On strategy of relativistic quantum theory construction

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Abstract

Two different strategies of the relativistic quantum theory construction are considered and evaluated. The first strategy is the conventional strategy, based on application of the quantum mechanics technique to relativistic systems. This approach cannot solve the problem of pair production. The apparent success of QFT at solution of this problem is conditioned by the inconsistency of QFT, when the commutation relations are incompatible with the dynamic equations. (The inconsistent theory "can solve" practically any problem, including the problem of pair production). The second strategy is based on application of fundamental principles of classical dynamics and those of statistical description to relativistic dynamic systems. It seems to be more reliable, because this strategy does not use quantum principles, and the main problem of QFT (join of nonrelativistic quantum principles with the principles of relativity) appears to be eliminated.

1 Introduction

The conventional quantum theory has been tested very well only for nonrelativistic physical phenomena of microcosm. The quantum theory is founded on the nonrelativistic quantum principles. Application of the quantum theory to relativistic phenomena of microcosm meets the problem of join of the nonrelativistic quantum principles with the principles of the relativity theory. Many researchers believe that such a join has been carried out in the relativistic quantum field theory (QFT). Unfortunately, it is not so, and the main difficulty lies in the fact that we do not apply properly the relativity principles. Writing dynamic equations in the relativistically covariant form, we believe that we have taken into account all demands of the relativity theory.

Unfortunately, it is not so. The relativistic invariance of dynamic equations is the necessary condition of true application of the relativistic principles, but it is not sufficient. Besides, it is necessary to use the relativistic concept of the state of the considered physical objects. For instance, describing a particle in the nonrelativistic mechanics, we consider the pointlike particle in the three-dimensional space to be a physical object, whose state is described by the particle position $\mathbf{x}$ and momentum $\mathbf{p}$. The world line of the particle is considered to be a history of the particle, but not a physical object. However, in the relativistic mechanics the particle world line $\mathcal{L}$ is considered to be a physical object (but not its history). In this case the pointlike particle is an intersection $\mathcal{L}\cap\mathcal{T}_C$ of the world line $\mathcal{L}$ with the hyperplane $\mathcal{T}_C$: $t=C$ =const. The hyperplane $\mathcal{T}_C$ is not invariant in the sense, that the set $\mathcal{S}_T$ of all hyperplanes $\mathcal{T}_C$ is not invariant with respect to the Lorentz transformations. If we have several world lines $\mathcal{L}_1$, $\mathcal{L}_2$, ... $\mathcal{L}_n$, their intersections $\mathcal{L}_k\cap\mathcal{T}_C$ with the hyperplane $\mathcal{T}_C$ form a set $\mathcal{S}_C$ of particles $\mathcal{P}_k=\mathcal{L}_k\cap\mathcal{T}_C$ in some coordinate system K. The set $\mathcal{S}_C$ of particles $\mathcal{P}_k$ depends on the choice of the coordinate system K. In the coordinate system K', moving with respect to the coordinate system K, we obtain another set $\mathcal{S}'_{C'}$ of particles $\mathcal{P}'_k=\mathcal{L}_k\cap\mathcal{T}'_{C'}$, taken at some time moment $t'=C'$ =const. If we have only one world line $\mathcal{L}_1$, we may choose the constant C' in such a way, that $\mathcal{P}_1=\mathcal{L}_1\cap\mathcal{T}_C$ coincides with $\mathcal{P}'_1=\mathcal{L}_1\cap\mathcal{T}'_{C'}$. However, if we have many world lines coincidence of sets $\mathcal{S}_C$ and $\mathcal{S}'_{C'}$ is impossible at any choice of the constant C'. In other words, the particle is not an invariant object, and it cannot be considered as a physical object in the relativistic mechanics. In the case of one world line we can compensate the noninvariant character of a particle by a proper choice of the constant C', but at the statistical description, where we deal with many world lines, such a compensation is impossible.

In the nonrelativistic mechanics there is the absolute simultaneity, and the set $\mathcal{S}_C$ hyperplanes $\mathcal{T}_C: t=$const is the same in all inertial coordinate systems. In this case intersections of world lines $\mathcal{L}_1$, $\mathcal{L}_2$, ... $\mathcal{L}_n$ with the hyperplane $\mathcal{T}_C$ form the same set of events in all coordinate systems, and particles are invariant objects, which may be considered as physical objects. Strictly, the world line should be considered as a physical object also in the nonrelativistic physics, as far as the nonrelativistic physics is a special case of the relativistic one. But, in this case a consideration of a particle as a physical object is also possible, and this consideration is simpler and more effective, as far as the pointlike particle is a simpler object, than the world line.

The above statements are not new. For instance, V.A. Fock stressed in his book [1], that concept of the particle state is different in relativistic and nonrelativistic mechanics. As a rule researchers do not object to such statements, but they do not apply them in practice. The nonrelativistic quantum theory has been constructed and tested in many experiments. It is a starting point for construction of the relativistic quantum theory. In this paper we try to investigate different strategies of the relativistic quantum theory construction, in order to choose the true one. However, at first we consider interplay between the fundamental physical theory and the truncated physical theory.

The scheme of this interplay is shown in the figure. The fundamental theory is a logical structure. The fundamental principles of the theory are shown below. The experimental data, which are to be explained by the theory are placed on high. Between them there is a set of logical corollaries of the fundamental principles. It is possible such a situation, when for some conditions one can obtain a list of logical corollaries, placed near the experimental data. It is possible such a situation, when some circle of experimental data and of physical phenomena may be explained and calculated on the basis of this list of corollaries without a reference to the fundamental principles. In this case the list of corollaries of the fundamental principles may be considered as an independent physical theory. Such a theory will be referred to as the truncated theory, because it explains not all phenomena, but only a restricted circle of these phenomena (for instance, only nonrelativistic phenomena). Examples of truncated physical theories are known in the history of physics. For instance, the thermodynamics is such a truncated theory, which is valid only for the quasi-static thermal phenomena. The thermodynamics is an axiomatic theory. It cannot be applied to nonstationary thermal phenomena. In this case one should use the kinetic theory, which is a more fundamental theory, as far as it may be applied to both quasi-static and nonstationary thermal phenomena. Besides, under some conditions the thermodynamics can be derived from the kinetic theory as a partial case.

The truncated theory has a set of properties, which provide its wide application.

1. The truncated theory is simpler, than the fundamental one, because a part of logical reasonings and mathematical calculations of the fundamental theory are used in the truncated theory in the prepared form. Besides, the truncated theory is located near experimental data, and one does not need long logical reasonings for application of the truncated theory.

2. The truncated theory is a list of prescriptions, and it is not a logical structure in such extent, as the fundamental theory is a logical structure. The truncated theory is axiomatic, it contains more axioms, than the fundamental theory, as far as logical corollaries of the fundamental theory appear in the truncated theory as fundamental principles (axioms).

3. Being simpler, the truncated theory appears before the fundamental theory. It is a reason of conflicts between the advocates of the fundamental theory and advocates of the truncated theory, because the last consider the truncated theory to be the fundamental one. Such a situation took place, for instance, at becoming of the statistical physics, when advocates of the axiomatic thermodynamics oppugn against Gibbs and Boltzmann. Such a situation took place at becoming of the doctrine of Copernicus-Galileo-Newton, when advocates

of the Ptolemaic doctrine oppugn against the doctrine of Copernicus-Galileo-Newton. They referred that there was no necessity to introduce the Copernican doctrine, as far as the Ptolemaic doctrine is simple and customary. Only discovery of the Newtonian gravitation law and consideration of the celestial phenomena, which cannot be described in the framework of the Ptolemaic doctrine, terminated the contest of the two doctrines.

4. Constructing the truncated theory before the fundamental one, the trial and error method is used usually. In other words, the truncated theory is guessed, but not constructed by a logical way.

The main defect of the truncated theory is an impossibility of its expansion over wider circle of physical phenomena. For instance, let the truncated theory explain nonrelativistic physical phenomena. It means, that the basic propositions of the truncated theory are obtained as corollaries of the fundamental principles and of nonrelativistic character of the considered phenomena. To expand the truncated theory on relativistic phenomena, one needs to separate, what in the principles of the truncated theory is a corollary of fundamental principles and what is a corollary of nonrelativistic character of the considered phenomena. A successful separation of the two factors means essentially a perception of the theory truncation and construction of the fundamental theory. If the fundamental theory has been constructed, the expansion of the theory on the relativistic phenomena is obtained by an application of the fundamental principles to the relativistic phenomena. The obtained theory will describe the relativistic phenomena correctly. It may be distinguished essentially from the truncated theory, which is applicable for description of only nonrelativistic phenomena.

The conventional nonrelativistic quantum theory is a truncated theory, which is applicable only for description of nonrelativistic phenomena. It has formal signs of the truncated theory (long list of axioms, simplicity, nearness to experimental data). Truncated character of the nonrelativistic quantum theory is called in question usually by researchers working in the field of the quantum theory. The principal problem of the relativistic quantum theory is formulated usually as a problem of unification of the nonrelativistic quantum principles with the principles of the relativity theory.

Conventionally the nonrelativistic quantum theory is considered to be a fundamental theory. The relativistic quantum theory is tried to be constructed without puzzling out, what in the nonrelativistic quantum theory is conditioned by the fundamental principles and what is conditioned by its nonrelativistic character. It is suggested that the linearity is the principal property of the quantum theory, and it is tried to be saved. However, the analysis shows that the linearity of the quantum theory is some artificial circumstance [2], which simplifies essentially the description of quantum phenomena, but it does not express the essence of these phenomena. The conventional approach to construction of the relativistic quantum theory is shown by the dashed line in the scheme. Following this line, the construction of the true relativistic quantum theory appears to be as difficult, as a discovery of the Newtonian gravitation law on the basis of the Ptolemaic conception, because in this case only the trial and errors method can be used. Besides, even if we succeeded to construct such a theory, it will be very difficult to choose the valid version of the theory, because it has no logical foundation. In other words, the conventional approach to construction of the relativistic quantum theory (invention of new hypotheses and fitting) seems to lead to blind alley, although one cannot eliminate the case that it appears to be successful. (the trial and error method appeared to be successful at construction of the nonrelativistic quantum mechanics).

Alternative way of construction of the relativistic theory of physical phenomena in the microcosm is shown in figure by the solid line. It supposes a derivation of fundamental principles and their subsequent application to the relativistic physical phenomena. Elimination of the nonrelativistic quantum principles is characteristic for this approach. This elimination is accompanied by elimination of the problem of an unification of the nonrelativistic quantum principles with the relativity principles. Simultaneously one develops dynamic methods of the quantum system investigation, when the quantum system is investigated simply as a dynamic system. These methods are free of application of quantum principles. They are used for investigation of both relativistic and nonrelativistic quantum systems. A use of logical constructions is characteristic for this approach. One does not use an invention of new hypotheses and fitting (the trial and error method).

It is assumed usually that quantum systems contain such a specific nonclassical object as the wave function. Quantum principles is a list of prescriptions, how to work with the wave functions. In reality the wave function is not a specific nonclassical object. The wave function is a complex hydrodynamic potential. Any ideal fluid can be described in terms the hydrodynamic potentials (Clebsch potentials [3, 4]). In particular, it can be described in terms of the wave functions [5]. Prescriptions for work with description in terms of wave functions follows directly from definition of the wave function and from prescriptions for work with the dynamic systems of hydrodynamic type. Quantum systems are such dynamic systems of hydrodynamic type, for which the dynamic equations are linear, if they are written in terms of the wave function. Statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$ of stochastic particles $\mathcal{S}_{\rm st}$ is a dynamic system of hydrodynamic type. Under some conditions dynamic equations for the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$ become linear, if they are written in terms of the wave function. In this case the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$ may be considered as a quantum system in the sense, that quantum principles (the prescriptions for work with wave function) may be applied to the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$.

Thus, the quantum systems are not enigmatic systems, described by a specific nonclassical object (wave function). Quantum systems are a partial case of dynamic systems, which may and must be investigated by conventional dynamic methods, applied in the fluid dynamics. The classical principles of dynamics and those of statistical description are fundamental principles of any dynamics and, in particular, of the quantum mechanics, considered to be a dynamics of stochastic particles. In other words, the nonrelativistic quantum theory is truncated theory with respect to dynamics of the stochastic systems.

Transition to relativistic quantum mechanics means that one should apply the general principles of mechanics to the statistical ensembles of stochastic particles, whose regular component of velocity is relativistic. (Stochastic component of velocity is always relativistic, even in the case, when the regular component is nonrelativistic). Such a statistical description can be carried out in terms of the wave function. However, we cannot state previously that dynamic equations will be linear, because in the relativistic case there is such a phenomenon as the particle production, which is absent in the classical relativistic mechanics and in the nonrelativistic quantum theory.

At first sight, the direct way of transition from nonrelativistic quantum theory to the relativistic one seems to be more attractive, because it is simpler and it does not need a discovery of fundamental concepts. Besides, it seems to be an unique way, if we believe that the nonrelativistic quantum theory is a fundamental theory (but not a truncated one). Unfortunately, following the quantum principles and this way, we come to a blind alley. This circumstance forces us to question, whether the nonrelativistic quantum theory is a fundamental theory (but not a truncated one).

We shall refer to the path, shown by the dashed line as the direct path (direct approach). The path, shown by the solid line will be referred to as the logical path (logical approach). Investigation of the two approaches and of investigation strategies connected with them is the main goal of this paper. The logical path seems to be more adequate, but at the same time it seems to be more difficult. There are two different methods of presentations of our investigation: (1) description of problems of the direct path from the viewpoint of the logical one, (2) description of those problems of the direct path which have lead to a refusal from the direct path in favour of the logical one. In this paper we prefer to use the second version.

2 Difficulties of the quantum principles application to the relativistic phenomena

The particle production is the physical phenomenon, which is characteristic only for quantum relativistic physics. This phenomenon has no classical analog, because it is absent in the classical relativistic physics. This phenomenon is absent in the nonrelativistic quantum physics. At the classical description the particle production is a turn of the world line in the time direction. According to such a conception the particles are produced by pairs particle – antiparticle. In classical physics there is no force field, which could produce or annihilate such pairs. If the world line describes the pair production, some segment of this world line is to be spacelike. At this segment we have

$$g_{ik}\frac{dx^i}{d\tau}\frac{dx^k}{d\tau} < 0 \tag{2.1}$$

where g_{ik} is the metric tensor and $x^k = x^k(\tau)$ is the equation of the world line $\mathcal{L}$. On the other hand, the action for the free classical relativistic particle has the form

$$\mathcal{A}[x] = -\int mc\sqrt{g_{ik}\frac{dx^i}{d\tau}\frac{dx^k}{d\tau}}d\tau \tag{2.2}$$

Relations (2.1) and (2.2) are incompatible. They become compatible, if there is such a force field, which changes the particle mass. For instance, if instead of the action (2.2) we have

$$\mathcal{A}[x] = \int L(x, \dot{x})\, d\tau, \qquad L = -m_{\text{eff}}c\sqrt{g_{ik}\frac{dx^i}{d\tau}\frac{dx^k}{d\tau}}, \qquad m_{\text{eff}} = m\sqrt{1+f} \tag{2.3}$$

where m_{eff} is the effective particle mass, and $f = f(x)$ is some external force field, which changes the effective particle mass m_{eff}. If $f < -1$, the effective mass is imaginary, the condition (2.1) takes place in the region, where $f < -1$, and the interdict on the pair production, or on the pair annihilation is violated.

Further we shall use the special term WL for the world line considered as a physical object. The term "WL" is the abbreviation of the term "world line". Along with the term WL we shall use the term "emlon", which is the perusal of Russian abbreviation "ML", which means world line. Investigation of the emlon, changing its direction in the time direction and describing pair production (or annihilation), shows [6, 7], that some segments of the emlon describe a particle, whereas another segments describe an antiparticle. The particle and antiparticle have opposite sign of the electric charge. The energy $E = \int T^{00}d\mathbf{x}$ of the particle and that of the antiparticle is always positive. The time components $p_0 = \partial L/\partial\dot{x}^0$, $\dot{x}^k \equiv dx^k/d\tau$ of the canonical momentum p_k of the particle and that of the antiparticle have opposite sign, if the world line (WL) is considered as a single physical object (single dynamic system). They may have the same sign and coincide with $-E$, if different segments of the emlon are associated with different dynamic systems (particles and antiparticles).

Description of the annihilation process as an evolution of two different dynamic systems (particle and antiparticle), which cease to exist after collision, is incompatible with the conventional formalism of classical relativistic dynamics, where dynamic systems may not disappear. However, description of this process as an evolution of some pointlike object SWL moving along the world line in the direction of increase of the evolution parameter τ is possible from viewpoint of the conventional formalism of the relativistic physics. The object SWL is the abbreviation of the term "section of world line". Along with the term "SWL" we shall use also the term "esemlon". It is the perusal of Russian abbreviation "SML", which means "Section of the world line". The esemlon is the collective concept with respect to concepts of particle and antiparticle. In the process of evolution the esemlon may change its state (particle or antiparticle). Such an approach is compatible with the relativistic kinematics.

The investigation [6] shows that the energy E and the temporal component of the canonical momentum $-p_0$ are different quantities, which may coincide, only if there is no pair production. In the presence of pair production the equality $E = -p_0$ for the whole world line is possible also in the case, when the whole world line is cut into segments, corresponding to particles and antiparticles, and each segment is considered to be a single dynamic system.

It is generally assumed that the perturbation theory and the divergencies are the main problems of QFT. In reality, it is only a vertex of iceberg. The main problem lies in the definition of the commutation relations. We demonstrate this in the example of the dynamic equation

$$(\partial_i \partial^i + m^2)\varphi = \lambda \varphi^* \varphi \varphi \tag{2.4}$$

Here φ is the scalar complex field and φ^* is the Hermitian conjugate field, λ is the self-action constant. There are two different schemes of the second quantization: (1) PA-scheme, where particle and antiparticle are considered as different physical objects and (2) WL-scheme, where world line (WL) is considered as a physical object. A particle and an antiparticle are two different states of SWL (or WL). These two schemes distinguish in the commutation relations, imposed on the operators φ and φ^* (see for details [8]).

In the PA-scheme there is indefinite number of objects (particles and antiparticles) which can be produced and annihilated. The commutators $[\varphi(x), \varphi(x')]_-$ and $[\varphi(x), \varphi^*(x')]_-$ vanish

$$[\varphi(x), \varphi(x')]_- = 0, \qquad [\varphi(x), \varphi^*(x')] = 0, \qquad |x - x'|^2 < 0 \tag{2.5}$$

if interval between the points x and x' is spacelike. The PA-scheme tried to describe the entire picture of the particle motion and their collision. It is a very complicated picture. It can be described only in terms of the perturbation theory, because the number of physical objects (objects of quantization) is not conserved. The commutation relation, which are used in the PA-scheme are *incompatible with the dynamic equations*. As a result the PA-scheme of the second quantization appears to be inconsistent.

In the WL-scheme of the second quantization the number of objects of quantization (WL) is conserved, and one can divide the whole problem into parts, containing one WL, two WLs, three WLs etc. Each of parts may be considered and solved independently. The statement of the problem reminds that of the nonrelativistic quantum mechanics, where the number of particles is conserved. As a result the whole problem may be divided into one-particle problem, two-particle problem, etc, and each problem can be solved independently. According to such a division of the whole problem into several simpler problems, the problem of the second quantization in WL-scheme is reduced to several simpler problems. As a result it may be formulated without a use of the perturbation theory (see for details [8]). Commutation relations in the WL-scheme do not satisfy the condition (2.5). This circumstance is connected with the fact that the objects of quantization (WLs) are lengthy objects. If there is the particle production, WLs are spacelike in the sense that they may contain points x and x', separated by the spacelike interval. There are such dynamic variables at x and at x', lying on the same WL, for which the commutator does not vanish, and it is a reason for violation of conditions (2.5) in the WL-scheme of quantization. The commutation relations in WL-scheme are compatible with dynamic equations. Besides, simultaneous commutation relations depend on the self-action constant λ. The WL-scheme of the second quantization is consistent and compatible with dynamic equations. It can be solved by means of nonperturbative methods. However, the pair production is absent in the WL-scheme, even if the self-action constant $\lambda \neq 0$ [8].

One believes, that there is the pair production in the PA-scheme. However, the PA-scheme is inconsistent, and the pair production is a corollary of this inconsistency [8]. Thus, neither PA-scheme nor WL-scheme of quantization can derive the pair production effect. It is connected with the fact, that the self-action of the form (2.4), as well as other power interactions cannot generate pair production. To generate the pair production, one needs interaction of special type [7].

Advocates of the PA-scheme state that in the WL-scheme the causality principle is violated, because the conditions (2.5) are not fulfilled. It is not so, because the causality principle has the form (2.5) only for pointlike objects. For lengthy objects (spacelike world lines) the causality principle has another form [8]. The condition (2.5) states simply that the dynamic variables of different dynamic systems commutate. But in the case of spacelike WL the points x and x', separated by the spacelike interval, may belong to the same dynamic system. In this case the condition (2.5) has to be violated. But independently of whether or not the advocates of the PA-scheme are right, the dynamic equation (2.4) does not describe the pair production, and appearance of the pair production [9, 10, 11, 12] is a result of incompatibility of the commutation relations with the dynamic equation.

Conventionally one considers the commutation relations as a kind of initial conditions for the dynamic equations. As a result one does not see a necessity to test the compatibility of the commutation relations with the dynamic equations. In reality the analogy between the commutation relations and initial conditions is not true. The commutation relations are additional constraints imposed on the dynamic variables. *Compatibility of additional constraints with the dynamic equations is to be tested.* Dependence of the simultaneous commutation relations on the self-action constant λ in the WL-scheme, where such a compatibility takes place, confirms the necessity of such a test.

Thus, a direct application of the quantum mechanics formalism to a relativistic dynamic systems leads to impossibility of the particle production description. It means that we should understand the essence of the quantum mechanics formalism and revise its form in accordance with the revised understanding of the quantum mechanics.

3 Linearity of quantum mechanics

To show that the linearity of quantum mechanics formalism is not an essential inherent property of the fundamental theory, we consider the Schrödinger particle, which is the dynamic system $\mathcal{S}_{\mathrm{S}}$ described by the action

$$\mathcal{S}_{\mathrm{S}}: \qquad \mathcal{A}_{\mathrm{S}}[\psi] = \int \left\{ \frac{i\hbar}{2} \left(\psi^* \partial_0 \psi - \partial_0 \psi^* \cdot \psi \right) - \frac{\hbar^2}{2m} \boldsymbol{\nabla} \psi^* \boldsymbol{\nabla} \psi \right\} d^4 x \tag{3.1}$$

where $\psi = \psi(t, \mathbf{x})$ is a complex wave function. The meaning of the wave function (connection between the particle and the wave function) is described by the relations.

$$\langle F(\mathbf{x}, \mathbf{p}) \rangle = B \int \mathrm{Re} \left\{ \psi^* F(\mathbf{x}, \hat{\mathbf{p}}) \psi \right\} d\mathbf{x}, \qquad \hat{\mathbf{p}} = -i\hbar \boldsymbol{\nabla}, \qquad B = \left(\int \psi^* \psi d\mathbf{x} \right)^{-1} \tag{3.2}$$

which define the mean value $\langle F(\mathbf{x}, \mathbf{p}) \rangle$ of any function $F(\mathbf{x}, \mathbf{p})$ of position $\mathbf{x}$ and momentum $\mathbf{p}$. We shall refer to the relation (3.2) together with the restrictions imposed on its applications as the quantum principles, because von Neumann [13] has shown, that all propositions of quantum mechanics can be deduced from relations of this type. Thus, the action (3.1) describes the quantum mechanics formalism (dynamics), whereas the relation (3.2) forms a basis for the conventional interpretation of the quantum mechanics.

Dynamic equation

$$i\hbar \partial_0 \psi = -\frac{\hbar^2}{2m} \boldsymbol{\nabla}^2 \psi, \tag{3.3}$$

generated by the action (3.1) is linear, and one believes that this linearity together with the linear operators, describing all observable quantities, is the inherent property of the quantum mechanics.

The quantum constant $\hbar$ is supposed to describe the quantum properties in the sense, that setting $\hbar = 0$ in the quantum description, we are to obtain the classical description. However, setting $\hbar = 0$ in the action (3.1), we obtain no description. All terms in the action contain $\hbar$, and it seems that the description by means of the action (3.1) is quantum from the beginning to the end. In reality the principal part of the dynamic system $\mathcal{S}_S$ is classical, and the quantum description forms only a part of the general description. In other words, description in terms of the action (3.1) is an artificial description.

To show this, we transform the wave function ψ, changing its phase

$$\psi \to \Psi_b: \quad \psi = |\Psi_b| \exp\left(\frac{b}{\hbar} \log \frac{\Psi_b}{|\Psi_b|} \right) \qquad b = \mathrm{const} \neq 0 \tag{3.4}$$

Substituting (3.4) in the action (3.1), we obtain

$$\mathcal{A}_{\mathrm{S}}[\Psi_b] = \int \left\{ \frac{ib}{2} \left(\Psi_b^* \partial_0 \Psi_b - \partial_0 \Psi_b^* \cdot \Psi_b \right) - \frac{b^2}{2m} \boldsymbol{\nabla} \Psi_b^* \boldsymbol{\nabla} \Psi_b \right.$$

$$+\frac{b^2}{2m}\left(\nabla\left|\Psi_b\right|\right)^2-\frac{\hbar^2}{2m}\left(\nabla\left|\Psi_b\right|\right)^2\Bigg\}dt\,d\mathbf{x} \tag{3.5}$$

This change of variables leads to the replacement $\hbar\to b$ and to appearance of two nonlinear terms which compensate each other, if $b=\hbar$. The change of variable does not change the dynamic system $\mathcal{S}_{\mathrm{S}}$, although the dynamic equation becomes nonlinear, if $b^2\neq\hbar^2$

$$ib\partial_0\Psi_b=-\frac{b^2}{2m}\nabla^2\Psi_b-\frac{\hbar^2-b^2}{8m}\left(\frac{(\nabla\rho)^2}{\rho^2}+2\nabla\frac{\nabla\rho}{\rho}\right)\Psi_b, \tag{3.6}$$

However, the description in terms of Ψ_b appears to be natural in the sense, that after setting $\hbar=0$, the action $\mathcal{A}_{\mathrm{S}}\left[\Psi_b\right]$ turns into the action

$$\mathcal{A}_{\mathrm{Scl}}\left[\Psi_b\right]=\int\left\{\frac{ib}{2}\left(\Psi_b^*\partial_0\Psi_b-\partial_0\Psi_b^*\cdot\Psi_b\right)-\frac{b^2}{2m}\nabla\Psi_b^*\nabla\Psi+\frac{b^2}{2m}\left(\nabla\left|\Psi_b\right|\right)^2\right\}dt\,d\mathbf{x} \tag{3.7}$$

which carries out the classical description. It describes the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$ of free classical particles $\mathcal{S}_{\mathrm{d}}$. The action $\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{d}}]}$ for this statistical ensemble can be represented in the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{d}}]}\left[\mathbf{x}\right]=\int\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2dt\,d\boldsymbol{\xi} \tag{3.8}$$

where $\mathbf{x}=\mathbf{x}\left(t,\boldsymbol{\xi}\right)$ is a 3-vector function of independent variables $t,\boldsymbol{\xi}=\{\xi_1,\xi_2,\xi_3\}$. The variables (Lagrangian coordinates) $\boldsymbol{\xi}$ label particles $\mathcal{S}_{\mathrm{d}}$ of the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$. The statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$ is a dynamic system of the hydrodynamic type. One can show that the dynamic system, described by the action $\mathcal{A}_{\mathrm{Scl}}\left[\Psi_b\right]$ (3.7) is a partial case (irrotational flow) of the dynamic system $\mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$ [5].

Connection between the Schrödinger equation and hydrodynamic description is well known [14, 15]. But a connection between the description in terms of wave function and the hydrodynamic description was one-way. One can transit from the Schrödinger equation to the hydrodynamic equations, but one cannot transit from hydrodynamic equations to the description in terms of the wave function, because one *needs* to *integrate* hydrodynamic equations. Indeed, the Schrödinger equation consists of two real first order equations for the density ρ and the phase φ, whereas the system of the hydrodynamic equations consists of four first order equations for the density ρ and for the velocity $\mathbf{v}$. To obtain four hydrodynamic equations one needs to take gradient of the equation for the phase φ and introduce the velocity $\mathbf{v}=m^{-1}\nabla\varphi$. On the contrary, if we transit from the hydrodynamic description to the description in terms of the wave function, we are to integrate hydrodynamic equations. In the general case this integration was not known for a long time.

Change of variables, leading from the action $\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{d}}]}\left[\mathbf{x}\right]$ to the action $\mathcal{A}_{\mathrm{Scl}}\left[\Psi_b\right]$ contains integration (see [5] or mathematical appendices to papers [16, 17]). The constant b in the action $\mathcal{A}_{\mathrm{Scl}}\left[\Psi_b\right]$ is an arbitrary constant of integration (gauge constant). Arbitrary integration functions are "hidden" inside the wave function Ψ_b.

Thus, the limit of Schrödinger particle (3.5) at $\hbar \to 0$ is a statistical ensemble $\mathcal{E}[\mathcal{S}_{\rm d}]$, but not an individual particle $\mathcal{S}_{\rm d}$. It means, that the *wave function describes a statistical ensemble of particles, but not an individual particle*, and the Copenhagen interpretation, where the wave function describes an individual particle, is *incompatible* with the quantum mechanics formalism.

Dynamic system $\mathcal{S}_{\rm S}$ is described by the action (3.5) as well as by the action (3.1). Interpretation (3.2) of the wave function ψ may be also rewritten as interpretation of Ψ_b by means of transformation (3.4). In the action (3.5) only one term contains the quantum constant $\hbar$, and this term is responsible for quantum effects. The linearity of the Schrödinger equation (3.3) may be considered as a result of the special choice of the arbitrary constant $b = \hbar$.

Such a choice is justified, because it transforms the natural dynamic equation (3.6) into the linear dynamic equation (3.3), which is very convenient for solution and for investigation of the dynamic system $\mathcal{S}_{\rm S}$. However, the dynamic equation (3.3) remains to be an artificial, because the linear property of dynamic equation is not an essential property. It is appears as a result of the special choice of the integration constant, and the linearity may not be used for the generalization of the nonrelativistic quantum theory on the relativistic case.

The fact that the classical approximation $\mathcal{S}_{\rm cl}$ of the Schrödinger particle $\mathcal{S}_{\rm S}$ is a statistical ensemble $\mathcal{E}[\mathcal{S}_{\rm d}]$ of free classical (deterministic) particles $\mathcal{S}_{\rm d}$ suggests the idea, that the Schrödinger particle $\mathcal{S}_{\rm S}$ is in reality a statistical ensemble $\mathcal{E}[\mathcal{S}_{\rm st}]$ of free stochastic particles $\mathcal{S}_{\rm st}$. This idea is the old reasonable idea, which has been suggested by different authors, for instance [18]. However, the mathematical realization of this idea met *difficulties, conditioned by incorrect application of the relativity principles.*

4 Statistical description of relativistic particles

Any statistical description is a description of *physical objects*. As we have mentioned above, in the nonrelativistic case the physical objects are points of the three-dimensional space. In the relativistic case the physical objects are lengthy objects: emlons. Statistical description of nonrelativistic particles distinguishes from that of relativistic particles in the sense, that the state density ρ at the nonrelativistic description is defined by the relation

$$dN = \rho dV \tag{4.1}$$

where dN is the number of particles in the infinitesimal 3-volume dV. In the relativistic case the state density j^k is described by the relation

$$dN = j^k dS_k \tag{4.2}$$

where dN is the flux of world lines through the infinitesimal area dS_k. It follows from the relations (4.1), (4.2) that in the nonrelativistic case one can introduce the concept of the probability density of the state on the basis of the nonnegative quantity ρ, whereas in the relativistic case it is impossible, because one cannot construct the probability density on the basis of the 4-vector j^k.

Statistical description is a description of the statistical ensemble, which is the dynamic system consisting of many identical independent systems. These systems may be dynamic or stochastic. However, the statistical ensemble is a dynamic system in any case. It means, that there are dynamic equations, which describe the evolution of the statistical ensemble state. Investigation of the statistical ensemble as a dynamic system admits one to investigate the mean characteristics of the stochastic systems, constituting this ensemble. Besides, in the nonrelativistic case the statistical ensemble is a tool for calculation of different mean quantities and distributions, because in this case the ensemble state may be interpreted as the probability density of the fact, that the system state is placed at some given point of the phase space.

The statistical ensemble is used usually in the statistical physics, where the statistical description of the deterministic nonrelativistic systems is produced. The principal property of the statistical ensemble (*to be a dynamic system*) is perceived as some triviality, and the statistical ensemble is considered usually as a tool for calculation of mean values of different functions of the state. When one tries to apply this conception of the statistical ensemble to description of relativistic stochastic particles, it is quite natural that one fails, because the probabilistic conception of the statistical ensemble (statistical ensemble as a tool for calculation of mean values) cannot be applied here. The problem of construction of the dynamic system (statistical ensemble) from stochastic systems is not stated in the statistical physics.

We display in the example of free nonrelativistic particles, how the statistical ensemble is constructed without a reference to the probability theory. The action $\mathcal{A}_{\mathcal{S}_{\rm d}}$ for the free deterministic particle $\mathcal{S}_{\rm d}$ has the form

$$\mathcal{A}_{\mathcal{S}_{\rm d}}\left[\mathbf{x}\right]=\int\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^{2}dt \tag{4.3}$$

where $\mathbf{x}=\mathbf{x}\left(t\right)$.

For the pure statistical ensemble $\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\rm d}]}$ of free deterministic particles we obtain the action

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\rm d}]}\left[\mathbf{x}\right]=\int\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^{2}dtd\boldsymbol{\xi} \tag{4.4}$$

where $\mathbf{x}=\mathbf{x}\left(t,\boldsymbol{\xi}\right)$ is a 3-vector function of independent variables $t,\boldsymbol{\xi}=\{\xi_1,\xi_2,\xi_3\}$. The variables (Lagrangian coordinates) $\boldsymbol{\xi}$ label particles $\mathcal{S}_{\rm d}$ of the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm d}\right]$. The statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm d}\right]$ is a dynamic system of hydrodynamic type.

The statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm st}\right]$ of free *stochastic* particles $\mathcal{S}_{\rm st}$ is a dynamic system, described by the action

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\rm st}]}\left[\mathbf{x},\mathbf{u}_{\rm df}\right]=\int\left\{\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^{2}+\frac{m}{2}\mathbf{u}_{\rm df}^{2}-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}_{\rm df}\right\}dtd\boldsymbol{\xi} \tag{4.5}$$

where $\mathbf{u}_{\mathrm{df}} = \mathbf{u}_{\mathrm{df}}(t, \mathbf{x})$ is a diffusion velocity, describing the mean value of the stochastic component of velocity, whereas $\frac{d\mathbf{x}}{dt}(t, \boldsymbol{\xi})$ describes the regular component of the particle velocity, and $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$ is the 3-vector function of independent variables $t, \boldsymbol{\xi} = \{\xi_{1,}\xi_2, \xi_3\}$. The variables $\boldsymbol{\xi}$ label particles $\mathcal{S}_{\mathrm{st}}$, substituting the statistical ensemble. The operator

$$\boldsymbol{\nabla} = \left\{ \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^2} \right\}$$

is defined in the coordinate space of $\mathbf{x}$. Note that the transition from the statistical ensemble (4.4) to the statistical ensemble (4.5) is purely dynamic. The concept of probability is not used. The character of stochasticity is determined by the form of two last terms in the action (4.5). For instance, if we replace $\boldsymbol{\nabla}\mathbf{v}_{\mathrm{df}}$ in (4.5) by some function $f(\boldsymbol{\nabla}\mathbf{v}_{\mathrm{df}})$, we obtain another type of stochasticity, which does not coincide with the quantum stochasticity.

The action for the single stochastic particle is obtained from the action (4.5) by omitting integration over $\boldsymbol{\xi}$. We obtain the action

$$\mathcal{A}_{\mathcal{S}_{\mathrm{st}}}[\mathbf{x}, \mathbf{u}_{\mathrm{df}}] = \int \left\{ \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 + \frac{m}{2}\mathbf{u}_{\mathrm{df}}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}_{\mathrm{df}} \right\} dt \tag{4.6}$$

were $\mathbf{x} = \mathbf{x}(t)$, $\mathbf{u}_{\mathrm{df}} = \mathbf{u}_{\mathrm{df}}(t, \mathbf{x})$. However, the action (4.6) has only a symbolic sense, as far as the operator $\boldsymbol{\nabla}$ is defined in some vicinity of the point $\mathbf{x}$, but not at the point $\mathbf{x}$ itself. It means, that the action (4.6) does not determine dynamic equations for the particle $\mathcal{S}_{\mathrm{st}}$, and the particle appears to be stochastic, although dynamic equations exist for the statistical ensemble of such particles. They are determined by the action (4.5). Thus, the particles described by the action (4.5) are stochastic, because there are no dynamic equations for a single particle. In the case, when the quantum constant $\hbar = 0$, the actions (4.6) and (4.3) coincide, because in this case it follows from (4.6), that $\mathbf{u}_{\mathrm{df}} = 0$.

Variation of action (4.5) with respect to variable $\mathbf{u}_{\mathrm{df}}$ leads to the equation

$$\mathbf{u}_{\mathrm{df}} = -\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho, \tag{4.7}$$

where the particle density ρ is defined by the relation

$$\rho = \left[\frac{\partial(x^1, x^2, x^3)}{\partial(\xi_1, \xi_2, \xi_3)}\right]^{-1} = \frac{\partial(\xi_1, \xi_2, \xi_3)}{\partial(x^1, x^2, x^3)} \tag{4.8}$$

The relation (4.7) is the expression for the mean diffusion velocity in the Brownian motion theory.

Eliminating $\mathbf{u}_{\mathrm{df}}$ from the dynamic equation for $\mathbf{x}$, we obtain dynamic equations of the hydrodynamic type.

$$m\frac{d^2\mathbf{x}}{dt^2} = -\boldsymbol{\nabla}U(\rho, \boldsymbol{\nabla}\rho), \qquad U(\rho, \boldsymbol{\nabla}\rho) = \frac{\hbar^2}{8m}\left(\frac{(\boldsymbol{\nabla}\rho)^2}{\rho^2} - 2\frac{\boldsymbol{\nabla}^2\rho}{\rho}\right) \tag{4.9}$$

By means of the proper change of variables these equations can be reduced to the Schrödinger equation [5].

However, there is a serious mathematical problem here. The fact is that the hydrodynamic equations are to be integrated, in order they can be described in terms of the wave function. The fact, that the Schrödinger equation can be written in the hydrodynamic form, is well known [14, 15]. However, the inverse transition from the hydrodynamic equations to the wave function was not known until the end of the 20th century [5], and the necessity of integration of hydrodynamic equations was a reason of this fact.

Derivation of the Schrödinger equation as a partial case of dynamic equations, describing the statistical ensemble of random particles (4.5), shows that the wave function is simply a method of description of hydrodynamic equations, but not a specific quantum object, whose properties are determined by the quantum principles. At such an interpretation of the wave function the quantum principles appear to be superfluous, because they are necessary only for explanation, what is the wave function and how it is connected with the particle properties. All remaining information is contained in the dynamic equations. It appears that the quantum particle is kind of stochastic particle, and all its exhibitions can be interpreted easily in terms of the statistical ensemble of stochastic particles (4.5).

The idea of that, the quantum particle is simply a stochastic particle, is quite natural. It was known many years ago [18]. However, the mathematical form of this idea could not be realized for a long time because of the two problems considered above (incorrect conception on the statistical ensemble of relativistic particles and necessity of integration of the hydrodynamic equations).

Thus, we see in the example of the Schrödinger particle, that quantum systems are a special sort of dynamic systems, which could be obtained from the statistical ensemble of classical dynamic systems by means of a change of parameters P of the dynamic system by its effective value P_{eff}. In particular, the free uncharged particle is described by an unique parameter: its mass m.

Statistical ensemble of free classical relativistic particles is described by the action

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\text{d}}]}[x] = -\int mc\sqrt{g_{ik}\dot{x}^i\dot{x}^k}d\tau d\boldsymbol{\xi}, \qquad \dot{x}^k \equiv \frac{dx^k}{d\tau} \tag{4.10}$$

where $x^k = x^k(\tau, \boldsymbol{\xi})$. To obtain the quantum description, we are to consider the statistical ensemble $\mathcal{E}[\mathcal{S}_{\text{st}}]$ of free stochastic relativistic particles $\mathcal{S}_{\text{st}}$, which is the dynamic system described by the action

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\text{st}}]}[x, u] = -\int m_{\text{eff}}c\sqrt{g_{ik}\dot{x}^i\dot{x}^k}d\tau d\boldsymbol{\xi}, \qquad \dot{x}^k \equiv \frac{dx^k}{d\tau} \tag{4.11}$$

where $x^k = x^k(\tau, \boldsymbol{\xi})$, $u^k = u^k(x)$, $k = 0, 1, 2, 3$. Here the effective mass is obtained from the mass m of the deterministic (classical) particle by means of the change

$$m^2 \to m_{\text{eff}}^2 = m^2\left(1 + g_{ik}\frac{u^i u^k}{c^2} + \frac{\hbar}{mc^2}\partial_k u^k\right) \tag{4.12}$$

where $u^k = u^k(x)$ the mean value of the 4-velocity stochastic component. Using the relation

$$\kappa^k = \frac{m}{\hbar} u^k, \tag{4.13}$$

it is convenient to introduce the 4-velocity $\kappa = \{\kappa^0, \boldsymbol{\kappa}\}$, with $\boldsymbol{\kappa}$ having dimensionality of the length. The action (4.11) takes the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\rm st}]}[x, \kappa] = -\int mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k} d\tau d\boldsymbol{\xi}, \qquad K = \sqrt{1 + \lambda^2 \left(g_{ik}\kappa^i\kappa^k + \partial_k\kappa^k\right)} \tag{4.14}$$

where $\lambda = \frac{\hbar}{mc}$ is the Compton wave length of the particle and the metric tensor g_{ik} =diag$\{c^2, -1, -1, -1\}$. In the nonrelativistic approximation the action (4.14) turns in the action (4.5), which takes the form

$$\mathcal{A}_{\mathcal{S}_{\rm st}}[\mathbf{x}, \mathbf{u}] = \int \left\{-mc^2 + \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right\} dt d\boldsymbol{\xi} \tag{4.15}$$

Deriving (4.15), we choose the parameter $\tau = t = x^0$, take into account the relation (4.13) and neglect $\partial_0\kappa^0$ as compared with $\boldsymbol{\nabla\kappa}$.

In the general case (4.14) the κ-field κ^k may be also represented in the form of gradient as well as in the case (4.7)

$$\kappa^k = g^{kl}\partial_l\kappa \tag{4.16}$$

Here κ is the scalar field of such a form, that e^κ satisfies the inhomogeneous wave equation.

Using proper change of variables, one can introduce the wave function, satisfying the Klein-Gordon equation. At such a change of variables the κ-field appears to be hidden in the wave function and its remarkable properties appear to be hidden. As well as the diffusion velocity $\mathbf{u}_{\rm df}$ in (4.6) the κ-field κ^k describes the mean value of the stochastic component of the particle velocity. In the non-relativistic case (4.6) the 3-velocity $\mathbf{u}_{\rm df}$ is determined uniquely by its source (the density ρ of particles). But the κ-field is a relativistic field, which may escape from its source and exist separately from its source. Besides, the κ-field can change the effective particle mass, as one can see from the expression (4.14) for the action. In particular, if

$$\lambda^2\left(g_{ik}\kappa^i\kappa^k + \partial_k\kappa^k\right) < -1, \qquad K^2 < 0 \tag{4.17}$$

the particle mass becomes imaginary. In this case the mean world line of the particle is spacelike, and the pair production becomes to be possible. In other words, the κ-field can produce pairs [7].

The property of pair production is a crucial property of the quantum relativistic physics. The classical fields (electromagnetic, gravitational) do not possess this property. As we have seen in the second section the description in framework of the conventional QFT has problems with the pair production description. There is a hope, that the proper statistical description of several relativistic stochastic particle will admit one to obtain the pair production effect. For instance, maybe, two colliding relativistic particles can produce pairs by means of their common κ-field. We hope that such a program may appear to be successful, provided the proper formalism of the statistical ensemble will be developed. Today we have only the developed formalism for statistical description of stochastic system consisting of one emlon (WL). Formalism for statistical description of stochastic system consisting of several emlons (WLs) is not yet developed properly.

5 Epistemological problems of quantum theory

Construction of the relativistic quantum theory is a very difficult problem. But solution of this problem depends not only on the difficulty of the problem in itself. It depends also on qualification of researchers, investigating this problem, on effectiveness of the applied investigation methods, on capability of researchers to logical reasonings and on other factors. In this section we shall try analyze the character of appearing difficulties. We shall separate them into two parts: objective difficulties and subjective difficulties. The objective difficulties have been discussed. Further we shall try to discuss subjective difficulties and mistakes. Discovery and correction of these mistakes is difficult because of their subjective character.

In our opinion, the main difficulty is a deficit of logic (predominance of the trial and error method over logic) at the construction of the quantum relativistic theory. In particular, this deficit of logic is displayed in disregard of demands, imposed by the relativity principles. Let us consider briefly the history of the question. In the beginning of the 20th century there were attempts of constructon of the nonrelativistic quantum mechanics as a statistical description of stochastic particles. In these attempts the statistical description was considered to be a probabilistic description. Incompatibility of the probabilistic description with the relativity principles was not known, because one ignored the circumstance that the world line (but not the particle) was the physical object. Because of this mistake one could not construct the statistical conception of the quantum mechanics. One succeeded to construct the axiomatic conception of quantum mechanics by means of the trial and error method. After this success the trial and error method became the principal investigation method in the quantum theory. The trial and error method had the success and became to be predominant, because it was insensitive to mistakes in the fundamental physical principles, whereas the classical investigation method, which run back to Isaac Newton, was founded on the logic. The method, founded on logic could not lead to correct results, if the fundamental physical principles contained mistakes, or these principles were applied incorrectly.

In the first half of the 20th century there were classics of physics, who knew and used the classical logical method of investigation. In the last half of the 20th century, there were only researchers, using the trial and error method. The predominancy of the trial and error method is explained by two factors. Firstly, it appeared to be successful in application to construction of the nonrelativistic quantum theory.

Secondly, the classical logical method appeared to be forgotten, because new generations of the researcher were educated on the example of the successful application of the trial and error method, which was perceived as the only possible method of investigation. Any ambitious theorist dreamed to invent such hypothesis (maybe, very exotic), which could be explain at once the mass spectrum of elementary particles and solve other problems of QFT. Development of the microcosm physics turned into competition of such hypotheses.

The circumstance, that the correct application of the relativity principles (the correct application of the fact that the world line is a physical object) may be important from the practical viewpoint became to be clear for the author of this paper after investigation of the world line properties [6]. Two important results followed from this paper: (1) the quantum mechanics as a statistical theory can be constructed, if one uses the relativistic concept of the state and construct the statistical description without a use the probability theory [22, 23, 24], (2) the perturbation theory at the second quantization may be eliminated, if the conservation law of physical objects (world lines) is taken into account) [25]. The paper [6] was reported at the seminar of the theoretical department of the Lebedev physical institute. Relation to the paper was sceptical as far as results were obtained without any additional suppositions (and this was unusual). Besides, many researchers did not believe, that it was possible a classical description of the world line, making a zigzag in the time direction. Further the calculation were tested, and all objections were eliminated.

It was clear to the author of paper [6], that the first and the second results were incompatible. He believed that the quantum mechanics is the statistical description of random world lines and the quantum principles are to be corollaries of this description. However, in that time the integration of hydrodynamic equations was not known, and from the mathematical viewpoint the statistical description could not be considered as a starting point of the quantum description. The second result admitted one to separate the problem of the second quantization in to parts and to solve exactly the one-emlon problem and the two-emlon problem without a use of the perturbation theory. The author expected that the further development of the second quantization led the problem into the blind alley, provided the fitting is not used. From his viewpoint it should prove that development of QFT in the direction of the second quantization were erroneous.

No additional hypotheses were used, to avoid a charge in a use of erroneous hypotheses, leading to a strange result (absence of the pair production). In particular, one uses neither the perturbation theory, nor cut off the self-action at the time tending to infinity. (Usually the two hypotheses are always used). Under these conditions the absence of the pair productions meant, that *the strategy of the second quantization in itself is erroneous*, as far as the relativistic quantum field theory, where there is no pair production, cannot be true. When the corresponding paper was submitted to a scientific journal, it was rejected on the basis of the review of the referee, who wrote: " I do not recommend the paper for publication, because the author himself stated that in his method of quantization the pair production is absent." (The paper has not been published, and now it can be found only at the site [26])

We see here a sample of logic, based on the trial and error method, which does not accept the papers with a negative result. The referee does not admit existence of other investigation methods other, than the trial and error method. Indeed, as far as in the trial and error method all hypotheses are random, the tests leading to a negative result are of no interest. In the logical investigation method the negative result means that the starting premises are false (of course, if there are no mistakes in the executed investigation). Unfortunately, the approach of the referee is typical. Most researchers are apt to use only the trial and error method and they cannot imagine anything other than this method. During thirty years the author of this paper had a chance to discuss the correctness of the second quantization problem with his colleagues dealing with QFT. Some of them agreed that, maybe, the commutation relation are incompatible with the dynamic equations. But at the same time they stated that it means nothing, because QFT agrees with the experimental data very well. The circumstance, that the experimental data are explained by means of the inconsistent theory did not lead to objections from them. Such an approach is a corollary of the predominant method of trial and error. We think that this method is the main obstacle on the path of the relativistic quantum theory construction. *One can find and correct mistakes in the theory, but a change of mentality needs some time. This time may be rather long.*

We have seen that the nonrelativistic quantum theory could be presented as a statistical description of stochastic particles, if we apply the relativity principles correctly and use the *dynamic conception* of the statistical description. In fact, the nonrelativistic quantum theory was developed mainly by the trial and error method. Appearance of quantum principles was a result of application of this method. The trial and error method is an effective method for investigation of single physical phenomena of unknown nature, because it admits one to discover new concepts, which are adequate to the considered phenomenon. However, the trial and error method is inadequate for construction of a fundamental theory, because the fundamental theory is a logical structure, which systematizes our knowledge. The systematization needs a long logical reasonings, because it is based on several fundamental propositions. The systematization as well as the fundamental theory is very sensitive to possible mistakes in the logical reasonings and in the mathematical calculations. Any mistake should be analyzed and eliminated.

On the contrary, the trial and error method is insensitive to mistakes. It is multiple-path. Usually before obtaining a correct solution one suggests and tests many different hypotheses. Only one of them may appear to be true. As far as the hypotheses are suggested occasionally, it is useless to analyze the erroneous hypotheses. Such an analysis gives nothing, because the hypotheses do not connect between themselves and with the obtained correct result.

If we use logical reasonings, based on the fundamental principles, and obtain an incorrect result, it means that either we make a mistake, or the fundamental principles are incorrect. Thus, at the logical approach we should discover and analyze our mistakes. It is useful for a correction of our reasonings. At the engineering approach to the construction of the fundamental theory, when one uses the trial and error method, a discovery and an analysis of the possible mistake is useless. Furthermore, it is possible such a case, when the obtained result is incorrect, although it agrees with the experimental data. Let us illustrate this in the example, which relates to the problem of the relativistic quantum theory construction.

We discuss the problem, whether the Dirac particle $\mathcal{S}_{\rm D}$ is pointlike or it has an internal structure. The Dirac particle $\mathcal{S}_{\rm D}$ is the dynamic system, described by the action of the form

$$\mathcal{S}_{\rm D}: \qquad \mathcal{A}_{\rm D}[\bar{\psi},\psi]=c^2\int(-mc\bar{\psi}\psi+\frac{i}{2}\hbar\bar{\psi}\gamma^l\partial_l\psi-\frac{i}{2}\hbar\partial_l\bar{\psi}\gamma^l\psi-\frac{e}{c}A_l\bar{\psi}\gamma^l\psi)d^4x \quad (5.1)$$

where m and e are respectively the mass and the charge of the Dirac particle, and c is the speed of the light. Here ψ is four-component complex wave function, ψ^* is the Hermitian conjugate wave function, and $\bar{\psi}=\psi^*\gamma^0$ is the conjugate one. The quantities γ^i, $i=0,1,2,3$ are 4×4 complex constant matrices, satisfying the relation

$$\gamma^l\gamma^k+\gamma^k\gamma^l=2g^{kl}I, \qquad k,l=0,1,2,3. \quad (5.2)$$

where I is the 4×4 identity matrix, and g^{kl} =diag$(c^{-2},-1,-1,-1)$ is the metric tensor. The quantity A_k, $k=0,1,2,3$ is the electromagnetic potential. The action (5.1) generates the dynamic equation for the dynamic system $\mathcal{S}_{\rm D}$, known as the Dirac equation

$$\gamma^l\left(-i\hbar\partial_l+\frac{e}{c}A_l\right)\psi+mc\psi=0 \quad (5.3)$$

and expressions for physical quantities: the 4-flux j^k of particles and the energy-momentum tensor T_l^k

$$j^k=c^2\bar{\psi}\gamma^k\psi, \qquad T_l^k=\frac{ic^2}{2}\left(\bar{\psi}\gamma^k\partial_l\psi-\partial_l\bar{\psi}\cdot\gamma^k\psi\right) \quad (5.4)$$

If the Dirac particle $\mathcal{S}_{\rm D}$ is not pointlike and has an internal structure, described by some additional degrees of freedom, this structure is to be present also in the nonrelativistic approximation. Conventionally one assumes that the Pauli particle $\mathcal{S}_{\rm P}$ is the nonrelativistic approximation of the Dirac particle $\mathcal{S}_{\rm D}$ (see, for instance, [19]).

In the partial case, when $A_0=0$, the Pauli particle $\mathcal{S}_{\rm P}$ is the dynamic system, described by the dynamic equation

$$i\hbar\partial_0\psi_1=\left(\frac{\pi_\mu\pi_\mu}{2m}+\frac{ie\hbar}{2mc}\varepsilon_{\nu\mu\alpha}\partial_\nu A_\mu\sigma_\alpha\right)\psi_1, \qquad \pi_\alpha\equiv i\hbar\partial_\alpha-\frac{e}{c}A_\alpha, \qquad \alpha=1,2,3 \quad (5.5)$$

where ψ_1 is the two-component complex wave function, $\varepsilon_{\mu\nu\alpha}$ is the Levi-Chivita pseudotensor, and $\boldsymbol{\sigma}=\{\sigma_1,\sigma_2,\sigma_3\}$ are the 2×2 Pauli matrices.

The Pauli particle $\mathcal{S}_{\rm P}$ is the pointlke particle, which has no internal structure. This fact agrees with the experimental data. Hence, if the Pauli particle $\mathcal{S}_{\rm P}$ is the nonrelativistic approximation of the Dirac particle $\mathcal{S}_{\rm D}$, the Dirac particle is pointlike also and has no internal structure. The Pauli equation (5.5) has a lower order (four first order real equations), than the Dirac equation (5.3) (eight first order real equations). It means that at transition from the Dirac equation to the Pauli equation the order of the system of dynamic equation reduces, and several degrees of freedom were lost.

The equation (5.5) is obtained from equation (5.3) as the limit $c \to \infty$. Some temporal derivatives ∂_0 in the Dirac equation (5.3) have small coefficients of the order c^{-1} and c^{-2}. These terms are neglected and the order of the system of dynamic equations reduces. However, the neglected terms are the terms with the small parameters before the highest derivative. One cannot neglect these terms, because at very high frequencies (of the order $\Omega = mc^2/\hbar$) these terms become to be of the same order as the remaining terms. Neglecting these terms, we neglect the high frequency degrees of freedom.

In reality, the states of the Dirac particle, which are linear superposition of conventional low frequency state with the high frequency state are unstable, because in this case the 4-current $j^k = c^2\bar{\psi}\gamma^k\psi$ oscillates with the frequency of the order $\Omega = mc^2/\hbar$. The Dirac particle is charged. As a result the energy of the high frequency excitation is radiated in the form of electromagnetic waves, and the Dirac particle appears at the state, where the 4-current $j^k = c^2\bar{\psi}\gamma^k\psi$ is constant. Thus, from the experimental viewpoint the additional high frequency degrees of freedom of the Dirac particle do not exist, because they are not observable.

Can one discover these degrees of freedom theoretically from the analysis of the Dirac particle? Yes, one can discover them at the scrupulous analysis in the framework of the conventional quantum mechanics [20]. But they have not been discovered. We are not sure, whether the theory of differential equations with small parameter before the highest derivative was known in the first half of the 20th century, but it was definitely known in the last half of the 20th century. Nevertheless the necessary analyses has not been produced, and the Dirac particle was considered to be pointlike.

It is true that the high frequency degrees of freedom are not observable at low energies, and they give no contribution to description of the Dirac particle in the nonrelativistic case. One can assume that these degrees of freedom absent in the nonrelativistic case, and this assumption agrees with the experimental data. However, at the high energy collisions of Dirac particles these degrees of freedom may be excited and make an essential contribution to description of the high energy collision process.

Why has one not discovered these degrees of freedom theoretically? Because researchers used the trial and error method, where the only criterion of validity of the theory is the agreement with experiment. The logical reasonings and mistakes in consideration are of no importance, provided they do not violate agreement with experiment. If we take into account that the nonrelativistic quantum theory was created by means of the trial and error method, and three generations of the microcosm researchers have been educated on the example of this method, we recognize that the internal degrees of freedom of the Dirac particle could not be discovered before the execution of the proper high energy experiments with Dirac particles.

The internal degrees of freedom of the Dirac particle were discovered at investigation of the Dirac particle by dynamic methods [16], which use the logical approach to investigation. The dynamic methods are attentive to the logic and to mistakes of investigation. They are not oriented to the trial and error method and to agreement with experiment.

Besides, investigation of the Dirac equation by the dynamic methods has shown [17] that the description of internal degrees of freedom is nonrelativistic. It means that the whole Dirac equation (5.1) is a nonrelativistic equation, although it is written in the relativistically covariant form. Nonrelativistic character of the Dirac equation means mathematically, that the set of all solutions of the Dirac equation is not invariant, in general, under some Poincaré transformations.

From viewpoint of researchers, who believed only in experimental test (but not in logic reasonings) the Dirac equation is relativistic, because only internal degrees of freedom are described nonrelativistically, but these degrees of freedom are ignored at the conventional approach. Publishing of the papers [16, 17] in the reviewed journal appears to be impossible, because the referee declared that he cannot believe that the Dirac equation be nonrelativistic. This declaration appears to be sufficient for rejection of the paper. I think that the opinion of the referee reflects the viewpoint of the statistical average researcher, and this opinion is erroneous.

Problem of the relativistic invariance of the Dirac equation is discussed in [17] in details. Here we shall not go in details. We refer only to the theorem, formulated by Anderson [21]. This theorem states: *The symmetry group of dynamic equations, written in the relativistically covariant form, coincides with the symmetry group of absolute objects.* The absolute objects are quantities, which are the same for all solutions of the dynamic equations. In the Dirac equation (5.3) the quantities A_k and matrices γ^k are absolute objects. We set for simplicity $A_k = 0$. Then the symmetry group of 4-vector $A_k = 0$ and of the unit matrix 4-vector γ^k is the group of translation and the group of rotation around the direction defined by the unit 4-vector γ^k. This 7-parametric group is a subgroup of the 10-parametric Poincarè group. Hence, the Dirac equation is not relativistic.

The Dirac equation distinguishes from the Klein-Gordon equation in the sense, that the Klein-Gordon equation contains the absolute object g^{kl} =diag$\{c^{-2}, -1, -1, -1\}$, which has the 10-parametric Poincarè group as its symmetry group.

As an illustration of the role of unit constant vector in the relativistically covariant equation, we note the dynamic equation for the free nonrelativistic classical particle

$$m\frac{d^2\mathbf{x}}{dt^2} = 0 \tag{5.6}$$

can be written in the relativistically covariant form, if one introduces the unit timelike constant 4-vector l_k $\quad (l_k g^{kl} l_l = 1)$. We obtain instead of (5.6)

$$\frac{mc^2}{(l_n \dot{x}^n)}\frac{d}{d\tau}\left[\frac{\dot{x}^i}{(l_k \dot{x}^k)} - \frac{1}{2}g^{ik}l_k\frac{\dot{x}^s g_{sl}\dot{x}^l}{(l_j \dot{x}^j)^2}\right] = 0, \qquad \dot{x}^i \equiv \frac{dx^i}{d\tau}, \qquad i = 0, 1, 2, 3 \tag{5.7}$$

Indeed, setting $l_k = \{c, 0, 0, 0\}$ in (5.7), we obtain for $i = 1, 2, 3$ the equation (5.6), because $\left(l_k \dot{x}^k\right) d\tau = cdt = cdx^0$. For $i = 0$ we obtain the identity

$$mc^2 \frac{d}{cdt}\left[\frac{dt}{cdt} - \frac{1}{2c}\right] \equiv 0$$

The Newtonian space of events contains two invariants $dt = dx^0$ and $dr = \sqrt{d\mathbf{x}^2}$, whereas the Minkowski space-time contains only one invariant $ds = \sqrt{c^2dt^2 - dr^2}$. Introduction of the constant unit 4-vector l_k admits one to construct two invariants dt and dr from one invariant ds and 4-vector dx^k by means of relations

$$cdt = l_k dx^k, \qquad dr = \sqrt{c^2dt^2 - ds^2} = \sqrt{\left(l_k dx^k\right)^2 - g_{ik} dx^i dx^k}$$

Thus, if the dynamic equations written in the relativistically covariant form contain the unit timelike constant vector, we should suspect that dynamic equations are not relativistic.

6 Necessity of the next modification of the space-time model

Thus, the quantum mechanics can be founded as a mechanics of stochastic particles. However, it is not known, why the motion of free particles is stochastic and from where the quantum constant does appear. There are two variants of answer to these questions.

1. The stochasticity of the free particle motion is explained by the space-time properties, and the quantum constant is a parameter, describing the space-time properties.

2. The stochasticity of the free particle motion is explained by the special quantum nature of particles. The motion of such a particle distinguishes from the motion of usual classical particle. There is a series of rules (quantum principles), determining the quantum particle motion. The universal character of the quantum constant is explained by the universality of the quantum nature of all particles and other physical objects. As to event space, it remains to be the same as at Isaac Newton.

It is quite clear that the first version of explanation is simpler and more logical, as far as it supposes *only a change of the space-time geometry.* The rest, including the principles of classical physics, remains to be unchanged. The main problem of the first version was an absence of the space-time geometry with such properties. In general, one could not imagine that such a space-time geometry can exist. As a result in the beginning of the 20th century one choose the second version. After a large work the necessary set of additional hypotheses (quantum principles) had been invented. One succeeded to explain all nonrelativistic quantum phenomena. However, an attempt of the quantum theory expansion to the relativistic phenomena lead to the problem, which is formulated as *join of nonrelativistic quantum principles with the principles of the relativity theory.*

In general, the question, why the motion of microparticles is stochastic, does not relate directly to the problem of the relativistic quantum theory construction. It relates only in the sense, that explanation of the stochasticity by the space-time properties creates an entire picture of the world, where the good old classical principles rule, and only the space-time properties are slightly changed. It is clear, that explanation of quantum properties by a slight correction of the space-time properties is more attractive, than the substitution of principles of classical physics by enigmatic quantum principles, which are incompatible with the relativity principles.

Besides, the correction of the space-time properties is very simple. It does not demand an introduction of additional exotic space-time properties such as a space-time stochasticity, or noncommutativity of coordinates in the space-time.

Correction of the space-time properties means a change of the world function σ [27] of the space-time. It consists of three points [28, 29, 30].

1. One proves that the proper Euclidean geometry has the σ-immanence property. It means that the proper Euclidean geometry is described entirely by its world function σ_{E}, and all Euclidean prescriptions for construction of geometrical objects and relations between them can be expressed in terms and only in terms of the Euclidean world function σ_{E}.

2. It is supposed that any space-time geometry $\mathcal{G}$ has the σ-immanence property. It means that all prescriptions of the geometry $\mathcal{G}$ for construction of geometrical objects and relations between them can be obtained from the Euclidean prescription by a proper deformation of the Euclidean geometry, i.e. by the change of the Euclidean world function σ_{E} by the world function σ of the space-time geometry $\mathcal{G}$ in all Euclidean prescriptions.

3. The world function σ_{d} of the space-time geometry $\mathcal{G}_{\mathrm{d}}$ is chosen in the form

$$\sigma_{\mathrm{d}} = \sigma_{\mathrm{M}} + D\left(\sigma_{\mathrm{M}}\right), \qquad D\left(\sigma_{\mathrm{M}}\right) = \begin{cases} \frac{\hbar}{2bc}, & \text{if} \quad \sigma_{\mathrm{M}} > \frac{\hbar}{2bc} \\ 0, & \text{if} \quad \sigma_{\mathrm{M}} < 0 \end{cases} \tag{6.1}$$

   where σ_{M} is the world function of the Minkowski space, c is the speed of the light and $b \leq 10^{-17}\mathrm{g/cm}$ is the constant, describing connection between the geometric mass μ and usual mass m by means of the relation $m = b\mu$.

In the space-time with nonvanishing distortion $D\left(\sigma_{\mathrm{M}}\right)$ the particle mass is geometrized [31], and motion of free particles is stochastic. The distortion function $D\left(\sigma_{\mathrm{M}}\right)$ describes the character of quantum stochasticity. Form of the distortion function $D\left(\sigma_{\mathrm{M}}\right)$ is determined by the demand that the stochasticity generated by distortion is the quantum stochasticity, i.e. the statistical description of the free stochastic particle motion is equivalent to the quantum description in terms of the Schrödinger equation [31].

7 Concluding remarks

We have considered two possible strategies of the relativistic quantum theory construction. The first strategy, founded on the application of the conventional quantum technique to relativistic systems, leads either to inconsistent conception or to the consistent theory, where the pair production does not appear for interactions of the degree type.

The second strategy is founded on the construction of the fundamental theory, which relates to the conventional nonrelativistic quantum theory approximately in such a way as the statistical physics relates to the axiomatic thermodynamics. The fundamental theory is the conventional relativistic classical theory in the space-time, whose geometry is slightly modified in such a way, that motion of free particles is primordially stochastic and the particle mass is geometrized. The quantum constant appears as a parameter of the space-time geometry. Statistical description of stochastic nonrelativistic particle motion appears to be equivalent to the conventional quantum description. There is no necessity to postulate the quantum principles, because they may be obtained as a corollary of such a statistical description of nonrelativistic stochastic particles.

There is a hope that the direct application of the statistical description to relativistic stochastic systems admits one to construct the relativistic quantum theory. The fundamental theory admits one to use only the logical investigation method of Isaac Newton. The fundamental theory is free of application of the trial and error method, which is the main obstacle on the path of the relativistic quantum theory construction. Predominance of the trial and error method in the 20th century generated a specific mentality of contemporary researchers, when the researcher tries to suggest new hypotheses and to guess the result but not to derive it by the logical way from the fundamental physical principles. This mentality is a very serious obstacle on the path of the relativistic quantum theory construction.

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