S(P,Q)$, which is a difference between of absolute spatial positions of points P and Q. The structure $S(P,Q)$ is not an independent structure. The spatial structure $S(P,Q)$ can be constructed of two structures σ and T. In any case *in the nonrelativistic physics there are two independent geometrical structures. In the relativistic physics there is only one structure* σ.

Usually one uses the time structure T and the spatial structure S in the nonrelativistic physics. However, one may use geometrical structures σ and T. In this case one can investigate additional restrictions, imposed by time structure T on the space-time geometry of Minkowski. Geometrical structures of the space-time determine a motion group of the space-time, and this motion group determines group of invariance of dynamic equations. Thus, the difference between the relativistic physics and nonrelativistic one is determined by the number of geometrical structures. This difference may be formulated in coordinateless form. The transformation laws of dynamic equations are only corollaries of these geometrical structures existence.

3 Statistical description of the stochastic particle motion

Statistical description of stochastic (indeterministic) particles was an origin of the physical geometry, because, it put the question on a nature of this indeterminism, which can be explained only by a more general uniform space-time geometry, than the geometry of Minkowski.

As we have mentioned in the introduction, a statistical description of indeterministic particles was made at first by means of the dynamical conception of statistical description (DCSD). This approach is founded on a use of relativistic concept of particle state [9, 10, 11]. Another method of the stochastic particles description has been used later, when a statistical ensemble (instead of a single particle) has been considered as a basic element of the particle dynamics [15]. The concept of a single particle and the concept of the phase space are not used in this method. This method goes around the nonrelativistic concept of the particle state. It does not use the concept of the particle state. It uses only concept of the ensemble state, which is insensitive to the problem of the limit (1.8) existence. From formal viewpoint this method uses DCSD, but not PCSD.

The action for the statistical ensemble $\mathcal{E}[\mathcal{S}_{\rm st}]$ of free indeterministic particles $\mathcal{S}_{\rm st}$ is written in the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\rm st}]}[\mathbf{x},\mathbf{u}] = \int\int_{V_\xi} \left\{\frac{m}{2}\dot{\mathbf{x}}^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right\} dtd\boldsymbol{\xi}, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \tag{3.1}$$

Independent variables $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$ label constituents $\mathcal{S}_{\mathrm{st}}$ of the statistical ensemble. The dependent variable $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$ describes the regular component of the particle motion. The variable $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$ describes the mean value of the stochastic velocity component, $\hbar$ is the quantum constant. The second term in (3.1) describes the kinetic energy of the stochastic velocity component. The third term describes interaction between the stochastic component $\mathbf{u}(t, \mathbf{x})$ and the regular component $\dot{\mathbf{x}}(t, \boldsymbol{\xi})$. The operator

$$\boldsymbol{\nabla} = \left\{ \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3} \right\} \tag{3.2}$$

is defined in the space of coordinates $\mathbf{x}$. Dynamic equations for the dynamic system $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ are obtained as a result of variation of the action (3.1) with respect to dynamic variables $\mathbf{x}$ and $\mathbf{u}$.

The action for a single indeterministic particle $\mathcal{S}_{\mathrm{st}}$ has the form

$$\mathcal{A}_{\mathcal{S}_{\mathrm{st}}}[\mathbf{x}, \mathbf{u}] = \int \left\{ \frac{m}{2}\dot{\mathbf{x}}^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u} \right\} dt, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \tag{3.3}$$

This action is not correctly defined, because operator $\boldsymbol{\nabla}$ is defined on 3D-space of coordinates $\mathbf{x} = \{x^1, x^2, x^3\}$, whereas in the action functional (3.3) the variable $\mathbf{x}$ is used only on one-dimensional set. It means that there are no dynamic equations for the particle $\mathcal{S}_{\mathrm{st}}$, and the particle $\mathcal{S}_{\mathrm{st}}$ is a stochastic (indeterministic) system. However, the action functional (3.1) is well defined, and dynamic equations exist for the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$, although dynamic equations do not exist for constituents of this statistical ensemble.

Variation of the action (3.1) leads to dynamic equations

$$\delta\mathbf{u}: \qquad m\rho\mathbf{u} + \frac{\hbar}{2}\boldsymbol{\nabla}\rho = 0, \qquad \mathbf{u} = -\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho \tag{3.4}$$

$$\delta\mathbf{x}: \qquad m\frac{d^2\mathbf{x}}{dt^2} = \boldsymbol{\nabla}\left(\frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right) \tag{3.5}$$

where

$$\rho = \frac{\partial(\xi_1, \xi_2, \xi_3)}{\partial(x^1, x^2, x^3)} = \left(\frac{\partial(x^1, x^2, x^3)}{\partial(\xi_1, \xi_2, \xi_3)}\right)^{-1} \tag{3.6}$$

After proper change of variables the dynamic equations are reduced to the equation [15]

$$i\hbar\partial_0\psi + \frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi + \frac{\hbar^2}{8m}\boldsymbol{\nabla}^2 s_\alpha \cdot (s_\alpha - 2\sigma_\alpha)\psi - \frac{\hbar^2}{4m}\frac{\boldsymbol{\nabla}\rho}{\rho}\boldsymbol{\nabla}s_\alpha\sigma_\alpha\psi = 0 \tag{3.7}$$

where ψ is the two-component complex wave function

$$\rho = \psi^*\psi, \qquad s_\alpha = \frac{\psi^*\sigma_\alpha\psi}{\rho}, \qquad \alpha = 1, 2, 3 \tag{3.8}$$

σ_α are 2×2 Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{3.9}$$

If components ψ_1 and ψ_2 are linear dependent $\psi = \begin{pmatrix} a\psi_1 \\ b\psi_1 \end{pmatrix}$, $a, b = \text{const}$, then $\mathbf{s} = \text{const}$. Two last terms of the equation (3.7) vanish, and the equation turns to the Schrödinger equation

$$i\hbar\partial_0\psi + \frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi = 0 \tag{3.10}$$

Thus, the Schrödinger equation and interpretation of the quantum mechanics appear from the dynamical system $\mathcal{E}[\mathcal{S}_{\text{st}}]$, described by the action functional (3.1). This fact seems rather unexpected, because in quantum mechanics the wave function is considered as a specific quantum object, which has no analog in classical physics. In reality, the wave function is simply a way of description of ideal continuous medium [16]. One may describe an ideal fluid in terms of hydrodynamic variables: density ρ and velocity $\mathbf{v}$. One may describe an ideal fluid in terms of the wave function. The statistical ensemble $\mathcal{E}[\mathcal{S}_{\text{st}}]$ is a dynamic system of the type of continuous medium. The two representations of dynamic equations for the dynamic system $\mathcal{E}[\mathcal{S}_{\text{st}}]$ can be transformed one into another.

Generalization of the action (3.3) on the stochastic relativistic charged particle, moving in an electromagnetic field, has the form [17]

$$\mathcal{A}[x,\kappa] = \int \left\{ -mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k} - \frac{e}{c}A_k\dot{x}^k \right\} d^4\xi, \qquad d^4\xi = d\xi_0 d\boldsymbol{\xi}, \tag{3.11}$$

$$K = \sqrt{1+\lambda^2\left(\kappa_l\kappa^l + \partial_l\kappa^l\right)}, \qquad \lambda = \frac{\hbar}{mc} \tag{3.12}$$

where $x = \{x^i(\xi_0, \boldsymbol{\xi})\}$, $i = 0,1,2,3$ are dependent variables. $\xi = \{\xi_0, \boldsymbol{\xi}\} = \{\xi_k\}$, $k = 0,1,2,3$ are independent variables, and $\dot{x}^i \equiv dx^i/d\xi_0$. The quantities $\kappa^l = \{\kappa^l(x)\}$, $l = 0,1,2,3$ are dependent variables, describing stochastic component of the particle motion, $A_k = \{A_k(x)\}$, $k = 0,1,2,3$ is the potential of electromagnetic field. The dynamic system, described by the action (3.11), (3.12) is a statistical ensemble of indeterministic particles, which looks as some continuous medium. The variables κ^l are connected with the stochastic component u^l of the particle 4-velocity by the relation

$$\kappa^l = \frac{m}{\hbar}u^l, \qquad l = 0,1,2,3 \tag{3.13}$$

In the nonrelativistic approximation one may neglect the temporal component $\kappa^0 = \frac{m}{\hbar}u^0$ with respect to the spatial one $\boldsymbol{\kappa} = \frac{m}{\hbar}\mathbf{u}$. Setting $\xi_0 = t = x^0$ and $A_k = 0$ in (3.11), (3.12), we obtain the action (3.1) instead of (3.11), (3.12).

After a proper change of variables one obtains dynamic equation for the action

(3.11), (3.12). This dynamic equation has the form [17]

$$
\begin{aligned}
&\left(-i\hbar\partial_k+\frac{e}{c}A_k\right)\left(-i\hbar\partial^k+\frac{e}{c}A^k\right)\psi-\left(m^2c^2+\frac{\hbar^2}{4}\left(\partial_l s_\alpha\right)\left(\partial^l s_\alpha\right)\right)\psi \\
=&\quad -\hbar^2\frac{\partial_l\left(\rho\partial^l s_\alpha\right)}{2\rho}\left(\sigma_\alpha-s_\alpha\right)\psi \qquad (3.14)
\end{aligned}
$$

where designations (3.8), (3.9) are used. In the case, when the wave function ψ is one-component, vector $\mathbf{s}$ =const, and the dynamic equation (3.14) turns to the Klein-Gordon equation

$$
\left(-i\hbar\partial_k+\frac{e}{c}A_k\right)\left(-i\hbar\partial^k+\frac{e}{c}A^k\right)\psi-m^2c^2\psi=0 \qquad (3.15)
$$

Transformation of hydrodynamic equations (3.4) into dynamic equations in terms of the wave function ψ is based on the fact, that a wave function is a method of description of hydrodynamic equations [16]. Transformation of hydrodynamic equations, described in terms of hydrodynamic variables (density ρ and velocity $\mathbf{v}$), to a description in terms the wave function rather is bulky, because it uses a partial integration of dynamic equations. These integration leads to appearance of arbitrary integration functions $g^a(\boldsymbol{\xi})$. The wave function is constructed of these integration functions [16].

One can explain the situation as follows. It is well known, that the Schrödinger equation can be written in the hydrodynamic form of Madelung-Bohm [19, 20]. The wave function ψ is presented in the form

$$
\psi=\sqrt{\rho}\exp\left(i\varphi/\hbar\right) \qquad (3.16)
$$

Substituting (3.16) in the Schrödinger equation (3.10), one obtains two real equations for dynamical variables ρ and φ. Taking gradient from the equation for φ and introducing designation

$$
\mathbf{v}=-\frac{\hbar}{m}\boldsymbol{\nabla}\varphi,\qquad \operatorname{curl}\mathbf{v}=0 \qquad (3.17)
$$

one obtains four equations of the hydrodynamic type

$$
\frac{\partial\rho}{\partial t}+\boldsymbol{\nabla}\left(\rho\mathbf{v}\right)=0,\qquad \frac{d\mathbf{v}}{dt}\equiv\frac{\partial\mathbf{v}}{\partial t}+\left(\mathbf{v}\boldsymbol{\nabla}\right)\mathbf{v}=-\frac{1}{m}\boldsymbol{\nabla}U_{\mathrm{B}} \qquad (3.18)
$$

where U_{B} is the Bohm potential, defined by the relation

$$
U_{\mathrm{B}}=U\left(\rho,\boldsymbol{\nabla}\rho,\boldsymbol{\nabla}^2\rho\right)=\frac{\hbar^2}{8m\rho}\left(\frac{\left(\boldsymbol{\nabla}\rho\right)^2}{\rho}-2\boldsymbol{\nabla}^2\rho\right)=-\frac{\hbar^2}{2m\sqrt{\rho}}\boldsymbol{\nabla}^2\sqrt{\rho} \qquad (3.19)
$$

Hydrodynamic equations (3.18) can be easily obtained from equations (3.4), (3.5). To obtain representation of equations (3.18), (3.19) in terms of wave function, one needs to integrate these equations, because they have been obtained by means of differentiation of the Schrödinger equation. This integration can be easily produced, if the condition (3.17) takes place and the fluid flow is non-rotational.

In the general case of vortical flow the integration is more complicated. Nevertheless this integration has been produced [16], and one obtains a more complicated equation (3.7), where two last terms describe vorticity of the flow. The Schrödinger equation (3.10) is a special case of the more general equation (3.7).

Note that the equation (3.7) is not linear, although it is invariant with respect to transformation

$$\psi \rightarrow \tilde{\psi} = A\psi, \qquad A = \text{const} \tag{3.20}$$

which admits one to normalize the wave function to any nonnegative quantity. This property describes independence of the statistical ensemble on the number of its constituents.

Representation of quantum mechanics as a statistical description of classical indeterministic particles admits one to interpret all quantum relations in terms of statistical description. This interpretation distinguishes in some clauses from conventional (Copenhagen) interpretation of quantum mechanics.

In any statistical description there are two different kinds of measurement, which have different properties. Massive measurement (M-measurement) is produced over all constituents of the statistical ensemble. A result of M-measurement of the quantity R is a distribution of the quantity R, which can be predicted as a result of solution of dynamic equations for the statistical ensemble.

Single measurement (S-measurement) is produced over one of constituents of the statistical ensemble. A result of S-measurement of the quantity R is some random value of the quantity R, which cannot be predicted by the theory. In the Copenhagen interpretation of the quantum mechanics the wave function is supposed to describe a single particle (but not a statistical ensemble of particles). As a result there is only one type of measurement, which is considered sometimes as a M-measurement and sometimes as a S-measurement. As far as M-measurement and S-measurement have different properties, such an identification is a source of numerous contradictions and paradoxes [21].

Representation of quantum mechanics as a statistical description of the indeterministic particles motion has two important consequences: (1) elimination of quantum principles as laws of nature, (2) problem of primordial stochastic motion of free particles.

4 Deformation principle

The idea, that a geometry is described completely by means of a distance function (or world function) is very old. At first it was a metric space, described by metric (distance). The metric has been restricted by a set of conditions such as the triangle axiom and nonnegativity of the metric. Condition of nonnegativity of metric does not permit to apply the metric space for description of the space-time. The main defect of the metric geometry and the distance geometry [13, 14] is impossibility of construction of geometrical objects in terms of the world function or in terms of the metric. Construction of geometrical objects in terms of the world function is to be possible, because it is supposed that the geometry is described completely by the world function and in terms of the world function. Furthermore, a physical geometry is to admit a coordinateless description.

Such a situation is possible, if one defines concepts of a geometry and those of a geometrical objects correctly.

Definition 4.1: The physical geometry $\mathcal{G} = \{\sigma, \Omega\}$ is a point set Ω with the single-valued function σ on it

$$\sigma: \qquad \Omega \times \Omega \rightarrow \mathbb{R}, \qquad \sigma(P,P) = 0, \qquad \sigma(P,Q) = \sigma(Q,P), \qquad P,Q \in \Omega \quad (4.1)$$

Definition 4.2: Two physical geometries $\mathcal{G}_1 = \{\sigma_1, \Omega_1\}$ and $\mathcal{G}_2 = \{\sigma_2, \Omega_2\}$ are equivalent ($\mathcal{G}_1 \mathrm{eqv} \mathcal{G}_2$) if the point set $\Omega_1 \subseteq \Omega_2 \wedge \sigma_1 = \sigma_2$, or $\Omega_2 \subseteq \Omega_1 \wedge \sigma_2 = \sigma_1$.

Remark: Coincidence of point sets Ω_1 and Ω_2 is not necessary for equivalence of geometries $\mathcal{G}_1$ and $\mathcal{G}_2$. If one demands coincidence of Ω_1 and Ω_2 in the case equivalence of $\mathcal{G}_1$ and $\mathcal{G}_2$, then an elimination of one point P from the point set Ω_1 turns the geometry $\mathcal{G}_1 = \{\sigma_1, \Omega_1\}$ into geometry $\mathcal{G}_2 = \{\sigma_1, \Omega_1 \backslash P\}$, which appears to be not equivalent to the geometry $\mathcal{G}_1$. Such a situation seems to be inadmissible, because a geometry on a part $\omega \subset \Omega_1$ of the point set Ω_1 appears to be not equivalent to the geometry on the whole point set Ω_1.

According to definition the geometries $\mathcal{G}_1 = \{\sigma, \omega_1\}$ and $\mathcal{G}_2 = \{\sigma, \omega_2\}$ on parts of Ω, $\omega_1 \subset \Omega$ and $\omega_2 \subset \Omega$ are equivalent $(\mathcal{G}_1 \mathrm{eqv} \mathcal{G}), (\mathcal{G}_2 \mathrm{eqv} \mathcal{G})$ to the geometry $\mathcal{G} = \{\sigma, \Omega\}$, whereas the geometries $\mathcal{G}_1 = \{\sigma, \omega_1\}$ and $\mathcal{G}_2 = \{\sigma, \omega_2\}$ are not equivalent, in general, if $\omega_1 \nsubseteq \omega_2$ and $\omega_2 \nsubseteq \omega_1$. Thus, the relation of equivalence is intransitive, in general. The space-time geometry may vary in different regions of the space-time. It means, that a physical body, described as a geometrical object, may evolve in such a way, that it appears in regions with different space-time geometry.

Definition 4.3: A geometrical object $g_{\mathcal{P}_n}$ of the geometry $\mathcal{G} = \{\sigma, \Omega\}$ is a subset $g_{\mathcal{P}_n} \subset \Omega$ of the point set Ω. This geometrical object $g_{\mathcal{P}_n}$ is a set of roots $R \in \Omega$ of the function $F_{\mathcal{P}_n}$

$$F_{\mathcal{P}_n}: \qquad \Omega \rightarrow \mathbb{R}$$

where

$$F_{\mathcal{P}_n} \quad : \qquad F_{\mathcal{P}_n}(R) = G_{\mathcal{P}_n}(u_1, u_2, ...u_s), \qquad s = \frac{1}{2}(n+1)(n+2) \qquad (4.2)$$

$$u_l = \sigma(w_i, w_k), \qquad i,k = 0,1,...n+1, \qquad l = 1,2,...\frac{1}{2}(n+1)(n+2) \qquad (4.3)$$

$$w_k = P_k \in \Omega, \qquad k = 0,1,...n, \qquad w_{n+1} = R \in \Omega \qquad (4.4)$$

Here $\mathcal{P}_n = \{P_0, P_1, ..., P_n\} \subset \Omega$ are $n+1$ points which are parameters, determining the geometrical object $g_{\mathcal{P}_n}$

$$g_{\mathcal{P}_n} = \{R | F_{\mathcal{P}_n}(R) = 0\}, \qquad R \in \Omega, \qquad \mathcal{P}_n \in \Omega^{n+1} \qquad (4.5)$$

$F_{\mathcal{P}_n}(R) = G_{\mathcal{P}_n}(u_1, u_2, ...u_s)$ is an arbitrary function of $\frac{1}{2}(n+1)(n+2)$ arguments u_s and of $n+1$ parameters $\mathcal{P}_n$. The set $\mathcal{P}_n$ of the geometric object parameters will be referred to as the skeleton of the geometrical object. The subset $g_{\mathcal{P}_n}$ will be referred to as the envelope of the skeleton. One skeleton may have many envelopes. When a particle is considered as a geometrical object, its motion in the space-time is described mainly by the skeleton $\mathcal{P}_n$. The shape of the envelope is of no importance in the first approximation.

Remark: Arbitrary subset of the point set Ω is not a geometrical object, in general. It is supposed, that physical bodies may have a shape of a geometrical object only, because only in this case one can identify identical physical bodies (geometrical objects) in different space-time geometries.

Existence of the same geometrical objects in different space-time regions, having different geometries, arises the question on equivalence of geometrical objects in different space-time geometries. Such a question was not arisen before, because one does not consider such a situation, when the physical body moves from one space-time region to another space-time region, having another space-time geometry. In general, mathematical technique of the conventional space-time geometry is not applicable for simultaneous consideration of several different geometries of different space-time regions.

We can perceive the space-time geometry only via motion of physical bodies in the space-time, or via construction of geometrical objects corresponding to these physical bodies. As it follows from the *definition 4.3* of the geometrical object, the function F as a function of its arguments (of world functions of different points) is the same in all physical geometries. It means, that a geometrical object $\mathcal{O}_1$ in the geometry $\mathcal{G}_1 = \{\sigma_1, \Omega_1\}$ is obtained from the same geometrical object $\mathcal{O}_2$ in the geometry $\mathcal{G}_2 = \{\sigma_2, \Omega_2\}$ by means of the replacement $\sigma_2 \rightarrow \sigma_1$ in the definition of this geometrical object.

As far as the physical geometry is determined by its geometrical objects construction, a physical geometry $\mathcal{G} = \{\sigma, \Omega\}$ can be obtained from some known standard geometry $\mathcal{G}_{\mathrm{st}} = \{\sigma_{\mathrm{st}}, \Omega\}$ by means a deformation of the standard geometry $\mathcal{G}_{\mathrm{st}}$. Deformation of the standard geometry $\mathcal{G}_{\mathrm{st}}$ is realized by the replacement $\sigma_{\mathrm{st}} \rightarrow \sigma$ in all definitions of the geometrical objects in the standard geometry. The proper Euclidean geometry is an axiomatizable geometry. It has been constructed by means of the Euclidean method as a logical construction. The proper Euclidean geometry is a physical geometry. It may be used as a standard geometry $\mathcal{G}_{\mathrm{st}}$. Construction of a physical geometry as a deformation of the proper Euclidean geometry will be referred to as the deformation principle. The most physical geometries are nonaxiomatizable geometries. They can be constructed only by means of the deformation principle.

Description of the elementary particle motion in the space-time contains only the particle skeleton $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$. The form of the function (4.2) is of no importance in the first approximation. In the elementary particle dynamics only equivalence of vectors $\mathbf{P}_i\mathbf{P}_k$, $i, k = 0, 1, ...n$ is essential. These vectors are defined by the particle skeleton $\mathcal{P}_n$.

The equivalence ($\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1$) of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined by the relations

$$(\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{Q}_0\mathbf{Q}_1): \qquad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \wedge |\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1| \tag{4.6}$$

where

$$|\mathbf{P}_0\mathbf{P}_1| = \sqrt{2\sigma(P_0, P_1)} \tag{4.7}$$

and the scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ is defined by the relation (1.10)

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_l) = \sigma(P_0, Q_l) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_l) \tag{4.8}$$

Skeletons $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$ and $\mathcal{P}'_n = \{P'_0, P'_1, ...P'_n\}$ may belong to the same geometrical object, if

$$|\mathbf{P}_i\mathbf{P}_k| = |\mathbf{P}'_i\mathbf{P}'_k|, \qquad i, k = 0, 1, ...n \tag{4.9}$$

i.e. lengths of all vectors $\mathbf{P}_i\mathbf{P}_k$ and $\mathbf{P}'_i\mathbf{P}'_k$ are equal. However, it is not sufficient for equivalence of skeletons $\mathcal{P}_n$ and $\mathcal{P}'_n$.

Skeletons $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$ and $\mathcal{P}'_n = \{P'_0, P'_1, ...P'_n\}$ are equivalent

$$(\mathcal{P}_n\text{eqv}\mathcal{P}'_n): \qquad \text{if} \quad \mathbf{P}_i\mathbf{P}_k\text{eqv}\mathbf{P}'_i\mathbf{P}'_k, \qquad i, k = 0, 1, , ...n \tag{4.10}$$

In other words, the equality of skeletons needs equality of the lengths of vectors $\mathbf{P}_i\mathbf{P}_k$ and $\mathbf{P}'_i\mathbf{P}'_k$ and equality of their mutual orientations.

5 Multivariance

The physical geometry has the property, called multivariance. It means that at the point P_0 there are many vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}'_1$, $\mathbf{P}_0\mathbf{P}''_1$, ...which are equivalent to the vector $\mathbf{Q}_0\mathbf{Q}_1$ at the point Q_0, but they are not equivalent between themselves. The proper Euclidean geometry has not the property of multivariance. In the proper Euclidean geometry there is only one vector $\mathbf{P}_0\mathbf{P}_1$ at the point P_0, which is equivalent to the vector $\mathbf{Q}_0\mathbf{Q}_1$ at the point Q_0.

Multivariance is connected formally with the definition of the vector equivalence via algebraic relations (4.6) - (4.8). If vector $\mathbf{Q}_0\mathbf{Q}_1$ is given, and it is necessary to determine the equivalent vector $\mathbf{P}_0\mathbf{P}_1$ at the point P_0, one needs to solve two equations (4.6) with respect to the point P_1. If the two equations have a unique solution, one has only one equivalent vector $\mathbf{P}_0\mathbf{P}_1$ (single-variance). If there are many solutions, one has many vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}'_1$, $\mathbf{P}_0\mathbf{P}''_1$, .., which are equivalent to vector $\mathbf{Q}_0\mathbf{Q}_1$ (multivariance). It is possible such a case, when there are no solutions. In this case one has zero-variance.

Multivariance of the space-time geometry leads to splitting of one world chain into many stochastic world chains. As a result the multivariance of the space-time geometry in microcosm leads to appearance of quantum effects.

Zero-variance appears in the case of many-point skeletons. It is interesting in that relation, that it may forbid existence of elementary particles with many-point skeletons.

6 Discreteness of the space-time geometry

The world function (1.3) describes a completely discrete geometry. However, the space-time geometry may discrete only partly. In the discrete geometry one may introduce the point density $\rho = d\sigma_{\mathrm{M}}/d\sigma_{\mathrm{d}}$ with respect to point density in the geometry of Minkowski. The discrete geometry may be described by the relative points density

$$\rho\left(\sigma_{\mathrm{d}}\right) = \frac{d\sigma_{\mathrm{M}}\left(\sigma_{\mathrm{d}}\right)}{d\sigma_{\mathrm{d}}} = \begin{cases} 0 & \text{if} \quad 0 < \left|\sigma_{\mathrm{d}}\right| < \frac{\lambda_0^2}{2} \\ 1 & \text{if} \quad \left|\sigma_{\mathrm{M}}\right| \geq \frac{\lambda_0^2}{2} \end{cases} \tag{6.1}$$

For close points the relative point density of the discrete geometry vanishes, and this circumstance is considered as a discreteness of the geometry. However, the discreteness may not be complete.

Let us consider the space-time geometry with the world function σ_{g}

$$\sigma_{\mathrm{g}} = \sigma_{\mathrm{M}} + \frac{\lambda_0^2}{2} \begin{cases} \operatorname{sgn}\left(\sigma_{\mathrm{M}}\right) & \text{if} \quad \left|\sigma_{\mathrm{M}}\right| \geq \sigma_0 \\ \frac{\sigma_{\mathrm{M}}}{\sigma_0} & \text{if} \quad \left|\sigma_{\mathrm{M}}\right| < \sigma_0 \end{cases}, \qquad \lambda_0, \sigma_0 = \text{const} \tag{6.2}$$

The relative point density in the geometry (6.2) has the form

$$\rho\left(\sigma_{\mathrm{g}}\right) = \frac{d\sigma_{\mathrm{M}}\left(\sigma_{\mathrm{g}}\right)}{d\sigma_{\mathrm{g}}} = \begin{cases} 1 & \text{if} \quad \left|\sigma_{\mathrm{g}}\right| \geq \sigma_0 + \frac{\lambda_0^2}{2} \\ \frac{\sigma_0}{\sigma_0 + \frac{\lambda_0^2}{2}} & \text{if} \quad \left|\sigma_{\mathrm{g}}\right| < \sigma_0 + \frac{\lambda_0^2}{2} \end{cases} \tag{6.3}$$

If $\sigma_0 \ll \lambda_0^2$ the relative point density in the region, where $\left|\sigma_{\mathrm{g}}\right| \in \left(0, \sigma_0 + \frac{\lambda_0^2}{2}\right)$ is much less, than 1. If $\sigma_0 \rightarrow 0$, the relative point density (6.3) tends to (6.1). The geometry (6.2) should be qualified as a partly discrete space-time geometry. We shall refer to the geometry (6.2) as a granular geometry. In the granular space-time geometry the relative density of points, separated by small distance (less, than λ_0), is much less than the relative density of other points. The granular geometry, described by the world function

$$\begin{aligned} \sigma_{\mathrm{g}} &= \sigma_{\mathrm{M}} + \frac{\lambda_0^2}{2} \begin{cases} \operatorname{sgn}\left(\sigma_{\mathrm{M}}\right) & \text{if} \quad \left|\sigma_{\mathrm{M}}\right| > \sigma_0 \\ f\left(\frac{\sigma_{\mathrm{M}}}{\sigma_0}\right) & \text{if} \quad \left|\sigma_{\mathrm{M}}\right| \leq \sigma_0 \end{cases}, \qquad \lambda_0, \sigma_0 = \text{const} \\ f(x) &= -f(-x), \qquad f(1) = 1 \end{aligned} \tag{6.4}$$

is a generalization of the the geometry (6.2).

7 Elementary particle dynamics

Dynamics of elementary particles in the granular space-time geometry is considered in [22]. The state of an elementary particle is described by its skeleton $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$, consisting of $n+1$ space-time points. Such description of the particle state is complete in the sense, that it does not need parameters of the particle (mass, charge, spin, etc.). All this information is described by the disposition of points in the skeleton. It means a geometrization of parameters of the elementary particles. Besides, the conventional description of the particle state as a point in the phase space is nonrelativistic. The granular geometry is multivariant, in general. The particle motion is stochastic, and the limit (1.8), which determines the particle momentum, does not exist. Thus, *to satisfy the relativity principles, we are forced to describe the particle state by its skeleton.*

Evolution of the particle state is described by the world chain $\mathcal{C}$, consisting of connected skeletons $\mathcal{P}_n^{(s)} = \left\{P_0^{(s)}, P_1^{(s)}, ...P_n^{(s)}\right\}$, $s = ... - 1, 0, 1, ...$

$$\mathcal{C} = \bigcup_s \mathcal{P}_n^{(s)}, \quad P_1^{(s)} = P_0^{(s+1)}, \quad s = ... - 1, 0, 1, ... \tag{7.1}$$

Connection between skeletons of the world chain arises, because the second point $P_1^{(s)}$ of the sth skeleton coincides with the first point $P_0^{(s+1)}$ of the $(s+1)$th skeleton. The vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)} = \mathbf{P}_0^{(s)}\mathbf{P}_0^{(s+1)}$ will be referred to as the leading vector, determining the shape of the world chain. All skeletons of the chain are similar in the sense, that

$$\left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right| = \mu_{ik} = \text{const}, \quad i, k = 0, 1, ..n, \quad s = ... - 1, 0, 1, ... \tag{7.2}$$

Definition: Two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent ($\mathbf{P}_0\mathbf{P}_1 \text{eqv} \mathbf{Q}_0\mathbf{Q}_1$), if

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \wedge |\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1| \tag{7.3}$$

If the particle is free, then the skeleton motion is progressive (i.e. motion without rotation), and orientation of adjacent skeletons $\mathcal{P}_n^{(s)}$, $\mathcal{P}_n^{(s+1)}$ is the same.

$$\begin{aligned} \left(\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}.\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}\right) &= \left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right| \cdot \left|\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}\right| = \mu_{ik}^2, \\ i, k &= 0, 1, ..n, \quad s = ... - 1, 0, 1, ... \end{aligned} \tag{7.4}$$

Equations (7.2), (7.4) means that the adjacent skeletons of the world chain are equivalent $\mathcal{P}_n^{(s)} \text{eqv} \mathcal{P}_n^{(s+1)}$, $s = ... - 1, 0, 1, ...$The adjacent skeletons are equivalent, if corresponding vectors of adjacent skeletons are equivalent

$$\left(\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)} \text{eqv} \mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}\right), \quad i, k = 0, 1, ..n, \quad s = ... - 1, 0, 1, .. \tag{7.5}$$

One obtains $n(n+1)$ difference dynamic equations (7.5) (or (7.2), (7.4)), which describe evolution of the particle state. Introducing a coordinate system, one obtains nD dynamic variables, whose values are to be determined by dynamic equations (7.5). Here D is the dimension of the space-time (the number of coordinates, describing the point position). In particular, in the case of a pointlike particle, whose state is described by two points P_0, P_1, the number of dynamic equations $n_{\text{d}} = 2$, whereas in the 4D-space-time the number of variable $n_{\text{v}} = 4$. In the multivariant space-time the dynamic equations have many solutions. As a result the world chain appears to be multivariant (stochastic).

In the Riemannian space-time and in the space-time of Minkowski the world chain can be approximated by a world line, provided characteristic lengths of the problem are much larger, than the lengths of the world chain links. In this case the dynamic equations (7.5) are reduced to ordinary differential equations. If the world line is timelike [22], the solution of dynamic equations appears to be unique. If the vectors $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ are spacelike, dynamic equations have many solutions even in the Riemannian space-time. It is connected with the circumstance, that the Riemannian geometry as well as the geometry of Minkowski is multivariant with respect to spacelike vectors. At the conventional approach the spacelike world lines are not considered at all. Such world lines are inadmissible by definition (It is a postulate).

One attempted to obtain differential dynamic equations for a pointlike particle in [23]. At first one obtained equation for free pointlike particle in the space-time of Minkowski. It is only one equation, whereas in the conventional approach one has three equations for the velocity components $\boldsymbol{\beta} = \mathbf{v}/c$. This equation has the form

$$\dot{\boldsymbol{\beta}}^2 + \frac{\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}\right)^2}{1-\boldsymbol{\beta}^2} = 0, \quad \dot{\boldsymbol{\beta}} \equiv \frac{d\boldsymbol{\beta}}{dt} \tag{7.6}$$

Let us introduce designation

$$\boldsymbol{\beta}\dot{\boldsymbol{\beta}} = \sqrt{\boldsymbol{\beta}^2\dot{\boldsymbol{\beta}}^2}\cos\phi \tag{7.7}$$

where ϕ is the angle between vectors $\boldsymbol{\beta}$ and $\dot{\boldsymbol{\beta}}$. The equation (7.6) takes the form

$$\dot{\boldsymbol{\beta}}^2\left(1 + \frac{\boldsymbol{\beta}^2\cos^2\phi}{1-\boldsymbol{\beta}^2}\right) = 0 \tag{7.8}$$

If the world line is timelike $\boldsymbol{\beta}^2 < 1$, and $\cos^2\phi \leq 1$, then the bracket in (7.8) is positive and one concludes from (7.8), that

$$\dot{\boldsymbol{\beta}}^2 = 0 \tag{7.9}$$

One obtains three equations from one equation (7.9)

$$\dot{\boldsymbol{\beta}} \equiv c^{-1}\frac{d\mathbf{v}}{dt} = 0 \tag{7.10}$$

If the world line is spacelike, then $\boldsymbol{\beta}^2 > 1$, and the bracket in (7.9) vanishes at

$$\cos^2\phi = \frac{\boldsymbol{\beta}^2 - 1}{\boldsymbol{\beta}^2} < 1 \tag{7.11}$$

The acceleration $\dot{\mathbf{v}} = c\dot{\boldsymbol{\beta}}$ becomes indefinite at this value of the angle ϕ between $\dot{\boldsymbol{\beta}}$ and $\boldsymbol{\beta}$. It should be interpreted as impossibility of spacelike world lines. At the conventional approach such an impossibility of spacelike world lines is simply postulated.

Such a result is rather evident, because the space-time of Minkowski is single-variant with respect to timelike vectors and it is multivariant with respect to space-like vectors. For timelike vectors one can obtain three dynamic equations (7.10) from one equation (7.8). For spacelike particles it is impossible.

Another example is considered in the paper [23]. Motion of pointlike particle in the gravitational field of a massive sphere of the mass M is considered. In the Newtonian approximation the world function $\sigma(t,\mathbf{y};t',\mathbf{y}')$ between the points with coordinates $(t,\mathbf{y})$ and $(t',\mathbf{y}')$ has the form

$$\sigma(t,\mathbf{y};t',\mathbf{y}') = \frac{1}{2}\left(c^2 - \frac{2GM}{\sqrt{\mathbf{x}^2}}\right)(t-t')^2 - \frac{1}{2}(\mathbf{y}-\mathbf{y}')^2 \tag{7.12}$$

where G is the gravitational constant, and

$$\mathbf{x} = \frac{\mathbf{y}+\mathbf{y}'}{2} \tag{7.13}$$

Metric tensor has the conventional form

$$g_{ik} = g_{ik}(\mathbf{x}) = \mathrm{diag}\left(c^2 - \frac{2GM}{\sqrt{\mathbf{x}^2}}, -1, -1, -1\right) \tag{7.14}$$

but the space-time geometry, described by the world function (7.12) is non-Riemannian.

The Riemannian geometry is conceptually defective in the sense, the world function of the Riemannian geometry with metric tensor (7.14) is multivalued, whereas the world function is to be single-valued. But the Riemannian geometry is single-variant with respect to timelike vectors, having common origin. As a result the timelike world chains in the Riemannian space-time geometry are deterministic. They can be replaced by deterministic world lines.

The space-time geometry (7.12) is multivariant in general, but the world function (7.12) is single-valued. The world function (7.12) is obtained in the extended general relativity, when one eliminates the unfounded restriction that the space-time geometry is to be a Riemannian geometry [24].

To obtain differential dynamic equations for a free particle, one considers two connected links of the world chain, defined by the points P_0, P_1, P_2, having coordinates

$$P_0 = \{y - dy_1\}, \qquad P_1 = \{y\}, \qquad P_2 = \{y + dy_2\} \tag{7.15}$$

where

$$y = \{t, \mathbf{y}\}, \qquad dy_1 = \{dt_1, d\mathbf{y}_1\}, \qquad dy_2 = \{dt_2, d\mathbf{y}_2\} \tag{7.16}$$

are coordinates in some inertial coordinate system. Dynamic equations (7.2), (7.4) have the form

$$\sigma(y, y - dy_1) = \sigma(y, y + dy_2) \tag{7.17}$$

$$4\sigma(y, y - dy_1) = \sigma(y - dy_1, y + dy_2) \tag{7.18}$$

Let us introduce designations

$$\mathbf{v}_1 = \frac{d\mathbf{y}_1}{dt_1}, \qquad \mathbf{v}_2 = \frac{d\mathbf{y}_2}{dt_2}, \qquad \boldsymbol{\beta}_1 = \frac{\mathbf{v}_1}{c}, \qquad \boldsymbol{\beta}_2 = \frac{\mathbf{v}_2}{c} \tag{7.19}$$

$$\boldsymbol{\beta}_1 = \boldsymbol{\beta} - \frac{1}{2}\dot{\boldsymbol{\beta}}dt, \qquad \boldsymbol{\beta}_2 = \boldsymbol{\beta} + \frac{1}{2}\dot{\boldsymbol{\beta}}dt, \qquad \dot{\boldsymbol{\beta}} \equiv \frac{d\boldsymbol{\beta}}{dt}, \qquad dt = \frac{dt_1 + dt_2}{2} \tag{7.20}$$

where

$$\mathbf{v} = c\boldsymbol{\beta} \qquad \dot{\mathbf{v}} = c\dot{\boldsymbol{\beta}} \tag{7.21}$$

$$V = V(\mathbf{y}) = \frac{GM}{\sqrt{(\mathbf{y})^2}}, \qquad U = U(\mathbf{y}) = \frac{V(\mathbf{y})}{c^2} \tag{7.22}$$

Using designations (7.19) - (7.22), transforming two equations (7.17), (7.18), and considering $dt, d\mathbf{y}_1, d\mathbf{y}_2$ as infinitesimal quantities, one obtains after simplifications

$$\frac{1}{2}\dot{\boldsymbol{\beta}}^2 (dt)^2 - c\dot{\boldsymbol{\beta}}\boldsymbol{\nabla}U (dt)^2 + \frac{1}{2}\frac{\left(c\boldsymbol{\beta}\boldsymbol{\nabla}U + \boldsymbol{\beta}\dot{\boldsymbol{\beta}}\right)^2}{1 - 2U - \boldsymbol{\beta}^2}(dt)^2 + \frac{c^2}{2}\beta^\alpha\beta^\beta\partial_\alpha\partial_\beta U (dt)^2 = 0 \tag{7.23}$$

where

$$\partial_\alpha \equiv \frac{\partial}{\partial y^\alpha}$$

Note, that the terms of the order of dt disappear.

In terms of variables $\mathbf{v}, \dot{\mathbf{v}}, V$, defined by relations (7.21), (7.22) the relation (7.23) has the form

$$\frac{1}{2}\dot{\mathbf{v}}^2 - \dot{\mathbf{v}}\boldsymbol{\nabla}V + \frac{1}{2}\frac{(\mathbf{v}\boldsymbol{\nabla}V + \mathbf{v}\dot{\mathbf{v}})^2}{c^2(1 - 2c^{-2}V - c^{-2}\mathbf{v}^2)} + \frac{1}{2c^2}v^\alpha v^\beta\partial_\alpha\partial_\beta V = 0 \tag{7.24}$$

One obtains in the nonrelativistic approximation

$$\frac{1}{2}\dot{\mathbf{v}}^2 - \dot{\mathbf{v}}\boldsymbol{\nabla}V = 0 \tag{7.25}$$

It is evident, that one cannot determine three components of vector $\dot{\mathbf{v}}$ from one equation (7.25). One can determine only mean value $\langle\dot{\mathbf{v}}\rangle$ of vector $\dot{\mathbf{v}}$, choosing some principle of averaging.

Let us represent $\mathbf{v}$ in the form

$$\dot{\mathbf{v}} = \dot{\mathbf{v}}_\parallel + \dot{\mathbf{v}}_\perp, \qquad \dot{\mathbf{v}}_\parallel = \boldsymbol{\nabla}V\frac{(\dot{\mathbf{v}}\boldsymbol{\nabla}V)}{|\boldsymbol{\nabla}V|^2}, \qquad \dot{\mathbf{v}}_\perp = \dot{\mathbf{v}} - \boldsymbol{\nabla}V\frac{(\dot{\mathbf{v}}\boldsymbol{\nabla}V)}{|\boldsymbol{\nabla}V|^2} \tag{7.26}$$

where $\mathbf{v}_\parallel$ and $\mathbf{v}_\perp$ are components of $\mathbf{v}$, which are parallel to $\boldsymbol{\nabla}V$ and perpendicular to $\boldsymbol{\nabla}V$ correspondingly. It follows from (7.25)

$$\dot{\mathbf{v}}_\parallel^2 - 2\dot{\mathbf{v}}_\parallel\boldsymbol{\nabla}V + \dot{\mathbf{v}}_\perp^2 = 0 \tag{7.27}$$

Let

$$\dot{v}_{\parallel}=\frac{\dot{\mathbf{v}}\boldsymbol{\nabla}V}{|\boldsymbol{\nabla}V|}=\frac{\dot{\mathbf{v}}_{\parallel}\boldsymbol{\nabla}V}{|\boldsymbol{\nabla}V|},\qquad \dot{\mathbf{v}}_{\parallel}=\boldsymbol{\nabla}V\frac{(\dot{\mathbf{v}}\boldsymbol{\nabla}V)}{|\boldsymbol{\nabla}V|^{2}}=\frac{\boldsymbol{\nabla}V}{|\boldsymbol{\nabla}V|}\dot{v}_{\parallel}$$

The equation (7.27) may be rewritten in the form

$$\dot{v}_{\parallel}^{2}-2\dot{v}_{\parallel}\left|\boldsymbol{\nabla}V\right|+\dot{\mathbf{v}}_{\perp}^{2}=0 \tag{7.28}$$

or

$$\dot{v}_{\parallel}=\left|\boldsymbol{\nabla}V\right|\pm\sqrt{\left|\boldsymbol{\nabla}V\right|^{2}-\dot{\mathbf{v}}_{\perp}^{2}} \tag{7.29}$$

It follows from (7.29), that

$$0<\dot{\mathbf{v}}_{\perp}^{2}\leq\left|\boldsymbol{\nabla}V\right|^{2},\qquad 0<\dot{v}_{\parallel}<2\left|\boldsymbol{\nabla}V\right| \tag{7.30}$$

The quantity $\dot{v}_{\parallel}$ vibrates around its mean value $\left\langle\dot{v}_{\parallel}\right\rangle=|\boldsymbol{\nabla}V|$.

$$\left\langle\dot{\mathbf{v}}_{\parallel}\right\rangle=\frac{\boldsymbol{\nabla}V}{|\boldsymbol{\nabla}V|}\left\langle\dot{v}_{\parallel}\right\rangle=\boldsymbol{\nabla}V \tag{7.31}$$

Taking into account a symmetry and supposing that $\langle\dot{\mathbf{v}}_{\perp}\rangle=0$, one obtains, that

$$\langle\dot{\mathbf{v}}\rangle=\left\langle\dot{\mathbf{v}}_{\parallel}\right\rangle=\boldsymbol{\nabla}V=\boldsymbol{\nabla}\frac{GM}{r},\quad r=|\mathbf{y}| \tag{7.32}$$

In the general case one obtains instead of (7.27)

$$\begin{aligned}&\dot{v}_{\parallel}^{2}-2\dot{v}_{\parallel}\left|\boldsymbol{\nabla}V\right|+\dot{\mathbf{v}}_{\perp}^{2}\\ =\;&-\frac{(\mathbf{v}\boldsymbol{\nabla}V)^{2}+2\left(\mathbf{v}\boldsymbol{\nabla}V\right)\left(\mathbf{v}\dot{\mathbf{v}}_{\parallel}\right)+\left(v_{\parallel}\dot{v}_{\parallel}+\mathbf{v}_{\perp}\dot{\mathbf{v}}_{\perp}\right)^{2}}{c^{2}-2V-\mathbf{v}^{2}}-\frac{1}{c^{2}}v^{\alpha}v^{\beta}\partial_{\alpha}\partial_{\beta}V\end{aligned} \tag{7.33}$$

This result distinguishes from the conventional result of the general relativity, because it depends on the second derivatives $\partial_{\alpha}\partial_{\beta}V$ of the gravitational potential. Equation (7.33) can be written in the form of quadratic equation with respect to $\dot{v}_{\parallel}$

$$\begin{aligned}&\dot{v}_{\parallel}^{2}\left(1+\frac{v_{\parallel}^{2}}{c^{2}-2V-\mathbf{v}^{2}}\right)-2\dot{v}_{\parallel}\left(\left|\boldsymbol{\nabla}V\right|-\frac{\left(\mathbf{v}\boldsymbol{\nabla}V\right)v_{\parallel}}{c^{2}-2V-\mathbf{v}^{2}}\right)+\left\langle\dot{\mathbf{v}}_{\perp}^{2}\right\rangle\\ =\;&-\frac{(\mathbf{v}\boldsymbol{\nabla}V)^{2}+(\mathbf{v}_{\perp}\dot{\mathbf{v}}_{\perp})^{2}}{c^{2}-2V-\mathbf{v}^{2}}-\frac{1}{c^{2}}v^{\alpha}v^{\beta}\partial_{\alpha}\partial_{\beta}V\end{aligned} \tag{7.34}$$

8 Fluidity of boundary between the particle dynamics and space-time geometry

In the space-time geometry $\mathcal{G}$ the dynamic equations (7.2), (7.4) are written in the form

$$\left(\mathbf{P}_{i}^{(s)}\mathbf{P}_{k}^{(s)}.\mathbf{P}_{i}^{(s+1)}\mathbf{P}_{k}^{(s+1)}\right)=\left|\mathbf{P}_{i}^{(s)}\mathbf{P}_{k}^{(s)}\right|^{2},\qquad i,k=0,1,...n \tag{8.1}$$

$$\left|\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}\right|^2 = \left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|^2, \qquad i,k=0,1,...n \tag{8.2}$$

The difference dynamic equations (8.1), (8.2) can be written in the form, which is close to the conventional description in the Kaluza-Klein space-time [22]. Let σ_{K_0} be the world function in the space-time geometry $\mathcal{G}_{\mathrm{K}_0}$. The geometry $\mathcal{G}_{\mathrm{K}_0}$ is the 5D pseudo-Euclidean geometry of the index 1 with the compactificied coordinate x^5. In other words, the space-time geometry $\mathcal{G}_{\mathrm{K}_0}$ is the Kaluza-Klein geometry with vanishing gravitational and electromagnetic fields. Let us represent the world function σ of the space-time geometry $\mathcal{G}$ in the form

$$\sigma(P,Q) = \sigma_{\mathrm{K}_0}(P,Q) + d(P,Q) \tag{8.3}$$

where the function d describes the difference between the true world function σ of the real space-time geometry and the world function σ_{K_0} of the standard geometry $\mathcal{G}_{\mathrm{K}_0}$, where the description will be produced. Then one obtains

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)_{\mathrm{K}_0} + d(P_0,Q_1) + d(P_1,Q_0) - d(P_0,Q_0) - d(P_1,Q_1) \tag{8.4}$$

$$|\mathbf{P}_0\mathbf{P}_1|^2 = |\mathbf{P}_0\mathbf{P}_1|^2_{\mathrm{K}_0} + 2d(P_0,P_1) \tag{8.5}$$

where index "K_0" means, that the corresponding quantities are calculated in the geometry $\mathcal{G}_{\mathrm{K}_0}$ by means of the world function σ_{K_0}.

By means of (8.4), (8.5) the dynamic equations (8.1), (8.2) can be written in the form

$$\left(\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}.\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}\right)_{\mathrm{K}_0} - \left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|^2_{\mathrm{K}_0} = w\left(P_i^{(s)},P_k^{(s)},P_i^{(s+1)},P_k^{(s+1)}\right), \qquad i,k=0,1,...n \tag{8.6}$$

$$\left|\mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}\right|^2_{\mathrm{K}_0} - \left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|^2_{\mathrm{K}_0} = 2d\left(P_i^{(s)},P_k^{(s)}\right) - 2d\left(P_i^{(s+1)},P_k^{(s+1)}\right), \qquad i,k=0,1,...n \tag{8.7}$$

where

$$\begin{aligned} w\left(P_i^{(s)},P_k^{(s)},P_i^{(s+1)},P_k^{(s+1)}\right) &= 2d\left(P_i^{(s)},P_k^{(s)}\right) - d\left(P_i^{(s)},P_k^{(s+1)}\right) \\ &\quad -d\left(P_k^{(s)},P_i^{(s+1)}\right) + d\left(P_i^{(s)},P_i^{(s+1)}\right) \\ &\quad +d\left(P_k^{(s)},P_k^{(s+1)}\right) \end{aligned} \tag{8.8}$$

Equations (8.6), (8.7) are dynamic difference equations, written in the geometry $\mathcal{G}_{\mathrm{K}_0}$. Rhs of these equations can be interpreted as some geometric force fields, generated by the fact that the space-time geometry $\mathcal{G}$ is described in terms of some standard geometry $\mathcal{G}_{\mathrm{K}_0}$. These force fields describe deflection of the granular geometry $\mathcal{G}$ from the Kaluza-Klein geometry $\mathcal{G}_{\mathrm{K}_0}$. Such a possibility is used at the description of the gravitational field, which can be described as generated by the curvature of the curved space-time, or as a gravitational field in the space-time geometry of Minkowski. In dynamic equations (8.6), (8.7) such a possibility is realized for arbitrary granular space-time geometry $\mathcal{G}$.

Evolution of the leading vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ is of most interest. These equations are obtained from equations (8.6), (8.7) at $i=0, k=1$. One obtains form equations (8.6), (8.7)

$$\left|\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right|_{\mathrm{K}_0}^2 - \left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{K}_0}^2 = 2d\left(P_0^{(s)},P_1^{(s)}\right) - 2d\left(P_1^{(s)},P_1^{(s+1)}\right) \tag{8.9}$$

$$\begin{aligned}&\left(\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}.\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right)_{\mathrm{K}_0} - \left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{K}_0}^2 \\ =\ & 3d\left(P_0^{(s)},P_1^{(s)}\right) - d\left(P_0^{(s)},P_1^{(s+1)}\right) + d\left(P_1^{(s)},P_1^{(s+1)}\right)\end{aligned} \tag{8.10}$$

where one uses, that $P_1^{(s)} = P_0^{(s+1)}$.

In the case, when the space-time is uniform, and the function

$$d(P,Q) = D\left(\sigma_{\mathrm{K}_0}(P,Q)\right) \tag{8.11}$$

the equations (8.9), (8.10) take the from

$$\left|\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right|_{\mathrm{K}_0}^2 - \left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{K}_0}^2 = 0 \tag{8.12}$$

$$\left(\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}.\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right)_{\mathrm{K}_0} - \left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{K}_0}^2 = 4d\left(P_0^{(s)},P_1^{(s)}\right) - d\left(P_0^{(s)},P_1^{(s+1)}\right) \tag{8.13}$$

In the case, when the leading vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ is timelike, one can introduce the angle $\phi_{01}^{(s)}$ between the vectors $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ and $\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}$ in the standard geometry $\mathcal{G}_{\mathrm{K}_0}$. By means of (8.12) it is defined by the relation

$$\cosh\phi_{01}^{(s)} = \frac{\left(\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}.\mathbf{P}_0^{(s+1)}\mathbf{P}_1^{(s+1)}\right)_{\mathrm{K}_0}}{\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{K}_0}^2} \tag{8.14}$$

Then in the uniform geometry $\mathcal{G}_{\mathrm{K}_0}$ the equation (8.13) has the form

$$\sinh\frac{\phi_{01}^{(s)}}{2} = \frac{\sqrt{4d\left(P_0^{(s)},P_1^{(s)}\right) - d\left(P_0^{(s)},P_1^{(s+1)}\right)}}{\sqrt{2}\left|\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}\right|_{\mathrm{K}_0}} \tag{8.15}$$

Thus, relativistic dynamics of particles can be generalized on the case of the granular space-time geometry.

Application of the world function technique admits one realize the boundary shift between the particle dynamics and the space-time geometry. A conceptual development of a theory seems to be more effective in the monistic conception, having the only basic quantity (world function). The monistic conception is more sensitive to possible mistakes. This circumstance admits one to find mistakes and correct them. In the conception, where there are several basic quantities (concepts), connection between these quantities may be quite different. This circumstance embarrasses a choice of correct connection between different basic concepts. Construction of a conceptual theory, based on physical principles, should be realized in a form of monistic conception. However, it does not exclude the fact, that non-monistic theory with a simple space-time geometry may be simpler in calculations of concrete physical phenomena.

Non-monistic conception is not so sensitive to mistakes in a theory, because influence of possible mistakes can be compensated in calculations of concrete physical phenomena by introduction of new hypotheses (and sometimes by invention of new principles, having a restricted meaning). As far as the calculations agree with experimental data concerning this physical phenomenon, the theory is considered as a true theory, confirmed by several experiments. Such an approach admits one to explain and to calculate new physical phenomena. However, this approach prevents from construction of consistent physical theory, which explains all physical phenomena. Undiscovered mistakes may appear at calculations of other physical phenomena.

Fluidity of boundary between the particle dynamics and the space-time geometry resolves debate between the adherents of the general relativity and adherents of the relativistic theory of gravitation.

9 Pair production

The pair production effect is considered in the paper on the physics geometrization, because it was the first appearance of the quantum field theory (QFT) inconsistency. Impossibility of the conventional QFT to explain the pair production consistently was the first step, which generated the idea of the quantum principles revision.

It is considered in the contemporary physics, that the pair production effect is a special quantum effect, which has no classical analog. It is the most important evidence in favour of quantum nature of microcosm. However, it is not so [17]. In reality, either classical or quantum mechanism of pair production is absent in the contemporary quantum field theory, if it is developed consistently according to quantum principles. Unfortunately, the quantum field theory was developed inconsistently. Here we describe only reasons of this inconsistency, referring to original papers for details .

The Klein-Gordon equation (3.15) describes an evolution of a free quantum object (world line). In the absence of electromagnetic field the equation (3.15) has the form

$$\hbar^2 \partial_k \partial^k \psi + m^2 c^2 \psi = 0 \tag{9.1}$$

Stationary states of this quantum object have the form

$$\psi = A \exp\left(-ik_0 t + i\mathbf{k}\mathbf{x}\right), \qquad k_0 = \pm\sqrt{\mathbf{k}^2 + \left(\frac{mc}{\hbar}\right)^2} \tag{9.2}$$

If $k_0 > 0$, this quantum object is at the state "particle". If $k_0 < 0$, this quantum object is at the state "antiparticle". The quantum object is called "semlon". This term is a reading of the abbreviation "SML", which is an abbreviation of Russian term "section of world line". Thus, a particle and an antiparticle are two different states of semlon, but not independent objects. The semlon has two different states: particle and antiparticle.

Pair production is a generation of a particle and of an antiparticle at some point of the space-time. The particle and the antiparticle are two different states of one dynamical system (world line). They cannot be two different dynamical systems, because two different dynamical objects cannot annihilate at some space-time point. Effect of pair production or of pair annihilation appears, when world line changes its direction in the time direction.

Let the world line of a particle be described by the equations

$$x^k = x^k(\tau), \qquad k = 0, 1, 2, 3 \tag{9.3}$$

where τ is some parameter (evolution parameter) along the world line. For the particle $dx^0/d\tau > 0$. For the antiparticle $dx^0/d\tau < 0$. The point, where the derivative $dx^0/d\tau$ changes its sign, is a point of production or a point of annihilation of a pair particle-antiparticle.

The quantity $p_0 = \hbar k_0$ is an eigenvalue of the temporal component $\hat{p}_0 = -i\hbar\partial_0$ of the 4-momentum operator

$$\hat{p}_k = -i\hbar\partial_k \qquad k = 0, 1, 2, 3 \tag{9.4}$$

Particle and antiparticle have different signs of temporal component $p_0 = \hbar k_0$ of 4-momentum. The energy E is positive at all semlon states.

$$E = \int T_0^0 d\mathbf{x} = \int \left(\hbar^2 \left(\partial_0\psi^* \cdot \partial^0\psi\right) + m^2c^2\psi^*\psi\right) d\mathbf{x} > 0 \tag{9.5}$$

Here T_0^0 is a component of the energy-momentum tensor for the Klein-Gordon $\mathcal{S}_{\mathrm{KG}}$ dynamic system, having the Klein-Gordon equation (9.1) as a dynamic equation. Thus, the evolution operator $\hat{H} = \hat{p}_0$ does not coincide in general, with the energy operator $\hat{E}$, which arises from the expression (9.5) at the second quantization. The same difference takes place at a classical description of a relativistic particle [26].

In the nonrelativistic approximation, when there is no pair production, the evolution operator H (Hamiltonian) coincides with the particle energy E (or with $-E$). This coincidence is transmitted to relativistic theory, where there is a pair production, from nonrelativistic theory, where such a pair production is absent. The relation

$$\partial_0\psi = \frac{1}{i\hbar}\left[\psi, \int T_0^0 d\mathbf{x}\right]_- \tag{9.6}$$

where $[...]_-$ denotes a commutator, is used in the second quantized theory for determination of the commutation relations.

These commutation relations lead to consideration of particle and antiparticle as two different dynamic systems (but not as different states of the same dynamic system). Formally it means, that the operator ψ contains both creation operators and annihilation operators. It is necessary to satisfy the relation (9.6). At such a way of the second quantization the particle and antiparticle are considered as independent objects. The vacuum state appears to be nonstationary. In general, if the vacuum state is a state without particles and antiparticles, it has to be stationary, because in this case the space-time is empty. However, it is supposed, that the vacuum state contains virtual particles, which may be converted to real particles and antiparticles, if there is some interaction, described by nonlinear term, added to the dynamic equation (9.1).

For instance, the pair production appears in the case of nonlinear equation [27, 28, 29, 30]

$$\hbar^2 \partial_k \partial^k \psi + m^2 c^2 \psi = g \psi^* \psi \psi \tag{9.7}$$

where g is a constant of self-action. Corresponding dynamic equations are written in form of expansion over the self-action constant g. Solving these equations, one uses a perturbation theory.

There is an alternative representation [31] of nonlinear equation (9.7), when particle and antiparticle are considered as different states of a semlon, but not as independent objects. In this case the wave function ψ contains only annihilation operators, and ψ^* contains only creation operators. In this case the energy operator $\hat{E}$ has only nonnegative eigenvalues. The energy E does not coincide with the Hamiltonian $\hat{H}$, as it takes place in the classical case [26]. The vacuum state is stationary, and there is no necessity for introduction of virtual particles. The dynamic equations can be written and solved without expansion over the self-action constant g and without a use of the perturbation theory. However, in this case there is no pair production.

Absence of the pair production effect means only, that the nonlinear term of type (9.7) cannot generate the pair production. The pair production effect is generated by a more complicated interaction, as it follows from (3.11), (3.12), or from [17], where the problem of pair production is investigated more elaborate. The pair production is connected with a change of an effective mass Km of a particle, but not with the virtual particles. It is not clear, how to take into account the change of effective mass in the framework of quantum theory, although the Klein-Gordon equation (3.15) takes into account the factor K (3.12), responsible for a change of effective mass.

The problem of pair production is not yet geometrized, although the way of the pair production geometrization it is rather clear.

10 Skeleton conception of elementary particles

After the paper [6] publication the role of the space-time geometry increased in the theory of elementary particles, because in fact the quantum principles were replaced by the multivariant space-time geometry. It became clear, that constructing a theory of elementary particles, one should use relativistic concept of the particle state.

In the case, when the particle is not pointlike, its state is described by its skeleton $\mathcal{P}_n = \{P_0, P_1, ..., P_n\}$, which is a set of $(n+1)$ space-time points. These points are connected rigidly. In the case of a pointlike particle the skeleton consists of two points. The skeleton $\mathcal{P}_n$ is a natural generalization of the skeleton of a pointlike particle on the case of a composite particle. Motion of any particle is described by the world chain, consisting of connected skeletons [25]. $...\mathcal{P}_n^{(0)}, \mathcal{P}_n^{(1)}, ..., \mathcal{P}_n^{(s)}...$

$$\mathcal{P}_n^{(s)} = \left\{P_0^{(s)}, P_1^{(s)}, ..P_n^{(s)}\right\}, \qquad s = ...0, 1, ... \tag{10.1}$$

The adjacent skeletons $\mathcal{P}_n^{(s)}, \mathcal{P}_n^{(s+1)}$ of the chain are connected by the relations $P_1^{(s)} = P_0^{(s+1)}$, $s = ...0, 1, ...$ The vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)} = \mathbf{P}_0^{(s)}\mathbf{P}_0^{(s+1)}$ is the leading vector, which determines the world chain direction.

Dynamics of free elementary particle is determined by the relations

$$\mathcal{P}_n^{(s)} \mathrm{eqv} \mathcal{P}_n^{(s+1)} : \qquad \mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)} \mathrm{eqv} \mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}, \qquad i, k = 0, 1, ..n; \qquad s = ...0, 1, ... \tag{10.2}$$

which describe equivalence of adjacent skeletons. Equivalence of vectors is defined by the relations (7.3).

Thus, dynamics of a free elementary particle is described by a system of algebraic equations (10.2). Specific of dynamics depends on the elementary particle structure (mutual disposition of points inside the skeleton) and on the space-time geometry. Lengths $\left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|$ of vectors $\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}$ are constant along the whole world chain. These $n\,(n+1)\,/2$ quantities may be considered as characteristics of a particle. In the case of pointlike particle the length $|\mathbf{P}_s\mathbf{P}_{s+1}|$ of the link $\mathbf{P}_s\mathbf{P}_{s+1}$ is the geometrical mass of the particle. In the case of a more complicated skeletons the meaning of parameters $\left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|$ should be investigated.

The system of dynamic equations (10.2) consists of $n\,(n+1)$ algebraic equations for nD dynamic variables, where D is the dimension of the space-time (the number of coordinates, which are necessary for labeling of all points of the space-time). If $n \leq D$, the number of dynamic variables is more, than the number of dynamic equations. In this case we have a discrimination mechanism, which forbids some skeletons. This mechanism admits one to explain discrete parameters of elementary particles. If $n > D+1$, the number of dynamic equations is more than the number of dynamic variables. In this case there may exist many solutions, and the particle motion becomes multivariant (stochastic). Both cases may take place in the theory of elementary particles.

Dynamic equations (10.2) are written in the coordinateless form, and this fact is a worth of the dynamic equations (10.2), as far as it saves from a necessity to consider the coordinate transformations. Dynamic equations (10.2) are algebraic equations (not differential), and this fact is also a worth of the theory, because the algebraic equations are insensitive to a possible discreteness of the space-time geometry.

The first (nontrivial) attempt of a use of the relativistic concept of the particle state was made. One considered the structure of the Dirac particle (fermion) [32]. It was a conceptual step, because a possibility of spacelike world chains was considered. Spacelike world lines of real particles are absent in the conventional conception of elementary particle (they are possible for virtual particles, but it is a special problem). In the skeleton conception of elementary particles such a restriction is absent.

The Dirac particle is a dynamic system $\mathcal{S}_{\mathrm{D}}$, whose dynamic equation is the Dirac equation

$$i\gamma^k \partial_k \psi + mc\psi = 0 \tag{10.3}$$

It appeared that the skeleton of the Dirac particle consists of n points ($n \geq 3$). Its world chain is a spacelike helix with a timelike axis.

In our calculations we used the mathematical technique [33, 34], where γ-matrices are represented as hypercomplex numbers. Using designations

$$\gamma_5 = \gamma^{0123} \equiv \gamma^0\gamma^1\gamma^2\gamma^3, \tag{10.4}$$

$$\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3, \} = \{-i\gamma^2\gamma^3, -i\gamma^3\gamma^1, -i\gamma^1\gamma^2\} \tag{10.5}$$

we make the change of variables

$$\psi = Ae^{i\varphi + \frac{1}{2}\gamma_5\kappa} \exp\left(-\frac{i}{2}\gamma_5 \boldsymbol{\sigma\eta}\right) \exp\left(\frac{i\pi}{2}\boldsymbol{\sigma}\mathbf{n}\right) \Pi \tag{10.6}$$

$$\psi^* = A\Pi \exp\left(-\frac{i\pi}{2}\boldsymbol{\sigma}\mathbf{n}\right) \exp\left(-\frac{i}{2}\gamma_5 \boldsymbol{\sigma\eta}\right) e^{-i\varphi - \frac{1}{2}\gamma_5\kappa} \tag{10.7}$$

where (*) means the Hermitian conjugation, and

$$\Pi = \frac{1}{4}(1+\gamma^0)(1+\mathbf{z}\boldsymbol{\sigma}), \qquad \mathbf{z} = \{z^\alpha\} = \text{const}, \qquad \alpha = 1,2,3; \qquad \mathbf{z}^2 = 1 \tag{10.8}$$

is a zero divisor. The quantities A, κ, φ, $\boldsymbol{\eta} = \{\eta^\alpha\}$, $\mathbf{n} = \{n^\alpha\}$, $\alpha = 1,2,3$, $\mathbf{n}^2 = 1$ are eight real parameters, determining the wave function ψ. These parameters may be considered as new dependent variables, describing the state of dynamic system $\mathcal{S}_{\mathrm{D}}$. The quantity φ is a scalar, and κ is a pseudoscalar. Six remaining variables A, $\boldsymbol{\eta} = \{\eta^\alpha\}$, $\mathbf{n} = \{n^\alpha\}$, $\alpha = 1,2,3$, $\mathbf{n}^2 = 1$ can be expressed through the flux 4-vector

$$j^l = \bar{\psi}\gamma^l\psi, \qquad l = 0,1,2,3 \tag{10.9}$$

and spin 4-pseudovector

$$S^l = i\bar{\psi}\gamma_5\gamma^l\psi, \qquad l = 0,1,2,3 \tag{10.10}$$

Because of two identities

$$S^l S_l \equiv -j^l j_l, \qquad j^l S_l \equiv 0. \tag{10.11}$$

there are only six independent components among eight components of quantities j^l, and S^l. Now we can write the action for the dynamic equation(10.3) in the hydrodynamical form

$$\mathcal{S}_{\rm D}: \qquad \mathcal{A}_D[j,\varphi,\kappa,\boldsymbol{\xi}] = \int \mathcal{L} d^4x, \qquad \mathcal{L} = \mathcal{L}_{\rm cl} + \mathcal{L}_{\rm q1} + \mathcal{L}_{\rm q2} \tag{10.12}$$

$$\mathcal{L}_{\rm cl} = -m\rho - \hbar j^i\partial_i\varphi - \frac{\hbar j^l}{2\left(1+\boldsymbol{\xi}\mathbf{z}\right)}\varepsilon_{\alpha\beta\gamma}\xi^\alpha\partial_l\xi^\beta z^\gamma, \qquad \rho \equiv \sqrt{j^l j_l} \tag{10.13}$$

$$\mathcal{L}_{\rm q1} = 2m\rho\sin^2(\frac{\kappa}{2}) - \frac{\hbar}{2}S^l\partial_l\kappa, \tag{10.14}$$

$$\mathcal{L}_{\rm q2} = \frac{\hbar(\rho+j_0)}{2}\varepsilon_{\alpha\beta\gamma}\partial^\alpha\frac{j^\beta}{(j^0+\rho)}\xi^\gamma - \frac{\hbar}{2(\rho+j_0)}\varepsilon_{\alpha\beta\gamma}\left(\partial^0 j^\beta\right)j^\alpha\xi^\gamma \tag{10.15}$$

Lagrangian is a function of 4-vector j^l, scalar φ, pseudoscalar κ, and unit 3-pseudovector $\boldsymbol{\xi}$, which is connected with the spin 4-pseudovector S^l by means of the relations

$$\xi^\alpha = \rho^{-1}\left[S^\alpha - \frac{j^\alpha S^0}{(j^0+\rho)}\right], \qquad \alpha = 1,2,3; \qquad \rho \equiv \sqrt{j^l j_l} \tag{10.16}$$

$$S^0 = \mathbf{j}\boldsymbol{\xi}, \qquad S^\alpha = \rho\xi^\alpha + \frac{(\mathbf{j}\boldsymbol{\xi})j^\alpha}{\rho + j^0}, \qquad \alpha = 1,2,3 \tag{10.17}$$

Let us produce dynamical disquantization [35, 36] of the action (10.12)–(10.15), making the change

$$\partial_k \rightarrow \frac{j_k j^s}{j^l j_l}\partial_s \tag{10.18}$$

The action (10.12)–(10.15) takes the form

$$\begin{aligned}\mathcal{A}_{\rm Dqu}[j,\varphi,\kappa,\boldsymbol{\xi}] \;\; &= \int \left\{ -m\rho\cos\kappa - \hbar j^i\left(\partial_i\varphi + \frac{\varepsilon_{\alpha\beta\gamma}\xi^\alpha\partial_i\xi^\beta z^\gamma}{2\left(1+\boldsymbol{\xi}\mathbf{z}\right)}\right)\right. \\ &\left. + \frac{\hbar j^k}{2(\rho+j_0)\rho}\varepsilon_{\alpha\beta\gamma}\left(\partial_k j^\beta\right)j^\alpha\xi^\gamma\right\} d^4x \end{aligned}\tag{10.19}$$

Note that the second term $-\frac{\hbar}{2}S^l\partial_l\kappa$ in the relation (10.14) is neglected, because 4-pseudovector S^k is orthogonal to 4-vector j^k, and the derivative $S^l\partial_{||l}\kappa = S^l\rho^{-2}j_l j^k\partial_k\kappa$ vanishes.

Although the action (10.19) contains a non-classical variable κ, in fact this variable is a constant. Indeed, a variation with respect to κ leads to the dynamic equation

$$\frac{\delta\mathcal{A}_{Dqu}}{\delta\kappa} = m\rho\sin\kappa = 0 \tag{10.20}$$

which has solutions

$$\kappa = n\pi, \qquad n = \text{integer} \tag{10.21}$$

Thus, the effective mass $m_{\text{eff}} = m\cos\kappa$ has two values

$$m_{\text{eff}} = m\cos\kappa = \kappa_0 m = \pm m \tag{10.22}$$

where κ_0 is a dichotomic quantity $\kappa_0 = \pm 1$ introduced instead of cos κ. The quantity κ_0 is a parameter of the dynamic system $\mathcal{S}_{\text{Dqu}}$. It is not to be varying. The action (10.19), turns into the action

$$\begin{aligned}\mathcal{A}_{\text{Dqu}}[j,\varphi,\boldsymbol{\xi}] &= \int\left\{-\kappa_0 m\rho - \hbar j^i\left(\partial_i\varphi + \frac{\varepsilon_{\alpha\beta\gamma}\xi^\alpha\partial_i\xi^\beta z^\gamma}{2\left(1+\boldsymbol{\xi}\mathbf{z}\right)}\right)\right.\\ &\quad\left.+\frac{\hbar j^k}{2(\rho+j_0)\rho}\varepsilon_{\alpha\beta\gamma}\left(\partial_k j^\beta\right)j^\alpha\xi^\gamma\right\}d^4x\end{aligned} \tag{10.23}$$

Let us introduce Lagrangian coordinates $\tau = \{\tau_0, \boldsymbol{\tau}\} = \{\tau_i\,(x)\}$, $i = 0,1,2,3$ as functions of coordinates x in such a way that only coordinate τ_0 changes along the direction j^l. The action (10.23) is transformed to the form

$$\mathcal{A}_{\text{Dqu}}[x,\boldsymbol{\xi}] = \int \mathcal{A}_{\text{Dcl}}[x,\boldsymbol{\xi}]d\boldsymbol{\tau}, \qquad \mathbf{d}\boldsymbol{\tau} = d\tau_1 d\tau_2 d\tau_3 \tag{10.24}$$

where

$$\mathcal{S}_{\text{Dcl}}: \qquad \mathcal{A}_{\text{Dcl}}[x,\boldsymbol{\xi}] = \int\left\{-\kappa_0 m\sqrt{\dot{x}^i\dot{x}_i} + \hbar\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{2(1+\boldsymbol{\xi}\mathbf{z})} + \hbar\frac{(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}}{2\sqrt{\dot{x}^s\dot{x}_s}(\sqrt{\dot{x}^s\dot{x}_s}+\dot{x}^0)}\right\}d\tau_0 \tag{10.25}$$

After dynamic disquantization the Dirac particle is a statistical ensemble of dynamic systems $\mathcal{S}_{\text{Dcl}}$, as it follows from (10.24) and (10.25). Any dynamic system $\mathcal{S}_{\text{Dcl}}$ has 10 degrees of freedom. 6 degrees of freedom describe a progressive motion of a particle and 4 degrees of freedom describe the rotational motion of the particle. It is a classical model of the Dirac particle, which contains the quantum constant. The quantum constant appears in classical dynamic equations, because these equations are to contain magnetic moment. But the magnetic moment, (classical quantity!) depends on the quantum constant.

The variables $\boldsymbol{\xi}$ describe rotation, which is a classical analog of so-called "zitterbewegung". The Dirac particle is not a pointlike particle [37]. Description of internal degrees of freedom in terms of $\boldsymbol{\xi}$ appears to be nonrelativistic [38, 39], although the translational degrees of freedom x are described relativistically.

One succeeds to describe the classical model $\mathcal{S}_{\text{Dcl}}$ of Dirac particle in the framework of the skeleton conception of the elementary particles. The discrete space-time geometry (1.3) is replaced by half-discrete space-time geometry, described by the world function σ_{d}

$$\sigma_{\text{d}} = \sigma_{\text{M}} + \lambda_0^2\left\{\begin{array}{lll}\text{sgn}\,(\sigma_{\text{M}}) & \text{if} & |\sigma_{\text{M}}| > \sigma_0 > 0\\ f\,(\sigma_{\text{M}}) & \text{if} & |\,\sigma_{\text{M}}| < \sigma_0\end{array}\right. \qquad \lambda_0^2 = \frac{\hbar}{2bc} \tag{10.26}$$

where σ_{M} is the world function of the space-time of Minkowski, b is some universal constant and σ_0 is some constant. The function f is a monotone nondecreasing function, having properties $f\,(-\sigma_0) = -1$, $f\,(\sigma_0) = 1$.

The space-time geometry, described by the world function (10.26) is uniform and isotropic. The part of the world function corresponding to $|\sigma_{\mathrm{M}}| > \sigma_0$ is responsible for quantum effects of pointlike particle (Schrödinger equation [6]). The part of the world function (10.26), corresponding to $|\sigma_{\mathrm{M}}| < \sigma_0$ is responsible for the structure of a particle with the skeleton consisting of more, than two points. If $|f(\sigma_{\mathrm{M}})| < |\sigma_{\mathrm{M}}/\sigma_0|$, the spacelike world chain may have a shape of a helix with a timelike axis.

The case, when

$$f(\sigma_{\mathrm{M}}) = \left(\frac{\sigma_{\mathrm{M}}}{\sigma_0}\right)^3 \tag{10.27}$$

has been investigated [32]. Such a choice of the world function does not pretend to description of the real space-time. It is only some model, which correctly describes quantum effects connected with pointlike particles and tries to investigate, whether spacelike world chain may have a shape of a helix with a timelike axis. According to semiclassical approximation of the Dirac equation [40, 37, 39] the world line of a *free classical* Dirac particle has the shape of a helix. Such a shape of the world line explains existence of a spin. It was interesting, whether the spin of the Dirac particle can be obtained in the skeleton conception of elementary particles.

Consideration in [32] confirmed the supposition on the helix world chain of the Dirac particle (fermion). The skeleton of a fermion is to contain more, than two points. Besides, some restrictions on disposition of the skeleton points were obtained. It means that in the skeleton conception there is a discrimination mechanism responsible for discrete values of parameters of the elementary particles. Such a discrimination mechanism is absent in the conventional approach, based on a use of quantum principles. The obtained results are preliminary, because the simple restriction (10.27) on the world function has been used. Nevertheless these results show, that the skeleton conception admits one to investigate the structure of elementary particles.

The conventional approach, based on quantum principles, admits one only to ascribe to elementary particles such phenomenological properties as mass, spin, color, flavour and other, without explanation how these properties relate to the elementary particle structure. Conventional approach admits one only to classify elementary particles by their phenomenological properties and to predict reaction between the elementary particles on the basis of this classification.

Such a situation reminds situation with investigation of chemical elements. Periodic system of chemical elements is a phenomenological construction. It is an attribute of chemistry. Arrangement of atoms of chemical elements is investigated by physics (quantum mechanics). The periodic system of chemical elements had been discovered earlier, than one began to investigate atomic structures. However, the periodic system did not help us to create quantum mechanics and to investigate the atomic structure. The periodic system and the quantum mechanics are attributes of different sciences. In the same track the skeleton conception of elementary particles and the conventional phenomenological approach are essentially attributes of different sciences, investigating different sides of the elementary particles.

11 Conclusions

Thus, in the twentieth century a transition from the nonrelativistic physics to the relativistic one has been produced only in dynamic equations, but not in the concept of the particle state. The particle state as a point of the phase space is inadequate in application to indeterministic particles. In the nonrelativistic physics the particle state is described as a point in the phase space. Existence of primordially indeterministic particles in the microcosm does not admit a use of phase space, because the limit (1.8), determining the particle momentum, does not exist for indeterministic particles. One is forced to describe the particle state without limits of the type (1.8).

The relativistic concept of the particle state is realized by means of the particle skeleton. The skeleton consists of several space-time points. Such a concept of the particle state can be applied both for deterministic and indeterministic particles. The number of the skeleton points depends on the structure of the elementary particle. It is important, that the skeleton describes all characteristics of the particle, including its mass, charge, momentum and other characteristics, if they take place, (spin, flower, etc.). As a result one obtains a monistic conception, where all fundamental physical phenomena (including electromagnetic and gravitational interactions) are described in terms of points of the event space and of world functions between them.

Dynamic equations are algebraic equations, formulated in the coordinateless form. These equations are simpler and more universal, than equations, used in conventional theory of elementary particles.

The obtained skeleton conception is not yet a theory of elementary particles. It is only a conception, which deals with physical and geometrical principles. It is supposed that the skeleton conception can be applied for any space-time geometry and for any skeletons, which are compatible with this space-time geometry. In reality, there is a real space-time geometry, and there are only those skeletons which are admitted by this space-time geometry. The skeleton conception turns to a theory of elementary particles, only when this real space-time geometry will be determined. This real space-time geometry and skeletons, which are compatible with this geometry are to agree with experimental data.

The conception (physical principles) $\mathcal{C}_{\mathrm{con}}$ of the conventional theory of elementary particles is inconsistent, because it uses nonrelativistic concept of the particle state, which cannot be used at description of indeterministic particles. Any inconsistent theory has a very useful property. *Such a theory admits one to obtain any desirable statement. One needs only to invent a proper hypothesis.* A consistent theory admits one to obtain only those statements, which follow from basic statements of the theory, even if anybody wants to obtain another statements. The consistent theory admits one to introduce only those additional hypotheses, which are compatible with the theory.

Experimenters investigating elementary particle need some concepts for description of their experiments. They cannot describe their experiments without a use of some concepts. The experimenters take these concepts from their experience and from existing theories. Unfortunately, these concepts are adequate not always. System of these concepts is phenomenological. It useful for description of experiments. However, it is not always adequate for description of the elementary particles nature. One can see this in the example of atomic structure investigation. Chemists, who investigated experimentally properties of chemical elements, knew nothing on the atoms arrangement. The atom structure cannot be described in those phenomenological concepts, which were used by chemists.

The presented skeleton conception is only a conception (but not a theory) of elementary particles. It cannot be tested experimentally. One needs to determine a real space-time geometry and to investigate possible skeletons of elementary particles. Then the skeleton conception turns to a theory of elementary particles, and it can be tested experimentally

References

[1] Yu.A. Rylov, Geometry without topology *e-print math.MG/0002161.*

[2] Yu.A.Rylov, Geometry without topology as a new conception of geometry. *Int. Jour. Mat. & Mat. Sci.* **30**, *iss. 12, 733-760, (2002)*, (see also *e-print math.MG/0103002*).

[3] Yu. A.Rylov, Deformation principle as a foundations of physical geometry and its application to the space-time geometry". *Hypercomplex Numbers in Geometry and Physics*, **2**, *69-96, (2004)*, (in Russian). (In Enlish *e-print physics/0411103*)

[4] Rylov Yu. A. Multivariance as a crucial property of microcosm *Concepts of Physics* **6**, iss.1, 89 -117, (2009), see also *e-print 0806.1716*

[5] Yu. A. Rylov, Non-Euclidean method of the generalized geometry construction and its application to space-time geometry. in *Pure and Applied Differential geometry pp.238-246.* eds. Franki Dillen and Ignace Van de Woestyne. Shaker Verlag, Aachen, 2007. (see also *e-print math/0702552.*)

[6] Yu.A. Rylov, Non-Riemannian model of the space-time responsible for quantum effects. *Journ. Math. Phys.* **32(8)**, *2092-2098, (1991)*

[7] J. E.Moyal, *Proc. Cambr. Phil. Soc.,* **45**, 99, (1949).

[8] I. Fényes, *Zs. f. Phys.*, **132**, 81, (1952).

[9] Yu.A. Rylov, Quantum Mechanics as a theory of relativistic Brownian motion. *Ann. Phys. (Leipzig).* **27**, *1-11 (1971).*

[10] Yu.A. Rylov, Quantum mechanics as relativistic statistics.I: The two-particle case. *Int. J. Theor. Phys.* **8**, *65-83, (1973)*

[11] Yu.A. Rylov, Quantum mechanics as relativistic statistics.II: The case of two interacting particles. *Int. J. Theor. Phys.* **8**, *123-139. (1973).*

[12] Yu.A. Rylov (1990) "Extremal properties of Synge's world function and discrete geometry " . *J.Math. Phys.* **31**, *2876-2890 (1990)*

[13] K. Menger, Untersuchen über allgemeine Metrik, *Mathematische Annalen,* **100**, 75-113, (1928)

[14] L. M. Blumenthal, *Theory and Applications of Distance Geometry*, Oxford, Clarendon Press, 1953.

[15] Yu. A. Rylov Uniform formalism for description of dynamic, quantum and stochastic systems. *e-print/physics/0603237v6*

[16] Yu.A. Rylov, Spin and wave function as attributes of ideal fluid. *Journ. Math. Phys.* **40**, *pp. 256 - 278, (1999).*

[17] Yu.A. Rylov, Classical description of pair production *e-print /physics/0301020.*

[18] Yu.A. Rylov, Quantum mechanics as a dynamic construction. *Found. Phys.* **28**, No.2, 245-271, (1998).

[19] E. Madelung, *Z.Phys.* **40,** *322, (1926).*

[20] D. Bohm, *Phys.Rev.* **85**, *166,(1952), 180,(1952).*

[21] Yu. A. Rylov, Incompatibility of the Copenhagen interpretation with quantum formalism and its reasons. *Concepts of Physics* **5**, *iss.2, 323-328, (2008).* ISSN1897-2357 See also *e-print /physics/0604111*

[22] Yu. A. Rylov, Generalization of relativistic particle dynamics on the case of non-Riemannian space-time geometry. *Concepts of Physics* **6**, iss.4, 605, (2009). See also *e-print 0811.4562.*

[23] Yu. A. Rylov, Necessity of the general relativity revision and free motion of particles in non-Riemannian space-time geometry. *e-print 1001.5362v1.*

[24] Yu. A. Rylov, General relativity extended to non-Riemannian space-time geometry. *e-print 0910.3582v7*

[25] Yu. A. Rylov, Generalization of relativistic particle dynamics on the case of non-Riemannian space-time geometry. *Concepts of Physics* **6**, iss.*4, 605, (2009).* ISSN1897-2357. (see also *e-print 0811.4562*).

[26] Yu.A. Rylov, On connection between the energy-momentum vector and canonical momentum in relativistic mechanics. *Teoretischeskaya i Matematischeskaya Fizika.* **2**, *333-337.*(in Russian). *Theor. and Math. Phys. (USA)*, **5**, *333, (1970)* (*translated from Russian*)

[27] J. Glimm and A. Jaffe, *Phys. Rev.* **176** (1968) 1945.

[28] J. Glimm and A. Jaffe, *Ann. Math.* **91** (1970) 362.

[29] J. Glimm and A. Jaffe, *Acta Math.* **125** (1970) 203.

[30] J. Glimm and A. Jaffe, *J. Math. Phys.* **13** (1972) 1568.

[31] Yu.A. Rylov, On quantization of non-linear relativistic field without recourse to perturbation theory. *Int. J. Theor. Phys.* **6**, *181-204.*

[32] Yu. A. Rylov, Geometrical dynamics: spin as a result of rotation with superluminal speed *e-print 0801.1913.*

[33] F.Sauter, *Zs.Phys.* **63**, 803, (1930), **64**, 295, (1930).

[34] A.Sommerfeld, *Atombau and Spektrallinien.* bd.2, Braunschweig, 1951.

[35] Yu.A.Rylov, Dynamic disquantization of Dirac equation. *e-print /quant-ph/0104060.*

[36] Yu. A. Rylov, Formalized procedure of transition to classical limit in application to the Dirac equation. *e-print physics/0507183*

[37] Yu. A. Rylov, Is the Dirac particle composite? *e-print physics /0410045*

[38] Yu. A. Rylov, Anderson's absolute objects and constant timelike vector hidden in Dirac matrices. *e-print quant-ph/0112091*

[39] Yu. A. Rylov, Is the Dirac particle completely relativistic? *e-print /physics /0412032.*

[40] Yu.A. Rylov, Dirac equation in terms of hydrodynamic variables. *Advances in Applied Clifford Algebras,* **5**, *pp 1-40, (1995))* (see also *e-print 1101.5868*)