STATISTICAL ENSEMBLE TECHNIQUE IN APPLICATION TO DESCRIPTION OF THE ELECTRON

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Abstract. A dynamic systems $\mathcal{S}_P$ and $\mathcal{S}_D$ described by the Pauli equation and the Dirac equation are investigated merely as a distributed dynamic systems. No quantum principles are used. The system $\mathcal{S}_P$ is shown to be a statistical ensemble of non-relativistic stochastic pointlike particles. The electron spin is shown to have a classical analog which is a collective (statistical) property of the ensemble (not a property of a single electron). Magnetic moment of the electron is a quantum property which has no classical analog. The magnetic moment is parallel to the spin only in the stationary state. In the arbitrary state the magnetic moment is not connected with the spin direction. The dynamic system $\mathcal{S}_D$ described by the Dirac equation is shown to be non-relativistic, because after transforming to hydrodynamic variables and eliminating γ-matrices, a constant timelike vector f^i appears. This vector describes a split of the space-time into space and time.

Key words: electron spin, magnetic moment, statistical ensemble, quantum mechanics principles, space-time splitting

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Contents

1. Introduction

Usually the electron spin is considered as a display of some internal electron degrees of freedom [Young, 1976; Marshall, 1963; Barut and Bracken, 1981; Barut and Zanghi, 1984; Rueda and Cavalleri, 1983]. Some authors consider the electron spin as a result of interaction with stochastic electromagnetic radiation [Peña and Jauregui, 1982; Spavieri, 1990]. In both cases a spin is considered as conditioned by the single electron structure. There are no classical characteristics of a pointlike particle corresponding to the spin operator described by the Pauli matrices $\boldsymbol{\sigma}$, and it is a basis for the statement about additional degrees of freedom.

At the same time an investigation of the Pauli's electron and the Dirac's electron considered simply as distributed dynamic systems leads to the following results.
1. Under some conditions the action functional for the Pauli equation turns into the action functional for a statistical ensemble of classical particles (not for a single particle). It means that the Pauli equation describes a statistical ensemble (but not a single particle). This statement is a mathematical result, but not a supposition.
2. Spin of the Pauli's electron is a statistical property, i.e. it is a property of the statistical ensemble (but not of a single particle).
3. Magnetic moment of the Pauli's electron is connected with the spin only at the stationary state. At an arbitrary state, in general, they are independent.
4. Eliminating the γ-matrices and writing the Dirac equation in terms of tensor variables (not spinors), one separates out a pecular timelike direction in the space-time. It means that the Dirac equation is not relativistic.

These results seem unexpected and incompatible with QM principles. Such an incompatibility is possible, because the QM principles were not used at derivation of these results.

CRUCIAL EVOLUTION POINTS

Thermal Phenomena		Quantum Phenomena
Thermodynamics (TD)	Axiomatic stage	Quantum Mechanics (QM)
1. Statistical technique **without** usage of TD principles. 2. Nature of the heat stochasticity (chaotic motion of molecules)	Statistical stage	1. Statistical technique **without** usage of QM principles. 2. Nature of the quantum stochasticity (distortion of the space-time)
Restriction of TD principles (TD as a chapter of Statistical Physics)	Result	Restriction of QM principles? (QM as a chapter of some more general theory?)

Fig.1. *Analogy between the theory evolution of the termal phenomena and that of the quantum phenomena.*

To understand what is the matter let us compare the evolution of the quantum phenomena theory with that of the thermal phenomena theory. (see Fig.1) In the theory of thermal phenomena one can recognize two stages: (1) axiomatic stage, known as thermodynamics (TD), and (2) statistical stage, known as statistical physics (SP). Transition from the axiomatic stage (TD) to the statistical stage (SP) arose, because not all thermal phenomena could be explained and understood in frames of thermodynamics. For instance, the Brownian motion cannot be explained by thermodynamics. Transition to the statistical stage is connected with appearance of two crucial points: (1) statistical technique *without* usage of TD principles, and (2) a model of the thermal stochasticity (chaotic motion of molecules).

Application of the statistical technique to the model of the heat stochasticity led to a restriction of the frame of scope of the TD principles. As a result some phenomena can be understood from the viewpoint of SP, but cannot from the viewpoint of TD. For instance, the statement of SP that the temperature is a statistical property, and one molecule has not a temperature cannot be understood from the viewpoint of axiomatic thermodynamics, where any portion of the matter has a temperature.

Now the theory of quantum phenomena is at the axiomatic stage, because the quantum mechanics (QM) is an axiomatic construction based on the QM principles. Unsuccessful attempts of solving the pair production problem (a crucial problem of the high energy physics) show that, apparently, a usage of the QM principles is to be restricted. At any rate two crucial points in evolution of the quantum phenomena theory are found out. These two points are: (1) a statistical technique *without* usage of QM principles [Rylov, 1991; Rylov, 1995], and (2) a model of quantum stochasticity [Rylov, 1991a].

It is possible that application of this statistical technique to the model of the quantum stochasticity permits to create such a theory of quantum phenomena, where application of the QM principles will be restricted, and this theory will relate to the conventional QM in the same way as the statistical physics relates to the thermodynamics.

The general statistical technique which does not use QM principles will be referred to as the statistical ensemble technique (SET). The idea of SET can be explained in the simple example [Rylov, 1991; Rylov, 1995]. An electron flies out with a fixed energy from an electron gun, flies through a narrow hole at the diafragm and hits the screen at the point x_1. Repeating the experiment, one obtains another points of impact x_2, $x_3, \ldots x_n$. These points do not coincide. The experiment with a single electron is irreproducible. Then by definition a single electron is a stochastic system. Let the number N of experiments is very large ($N \gg 1$). The points of impact x_1, $x_2, \ldots x_N$ form some distribution $d_1(x)$ of points over the screen.

Let us produce another series of N experiments. If N is large enough, the disrtibution $d_2(x)$ of points over the screen coincides with $d_1(x)$. It means that experiments with many single electrons are reproducible, although an experiment with a single electron is irreproducible. In other words, many independent stochastic electrons form a deterministic dynamic system (statistical ensemble). It is difficult to say what is a reason of such a change, but this fact takes place.

This statistical ensemble of stochastic electrons may be imagined as follows. Let there be a set of N independent similar arrangements, where one produces simultaneously N similar experiments with electrons. Let us project mentually all arrangements into one of them. One obtains a set of independent and non-interacting electrons moving along stochastic trajectories. These stochastic electrons form something like a continuous medium (fluid). This fluid is *a deterministic dynamic system.* A time evolution of this fluid is described by some dynamic equations. Although electrons of the fluid are charged, they do not interact between themselves via electromagnetic field, because they are a result of projection of electrons from spatially separated independent arrangements. At this stage it is of no importance what is a nature of the stochasticity, quantum or classical.

Thus, although one cannot follow the course of an evolution of a single stochastic system, it is possible to track the evolution of many independent similar stochastic systems, and by definition it is a statistical description of a stochastic system. The statistical ensemble $\mathcal{E}$, as any dynamic system, has a Lagrangian, an energy-momentum tensor, an angular momentum tensor, a flux of the particle, and, maybe, some other additive quantities characteristic for distributed dynamic systems. Calculating these quantities for the ensemble and dividing by the number of systems in the ensemble, one can obtain corresponding mean values for a single stochastic system $\mathcal{S}_{st}$.

It should note that such a statistical description does not use the QM principles in explicit way. All above considerations are valid for the ensemble of any stochastic systems (including the quantum ones and the Brownian particle), because they are based on the properties of an ensemble considered as a dynamic system.

The statistical ensemble $\mathcal{E}$ has two basic properties:

(1) The statistical ensemble $\mathcal{E}$ is a distributed dynamic system.

(2) The statistical ensemble $\mathcal{E}$ is an instrument for calculation of average values concerning the stochastic system $\mathcal{S}_s$ (interpretation).

Conventionally only the second property is used effectively. The first one is ignored usually. The rules of calculating averages are different for quantum and classical systems, because the associated statistical ensembles are different dynamic systems in the two cases. Being axiomatized and considered as principles, these rules devide the physical phenomena into two conceptually different parts: quantum phenomena and classical ones.

A stochastic system is not an object of dynamics, whereas a statistical ensemble of stochastic systems is an object of dynamics. The SET-scheme of the statistical description distinguishes from the conventional scheme of statistical description by a direct connection between the stochastic system and dynamics (see Fig.2). This connection does not depend on the type of interpretation (a set of rules for calculation of average values). In the conventional scheme of the statistical description the dynamics is connected with the stochastic system through an intermediate link (interpretation). As a result a conceptual boundary between the classical and quantum dynamics arises. This boundary is generated by different types of interpretation (rules for calculation of average values), used at description of classical and quantum phenomena. But the quantum interpretation cannot be tested experimentally [Rylov, 1977]. In fact, there are many correct interpretations of QM.

Let us consider application of the SET approach to the electron, described by the Pauli equation. Quantum dynamics finds out a meaning of quantities establishing a correspondence between the quantities and their operators. SET finds out a meaning of quantities, separating out a classical part $\mathcal{A}_{cl}$ of the action $\mathcal{A}$. $\mathcal{A}_{cl}$ describes a statistical ensemble of classical systems $\mathcal{S}_d$, where one can separate properties of a single dynamic system from those of the ensemble as a whole. Such a separation cannot be carried out for an ensemble of stochastic systems.

Such a separation can be realized formally by introducing special quantum variables $\boldsymbol{\kappa}$ responsible for quantum effects, because at $\boldsymbol{\kappa} \equiv 0$ the action $\mathcal{A}$ converts into its classical part $\mathcal{A}_{cl}$.

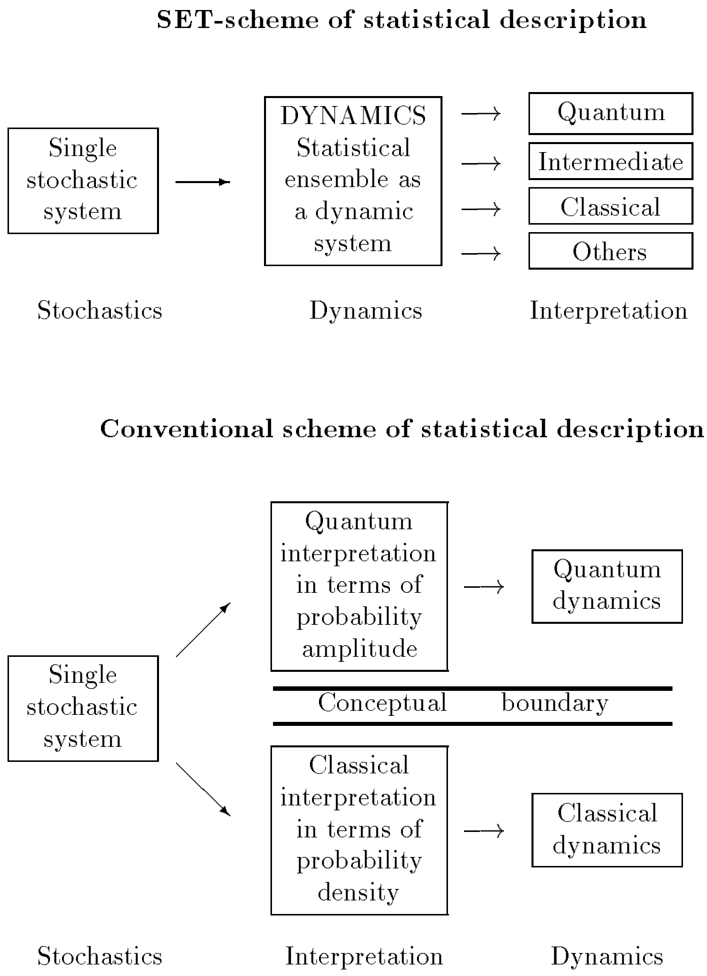

Fig.1. *Schemes of statistical description. Interpretation means a set of rules for calcultion of average values of quantities relating to the single stochastic system.*

One considers a quantum system $\mathcal{E}_P$ described by the Pauli equation, or by the action

$$\mathcal{A}_P[\psi^*,\psi] = \int [\frac{i\hbar}{2}(\psi^*\frac{\partial\psi}{\partial t} - \frac{\partial\psi^*}{\partial t}\psi) - \frac{\hbar^2}{2m}\nabla\psi^*\nabla\psi]d^4x, \tag{1.1}$$

where ψ is the wave function

$$\psi = \binom{\psi_1}{\psi_2}, \qquad \psi^* = (\psi_1^*, \psi_2^*) \tag{1.2}$$

ψ_1 and ψ_2 are complex functions, and ψ_1^* and ψ_2^* are complex conjugate functions.

The investigation is based on the supposition that the hydrodynamic variables $j = \{\rho, \mathbf{j}\}$, describe four-flux of particles, and the variables $\mathbf{s}$ are connected with the electron spin. These quantities are defined by the relations

$$j^0 = \rho = \psi^*\psi, \qquad \mathbf{j} = -\frac{i\hbar}{2m}(\psi^*\nabla\psi - \nabla\psi^*\psi), \qquad \mathbf{s} = \psi^*\boldsymbol{\sigma}\psi/(\psi^*\psi), \tag{1.3}$$

where $\boldsymbol{\sigma} = \{\sigma_\alpha\}, \alpha = 1, 2, 3$ are the Pauli matrices.

Let us investigate whether there is a classical analog of the electron spin, and if yes, then what is it. The action (1.1) can be considered as an action for the ensemble of non-relativistic electrons (or for a single electron). There is different viewpoints about this subject. We shall *not predetermine* this. One supposes only that the action (1.1) relates to non-relativistic electrons and $j = \{\rho, \mathbf{j}\}$ describes four-flux of electrons.

2. Classical Part of the Action

Using the change of variables

$$\begin{gathered}\psi = \rho^{1/2} e^{i\varphi}\boldsymbol{\sigma\eta}\chi, \qquad \psi^* = \chi^*\boldsymbol{\sigma\eta}e^{-i\varphi}\rho^{1/2} \\ \chi = \binom{\chi_1}{\chi_2} = \text{ const}, \qquad \chi^* = (\chi_1^*, \chi_2^*) = \text{ const} \\ \chi^*\chi = 1, \qquad \chi^*\boldsymbol{\sigma}\chi = \mathbf{b}, \qquad \mathbf{b}^2 = 1, \qquad \boldsymbol{\eta}^2 = 1,\end{gathered} \tag{2.1}$$

one obtains the action (1.1) in terms of hydrodynamical variables

$$\mathcal{A}_P[\rho, \mathbf{j}, \varphi, \boldsymbol{\xi}] = \mathcal{A}_{Pc}[\rho, \mathbf{j}, \varphi, \boldsymbol{\xi}] + \mathcal{A}_{Pq}[\rho, \mathbf{j}, \varphi, \boldsymbol{\xi}] \tag{2.2}$$

$$\mathcal{A}_{Pc}[\rho, \mathbf{j}, \varphi, \boldsymbol{\xi}] = \int (\frac{m}{2\rho}\mathbf{j}^2 - \hbar j^i\partial_i\varphi - \hbar j^i\varepsilon_{\alpha\beta\gamma}\eta_\alpha\partial_i\eta_\beta b_\gamma)d^4x, \qquad j^0 \equiv \rho, \tag{2.3}$$

$$\mathcal{A}_{Pq}[\rho, \mathbf{j}, \varphi, \boldsymbol{\xi}] = \int \frac{\hbar^2\rho}{8m}[4(\varepsilon_{\alpha\beta\gamma}\eta_\alpha\nabla\eta_\beta b_\gamma)^2 - 4(\partial_\alpha\eta_\beta)(\partial_\alpha\eta_\beta) - \frac{(\nabla\rho)^2}{\rho^2}]d^4x \tag{2.4}$$

where $\varepsilon_{\alpha\beta\gamma}$ is the Levi-Chivita pseudo-tensor, and $\boldsymbol{\xi} = \{\xi_\alpha\}, \alpha = 1, 2, 3$ is a variable defined by the relation

$$\boldsymbol{\xi}/\mid\boldsymbol{\xi}\mid = \boldsymbol{\eta} = (\mathbf{b} + \mathbf{s})[2(1 + \mathbf{bs})]^{-1/2} \tag{2.5}$$

The unit spin vector $\mathbf{s}$ is expressed through the unit vector $\boldsymbol{\eta}$ by means of the relation

$$\mathbf{s} = 2\boldsymbol{\eta}(\mathbf{b}\boldsymbol{\eta}) - \mathbf{b}, \tag{2.6}$$

Equivalency of the action (2.2)–(2.4) with the action (1.1) can be established as follows. Substitution of ψ from Eq.(2.1) into the action (1.1) leads to an action

$\mathcal{A}_P[\rho,\varphi,\boldsymbol{\eta}]$. On the other side, eliminating $\mathbf{j}$ from the action (2.2)–(2.4) by means of dynamic equations $\delta\mathcal{A}_P/\delta\mathbf{j}=0$, one obtains the same expression $\mathcal{A}_P[\rho,\varphi,\boldsymbol{\eta}]$.

Eq.(2.3) describes a classical part of the action (2.2) which can be reduced to the form

$$\mathcal{A}_{cl}[\mathbf{x}]=\int\frac{m}{2}(\frac{d\mathbf{x}}{dt})^2 dtd\boldsymbol{\xi}, \qquad \mathbf{x}=\mathbf{x}(t,\boldsymbol{\xi}). \tag{2.7}$$

Indeed, the Jacobians

$$j^i=\frac{\partial(x^i,\xi_1,\xi_2,\xi_3)}{\partial(x^0,x^1,x^2,x^3)}, \qquad i=0,1,2,3 \tag{2.8}$$

are solutions of dynamic equations $\delta\mathcal{A}_{Pc}/\delta\varphi=0$, $\delta\mathcal{A}_{Pc}/\delta\xi_\alpha=0$, $\alpha=1,2,3$. Adding Eq.(2.8) to the action (2.3) as additional constraints, one does not change the variational problem. Then eliminating j^i from Eq.(2.3) by means of Eq.(2.8) and reversing relations $\boldsymbol{\xi}=\boldsymbol{\xi}(t,\mathbf{x})$, one obtains Eq.(2.7), where $\mathbf{x}=\mathbf{x}(t,\boldsymbol{\xi})$.

The action (2.7) describes a statistical ensemble $\mathcal{E}_{Pc}$ of independent classical pointlike particles, labelled by Lagrangian coordinates $\boldsymbol{\xi}$. It means that the action (2.2) describes the statistical ensemble of quantum particles. Indeed, supposing that the wave function ψ and the action $\mathcal{A}_P$ describe a single quantum electron, one cannot explain, why in the quasi-uniform limit

$$\mid\frac{\hbar\rho}{m\mid\mathbf{j}\mid}\nabla\log u\mid\ll 1, \qquad u=\rho,\eta_1,\eta_2,\eta_3, \tag{2.9}$$

when one can neglect $\mathcal{A}_{Pq}$ as compared with $\mathcal{A}_{Pc}$, the action $\mathcal{A}_P=\mathcal{A}_{Pc}$ describes a classical statistical ensemble, but not a single classical particle. The statement that the dynamic system $\mathcal{E}_P$ is a statistical ensemble is the crucial point, where statistics penetrates into quantum dynamics.

The quantum ensemble $\mathcal{E}_S$ of spinless particle is also described by the action (1.1), but in this case ψ is one-component wave function. Then instead of Eqs.(1.3), (2.1) one uses a change of variables (1.3) and

$$\psi=\sqrt{\rho}\exp(i\varphi), \qquad \psi^*=\sqrt{\rho}\exp(-i\varphi), \tag{2.10}$$

Instead of Eqs.(2.2)–(2.4) one obtains

$$\mathcal{A}_S[\rho,\mathbf{j},\varphi]=\int(\mathcal{L}_{Sc}+\mathcal{L}_{Sq})d^4x, \tag{2.11}$$

$$\mathcal{L}_{Sc}=\frac{m}{2\rho}\mathbf{j}^2-\hbar j^i\partial_i\varphi, \qquad j^0\equiv\rho \tag{2.12}$$

$$\mathcal{L}_{Sq}=-\frac{\hbar^2}{8m}\frac{(\nabla\rho)^2}{\rho} \tag{2.13}$$

Classical ensembles $\mathcal{E}_{Pc}$ and $\mathcal{E}_{Sc}$ described correspondingly by Lagrangians (2.3) and (2.12) are different, although they consist of similar classical systems (free pointlike particles). This situation can be treated in the sole way. The electron spin is a *classical* property of the pure statistical ensemble, (not that of a single electron).

Spin is a *collective* (statistical property). Ensembls $\mathcal{E}_{Pc}$ and $\mathcal{E}_{Sc}$ distinguish by their vorticity $\mathbf{V}$

$$\mathbf{V}_{Sc} = \nabla \times \mathbf{v} \equiv \nabla \times (\mathbf{j}/j^0) = 0$$

$$\mathbf{V}_{Pc} = \nabla \times \mathbf{v} = \frac{\hbar}{m}\varepsilon_{\alpha\beta\gamma}\nabla\eta_\alpha \times \nabla\eta_\beta b_\gamma$$

where $\times$ means the vector product.

Separation of the action (1.1) into classical and quantum parts can be described by introduction of some quantum variables $\boldsymbol{\kappa}_i = \{\kappa_i^1, \kappa_i^2, \kappa_i^3\}$ $\quad (i = 0, 1, 2, 3)$ responsible for quantum effects. Let us rewrite the action (2.4) in the form

$$\mathcal{A}_{Pq}[\rho, \mathbf{j}, \varphi, \boldsymbol{\xi}, \boldsymbol{\kappa}] = -\int \frac{\hbar^2\rho}{2m}[(\varepsilon_{\alpha\beta\gamma}\eta_\alpha\boldsymbol{\kappa}_\beta b_\gamma)\varepsilon_{\mu\nu\sigma}\eta_\mu b_\sigma(\boldsymbol{\kappa}_\nu - 2\nabla\eta_\nu) -$$

$$-\boldsymbol{\kappa}_\beta(\boldsymbol{\kappa}_\beta - 2\nabla\eta_\beta) - \frac{1}{4}\boldsymbol{\kappa}_0(\boldsymbol{\kappa}_0 - 2\nabla\log\rho)]d^4x \tag{2.14}$$

Variation of Eq.(2.14) with respect to $\boldsymbol{\kappa}_i$, $i = 0, 1, 2, 3$ leads to the equations

$$\boldsymbol{\kappa}_0 = \nabla\log\rho, \qquad (a_\mu a_\alpha - \delta_{\mu\alpha})(\boldsymbol{\kappa}_\alpha - \nabla\eta_\alpha) = 0, \qquad \alpha = 1, 2, 3 \tag{2.15}$$

$$a_\mu \equiv \varepsilon_{\alpha\mu\gamma}\eta_\alpha b_\gamma$$

Resolving Eq.(2.15) with respect to $\boldsymbol{\kappa}$ and substituting the solutions into Eq.(2.14), one obtains the quantum part (2.4) of the action. Suppressing variables $\boldsymbol{\kappa}$ ($\boldsymbol{\kappa}_i \equiv 0$, $i = 0, 1, 2, 3$), the action (2.14) vanishes. It means that the variables $\boldsymbol{\kappa}$ are responsible for quantum effects, because vanishing $\boldsymbol{\kappa}$ lead to vanishing quantum part of the action. Separation of the action (2.2), carried out by the quantum variables $\boldsymbol{\kappa}$, is invariant with respect to any change of dependent variables which does not contain $\boldsymbol{\kappa}$. For instance, returning to variables $\psi^*, \psi, \boldsymbol{\kappa}_i$ in the action (2.2) with $\mathcal{A}_{Pc}$ and $\mathcal{A}_{Pq}$ defined respectively by Eqs. (2.3) and (2.14), one obtains instead of Eq.(1.1)

$$\mathcal{A}_P[\psi^*, \psi, \boldsymbol{\kappa}] = \int\{\frac{i\hbar}{2}(\psi^*\frac{\partial\psi}{\partial t} - \frac{\partial\psi^*}{\partial t}\psi) - \frac{\hbar^2}{2m}\nabla\psi^*\nabla\psi\}d^4x -$$

$$-\frac{\hbar^2}{2m}\int\psi^*\psi\{[\varepsilon_{\alpha\beta\gamma}\eta_\alpha(\boldsymbol{\kappa}_\beta - \nabla\eta_\beta)b_\gamma]^2 -$$

$$-(\boldsymbol{\kappa}_\alpha - \nabla\eta_\alpha)(\boldsymbol{\kappa}_\alpha - \nabla\eta_\alpha) - \frac{1}{4}[\boldsymbol{\kappa}_0 - \nabla\log(\psi^*\psi)]^2\}dx^4, \tag{2.16}$$

where

$$\boldsymbol{\eta} = (\psi^*\boldsymbol{\sigma}\psi + \mathbf{b}\psi^*\psi)\{2[(\psi^*\psi)^2 + \mathbf{b}(\psi^*\boldsymbol{\sigma}\psi)]\}^{-1/2}$$

and $\mathbf{b}$ is a constant unit vector. Setting $\boldsymbol{\kappa}_i \equiv 0$ in Eq.(2.16), one obtains an action for the statistical ensemble of classical electrons in terms of wave functions.

3. Connection between the Spin and the Magnetic Moment

As it is well known the electromagnetic field is taken into account by means of the substitution

$$\partial_k\psi \to (\partial_k - \frac{ie}{\hbar c}A_k)\psi, \qquad \partial_k\psi^* \to (\partial_k + \frac{ie}{\hbar c}A_k)\psi^*, \qquad k = 0, 1, 2, 3 \qquad (3.1)$$

in Eqs.(1.1), (1.3). This substitution leads to appearance of two additional terms in the action (2.2).

$$\mathcal{A}_{Pel}[\rho, \mathbf{j}, \varphi, \boldsymbol{\xi}] = \int (-\frac{e}{c}j^i A_i - \frac{e\hbar}{2mc}\rho\mathbf{s}\mathbf{H})d^4x, \qquad \mathbf{H} = \nabla\times\mathbf{A} \qquad (3.2)$$

Let us consider a quasi-uniform (quasi-classical) electron ensemble in a given electromagnetic field. In such an ensemble the condition (2.9) takes place, and the terms containing the second power of derivatives (proportional to $\hbar^2$) can be neglected. It is equivalent formally to a calculation to within $\hbar^2$. Let us make change of variables (2.1) in the action (2.2) with additional terms taking into account the external electromagnetic field. Then, neglecting the terms proportional to $\hbar^2$, one obtains the action of the form.

$$\mathcal{A}_{Pm}[\rho, a, \boldsymbol{\xi}] = \int \{-\rho(\partial_0 a + \hbar f_0) - \frac{e}{c}\rho A_0 - \frac{\rho}{2m}(\nabla a + \frac{e}{c}\mathbf{A})^2 -$$
$$-\frac{\hbar}{m}(\nabla a + \frac{e}{c}\mathbf{A})[\rho\mathbf{f} + \frac{1}{2}\nabla\times(\mathbf{s}\rho)]\}d^4x + O(\hbar^2), \qquad \boldsymbol{\eta} = \boldsymbol{\xi}/\mid\boldsymbol{\xi}\mid \qquad (3.3)$$

Here a is defined by the relation $a = \hbar\varphi$. $\mathbf{s}$ and $\mathbf{f}$ are functions of $\boldsymbol{\eta}$ defined by Eqs.(2.6), and

$$f_i = \varepsilon_{\alpha\beta\gamma}\eta_\alpha\partial_i\eta_\beta b_\gamma, \qquad i = 0, 1, 2, 3; \qquad f = \{f_0, \mathbf{f}\},$$

The action $\mathcal{A}_{Pm}$ is not an action of a classical statistical ensemble. Indeed, the action (3.3) can reduce to the form of the type (2.7):

$$\mathcal{A}[\mathbf{x}] = \int [\frac{m}{2}(\frac{d\mathbf{x}}{dt})^2 - \frac{e_0}{c}A_0(t, \mathbf{x}) - \frac{e_0}{c}\mathbf{A}(t, \mathbf{x})\frac{d\mathbf{x}}{dt} - \frac{e\hbar}{2mc}\mathbf{s}\mathbf{H}(t, \mathbf{x})]dtd\boldsymbol{\xi}, \qquad (3.4)$$

where $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$, and $\mathbf{s}$ is an explicit function of Lagrangian coordinates $\boldsymbol{\xi}$ defined by relations (2.5), (2.6).

The action (3.4) describes a set of different classical dynamic systems, because the last term depends explicitly on the parameters $\boldsymbol{\xi}$, and it is different for different systems of the set. The dynamic system (3.4) is not a classical statistical ensemble, because it is not a set of *similar* classical systems.

For the action (3.3) the dynamic equations have the form

$$\frac{\delta\mathcal{A}_{Pm}}{\delta a} = \partial_0\rho + \nabla[\frac{\rho}{m}(\nabla a + \frac{e}{c}\mathbf{A}) + \frac{\hbar}{m}\rho\mathbf{f}] = O(\hbar^2) \qquad (3.5)$$

$$\frac{\delta\mathcal{A}_{Pm}}{\delta\rho} = -\partial_0 a - \frac{e}{c}A_0 - \hbar f_0 - \frac{1}{2m}(\nabla a + \frac{e}{c}\mathbf{A})^2 -$$

$$-\frac{\hbar}{m}(\nabla a+\frac{e}{c}\mathbf{A})\mathbf{f}+\frac{e\hbar}{2mc}\mathbf{s}\nabla\times\mathbf{A}=O(\hbar^2) \tag{3.6}$$

$$\frac{\delta\mathcal{A}_{Pm}}{\delta\xi_\alpha}=(\delta_{\alpha\mu}-\eta_\alpha\eta_\mu)\{-2\hbar\rho\varepsilon_{\mu\beta\gamma}[\partial_0\eta_\beta+\frac{1}{m}(\nabla a+\frac{e}{c}\mathbf{A})\nabla\eta_\beta]b_\gamma-$$

$$-\frac{e\hbar\rho}{mc}\varepsilon_{\nu\beta\gamma}\partial_\beta A_\gamma[\delta_{\mu\nu}(\mathbf{b}\boldsymbol{\eta})+\eta_\nu b_\mu]\}=O(\hbar^2) \tag{3.7}$$

One uses Eqs.(3.5) and (2.6) for obtaining Eq.(3.7).

Let us introduce the following designations

$$\dot\eta_\beta=\partial_0\eta_\beta+\frac{1}{m}(\nabla a+\frac{e}{c}\mathbf{A})\nabla\eta_\beta+O(\hbar),\qquad \dot{\boldsymbol{\eta}}=\{\dot\eta_\beta\}\qquad \beta=1,2,3; \tag{3.8}$$

$$\mathbf{H}=\nabla\times\mathbf{A} \tag{3.9}$$

where the dot means a substantional derivative, and $\mathbf{H}$ is a magnetic field. The equation (3.7) reduces to the form

$$\dot{\boldsymbol{\eta}}\times\mathbf{b}-\boldsymbol{\eta}[f_0+\frac{1}{m}(\nabla a+\frac{e}{c}\mathbf{A})\mathbf{f}]+\frac{e}{2mc}[\mathbf{H}(\mathbf{b}\boldsymbol{\eta})+\mathbf{b}(\boldsymbol{\eta}\mathbf{H})-2\boldsymbol{\eta}(\mathbf{H}\boldsymbol{\eta})(\boldsymbol{\eta}\mathbf{b})]=O(\hbar) \tag{3.10}$$

Taking the scalar product of Eq.(3.10) and $\mathbf{b}$, one obtains

$$f_0+\frac{1}{m}(\nabla a+\frac{e}{c}\mathbf{A})\mathbf{f}=\frac{e}{2mc}[(\mathbf{H}\mathbf{b})-2(\mathbf{H}\boldsymbol{\eta})(\boldsymbol{\eta}\mathbf{b})+(\boldsymbol{\eta}\mathbf{H})/(\boldsymbol{\eta}\mathbf{b})]+O(\hbar) \tag{3.11}$$

Substituting Eq.(3.11) into Eq.(3.6) and using the relations (2.6), one obtains

$$-\partial_0 a-\frac{e}{c}A_0-\frac{1}{2m}(\nabla a+\frac{e}{c}\mathbf{A})^2+\frac{e\hbar}{2mc}(\boldsymbol{\eta}\mathbf{H})/(\boldsymbol{\eta}\mathbf{b})=O(\hbar^2) \tag{3.12}$$

It is the Jacobi-Hamilton equation for an electron moving in a given electromagnetic field. The last term describes an interaction of the magnetic moment $\boldsymbol{\mu}$ of the electron with the magnetic field, the magnetic moment being determined by the expression

$$\boldsymbol{\mu}=-\frac{e\hbar}{2mc}\boldsymbol{\eta}/(\boldsymbol{\eta}\mathbf{b})+\mathbf{q}=-\frac{e\hbar}{mc}\frac{\mathbf{s}+\mathbf{b}}{(\mathbf{s}+\mathbf{b})^2}+\mathbf{q} \tag{3.13}$$

where $\mathbf{q}$ is a vector orthogonal to $\mathbf{H}$. Substituting Eq.(3.11) into Eq.(3.10), solving Eq.(3.10) with respect to $\dot{\boldsymbol{\eta}}$ and taking into account that $\boldsymbol{\eta}\dot{\boldsymbol{\eta}}=0$, one obtains

$$\dot{\boldsymbol{\eta}}=-\frac{\Omega}{2}[\mathbf{n}+\mathbf{b}(\mathbf{n}\boldsymbol{\eta})/(\mathbf{b}\boldsymbol{\eta})]\times\boldsymbol{\eta} \tag{3.14}$$

where $\mathbf{n}$ is the unit vector directed along the magnetic field $\mathbf{H}$

$$\Omega=\frac{eH}{mc},\qquad \mathbf{H}=H\mathbf{n},\qquad \mathbf{n}^2=1,\qquad H=\sqrt{\mathbf{H}^2}. \tag{3.15}$$

If $\mathbf{H}$ is constant, the equation (3.14) can be solved. In this case $\mathbf{n}$=const, H=const.

Let $\mathbf{nb}=\cos\alpha\neq\pm1$. Let us project Eq.(3.14) onto vectors $\mathbf{b},\mathbf{n}$, and $\mathbf{b}\times\mathbf{n}$. Let us introduce the designations

$$u = (\mathbf{b}\boldsymbol{\eta}), \qquad v = (\mathbf{n}\boldsymbol{\eta}), \qquad w = (\mathbf{b}\times\mathbf{n})\boldsymbol{\eta} \tag{3.16}$$

Then Eq.(3.14) reduces to the system of equations

$$\dot{u} = -\frac{\Omega}{2}w, \qquad \dot{v} = \frac{\Omega}{2}vw/u, \qquad \dot{w} = -\frac{\Omega}{2}(v^2 - u^2)/\mathrm{u}. \tag{3.17}$$

The general solution of the system has the form

$$\begin{aligned} u &= \sqrt{d_1 - (d_1^2 - d_2^2)^{1/2}\sin(\Omega t + \psi)} \\ v &= d_2[d_1 - (d_1^2 - d_2^2)^{1/2}\sin(\Omega t + \psi_0)]^{-1/2} \\ w &= (d_1^2 - d_2^2)^{1/2}[d_1 - (d_1^2 - d_2^2)^{1/2}\sin(\Omega t + \psi_0)]^{-1/2}\cos(\Omega t + \psi_0) \end{aligned} \tag{3.18}$$

where $d_1 > 0$, d_2 and ψ_0 are constant along the lines tangent to the vector j^i. As far as $\boldsymbol{\eta}$ is an unit vector, the additional constraint on the constants d_1 and d_2 arises:

$$d_1 = d_2\cos\alpha + \frac{1}{2}\sin^2\alpha \tag{3.19}$$

As it follows from Eq.(3.18), if $d_1^2 \neq d_2^2$, the projections of the vector $\boldsymbol{\eta}$ onto directions $\mathbf{b}, \mathbf{n}, \mathbf{b}\times\mathbf{n}$ oscillate with the frequency $\Omega = eH/\mathrm{mc}$. The potential energy $\boldsymbol{\mu}\mathbf{H}$, connected with the magnetic moment is described by the expression

$$\boldsymbol{\mu}\mathbf{H} = -\frac{e\hbar H}{2mc}(\boldsymbol{\eta}\mathbf{n})/(\boldsymbol{\eta}\mathbf{b}) = -\frac{1}{2}\frac{\hbar\Omega d_2}{d_1 - (d_1^2 - d_2^2)^{1/2}\sin(\Omega t + \psi_0)} \tag{3.20}$$

$\boldsymbol{\mu}\mathbf{H}$ oscillates around the mean value

$$< \boldsymbol{\mu}\mathbf{H} > = -\frac{\Omega}{2\pi}\int\limits_{-\pi/2\Omega}^{\pi/2\Omega}\frac{\hbar\Omega d_2}{d_1 - (d_1^2 - d_2^2)^{1/2}\sin(\Omega t + \psi_0)}dt = -\hbar\frac{\Omega}{2}\,\mathrm{sgn}(d_2/d_1) \tag{3.21}$$

$$d_2 = uv = (\boldsymbol{\eta}\mathbf{n})(\boldsymbol{\eta}\mathbf{b})$$

Using expression (2.6) for the unit spin vector $\mathbf{s}$ and relations (3.16), (3.18), one can determine projections of $\mathbf{s}$ onto vectors $\mathbf{b}, \mathbf{n}$, and $[\mathbf{b}\times\mathbf{n}]$. One obtains

$$\begin{aligned} (\mathbf{sb}) &= 2u^2 - 1 = 2d_1 - 1 - 2(d_1^2 - d_2^2)^{1/2}\sin(\Omega t + \psi_0) \\ (\mathbf{sn}) &= 2uv - (\mathbf{nb}) = 2d_2 - \cos\alpha \\ \mathbf{s}(\mathbf{b}\times\mathbf{n}) &= 2wu = 2(d_1^2 - d_2^2)^{1/2}\cos(\Omega t + \psi_0) \end{aligned} \tag{3.22}$$

The solution (3.22) is stationary, if

$$d_1 = \varepsilon d_2, \qquad \varepsilon = \pm 1. \tag{3.23}$$

Substituting d_1 from Eq.(3.23) into Eq.(3.19), one obtains

$$d_2 = \frac{1}{2}(\varepsilon + \cos\alpha) \tag{3.24}$$

In the stationary case it follows from Eqs.(3.20), (3.21)

$$\boldsymbol{\mu}\mathbf{H} =< \boldsymbol{\mu}\mathbf{H} > = -\frac{1}{2}\hbar\Omega\varepsilon \tag{3.25}$$

Substituting Eq.(3.24) into Eq.(3.22), one obtains

$$(\mathbf{sb}) = \varepsilon\cos\alpha, \qquad (\mathbf{sn}) = \varepsilon, \qquad \mathbf{s}[\mathbf{b}\times\mathbf{n}] = 0 \tag{3.26}$$

It means that in the stationary case the spin vector $\mathbf{s}$ is parallel, or antiparallel to the magnetic field $\mathbf{H}$. In this case

$$\boldsymbol{\mu}\mathbf{H} = -\frac{e\hbar}{2mc}\mathbf{sH}, \qquad \boldsymbol{\mu} = -\frac{e\hbar}{2mc}\mathbf{s} \tag{3.27}$$

In the non-stationary case the relations (3.27) do not take place, in general, although the projection of the spin $\mathbf{s}$ onto the magnetic field direction does not oscillate in any case. One obtains the same result in the case, when $\mathbf{b} \parallel \mathbf{H}$.

Thus, the connection (3.27) between the spin and the magnetic moment is valid only for stationary states. In the non-stationary case there is no connection between $< \boldsymbol{\mu}\mathbf{H} >$ and $(\mathbf{sH})$. There is no connection even between sgn$< \boldsymbol{\mu}\mathbf{H} >$ and sgn$(\mathbf{sH})$.

4. Application of SET Approach to the Dirac Electron

SET was applied to the description of a relativistic electron in the paper [Rylov, 1995a]. The relativistic electron associates with the Dirac equation. Conventional form of the action for the Dirac equation

$$\mathcal{A}_D[\bar\psi,\psi] = \int(-m\bar\psi\psi + \frac{i}{2}\hbar\bar\psi\gamma^l\partial_l\psi - \frac{i}{2}\hbar\partial_l\bar\psi\gamma^l\psi - eA_l\bar\psi\gamma^l\psi)d^4x \tag{4.1}$$

can be written in terms of hydrodynamic variables φ, j^l, S^l, $(l = 0,1,2,3)$, κ, defined by the relations

$$\begin{aligned} j^l &= \bar\psi\gamma^l\psi, \qquad l = 0,1,2,3, \qquad \bar\psi = \psi^*\gamma^0; \\ S^l &= i\bar\psi\gamma_5\gamma^l\psi, \qquad l = 0,1,2,3, \qquad \gamma_5 = \gamma^{0123} \equiv \gamma^0\gamma^1\gamma^2\gamma^3; \\ \partial_l\varphi &= (\bar\psi\partial_l\psi - \partial_l\bar\psi\psi)(2i\bar\psi\psi)^{-1}, \qquad l = 0,1,2,3; \\ \cos\kappa &= \bar\psi\psi(j^l j_l)^{-1/2}; \end{aligned} \tag{4.2}$$

Here γ^l, $l = 0,1,2,3$ are the Dirac γ-matrices satisfying the commutation relation

$$\gamma^i\gamma^k + \gamma^k\gamma^i = 2g^{ik}, \qquad i,k = 0,1,2,3, \tag{4.3}$$

where g^{ik} =diag$(1,-1,-1,-1)$ is the metric tensor. Only two components of the pseudo-vector S^l are independent, because there are two identities

$$S^l S_l \equiv -j^l j_l, \qquad j^l S_l \equiv 0. \tag{4.4}$$

After a change of variables [Rylov, 1995a] the action (4.1) reduces to the form

$$\mathcal{A}_D[j,S,\kappa,\varphi,\kappa^i]=\int(\mathcal{L}_{cl}+\mathcal{L}_{q1}+\mathcal{L}_{q3})d^4x \tag{4.5}$$

$$\mathcal{L}_{cl}=-m\rho-ej^iA_i-\hbar j^i\{\partial_i\varphi+\varepsilon_{klsm}[\mu^k\partial_i\mu^l f^s z^m-\frac{j^k\partial_i j^l f^s\nu^m}{2\rho(\rho+j^jf_j)}]\} \tag{4.6}$$

$$\mathcal{L}_{q1}=2m\rho\sin^2(\frac{\kappa}{2})-\frac{\hbar}{2}S^l\partial_l\kappa, \tag{4.7}$$

$$\mathcal{L}_{q3}=\hbar\rho\varepsilon_{iklm}(\kappa^l\partial_\perp^k\kappa^i-q^l\partial_\perp^k\kappa^i+\kappa^i\partial_\perp^k q^l)\nu^m \tag{4.8}$$

where

$$\mu^k=[w^k-\frac{j^kw^mf_m}{\rho+j^sf_s}][2(1-w^lz_l+\frac{j_lz^lw^mf_m}{\rho+j^sf_s})]^{-1/2},\qquad w^i\equiv\frac{S^i}{\sqrt{-S^lS_i}} \tag{4.9}$$

$$q^i\equiv\frac{j^i+f^i\rho}{\sqrt{(j^l+f^l\rho)(j_l+f_l\rho)}}=\frac{j^i+f^i\rho}{\sqrt{2\rho(\rho+j^lf_l)}} \tag{4.10}$$

$$\nu^k=w^k-\frac{j^kw^mf_m}{\rho+j^sf_s},\qquad \rho\equiv\sqrt{j^ij_i} \tag{4.11}$$

The operator of the transversal differentiation $\partial_\perp^k$ is defined by the relation

$$\partial^kq^l=\partial_\perp^kq^l+\frac{j^kj^s}{\rho^2}\partial_sq^l,\qquad \partial_\perp^k\equiv\partial^k-\frac{j^kj^s}{\rho^2}\partial_s \tag{4.12}$$

Constant timelike vector f^i and pseudovector z^i satisfy the following constraints

$$f^lf_l=1,\qquad f^lz_l=0,\qquad z^lz_l=-1. \tag{4.13}$$

In the quasi-uniform state, when derivatives orthogonal to the current vector j^i are small and can be neglected. It is equivalent to vanishing variables κ and κ^i, $i=0,1,2,3$. It means that one can suppress them, setting $\kappa\equiv0$, $\kappa^i\equiv0$, $i=0,1,2,3$. In this case the Dirac system $\mathcal{S}_D$ described by the action (4.5) turns to a statistical ensemble $\mathcal{S}_{Dcu}$ of classical systems $\mathcal{S}_{dc}$ (see [Rylov, 1995a] for mathematical details). Any classical system $\mathcal{S}_{dc}$ has 11 degrees of freedom. $\mathcal{S}_{dc}$ can be considered as a guiding center moving as a charged classical particle with a particle spiralling around the guiding center. $\mathcal{S}_{dc}$ is a non-relativistic system, because there is *interaction on a distance* between the guiding center and the spiralling particle.

The Dirac equation is also non-relativistic, because the action (4.5) contains non-fictitious timelike vector f^i, describing a split of the space-time into space and time. Such a split is characteristic for a non-relativistic description of physical phenomena.

5. Concluding Remarks

Statistical physics enables to explain a Brownian motion, but the axiomatic thermodynamics cannot. It means that the area of application of statistical physics is wider, than that of thermodynamics. A similar question arises about SET and quantum dynamics technique (QDT). Whose area of application is wider? QDT is an axiomatic technique restricted by a linearity of dynamic equations formulated in terms of wave functions. SET does not contain any constraints of such a kind. It is to expect that the area of application of SET is wider, than that of QDT.

It counts in favor of this suggestion the misterious spin - statisitcs association. If the spin is a statistical (collective) property, this association seems more reasonable, than in the case, when the spin is a dynamic variable.

The problem of pair production is a principal problem of QFT. For the last fifty years the quantum theory cannot solve this problem. When the pair production is not a dominating process, and it can be considered as a correction (for instance, in quantum electrodynamics), QFT succedes. When the pair production is a dominating process, QFT fails. Maybe, this failure is connected with constraints imposed by the quantum principles. One can hope that usage of the more general SET permits to overcome the problem of pair production.

Statistical character of the electron spin generates doubts in dynamical character of other quantum numbers of elementary particles (such as isospin, strangeness, charm, beauty etc.). If some of other quantum numbers describe also statistical properties, then some of transmutations of elementary particles are in fact merely structure reconstructions of corresponding statistical ensembles. This is very important from the physical point of view.

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