Repeated dynamic quantization

Yuri A. Rylov

Institute for Problems in Mechanics, Russian Academy of Sciences
101-1,Vernadskii Ave., Moscow, 119526, Russia
email: rylov@ipmnet.ru
Web site: *http://rsfq1.physics.sunysb.edu/˜rylov/yrylov.htm*
or mirror Web site:
http://gasdyn-ipm.ipmnet.ru/˜rylov/yrylov.htm

Abstract

In conventional quantum mechanics the quantum particle is a special object, whose properties are described by special concepts and quantum principles. The quantization is a special procedure, which is accompanied by introduction of special concepts, and this procedure cannot be repeated. In the model conception of quantum phenomena (MCQP) the quantum particle is a partial case of a stochastic particle, and the quantum dynamics is a special case of the stochastic particle dynamics. In MCQP the quantization is a dynamical procedure, where a special quantum term is added to the Lagrangian of the statistical ensemble of (classical) particles. This procedure can be repeated many times, because it is not accompanied by introduction of new concepts. It is very convenient from the formal viewpoint, because the set of dynamic quantizations forms a one-parameter group, which allows one to separate the dynamical and statistical (stochastic) properties.

1 Introduction

Sometimes investigation of a new class of physical phenomena is carried out by two stages. At first, the simpler axiomatic conception based on simple empiric considerations arises. Next, the axiomatic conception is replaced by a more developed model conception, where axioms of the first stage are obtained as properties of the model. Theory of thermal phenomena was developed according to this scheme. At first, the thermodynamics (axiomatic conception of thermal phenomena) appeared. Next, the statistical physics (model conception of thermal phenomena) was developed. Axioms of thermodynamics and properties of the caloric fluid were obtained as properties of the chaotic molecular motion. A model conception offers few advantages over the axiomatic conception. *The investigation methods and mathematical* *technique of the model conception are more subtle and effective*, than those of the axiomatic conception. For instance, in the framework of the statistical physics we can calculate the heat capacity and other characteristics of the matter, whereas in the framework of thermodynamics we can only measure them empirically. The second law of thermodynamics is a postulate of thermodynamics. It is a formal evidence of the axiomatic character of thermodynamics. In the framework of the statistical physics the second law of thermodynamics is a corollary of the heat model (chaotic molecular motion) and of the statistical principles.

The contemporary quantum theory is the first (axiomatic) stage in the development of the microcosm physics. Formal evidences of this is an existence of quantum principles, which are additional to primary principles of classical physics. Appearance of the next (model) stage, where the quantum principles are consequences of the model, seems to be unavoidable, because only in this case the microcosm physics has a reliable foundation for its further development. The model conception is attractive, because it uses more subtle and effective mathematical methods of investigation. Besides, it gives boundaries of the axiomatic conception application. We can see this in example of interplay of statistical physics and thermodynamics.

Model conception of quantum phenomena (MCQP) looks as follows [1]. As a result of a generalization of the Riemannian geometry we obtain a class of such uniform and isotropic space-time geometries (nondegenerate geometries [2, 3, 4]), where the free motion of particles is primordially stochastic and the particle mass is geometrized. Experiments show that the free motion of particles of small mass is stochastic, and this stochasticity depends on the particle mass. Free motion of particles is to be determined only by the space-time geometry, and nondegenerate geometries with stochastic motion of free particle are more valid, than the Minkowski space-time geometry with deterministic motion of free particles. It is possible to choose such parameters of the nondegenerate space–time geometry, that the statistical description of the stochastically moving particles coincides with the quantum description (Schrödinger equation). The quantum constant $\hbar$ appears as an attribute of the space-time geometry. The quantum principles appear to be corollaries of the nonrelativistic approximation of the space-time model. Then the model conception of quantum phenomena (MCQP) arises.

Briefly, MCQP can be formulated as follows. The space-time geometry depends on the quantum constant $\hbar$ and generates the stochasticity, which depends on the particle mass m and on $\hbar$. Statistical ensemble of stochastically moving particles is a kind of fluid, depending on m and $\hbar$. Spin and wave function are attributes of any ideal fluid [4]. Describing the fluid (statistical ensemble) in terms of the wave function, we obtain the quantum description (Schrödinger equation).

Stochasticity of the free particle motion in the deterministic nondegenerate space-time geometry is explained as follows. In the nondegenerate geometry there are many vectors $\overrightarrow{P_0Q}$ of fixed length, which are parallel to the vector $\overrightarrow{P_0P_1}$, whereas in the degenerate geometry there is only one vector $\overrightarrow{P_0Q}$ of fixed length, which is parallel to the vector $\overrightarrow{P_0P_1}$. The momentum vector $\overrightarrow{p}$ of a free particle is transported in parallel along the particle world line. The momentum vector $\overrightarrow{p}$ is tangent to the world line and determines its direction. In the degenerate geometry (for instance, in Minkowski geometry) the momentum vector $\overrightarrow{p}$, transported in parallel, is unique, and the world line is determined uniquely. In the nondegenerate geometry there are many momentum vectors, transported in parallel, and the world line is not determined uniquely. In other words, the world line appears to be stochastic (random).

Replacement of the axiomatic conception of quantum phenomena (ACQP) by MCQP is a rather radical modification of the existing theory of microcosm, and for a correct evaluation of interplay between ACQP and MCQP one should take into account such a factor as the style of investigation, which is usually very conservative. Style of investigations influences strongly on the evaluations of the achievements of a theory. There are two styles of investigations: (1) classical style of investigations (C-style) and (2) pragmatic style of investigations (P-style).

The C-style is a deductive style of investigations. It is very sensitive to validity of the fundamental statements and principles of the physical conception. Deductive character of the C-style does not allow to produce investigations with false principles or fundamental statements, because all corollaries are deduced from the primary principles. The essence of the C-style is expressed by the Newton's slogan: "Hypotheses non fingo". If some primary principles are false, the C-style of investigations does not work.

On the contrary, the P-style of investigations is insensitive to the validity of the primary principles. P-style uses deduction slightly, and it can effectively work, when not all primary principles are valid. The P-style uses short logic (short logical chains). It cannot use long logical chains. It is unreliable, because one cannot be sure in the validity of primary principles. Explanation of experimental data is the unique criterion, used by the P-style. P-style is adequate for solution of small physical problems and for description of restricted cycle of physical phenomena. The P-style is inadequate for construction of fundamental physical conception, because for such a construction the primary principles are to be true. To explain new unknown physical phenomena, P-style uses new additional suppositions (hypotheses), which connect different physical phenomena and explain them in terms of the new hypotheses. P-style is effective and useful for determination of connection between different physical phenomena. P-style was used effectively at construction of the quantum mechanics.

Some contemporary investigators consider the P-style as a new investigation style of contemporary physics, playing off it against the old-fashioned style of classical physics (C-style). In reality the P-style is not new. Ptolemeus and his successors used P-style in explanation of celestial phenomena. Primary principles of celestial mechanics of that time contained a mistake (the Sun rotates around the Earth). Nevertheless, Ptolemeus succeeded to explain and to predict correctly the planet motion, because he compensated the invalid primary statement by additional suppositions. Such a conception, where the invalid primary statement is compensated by additional suppositions will be referred to as a compensating conception. Such a compensating conception is suitable for a correct description of special physical phenomena, but it is impossible to construct a well defined fundamental theory in framework of the compensating theory, which uses P-style. For instance, in framework of the Ptolemaic doctrine it was impossible to discover the Newton's gravitation law. This discovery took place more than a century ago after the mistake in interplay with the Sun and the Earth had been corrected by Copernicus.

A detailed investigation of interplay between P-style and C-style has been made in [5]. Here we restrict ourselves by the following statements. From viewpoint of P-style the only criterion of the quality of the scientific conception is an explanation of experimental data. Such categories as logical structure of conception and the number of additional hypotheses are not taken into account. Let two different conceptions A and B explain the same experimental data. If the conception A was posed earlier, from viewpoint of P-style it has the advantage of the conception B. In this case the logic of the P-style adherents is very simple: we have conception A and we do not need another conception B, which cannot explain new experiments. The P-style adherents does not interested in the relative quality of the conceptions A and B.

From viewpoint of C-style the well defined conception must not have additional hypotheses at all. If the conception has additional hypotheses, it is a compensating conception, which contains mistakes (delusions) in its primary principles. In the framework of C-style the criterion of explanation of experimental data is also valid, but a direct test of experimental data is difficult, because of the deductive character of the well defined conception. If the well defined conception appears in that time, when there is a competitive compensating conception, one needs a long time to deduce mathematical technique from primary principles and explain existing experimental data on the basis of this technique. In the case of competition between the Ptolemaic and Copernicus doctrines one needed more than a century. To demonstrate capacities of MCQP one needs to predict a new physical phenomenon on the basis of the mathematical technique, generated by MCQP. It is difficult to say what time do we need for such a prediction, because construction of mathematical technique of MCQP is a difficult problem.

Idea of the quantum mechanics foundation as a statistical description of stochastically moving particles is a very old idea, but one has failed to realize this idea because of mathematical problems. There were three serious problems in realization of this idea:

1. Construction of the adequate deterministic space-time geometry with stochastically moving particles

2. Construction of dynamical conception of the statistical description, where the concept of the probability is not used

3. Description of ideal fluid in terms of the wave function.

Necessity of solving these problems has been existing since the beginning of the quantum mechanics creation. To explain stochastic motion of particles, the stochastic space-time geometries were invented [7, 8, 9], but these stochastic geometries were fortified geometries, i.e. geometries with additional structures, given in the space-time, whereas one needs to describe the particle stochasticity in the framework of deterministic physical geometry, without introducing any additional structures (which are additional hypotheses). It was well known that the Schrödinger equation can be presented in the form of hydrodynamic equations for irrotational flow of some fluid [10], but nobody could describe the rotational flow of ideal fluid in terms of the wave function, because to make this, one needs to integrate hydrodynamic equation for the ideal fluid. As a result the wave function remains to be a primary object. After description of the ideal fluid in terms of wave function [4] the fluid becomes to by a primary object and the wave function may be considered to be an attribute of the fluid. Then quantum phenomena can be interpreted in hydrodynamic terms. The mentioned problems have been solved respectively in the papers [3, 2], [11, 6, 12] and [4].

It is characteristic and essential that the first two problems have been solved very simply only after discovery and correction of mistakes (delusions) in classical approach to these problems. From viewpoint of P-style one may suggest exotic additional suppositions, but a search of mistakes in foundation of geometry and in principles of statistical description is a hopeless undertaking, something like a scientific heresy. Only scientific dissidents can look for mistakes in classical conceptions. Nevertheless, the delusions (mistakes) have been found. These mistakes were not logical or mathematical. They were associative. The ancient Egyptians believed that all rivers flows towards the North, because they knew only one river the Nile, which flowed exactly towards the North. The ancient Egyptians associate direction of the river flow with the direction in the space, and it was an associative delusion, because the origin of this delusion is an incorrect association.

Impossibility of solution of the first problem (construction of a deterministic space-time geometry with primordially stochastic motion of free particles)s is connected with an associative delusion. We believe that the straight line (analog of the straight line) is a one-dimensional set of points in any physical geometry, because we know only such geometries, where this statement is valid. As far as any physical geometry is a result of deformation of the proper Euclidean geometry [3, 2], this one-dimensionality of the straight imposes unwarranted constraint on possible deformation of the Euclidean geometry and eliminates true space-time geometry from the list of possible space-time geometries.

Impossibility of solution of the second problem (construction of dynamical conception of the statistical description) is connected with another associative delusion. In the statistical physics the statistical description is produced in terms of distributions which are a kind of the probability density. On this basis many researchers believe that any statistical description is a probabilistic description and try to construct the statistical description of stochastic world lines as a probabilistic statistical description. Association of the statistical description with the probability theory is an associative delusion. Sometimes such a statistical description is possible, but not always. Dynamical conception of statistical description is possible always, although it is not so informative as the probabilistic statistical description.

Our strategy of the microcosm investigation is as follows. We look for mistakes in the classical approach to the microcosm description, find the mistakes and correct them. Such a strategy is a safe strategy, because the mistakes should be found and corrected independently of whether or not the correction of the mistakes helps us to explain experimental data.

ACQP and MCQP are different conception describing quantum phenomena, as well as thermodynamics and statistical physics are different conceptions, describing the thermal phenomena. There is some correspondence between procedures and methods in ACQP and in MCQP. This correspondence is described by the following scheme

ACQP	MCQP
1. Additional hypotheses are used (QM principles)	1. *No additional hypotheses are used*
2. One kind of measurement, as far as only one statistical average object $\langle \mathcal{S} \rangle$ is considered. It is referred to as a quantum system	2. Two kinds of measurement, because two kinds of objects (individual $\mathcal{S}_{\text{st}}$ and statistical average $\langle \mathcal{S} \rangle$) are considered
3. Quantization: procedure on the conceptual level: $\mathbf{p} \rightarrow -i\hbar\mathbf{\nabla}$ etc.	3. Dynamic quantization: relativistic procedure on the dynamic level $m^2 \rightarrow m_{\text{eff}}^2 = m^2 + \frac{\hbar^2}{c^2}\left(\kappa_l \kappa^l + \partial_l \kappa^l\right)$
4. Transition to classical description: procedure on conceptual level $\hbar \rightarrow 0 \qquad \psi \rightarrow (\mathbf{x}, \mathbf{p})$	4. Dynamic disquantization: relativistic procedure on dynamic level $\partial^k \rightarrow \frac{j^k j^l}{j_s j^s}\partial_l$
5. Combination of nonrelativistic quantum technique with principles of relativity	5. Consequent relativistic description at all stages
6. Interpretation in terms of wave function ψ	6. Interpretation in terms of statistical average world lines (WL) $\frac{dx^i}{d\tau} = j^i(x)$, $j^k = -\frac{i\hbar}{2}\left(\psi^* \partial^k \psi - \partial^k \psi^* \cdot \psi\right)$

Now the main goal of MCQP is further development of its mathematical technique, which distinguishes from the corresponding technique of ACQP. In the present paper we consider only properties of the quantization procedure. In conventional quantum mechanics (ACQP) the quantization procedure is a transition from the classical description of a particle to the quantum description of the particle. In ACQP the quantization is conceptual procedure, where concepts of classical physics (coordinate $\mathbf{x}$ and momentum $\mathbf{p}$) are replaced by operators ($\mathbf{x}$ and $-i\hbar\mathbf{\nabla}$) and the new concept – wave function ψ appears. The dynamical equations for variables $\mathbf{x}, \mathbf{p}$ are replaced by the Schrödinger (or Klein-Gordon) equation for the wave function ψ. In ACQP the wave function ψ is a fundamental object of the theory, and properties of ψ are described by quantum principles.

In the case of MCQP the quantization is a dynamical procedure. No new concepts appear. Dynamic equations for the statistical ensemble $\mathcal{E}_{\text{d}}\left[\mathcal{S}_{\text{d}}\right]$ of deterministic (classical) particles $\mathcal{S}_{\mathrm{d}}$ are transformed to the dynamic equations for the statistical ensemble $\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]$ of stochastic (quantum) particles $\mathcal{S}_{\mathrm{st}}$. These dynamic equations are equivalent to the Schrödinger (or Klein-Gordon) equation for the wave function ψ, which describes the state of this statistical ensemble $\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]$. In MCQP the wave function is simply a method of description of the statistical ensemble, which is a kind of fluid. The wave function may be used for description of the statistical ensemble $\mathcal{E}_{\mathrm{d}}[\mathcal{S}_{\mathrm{d}}]$ of deterministic (classical) particles $\mathcal{S}_{\mathrm{d}}$, as well as for description of the statistical ensemble $\mathcal{E}_{\mathrm{st}}[\mathcal{S}_{\mathrm{st}}]$ of stochastic particles $\mathcal{S}_{\mathrm{st}}$. In both cases the statistical ensemble is a fluidlike dynamic system.

In the framework of MCQP the quantization procedure (dynamic quantization) may be produced many times, because dynamic quantization is not accompanied by an introduction of new concepts. Formally, the dynamic quantization is an addition of an accessory term to the Lagrangian. This accessory term introduces a new field κ, describing the stochastic component of the stochastic particle $\mathcal{S}_{\mathrm{st}}$ motion. We may repeat the dynamic quantization many times, introducing any time a new field κ. It appears that any new field κ coincides with the existing field κ to within a factor, and the repeated quantization leads only to a change of the κ-field intensity. The κ-field is a very important field, because it is responsible for pair production, and the pair production mechanism can be described in terms of the κ-field [13]. Nothing of that kind cannot be obtained in the framework of ACQP, because in ACQP the quantization is a conceptual procedure, which can be carried out only once. Besides, in ACQP the κ-field is a constituent of the wave function. The κ-field cannot be separated from the wave function, because the wave function is a fundamental concept of ACQP. Mechanism of the pair production cannot be determined and described in the framework of ACQP. In a like way in thermodynamics we cannot obtain any information on the molecular structure of the matter, whereas we can do this in the framework of the statistical physics. In other words, MCQP carries out more detailed description of quantum objects. Such a description is impossible in the framework of ACQP.

Mathematical technique of MCQP distinguishes from that of ACQP as well as the mathematical technique of statistical physics distinguishes from that of thermodynamics. Now a development of mathematical technique of MCQP is the main problem of the MCQP construction. In the present paper we investigate mathematical properties of the repeated dynamic quantization. In the framework of MCQP the quantization procedure manifests group properties, and this may appear to be interesting and useful for further investigations.

2 Dynamic quantization

The action for the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{d}}]$ of deterministic free relativistic particles $\mathcal{S}_{\mathrm{d}}$ has the form

$$\mathcal{E}[\mathcal{S}_{\mathrm{d}}]: \qquad \mathcal{A}[x] = \int \left\{-mc\sqrt{g_{ik}\dot{x}^i\dot{x}^k}\right\} d\tau d\boldsymbol{\xi}, \qquad \dot{x}^k \equiv \frac{dx^k}{d\tau} \tag{2.1}$$

where coordinates $x = \{x^i(\xi)\}$, $i = 0,1,2,3$, $\xi = \{\tau, \boldsymbol{\xi}\} = \{\tau, \xi_1, \xi_2, \xi_3\}$ describe the particle position in the space-time. Lagrangian coordinates $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$ label particles of the statistical ensemble $\mathcal{E}[\mathcal{S}_\mathrm{d}]$. Here and in what follows a summation is produced over repeated Latin indices $(0-3)$. To produce dynamic quantization and to obtain the action for the statistical ensemble $\mathcal{E}_\mathrm{st}[\mathcal{S}_\mathrm{st}]$ of stochastic particles, it should make the change

$$m^2 \to m^2 + \frac{\hbar^2}{c^2}\left(\kappa_l\kappa^l + \partial_l\kappa^l\right), \qquad \partial_l \equiv \frac{\partial}{\partial x^l} \tag{2.2}$$

where $\kappa = \left\{\kappa^l(x)\right\}$, $l = 0,1,2,3$ are new dynamic variables, describing the mean intensity of the stochastic component of the particle motion (this component is absent for deterministic particles). Dynamic equations for the κ-field κ^l are determined from the variational principle by means of a variation with respect to κ^l. After the change (2.2) the action (2.1) turns into the action for the statistical ensemble $\mathcal{E}_\mathrm{st}[\mathcal{S}_\mathrm{st}]$

$$\mathcal{E}_\mathrm{st}[\mathcal{S}_\mathrm{st}]: \qquad \mathcal{A}[x,\kappa] = \int\left\{-mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k}\right\}d\tau d\boldsymbol{\xi}, \qquad K = \sqrt{1+\lambda^2\left(\kappa_l\kappa^l + \partial_l\kappa^l\right)} \tag{2.3}$$

where $\lambda = \hbar/mc$ is the Compton wave length of the particle.

Meaning of the change (2.2) becomes to be clear, if we write the action (2.3) in the nonrelativistic approximation, when $g_{ik} =$diag$\{c^2,-1,-1,-1\}$, $c^{-2}\left(\kappa^0\right)^2 \ll \boldsymbol{\kappa}^2$, and $c^2\left(\dot{x}^0\right)^2 \gg \dot{\mathbf{x}}^2$. We have in the nonrelativistic approximation instead of (2.3)

$$\mathcal{E}_\mathrm{st}[\mathcal{S}_\mathrm{st}]: \qquad \mathcal{A}[\mathbf{x},\mathbf{u}] = \int\left\{-mc^2 + \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right\}dtd\boldsymbol{\xi}, \tag{2.4}$$

where $\mathbf{x} = \mathbf{x}(t,\boldsymbol{\xi})$, $\mathbf{u} = \mathbf{u}(t,\mathbf{x}) = \frac{\hbar}{m}\boldsymbol{\kappa}$. The variable $\mathbf{u}$ describes the mean value of the stochastic component of velocity. Energy $m\mathbf{u}^2/2$ associated with this stochastic component is added to the energy associated with the regular velocity of the particle. The last term in (2.4) describes connection between the stochastic component of the velocity and the regular one.

Formally the change (2.2) with arbitrary parameter $a = \hbar^2$

$$m^2 \to m^2 + \frac{a}{c^2}\left(\kappa_l\kappa^l + \partial_l\kappa^l\right) \tag{2.5}$$

may be applied to the statistical ensemble (2.3) of stochastic particles (deterministic particles are considered as stochastic ones with vanishing stochasticity). Such a transformation changes the stochasticity intensity, and we obtain the stochastic particle dynamics with other kind of stochasticity. Such an approach allows one to obtain the stochastic particle dynamics with continuous dependence on the stochasticity intensity, described by the parameter $a = \hbar^2$. Such a dependence on the parameter allows one to separate dynamical properties from the statistical properties, conditioned by the particle motion stochasticity. Of course, results of description in the framework of ACQP depend also on the parameter $a = \hbar^2$, but in this case a change of the parameter $a = \hbar^2$ generates a change of quantum principles, which contains the parameter $a = \hbar^2$. Besides, setting $a = \hbar^2 = 0$ in the conventional quantum description, we do not obtain the classical description, because in ACQP the quantum description do not turn to the classical one at $\hbar \to 0$. In the conventional quantum description the dynamics is mixed with the stochasticity in such a way, that separation of them is not a simple problem. Mathematical reason of this tangle will be shown below.

Dynamic equation for the variables κ^l are obtained from the action (2.3) by means of variation with respect to κ^l

$$\frac{\delta \mathcal{A}}{\delta \kappa^l} = -\lambda^2 \kappa_l \frac{mcR}{K} + \lambda^2 \partial_l \frac{mcR}{2K} = 0, \qquad l = 0,1,2,3 \tag{2.6}$$

where

$$R = J\sqrt{g_{ik}\dot{x}^i \dot{x}^k}, \qquad J = \frac{\partial\left(\tau, \xi_1, \xi_2, \xi_3\right)}{\partial\left(x^0, x^1, x^2, x^3\right)} \tag{2.7}$$

Solution of equations (2.6) has the form

$$\kappa_l = \partial_l \kappa, \qquad l = 0,1,2,3, \qquad \kappa = \frac{1}{2}\ln\frac{mcR}{K} \tag{2.8}$$

After a series of changes of variables and some integration the action (2.3) is reduced to the form (See mathematical details in Appendix)

$$\mathcal{A}\left[\psi, \psi^*\right] = \int \left\{ b_0^2 \partial_k \psi^* \partial^k \psi - m^2 c^2 \rho - \frac{b_0^2}{4}\left(\partial_l s_\alpha\right)\left(\partial^l s_\alpha\right)\rho + \left(\hbar^2 - b_0^2\right)\frac{\partial_l \rho \partial^l \rho}{4\rho} \right\} d^4 x \tag{2.9}$$

where $\psi = \psi(x)$ is the two-component complex wave function, and ψ^* is the quantity complex conjugate to ψ

$$\psi = \binom{\psi_1}{\psi_2}, \qquad \psi^* = \left(\psi_1^*, \psi_2^*\right), \tag{2.10}$$

$$\rho = \psi^* \psi, \qquad s_\alpha = \frac{\psi^* \sigma_\alpha \psi}{\rho}, \qquad \alpha = 1,2,3 \tag{2.11}$$

where $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ are the Pauli matrices. The quantity b_0 is an arbitrary real constant ($b_0 \neq 0$). Here and in what follows, a summation is produced over repeated Greek indices ($1-3$). The dynamic system, described by the action, is an ideal fluid, where the 4-current j^i is described by the relation

$$j^l = \frac{ib_0}{2}\left(\psi^* \partial^l \psi - \partial^l \psi^* \cdot \psi\right) \tag{2.12}$$

The quantities s_α describe vorticity of the fluid flow. If $s_\alpha =$const, $\alpha = 1,2,3$, the fluid flow is irrotational.

In the case of the irrotational flow we should use linear dependent components of the wave function $\psi_1 = a\psi_2$, $a =$const. In this case $s_\alpha =$const, $\alpha = 1,2,3$ and the action (2.9) is reduced to the form

$$\mathcal{A}\left[\psi, \psi^*\right] = \int \left\{ b_0^2 \partial_k \psi^* \partial^k \psi - m^2 c^2 \rho + \left(\hbar^2 - b_0^2\right)\frac{\partial_l \rho \partial^l \rho}{4\rho} \right\} d^4 x \tag{2.13}$$

Setting $b_0 = \hbar$ in (2.13), we obtain the action for the Klein-Gordon equation

$$\mathcal{E}_{\mathrm{st}}\left[\mathcal{S}_{\mathrm{st}}\right]: \qquad \mathcal{A}\left[\psi,\psi^*\right] = \int \left\{\hbar^2\partial_k\psi^*\partial^k\psi - m^2c^2\psi^*\psi\right\} d^4x \tag{2.14}$$

Thus, the change (2.2) realizes quantization of dynamic equations for a free relativistic particle by means of dynamic methods, i.e. without a reference to the quantum principles.

In the action (2.13) b_0 is an arbitrary constant, and the actions (2.13) and (2.14) describe the same dynamic system for any value of $b_0 \neq 0$. But there is a difference in description of the statistical ensemble in terms of actions (2.13) and (2.14). The dynamic equation generated by the action (2.14) is linear, whereas the dynamic equation generated by the action (2.13) is linear only at $b_0^2 = \hbar^2$. On the other hand, if we set $\hbar = 0$ in the action (2.13), we obtain the classical description, whereas if we set $\hbar = 0$ in the action (2.14), we obtain no description at all. The fact is that the constant b_0 is connected with dynamics, whereas the constant $\hbar$ is connected with stochasticity. If we set $b_0 = 0$ in the action (2.13), we suppress the dynamics. If we set $\hbar = 0$ in the action (2.13), we suppress the stochasticity. In the action (2.14) $b_0 = \hbar$, and setting $\hbar = 0$ in the action (2.14), we suppress stochasticity and dynamics simultaneously. Thus, in (2.14) the dynamics is mixed with the stochasticity, and this mixture is a necessary condition of the dynamic equation linearity. A linearity of dynamic equation is very attractive. ACQP considers this linearity as a principle. The tangle of stochasticity and dynamics is a payment for this linearity.

Let us apply the repeated dynamic quantization to the action (2.3). We obtain instead of (2.3)

$$\mathcal{A}\left[x,\kappa_{(1)},\kappa_{(2)}\right] = \int \left\{-mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k}\right\} d\tau d\boldsymbol{\xi} \tag{2.15}$$

where now

$$K = \sqrt{1 + \lambda^2 \sum_{A=1,2}\left(\kappa_{(A)l}\kappa_{(A)}^l + \partial_l\kappa_{(A)}^l\right)}, \tag{2.16}$$

Dynamic equations for $\kappa_{(A)l}$ have the form

$$\frac{\delta\mathcal{A}}{\delta\kappa_{(A)}^l} = -\lambda^2\kappa_{(A)l}\frac{mcR}{K} + \lambda^2\partial_l\frac{mcR}{2K} = 0, \qquad A = 1,2 \tag{2.17}$$

$$R = \sqrt{g_{ik}\dot{x}^i\dot{x}^k}\frac{\partial\left(\tau,\xi_1,\xi_2,\xi_3\right)}{\partial\left(x^0,x^1,x^2,x^3\right)}$$

Solution of dynamic equations (2.17) gives

$$\kappa_{(1)l} = \kappa_{(2)l} = \frac{1}{2}\partial_l\kappa, \qquad \kappa = \ln\frac{mcR}{K} \tag{2.18}$$

Substitution of (2.18) in (2.16) leads to

$$K = \sqrt{1 + \lambda'^2\left(\kappa_l\kappa^l + \partial_l\kappa^l\right)}, \qquad \lambda'^2 = 2\lambda^2 = 2\left(\frac{\hbar}{mc}\right)^2 \tag{2.19}$$

Comparing (2.17) with (2.3), we conclude that two subsequent dynamic quantizations with intensity described by the parameter $\hbar^2$ are equivalent to one dynamic quantization with the intensity described by the parameter $\hbar'^2 = 2\hbar^2$.

Dynamic quantization does not depend on the form of the action representation. For instance, let us apply the repeated dynamic quantization to the action (2.13). Using replacement (2.2) in the action (2.13), we obtain additional term $\mathcal{A}_{\text{add}}[\psi, \psi^*, \kappa]$ in the action (2.13)

$$\mathcal{A}_{\text{add}}[\psi, \psi^*, \kappa] = -\int \left\{\hbar^2 \left(\kappa_l \kappa^l + \partial_l \kappa^l\right) \psi^* \psi\right\} d^4x \tag{2.20}$$

Dynamic equation for the κ-field have the form

$$\frac{\delta \mathcal{A}}{\delta \kappa^l} + \frac{\delta \mathcal{A}_{\text{add}}}{\delta \kappa^l} = \frac{\delta \mathcal{A}_{\text{add}}}{\delta \kappa^l} = -2\hbar^2 \kappa_l (\psi^* \psi) + \hbar^2 \partial_l (\psi^* \psi) = 0, \qquad l = 0, 1, 2, 3 \tag{2.21}$$

Solution of dynamic equations (2.21) can be written in the form

$$\kappa_l \equiv \frac{1}{2} \partial_l \ln \rho = \frac{1}{2} \partial_l \ln (\psi^* \psi) \tag{2.22}$$

After substitution of (2.22) in (2.20) we obtain

$$\mathcal{A}_{\text{add}}[\psi, \psi^*] = -\int \hbar^2 \left(\frac{\partial_l \rho \partial^l \rho}{4\rho} - \frac{\partial_l \rho \partial^l \rho}{2\rho} + \frac{1}{2} \partial_l \partial^l \rho\right) d^4x \tag{2.23}$$

The last term in (2.23) has the form of divergence. It does not contribute to dynamic equations and can be omitted. Uniting (2.13) and (2.23), we obtain

$$\begin{aligned} \mathcal{A}[\psi, \psi^*] + \mathcal{A}_{\text{add}}[\psi, \psi^*] \;\; = \;\; & \int \left\{ b_0^2 \partial_k \psi^* \partial^k \psi - \frac{b_0^2}{4} (\partial_l s_\alpha) \left(\partial^l s_\alpha\right) \rho \right. \\ & \left. - m^2 c^2 \rho + \left(2\hbar^2 - b_0^2\right) \frac{\partial_l \rho \partial^l \rho}{4\rho} \right\} d^4x \end{aligned} \tag{2.24}$$

The action (2.24), obtained as a result of the repeated dynamic quantization, distinguishes from the action (2.13) only in the sense that the quantum constant $\hbar$ is replaced by the quantum constant $\hbar' = \sqrt{2}\hbar$.

3 Discussion

The repeated dynamic quantization manifests the difference between the approach of ACQP and that of MCQP. This difference lies mainly in the interpretation of the κ-field. From the viewpoint of ACQP the κ-field does not exist at all, because according to (2.22) it is a constituent of the wave function, and the wave function is an attribute of the particle. In the framework of ACQP there is no necessity to consider the κ-field, it is sufficient to consider the corresponding wave function. In ACQP the wave function is a fundamental object of ACQP, whose properties are described by the quantum axiomatics, and it is useless to divide the wave function into its constituents.

In MCQP the wave function is only a method of description of the statistical ensemble $\mathcal{E}_{\rm st}[\mathcal{S}_{\rm st}]$, consisting of stochastic particles $\mathcal{S}_{\rm st}$. Regular component of the stochastic particle motion is described by the 4-current j^k, whereas the stochastic component is described by the κ-field κ^l. From formal viewpoint the κ-field is a relativistic force field, which is generated by the regular component of motion, and which can exist separately from its source [13]. The κ-field interacts with regular component of the particle motion. Two different stochastic particle can interact via their common κ-field in a like way, as two charged particles interact via their common electromagnetic field. Interaction of two particles via the κ-field takes place only in MCQP. This property is absent in ACQP, and it is a serious defect of ACQP, because the κ-field can produce the particle-antiparticle pairs. Neither electromagnetic field, nor gravitational one can produce pairs, because they do not change the particle mass, that is necessary for the pair production. Only κ-field can produce pairs, because the factor K in (2.3) can make the particle mass m to be imaginary, when $K^2 < 0$. It is necessary for the particle 4-velocity component $dx^0/d\tau$ can change its sign.

The pair production effect is the crucial effect of the high energy particle collision. Experiments show that the pair production is an essentially quantum effect. Now there is no satisfactory mechanism of the pair production. Apparently, this mechanism is connected with application of the κ-field. At any rate, the pair production by means of the given time-dependent κ-field is obtained [13], whereas the pair production at the collision of two relativistic particles is an unsolved problem. The conventional description of the pair production in the framework of the quantum field theory is unsatisfactory in some aspects (See for details [13]).

Appendix A. Transformation of the action

Let us transform the action (2.3) to the form (2.13). Instead of τ we introduce the variable ξ_0, and rewrite (2.3) in the form

$$\mathcal{A}[x,\kappa] = \int \left\{-mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k}\right\} d^4\xi, \qquad K = \sqrt{1+\lambda^2\left(\kappa_l\kappa^l + \partial_l\kappa^l\right)} \tag{A.1}$$

where $\xi = \{\xi_0, \boldsymbol{\xi}\} = \{\xi_k\}$, $k = 0,1,2,3$, $x = \left\{x^k(\xi)\right\}$, $k = 0,1,2,3$.

Let us consider variables $\xi = \xi(x)$ in (A.1) as dependent variables and variables x as independent variables. Let the Jacobian

$$J = \frac{\partial(\xi_0,\xi_1,\xi_2,\xi_3)}{\partial(x^0,x^1,x^2,x^3)} = \det||\xi_{i,k}||, \qquad \xi_{i,k} \equiv \partial_k\xi_i, \qquad i,k = 0,1,2,3 \tag{A.2}$$

be considered to be a multilinear function of $\xi_{i,k}$. Then

$$d^4\xi = Jd^4x, \qquad \dot{x}^i \equiv \frac{dx^i}{d\xi_0} \equiv \frac{\partial(x^i,\xi_1,\xi_2,\xi_3)}{\partial(\xi_0,\xi_1,\xi_2,\xi_3)} = J^{-1}\frac{\partial J}{\partial \xi_{0,i}}, \qquad i = 0,1,2,3 \tag{A.3}$$

After transformation to dependent variables ξ the action (A.1) takes the form

$$\mathcal{A}[\xi,\kappa]=-\int mcK\sqrt{g_{ik}\frac{\partial J}{\partial\xi_{0,i}}\frac{\partial J}{\partial\xi_{0,k}}}d^4x, \tag{A.4}$$

We introduce new variables

$$j^k=\frac{\partial J}{\partial\xi_{0,k}},\qquad k=0,1,2,3 \tag{A.5}$$

by means of Lagrange multipliers p_k

$$\mathcal{A}[\xi,\kappa,j,p]=\int\left\{-mcK\sqrt{g_{ik}j^ij^k}+p_k\left(\frac{\partial J}{\partial\xi_{0,k}}-j^k\right)\right\}d^4x, \tag{A.6}$$

Variation with respect to ξ_i gives

$$\frac{\delta\mathcal{A}}{\delta\xi_i}=-\partial_l\left(p_k\frac{\partial^2 J}{\partial\xi_{0,k}\partial\xi_{i,l}}\right)=0,\qquad i=0,1,2,3 \tag{A.7}$$

Using identities

$$\frac{\partial^2 J}{\partial\xi_{0,k}\partial\xi_{i,l}}\equiv J^{-1}\left(\frac{\partial J}{\partial\xi_{0,k}}\frac{\partial J}{\partial\xi_{i,l}}-\frac{\partial J}{\partial\xi_{0,l}}\frac{\partial J}{\partial\xi_{i,k}}\right) \tag{A.8}$$

$$\frac{\partial J}{\partial\xi_{i,l}}\xi_{k,l}\equiv J\delta_k^i,\qquad \partial_l\frac{\partial^2 J}{\partial\xi_{0,k}\partial\xi_{i,l}}\equiv 0 \tag{A.9}$$

one can test by direct substitution that the general solution of linear equations (A.7) has the form

$$p_k=b_0\left(\partial_k\varphi+g^\alpha(\boldsymbol{\xi})\,\partial_k\xi_\alpha\right),\qquad k=0,1,2,3 \tag{A.10}$$

where $b_0\neq 0$ is a constant, $g^\alpha(\boldsymbol{\xi})$, $\alpha=1,2,3$ are arbitrary functions of $\boldsymbol{\xi}=\{\xi_1,\xi_2,\xi_3\}$, and φ is the dynamic variable ξ_0, which ceases to be fictitious. Let us substitute (A.10) in (A.6). The term of the form $\partial_k\varphi\partial J/\partial\xi_{0,k}$ is reduced to Jacobian and does not contribute to dynamic equation. The terms of the form $\xi_{\alpha,k}\partial J/\partial\xi_{0,k}$ vanish due to identities (A.9). We obtain

$$\mathcal{A}[\varphi,\boldsymbol{\xi},\kappa,j]=\int\left\{-mcK\sqrt{g_{ik}j^ij^k}-j^kp_k\right\}d^4x, \tag{A.11}$$

where quantities p_k are determined by the relations (A.10)

Variation of (A.11) with respect to κ^l gives

$$\frac{\delta\mathcal{A}}{\delta\kappa^l}=-\frac{\lambda^2 mc\sqrt{g_{ik}j^ij^k}}{K}\kappa_l+\partial_l\frac{\lambda^2 mc\sqrt{g_{ik}j^ij^k}}{2K}=0 \tag{A.12}$$

It can be written in the form

$$\kappa_l=\partial_l\kappa=\frac{1}{2}\partial_l\ln\rho,\qquad e^{2\kappa}=\frac{\rho}{\rho_0}\equiv\frac{\sqrt{j_sj^s}}{\rho_0 mcK}, \tag{A.13}$$

where ρ_0 =const is the integration constant. Substituting expression for K from (A.1) in (A.13), we obtain dynamic equation for κ

$$\hbar^2 \left(\partial_l \kappa \cdot \partial^l \kappa + \partial_l \partial^l \kappa\right) = \frac{e^{-4\kappa} j_s j^s}{\rho_0^2} - m^2 c^2 \tag{A.14}$$

Variation of (A.11) with respect to j^k gives

$$p_k = -\frac{mcKj_k}{\sqrt{g_{ls} j^l j^s}} \tag{A.15}$$

or

$$p_k g^{kl} p_l = m^2 c^2 K^2 \tag{A.16}$$

Substituting the second equation (A.13) in (A.15), we obtain

$$j_k = -\rho_0 e^{2\kappa} p_k, \tag{A.17}$$

Now we eliminate the variables j^k from the action (A.11), using relation (A.17) and (A.13). We obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa] = \int \rho_0 e^{2\kappa} \left\{-m^2 c^2 K^2 + p^k p_k\right\} d^4 x, \tag{A.18}$$

where p_k is determined by the relation (A.10). Using expression (A.1) for K, the first term of the action (A.18) can be transformed as follows.

$$\begin{aligned} -m^2 c^2 e^{2\kappa} K^2 &= -m^2 c^2 e^{2\kappa} \left(1 + \lambda^2 \left(\partial_l \kappa \partial^l \kappa + \partial_l \partial^l \kappa\right)\right) \\ &= -m^2 c^2 e^{2\kappa} + \hbar^2 e^{2\kappa} \partial_l \kappa \partial^l \kappa - \frac{\hbar^2}{2} \partial_l \partial^l e^{2\kappa} \end{aligned}$$

Let us take into account that the last term has the form of divergence. It does not contribute to dynamic equations and can be omitted. Omitting this term, we obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa] = \int \rho_0 e^{2\kappa} \left\{-m^2 c^2 + \hbar^2 \partial_l \kappa \partial^l \kappa + p^k p_k\right\} d^4 x, \tag{A.19}$$

Instead of dynamic variables $\varphi, \boldsymbol{\xi}, \kappa$ we introduce n-component complex function

$$\psi = \{\psi_\alpha\} = \left\{\sqrt{\rho} e^{i\varphi} u_\alpha(\boldsymbol{\xi})\right\} = \left\{\sqrt{\rho_0} e^{\kappa + i\varphi} u_\alpha(\boldsymbol{\xi})\right\}, \qquad \alpha = 1, 2, \ldots n \tag{A.20}$$

Here u_α are functions of only $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, having the following properties

$$\sum_{\alpha=1}^{\alpha=n} u_\alpha^* u_\alpha = 1, \qquad -\frac{i}{2} \sum_{\alpha=1}^{\alpha=n} \left(u_\alpha^* \frac{\partial u_\alpha}{\partial \xi_\beta} - \frac{\partial u_\alpha^*}{\partial \xi_\beta} u_\alpha\right) = g^\beta(\boldsymbol{\xi}) \tag{A.21}$$

where $(^*)$ denotes complex conjugation. The number n of components of the wave function ψ is chosen in such a way, that equations (A.21) have a solution. Then we obtain

$$\psi^* \psi \equiv \sum_{\alpha=1}^{\alpha=n} \psi_\alpha^* \psi_\alpha = \rho = \rho_0 e^{2\kappa}, \qquad \partial_l \kappa = \frac{\partial_l (\psi^* \psi)}{2\psi^* \psi} \tag{A.22}$$

$$p_k = -\frac{ib_0 (\psi^* \partial_k \psi - \partial_k \psi^* \cdot \psi)}{2\psi^* \psi}, \qquad k = 0, 1, 2, 3 \tag{A.23}$$

Substituting relations (A.22), (A.23) in (A.19), we obtain the action, written in terms of the wave function ψ

$$\begin{aligned}\mathcal{A}[\psi,\psi^*] &= \int\left\{\left[\frac{ib_0\left(\psi^*\partial_k\psi-\partial_k\psi^*\cdot\psi\right)}{2\psi^*\psi}\right]\left[\frac{ib_0\left(\psi^*\partial^k\psi-\partial^k\psi^*\cdot\psi\right)}{2\psi^*\psi}\right]\right.\\ &\left.+\hbar^2\frac{\partial_l\left(\psi^*\psi\right)\partial^l\left(\psi^*\psi\right)}{4\left(\psi^*\psi\right)^2}-m^2c^2\right\}\psi^*\psi d^4x \qquad \text{(A.24)}\end{aligned}$$

Now we consider the case, when $n=2$, and the wave function has two components. In this case

$$\psi=\binom{\psi_1}{\psi_2},\qquad \psi^*=\left(\psi_1^*,\psi_2^*\right), \qquad \text{(A.25)}$$

and we have the following identity

$$\begin{aligned}&\frac{\left(\psi^*\partial_l\psi-\partial_l\psi^*\cdot\psi\right)\left(\psi^*\partial^l\psi-\partial^l\psi^*\cdot\psi\right)}{4\rho}-\frac{\left(\partial_l\rho\right)\left(\partial^l\rho\right)}{4\rho}\\ \equiv\ &-\partial_l\psi^*\partial^l\psi+\frac{1}{4}\left(\partial_l s_\alpha\right)\left(\partial^l s_\alpha\right)\rho \qquad \text{(A.26)}\end{aligned}$$

where 3-vector $\mathbf{s}=\{s_1,s_2,s_3,\}$ is defined by the relation

$$\rho=\psi^*\psi,\qquad s_\alpha=\frac{\psi^*\sigma_\alpha\psi}{\rho},\qquad \alpha=1,2,3 \qquad \text{(A.27)}$$

and Pauli matrices $\boldsymbol{\sigma}=\{\sigma_1,\sigma_2,\sigma_3\}$ have the form

$$\sigma_1=\left(\begin{array}{cc}0&1\\1&0\end{array}\right),\qquad \sigma_2=\left(\begin{array}{cc}0&-i\\i&0\end{array}\right),\qquad \sigma_1=\left(\begin{array}{cc}1&0\\0&-1\end{array}\right) \qquad \text{(A.28)}$$

Using identity (A.26), we obtain from (A.24)

$$\mathcal{A}[\psi,\psi^*]=\int\left\{b_0^2\partial_k\psi^*\partial^k\psi-m^2c^2\rho-\frac{b_0^2}{4}\left(\partial_l s_\alpha\right)\left(\partial^l s_\alpha\right)\rho+\left(\hbar^2-b_0^2\right)\frac{\partial_l\rho\partial^l\rho}{4\rho}\right\}d^4x \qquad \text{(A.29)}$$

which coincide with (2.9).

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