Statistical Conception of Quantum Field Theory

Yuri A.Rylov

Institute for Problems in Mechanics, Russian Academy of Science,
101 Vernadskii Ave., Moscow, 117526, Russia.

Published in *J. Math. Phys.* **35** (1994) 3922

Abstract

Two equivalent modes of statistical description of stochastic particle production are considered, exclusive and inclusive. Information about particle production processes is destributed inside exclusive and inclusive descriptions in different ways. The exclusive description realizes an "all or nothing" investigation strategy, whereas the inclusive description realizes the "step by step" investigation strategy. Conventional quantum field theory is shown to use essentially an exclusive description. A possibility of a transition to more reasonable investigation strategy (inclusive description) of particle production description is discussed.

PACS numbers: $02.50. + s; 03.70. + k$

1 Introduction.

Quantum field theory (QFT) is a kind of statistical theory, and problems of QFT are not only dynamical ones, some are statistical. The mode of statistical description (statistical conception) used in the conventional QFT is not the only one possible. There is at any rate one more statistical mode of description. Statistical description is a pre-dynamical description. First, one forms concepts and variables of a physical theory (its language), and afterwards one establishes dynamic relations between them. A change of the statistical conception of QFT can remove old dynamic problems (such as non-stationarity of the vacuum, ultraviolet and infrared divergences) and set new ones.

Speaking about statistical conception of QFT, one means in the first place the statistical mode of particle production description, because particle production is a principal problem of QFT. In that field of QFT, where this problem can be solved, or can be considered as a correction (for instance, in quantum electrodynamics), QFT succeeds.

The mode of statistical description (exclusive description) used in the conventional QFT (see, for instance, Schweber [1]) is effective, when the corresponding collision problem is solved completely. "All or nothing" is an investigation strategy of the statistical conception of conventional QFT. Non-effectivity of this strategy cannot be compensated on the following dynamical level. Only a different investigation strategy (a different statistical conception) can be effective.

In the second section two different modes of statistical descriptions are considered. Effectivity of both descriptions is discussed. The third section is devoted to the discussion of which statistical description is used in QFT. In the fourth section a possibility of a more effective investigation strategy is discussed.

2 Modes of statistical description of stochastic particle production.

Let us consider two different modes of the statistical description of identical pointlike particles that arise randomly inside some volume V. $\mathbf{x}$ denotes coordinates of a particle in some coordinate system. Let us use designations

$$X^n = \{\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_n\}, \qquad d({}^nX^l) = d\mathbf{x}_{n+1} \wedge d\mathbf{x}_{n+2} \wedge \ldots \wedge d\mathbf{x}_l,$$
$$dX^l = d({}^0X^l), \qquad {}^lV^n = \bigotimes_{k=l+1}^{k=n} V, \qquad V^n = {}^0V^n \tag{2.1}$$

where $\otimes$ means a direct product.

Let $p_n(X^n)$, $n = 0, 1, \ldots$ be the probability density of the event that inside V there are n and only n particles, one of them at $\mathbf{x}_1$, another at $\mathbf{x}_2, \ldots$ the nth at $\mathbf{x}_n$. The $p_n(X^n)$ will be referred to as the probability density of the state X^n. Any state X^n describes some possible spatial distribution of generated particles, and the quantity $p_n(X^n)$ describes probability density of this distribution. The $p_n(X^n)$ are considered as independent in the sense that the normalization condition takes place:

$$\sum_{n=0}^{\infty} \frac{1}{n!} \int_{V^n} p_n(X^n) dX^n = 1 \tag{2.2}$$

Another mode of description can be realized by means of quantities $w_n(X^n)$, $(n = 0, 1, \ldots)$ determined by relations

$$w_n(X^n) = \sum_{l=0}^{\infty} \frac{1}{l!} \int_{{}^nV^{n+l}} p_{n+l}(X^{n+l}) d({}^nX^{n+l}), \qquad n = 0, 1, \ldots \tag{2.3}$$

The $w_n(X^n)$ is the probability density of the event that there are at least n particles, one of them at $\mathbf{x}_1$, another at $\mathbf{x}_2, \ldots$ the nth at $\mathbf{x}_n$. Beyond this there can be more particles. The $w_n(X^n)$ will be referred to as the probability density of at least the state X^n. In particular, the probability w_0 of at least the state X^0, containing 0 particles is

$$w_0 = 1 \tag{2.4}$$

This relation follows from Eqs.(2.2), (2.3). Descriptions in terms of p_n and w_n are equivalent, because p_n $(n = 0, 1, \ldots)$ are expressed through w_n $(n = 0, 1, \ldots)$ by means of similar relations

$$p_n(X^n) = \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \int_{{}^nV^{n+l}} w_{n+l}(X^{n+l}) d({}^nX^{n+l}), \qquad n = 0, 1, \ldots \tag{2.5}$$

It is easy to verify that substituting Eq.(2.5) into Eq.(2.3), one obtains identity.

Thus, descriptions in terms of p_n and w_n are different statistical descriptions of the stochastic particle production. They will be referred to correspondingly as exclusive and inclusive descriptions. The term inclusive description (process) was suggested by Feynman [2] for such many-particle processes, where only one (or a few) particle of some kind is described statistically in detail, but all other particles remain outside the scope. Such a description is used in the cosmic ray theory and in the high energy collision theory, where a detailed description of all produced particles is impossible [3-6]. Many empiric laws of high energy physics were derived by means of investigation of inclusive processes, for instance, the quark-parton structure of a nucleon [7-9], scaling in electron-nucleus scattering [10,11].

Unlike the exclusive description the quantities $w_n(X^n)$ are not independent. For instance, the event that there is at least 0 particles (X^0) includes also the cases, when there are at least one particle (X^1), at least two particles (X^2) and so on.

There is another distinction between the two descriptions. The inclusive description is local, but the exclusive one is not. One keeps in mind the following. Let the volume $V_1 \subset$V. Then inside V_1 the mean value $\langle F \rangle_{V_1}$ of any quantity F can be expressed through $w_n(X^n)$, $X^n \in V_1^n$ $(n = 0, 1, \ldots)$. For instance, the mean number $\langle n(\mathbf{x}) \rangle$ of particles at the point $\mathbf{x}$ is determined by the relation

$$\langle n(\mathbf{x}_1) \rangle = \sum_{l=0}^{\infty} \frac{1}{l!} \int_{{}^1V^{1+l}} p_{1+l}(X^{1+l}) d({}^1X^{1+l}) = w_1(\mathbf{x}_1), \qquad \mathbf{x}_1 \in V_1 \tag{2.6}$$

But the $\langle F \rangle_{V_1}$ cannot be expressed through p_n $(n = 0, 1, \ldots)$. It can be expressed only through $p_n(X^n)$, $X^n \in V^n$ $(n = 0, 1, \ldots)$, as it can be seen from Eq.(2.6). The fact is that the volume V is referred to in the definiton of $p_n(X^n)$, i.e. $p_n(X^n)$ is defined inside V. This circumstance manifests itself formally in existence of the normalization condition (2.2), which refers to V. For w_0 the relation (2.2) turns into Eq.(2.4) which is an evident statement.

The dependence of p_n upon V can be expressed in a more explicit form. Let us express w_l and p_l through the mean value of particle concentration $n(\mathbf{x})$. One can show that (see Appendix)

$$w_l(X^l) = \langle \hat{w}_l(X^l) \rangle, \qquad \hat{w}_l(X^l) = \mathcal{N}_s \left\{ \prod_{k=1}^{k=l} n(\mathbf{x}_k) \right\}, \qquad l = 0, 1, \ldots \tag{2.7}$$

$$p_l(X^l) = \langle \hat{p}_l(X^l) \rangle, \qquad \hat{p}_l(X^l) = \mathcal{N}_s \left\{ \exp\left[-\int_V n(\mathbf{x}) d\mathbf{x} \right] \prod_{k=1}^{k=l} n(\mathbf{x}_k) \right\}, \qquad l = 0, 1, \ldots \tag{2.8}$$

where $\hat{w}_l(X^l)$ and $\hat{p}_l(X^l)$ are functions (functionals) of particle concentration $n(\mathbf{x})$ and parameters X^l. Here the angular brackets denote averaging, and $\mathcal{N}_s$ denotes a linear operator on the space of analytical functionals of $n(\mathbf{x})$. The action of the operator $\mathcal{N}_s$ on the basic vector $n(\mathbf{x}_1)n(\mathbf{x}_2)\dots n(\mathbf{x}_l)$ is defined as follows

$$\mathcal{N}_s\left\{\prod_{k=1}^{k=l} n(\mathbf{x}_k)\right\} = \prod_{k=1}^{k=l}\left\{n(\mathbf{x}_k) - \sum_{s=1}^{s=k}\delta(\mathbf{x}_k - \mathbf{x}_s)\right\} \tag{2.9}$$

If there are M kinds of particles, then the formulae (2.7),(2.8) are generalized in the evident way

$$w_{l_1 l_2 \dots l_M}(X_1^{l_1}, X_2^{l_2}, \dots X_M^{l_M}) = \left\langle \mathcal{N}_s\left\{\prod_{A=1}^{A=M}\prod_{k=1}^{k=l_A} n_A(\mathbf{x}_{Ak})\right\}\right\rangle$$

$$p_{l_1 l_2 \dots l_M}(X_1^{l_1}, X_2^{l_2}, \dots X_M^{l_M}) = \left\langle \mathcal{N}_s\left\{\prod_{A=1}^{A=M}\left[\exp\left(-\int_V n_A(\mathbf{x})d\mathbf{x}\right)\prod_{k=1}^{k=l_A} n_A(\mathbf{x}_{Ak})\right]\right\}\right\rangle \tag{2.10}$$

$$l_1, l_2, \dots l_M = 0, 1, \dots$$

Here $X_A^{l_A} = \{\mathbf{x}_{A1}, \mathbf{x}_{A2}, \dots \mathbf{x}_{Al}\}$ are coordinates of the particles of the sort A.

The locality of the inclusive description manifests itself also as follows. Let there be an inclusive description inside two adjacent volumes V_1 and V_2. Then the inclusive description inside the volume $V = V_1 \cup V_2$ is obtained, uniting the inclusive descriptions inside V_1 and inside V_2. Now let there be an exclusive description inside V_1 and inside V_2. To obtain the exclusive description inside $V = V_1 \cup V_2$, it is necessary to use inclusive descriptions inside V_1 and inside V_2, to combine them into an inclusive description inside $V = V_1 \cup V_2$, and then derive the exclusive description inside V by means of Eq.(2.5). In other words, the inclusive description is additive, but the exclusive one is not.

Let the inclusive description inside V_1 and inside V_2 be incomplete [i.e., not all $w_l(X^l)$, $X^l \in V_1^l$, $X^l \in V_2^l$, $(l = 0, 1, \dots)$ are known]. For instance, only $w_0 = 1$, $w_1(\mathbf{x}_1)$, $\mathbf{x}_1 \in V_1$, $w_1(\mathbf{x}_1)$, are known. Then the incomplete inclusive description inside $V = V_1 \cup V_2$ can be obtained in the form that $w_0 = 1$, $w_1(\mathbf{x}_1)$, $\mathbf{x}_1 \in V$ are known.

Let an exclusive description inside V_1 and inside V_2 be incomplete (i.e., not all $p_l(X^l)$, $X^l \in V_1^l$, $X^l \in V_2^l$, $(l = 0, 1, \dots)$ are known). Then one cannot obtain even an incomplete description inside $V = V_1 \cup V_2$ (either an inclusive description, or an exclusive one), because, using an incomplete exclusive description inside V_1, one cannot obtain even an incomplete inclusive description inside V_1.

Let us imagine that there are some dynamical equations describing the time evolution of $w_l(X^l)$. If this description is complete, then the equations can be transformed into equations for $p_l(X^l)$. In this case the inclusive description and the exclusive one are equivalent. But the situation seems to be more realistic, when only some first functions [for instance, $w_1(\mathbf{x}_1), w_2(\mathbf{x}_1, \mathbf{x}_2)$ in the case of the inclusive description, or $p_0, p_1(\mathbf{x}_1), p_2(\mathbf{x}_1, \mathbf{x}_2)$ in the case of the exclusive one] can be determined. Although the amount of information is similar in both cases, its value is quite different inside

V. Let the mean number $\langle N_{pr}\rangle$ of particles produced at the collision of two particles is small. Mathematically it means that

$$\langle N_{pr}\rangle = \sum_{n=3}^{\infty} \frac{1}{(n-1)!} \int_{V^n} p_n(X^n) dX^n \ll 1, \qquad \sum_{n=3}^{\infty} \frac{1}{n!} \int_{V^n} p_n(X^n) dX^n \ll 1$$

Then both the inclusive description and the exclusive one are almost equivalent. But, if $\langle N_{pr}\rangle \gg 1$,

$$\langle N_{pr}\rangle = \sum_{n=3}^{\infty} \frac{1}{(n-1)!} \int_{V^n} p_n(X^n) dX^n \gg 1, \qquad 1 - \sum_{n=3}^{\infty} \frac{1}{n!} \int_{V^n} p_n(X^n) dX^n \ll 1$$

then p_0, p_1, p_2 give little information about the whole process, whereas w_1, w_2 describe the mean concentration of particles and the correlation between concentrations of particles at two different points. Thus, for the large number N of particles, when the contribution of $p_n(X^n)$ with $n \gg 1$ is essential, the inclusive description is preferable. For $N \gg 1$ the exclusive description can be used effectively in those cases, where recurrent relations between densities p_n are known. For instance, it is effective for independent particles $p_n(X^n) = \prod_{l=1}^{l=n} p_1(\mathbf{x}_l)$. In this case the whole exclusive description is determined by $p_1(\mathbf{x})$.

Thus, if the total mean number of particles $N \gg 1$, the exclusive description realizes an "all or nothing" investigation strategy, whereas the inclusive description realizes a more reasonable strategy. It permits a description which is incomplete to different degrees. In general, adding next functions w_l, one can increase the completeness of the inclusive description.

It is necessary to stress that the constraints, arising in exclusive description, have nothing to do with dynamics. These constraints are those of description (but not of dynamics). The consideration of dynamic equations can only add new (dynamic) constraints, but cannot remove the description constraints.

In the gas kinetic theory and in the plasma theory, where there is no particle production, one uses conventionally either the inclusive description, or the exclusive description with some recurrent relations

$$p_n(X^n) = \prod_{l=1}^{l=n} p_1(\mathbf{x}_l), \qquad n = 0, 1, \ldots \tag{2.11}$$

In QFT, where the particle production is essential, the exclusive description is realized conventionally without using any recurrent relations, In other words, one realizes the "all or nothing" investigation strategy. Let us show this.

3 Statistical description in the quantum field theory

In QFT the particle concentration at the point $\mathbf{x}$ is described by the operator $n(\mathbf{x})$. It is presented in the form

$$n(\mathbf{x}) = a^{+}(\mathbf{x})a(\mathbf{x}) \tag{3.1}$$

where a($\mathbf{x}$) and $a^{+}(\mathbf{x})$ are Hermitian conjugate operators known as the annihilation and creation operators respectively. They *sa*tisfy the commutation relation

$$\begin{aligned}[a(\mathbf{x}), a^{+}(\mathbf{x}')]_{-} \equiv a(\mathbf{x})a^{+}(\mathbf{x}') - a^{+}(\mathbf{x}')a(\mathbf{x}) = \delta(\mathbf{x}-\mathbf{x}') \\ [a(\mathbf{x}), a(\mathbf{x}')]_{-} = [a^{+}(\mathbf{x}), a^{+}(\mathbf{x}')]_{-} = 0.\end{aligned} \tag{3.2}$$

for bosons, and

$$\begin{aligned}[a(\mathbf{x}), a^{+}(\mathbf{x}')]_{+} \equiv a(\mathbf{x})a^{+}(\mathbf{x}') + a^{+}(\mathbf{x}')a(\mathbf{x}) = \delta(\mathbf{x}-\mathbf{x}') \\ [a(\mathbf{x}), a(\mathbf{x}')]_{+} = [a^{+}(\mathbf{x}), a^{+}(\mathbf{x}')]_{+} = 0.\end{aligned} \tag{3.3}$$

for fermions. Here $\delta(\mathbf{x}-\mathbf{x}')$ means the Dirac δ function.

Operators $n(\mathbf{x})$ at different points commute between themselves. Besides, using Eq.(3.2), one can show that

$$a(\mathbf{x})F(n(\mathbf{x}'), n(\mathbf{x}''), \ldots) = F(n(\mathbf{x}') + \delta(\mathbf{x}-\mathbf{x}'), n(\mathbf{x}'') + \delta(\mathbf{x}-\mathbf{x}''), \ldots)a(\mathbf{x}) \tag{3.4}$$

$$a^{+}(\mathbf{x})F(n(\mathbf{x}'), n(\mathbf{x}''), \ldots) = F(n(\mathbf{x}') - \delta(\mathbf{x}-\mathbf{x}'), n(\mathbf{x}'') - \delta(\mathbf{x}-\mathbf{x}''), \ldots)a^{+}(\mathbf{x}) \tag{3.5}$$

where F is an arbitrary function of its arguments. The same results (3.4), (3.5) arise by usage of Eq.(3.3) instead of Eq.(3.2). Thus, the commutation relations between the operator $n(x)$ and operators $a(\mathbf{x}), a^{+}(\mathbf{x})$ do not depend on the type (3.2), (3.3) of the commutation relations.

In QFT the normal ordering operation $\mathcal{N}$ is defined as such an order of operators, when all creation operators are placed to the left of all annihilation operators [1]. In application to the operators $n(\mathbf{x})$ the normal ordering operation means

$$\mathcal{N}\{n(\mathbf{x}_1)n(\mathbf{x}_2)\ldots n(\mathbf{x}_l)\} = a^{+}(\mathbf{x}_1)a^{+}(\mathbf{x}\ldots a^{+}(\mathbf{x}_l)a(\mathbf{x}_l)a(\mathbf{x}_{l-1})\ldots a(\mathbf{x}_1) \tag{3.6}$$

and this result is the same for both commutation relations (3.2) and (3.3).

Using relation (3.1) and (3.4), (3.5), one can turn rhs of Eq. (3.6) into the form (2.9). In other words, the operator $\mathcal{N}_s$ on the space of analytical functionals of $n(\mathbf{x})$ coincides with normal ordering operation $\mathcal{N}$ when applied to $n(\mathbf{x})$.

In QFT the state of a dynamic system is described by means of the state vector $|\Psi\rangle$ in the Fock space.

$$\mid \Psi\rangle = \begin{pmatrix} \psi_0 \\ \psi_1(\mathbf{x}_1) \\ \ldots \\ \psi_n(X^n) \\ \ldots \end{pmatrix} = \sum_{k=0}^{\infty} \frac{1}{k!} \int_{V^k} \psi_k(X^k)\hat{a}_k^{+}(X^k) \mid 0\rangle dX^k, \tag{3.7}$$

$$\hat{a}_k^{+}(X^k) = \prod_{i=1}^{k} a^{+}(\mathbf{x}_i) = a^{+}(\mathbf{x}_1)a^{+}(\mathbf{x}_2)\ldots a^{+}(\mathbf{x}_k), \qquad k = 0, 1, ..$$

$$\langle \Psi \mid \Psi \rangle = 1, \qquad a(\mathbf{x}) \mid 0\rangle = 0, \tag{3.8}$$

where $a^{+}(\mathbf{x})$ is the creation operator, and $\mid 0\rangle$ is the vacuum state vector. $\psi_k(X^k)$ is a symmetrical function of $\mathbf{x}_i$ in the case (3.2), and an antisymmetrical function of $\mathbf{x}_i$ in the case (3.3). Description in terms of $\mid \Psi\rangle$ carries out the exclusive description.

Let us calculate the expression (2.8), where $n(\mathbf{x})$ is considered as the operator (3.1), and averaging is made over the state (3.7). One obtains

$$p_l(X^l) = \langle \Psi \mid \hat{a}_l^{+}(X^l) \mid \Psi\rangle = \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \sum_{s=0}^{\infty} \frac{(-1)^s}{s!} \int_{V^k} \int_{{}^lV^{l+s}} \int_{V^k} \langle 0 \mid \hat{a}_k(X'^k) \hat{a}_{l+s}^{+}(X^{l+s})$$

$$\times \hat{a}_{l+s}(X^{l+s}) \hat{a}_k^{+}(X''^k) \mid 0\rangle \psi_k^{*}(X'^k) \psi_k(X''^k) dX'^k d({}^lX^{l+s}) dX''^k$$

$$= \sum_{k=0}^{\infty} \frac{1}{(k-l)!} \sum_{s=0}^{k-l} \frac{(-1)^s (k-l)!}{s!(k-l-s)!} \int_{{}^lV^k} \mid \psi_k(X^k) \mid^2 d({}^lX^k). \tag{3.9}$$

Taking into account the identity

$$\sum_{s=0}^{k-l} \frac{(-1)^s (k-l)!}{s!(k-l-s)!} I^s I^{k-l-s} \equiv (I-I)^{k-l} = \delta_{kl}, \qquad k \geq l \tag{3.10}$$

where I^s means s-fold integral of a symmetrical function, one obtains from Eq.(3.9)

$$p_l(X^l_) = \mid \psi_l(X^l) \mid^2, \qquad l = 0, 1, \ldots \tag{3.11}$$

This result agrees with the conventional statement that $\mid \psi_l(X^l) \mid^2$ is a probability density of detecting the state X^l.

Under the condition (3.11) the normalization condition (3.8) takes the form (2.2)

$$\langle \Psi \mid \Psi\rangle = \sum_{n=0}^{\infty} \frac{1}{n!} \int_{V^n} \mid \psi_n(X^n) \mid^2 dX^n = \sum_{n=0}^{\infty} \frac{1}{n!} \int_{V^n} p_n(X^n) dX^n = 1. \tag{3.12}$$

Calculation of (2.7) leads to the following result

$$w_l(X^l) = \langle \Psi \mid \hat{w}_l(X^l) \mid \Psi\rangle = \sum_{k=0}^{\infty} \frac{1}{k!} \int_{{}^lV^{k+l}} \mid \psi_{k+l}(X^{k+l}) \mid^2 d({}^lX^{k+l}), \tag{3.13}$$

which coincides with the relation (2.3) due to Eq.(3.11).

The description in terms of the states (3.7) realizes the exclusive description in the sense that, if only first k functions $\psi_0, \psi_1(\mathbf{x}_1), \ldots \psi_k(X^k)$ are known, then one can determine k first probability densities $p_0, p_1(\mathbf{x}_1), \ldots p_k(X^k)$. Generally, the $w_l(X^l)$ $(l = 1, 2, \ldots)$ cannot be determined.

It is necessary to take into account that dynamical equations impose constraints on the form of the particle concentration operator $n(\mathbf{x})$. For instance, in the relativistic theory the $n(\mathbf{x})$ is a time component of a flux 4-vector. In this case the particle concentration operator has the form (3.1), (3.2) or (3.1), (3.3) only in the momentum space. In space-time the $n(\mathbf{x})$ has the form which does not coincide with the form (3.1), (3.2). In this case the result of the normal ordering (3.6) does not coincide with that of the operation $\mathcal{N}_s$, defined by Eq.(2.9). It can violate compatibility between relations (3.9), (3.11) and (3.13) which are corollaries of the identity of the operations $\mathcal{N}_s$ and $\mathcal{N}$. But it does not change the main conclusion that QFT realizes an exclusive description, because in the momentum space the relations (3.1), (3.2) remain valid.

4 Possibility of alternative investigation strategy. Discussion

Let us consider now what the inclusive description means in application to high energy collisions, where many particles are produced. Colliding particles are ultrarelativistic in this case, and both particle concentration $n(x)$ and flux $n(x)\mathbf{v}(x)$ are important. Let $n_A^i(x) = \{n_A(x), n_A(x)\mathbf{v}_A(x)\}$ be a 4-flux of particles of the kind A. $x = \{x^0, x^1, x^2, x^3\}$ are coordinates of a point in space-time. This flux $n_A^i(x)$ is a random quantity in the sense that repeating single experiments with two colliding particles leads to different numbers of produced particles and different momenta observed in similar experiments. In other words, a single experiment cannot be repeated in all details. Only an ensemble of many similar experiments is reproducible. Only mean quantities over the ensemble are reproducible.

Let us use the following designations.

$$j_A^i(x_A) = \langle n_A^i(x_A)\rangle,$$
$$j_{AB}^{ik}(x_A, x_B) = \langle n_A^i(x_A) n_B^k(x_B)\rangle - g^{ik}\delta_{AB}\delta(x_A - x_B)\langle n_A^i(x_A)\rangle \qquad (4.1)$$

The angular brackets denote some averaging over the ensemble.

A reasonable inclusive description arises, if dynamic equations can be reduced to the form

$$\mathcal{L}_1(j^i) = 0, \qquad j^i = \{j_A^i\}, \qquad A = 1, 2, \dots$$
$$\mathcal{L}_2(j^i, j^{ik}) = 0, \qquad j^{ik} = \{j_{AB}^{ik}\}, \qquad A, B = 1, 2, \dots \qquad (4.1)$$

The solution of the system $\mathcal{L}_1 = 0$ permits one to determine a space-time distribution of particles arising after collision. Then one can determine the mean number of particles generated by the collision, their mean velocities, mean energies and angular distributions. The solution of the second system $\mathcal{L}_2 = 0$ permits one to determine more detailed information including correlation of different types. Actually, the solutions are not limited by high energies of colliding particles, as it takes place in the exclusive description, as far as the number of produced particles increases with the increase of colliding particles energy.

Let us consider the following example. Let $\psi_A(x) = \psi_A(t, \mathbf{x})$, $A = 1, 2, \dots N$ be some wave functions describing identical particles and satisfying the equation

$$i\frac{\partial \psi_A}{\partial t} = H_0\psi_A, \qquad A = 1, 2, ..N, \qquad H_0 = -\frac{1}{2m}\nabla^2 + V(\mathbf{x}) \tag{4.3}$$

The Planck constant $\hbar$ is supposed to be equal to 1. $V(\mathbf{x})$ is the particle potential energy. Any of the functions ψ_A describes a wave packet moving with the velocity $\mathbf{v}_A$. It is supposed that at some time moment T these wave packets are far from each other, so that

$$\int_V \psi_A^*(t, \mathbf{x})\psi_B(t, \mathbf{x})d\mathbf{x} = \delta_{AB}, \qquad A, B = 1, 2, \dots \qquad t = T \tag{4.4}$$

where V means the whole space. Then due to Eq.(4.3) the relation (4.4) is valid at any time t.

Let us construct one-particle state vector

$$\Phi_1(t) = \int_V \Psi_1(t, \mathbf{x})a^+(\mathbf{x})d\mathbf{x} \mid 0\rangle \qquad \Psi_1(t, \mathbf{x}) = \sum_{A=1}^{N} \psi_A(t, \mathbf{x})$$

$$\langle\Phi_1(t) \mid \Phi_1(t)\rangle = N, \tag{4.5}$$

and N-particle state vector

$$\Phi_N(t) = \frac{1}{N!}\int_{V^N} \Psi_N(t, X^N)\hat{a}_N^+(X^N) \mid 0\rangle dX^N, \tag{4.6}$$

Here $\hat{a}_N^+(X^N)$ is defined by Eq.(3.7), Ψ_N means

$$\Psi_N(t, X^N) = \sum_{\mathcal{P}^A} \prod_{A=1}^{N} \psi_A(t, \mathbf{x}_A), \tag{4.7}$$

where summation is made over all transpositions $\mathcal{P}$ of particles. The normalization condition for the vector state $\Phi_N(t)$ has the form

$$\langle\Phi_N(t) \mid \Phi_N(t)\rangle = \frac{1}{N!}\int \mid \Psi_N(t, X^N) \mid^2 dX^N = 1 \tag{4.8}$$

Let us calculate the mean values of the flux 4-vector $n^k = \{n^0, \mathbf{n}\}$ in the states $\Phi_1(t)$ and $\Phi_N(t)$. Here $n^0 = n^0(\mathbf{x})$ is defined by Eq.(3.1), (3.2) and

$$\mathbf{n}(\mathbf{x}) = \frac{1}{2mi}(a^+(\mathbf{x})\nabla a(\mathbf{x}) - \nabla a^+(\mathbf{x})a(\mathbf{x})) \tag{4.9}$$

One obtains for $\langle n\rangle = \{\rho, \mathbf{j}\}$

$$\rho_{\rm in} = j_{\rm in}^0(t, \mathbf{x}) \equiv \langle\Phi_1(t) \mid n^0(\mathbf{x}) \mid \Phi_1(t)\rangle = \mid \Psi_1(t, \mathbf{x}) \mid^2 \tag{4.10}$$

$$\mathbf{j}_{\rm in}(t, \mathbf{x}) \equiv \langle\Phi_1(t) \mid \mathbf{n}(\mathbf{x}) \mid \Phi_1(t)\rangle = \frac{1}{2mi}(\Psi_1^*(t, \mathbf{x})\nabla\Psi_1(t, \mathbf{x}) - \nabla\Psi_1^*(t, \mathbf{x})\Psi_1(t, \mathbf{x}))$$

$$\rho_{\rm ex} = j_{\rm ex}^0(t, \mathbf{x}_1) \equiv \langle\Phi_N(t) \mid n^0(\mathbf{x}_1) \mid \Phi_N(t)\rangle = \frac{1}{(N-1)!}\int_{^1V^N} \mid \Psi_N(t, X^N) \mid^2 d(^1X^N)$$

$$= \sum_{A=1}^{N} \mid \psi_A(t, \mathbf{x}_1) \mid^2$$

$$\mathbf{j}_{\mathrm{ex}}(t, \mathbf{x}_1) \equiv \langle \Phi_N(t) \mid n(\mathbf{x}_1) \mid \Phi_N(t) \rangle = \frac{1}{(N-1)!} \int_{{}^1V^N} \frac{1}{2mi} [\Psi_N^*(t, X^N) \nabla_1 \Psi_N(t, X^N)$$

$$-\Psi_N^*(t, X^N) \nabla_1 \Psi_N(t, X^N)] d({}^1X^N)$$

$$= \frac{1}{2mi} \sum_{A=1}^{N} (\psi_A^*(t, \mathbf{x}_1) \nabla_1 \psi_A(t, \mathbf{x}_1) - \nabla_1 \psi_A^*(t, \mathbf{x}_1) \psi_A(t, \mathbf{x}_1)) \tag{4.11}$$

where the operator $\nabla_1 = \{\partial/\partial x_1^1, \partial/\partial x_1^2, \partial/\partial x_1^3\}$ acts only on the coordinates $\mathbf{x}_1$.

One can see from Eqs.(4.10), (4.11) that the mean values of the operator $n^k(\mathbf{x})$ over the one-particle state Φ_1 and over the N-particle state Φ_N coincide in those space-time regions, where the wave packets do not overlap, i.e.

$$\psi_A^*(t, \mathbf{x}) \psi_B(t, \mathbf{x}) = 0, \qquad \text{if } A \neq B \tag{4.12}$$

It is possible [11] to derive dynamic equations for j_{in}^k or j_{ex}^k. Dynamic equations for j_{in}^k and j_{ex}^k can be obtained by differentiation of Eq.(4.10), (4.11) with respect to t, using the relation (4.3) afterwards. One obtains

$$\frac{\partial j^k}{\partial t} = \langle \frac{\partial \Phi}{\partial t} \mid n^k(\mathbf{x}) \mid \Phi \rangle + \langle \Phi \mid n^k(\mathbf{x}) \mid \frac{\partial \Phi}{\partial t} \rangle = \langle \Phi \mid i[n^k(\mathbf{x}), H]_- \mid \Phi \rangle \tag{4.13}$$

Then two hydrodynamic formulations of the dynamic equation arise. The inclusive formulation has the form

$$\begin{gathered}
\frac{\partial \rho_{\mathrm{in}}}{\partial t} + \partial_\beta j_{\mathrm{in}}^\beta = 0, \qquad \rho_{\mathrm{in}} = j_{\mathrm{in}}^0 \\
\frac{\partial j_{\mathrm{in}}^\alpha}{\partial t} + \partial_\beta \frac{j_{\mathrm{in}}^\alpha j_{\mathrm{in}}^\beta}{\rho_{\mathrm{in}}} = -\partial_\alpha V(x) + \partial_\beta P_{\mathrm{in}}^{\alpha\beta}, \qquad \alpha = 1, 2, 3 \\
P_{\mathrm{in}}^{\alpha\beta} = \frac{1}{2m^2} (\partial_\alpha \rho_{\mathrm{in}}^{1/2} \partial_\beta \rho_{\mathrm{in}}^{1/2} - \rho_{\mathrm{in}}^{1/2} \partial_\alpha \partial_\beta \rho_{\mathrm{in}}^{1/2})
\end{gathered} \tag{4.14}$$

where $P_{\mathrm{in}}^{\alpha\beta}$ is the stress tensor of the fluid described by the flux vector j_{in}^k.

Another formulation has the form

$$\begin{gathered}
\rho_{\mathrm{ex}} = j_{\mathrm{ex}}^0 = \sum_{A=1}^{N} \rho_A, \qquad j_{\mathrm{ex}}^\alpha = \sum_{A=1}^{N} j_A^\alpha, \qquad \alpha = 1, 2, 3 \\
\frac{\partial \rho_A}{\partial t} + \partial_\beta j_A^\beta = 0, \qquad \rho_A = j_A^0, \qquad A = 1, 2, \ldots N \\
\frac{\partial j_A^\alpha}{\partial t} + \partial_\beta \frac{j_A^\alpha j_A^\beta}{\rho_W} = -\partial_\alpha V(x) + \partial_\beta P_A^{\alpha\beta}, \qquad \alpha = 1, 2, 3, \qquad A = 1, 2, \ldots N \\
P_A^{\alpha\beta} = \frac{1}{2m^2} (\partial_\alpha \rho_A^{1/2} \partial_\beta \rho_A^{1/2} - \rho_A^{1/2} \partial_\alpha \partial_\beta \rho_A^{1/2}), \qquad A = 1, 2, \ldots N
\end{gathered} \tag{4.15}$$

Eq.(4.14) describes the motion of a fluid described by the vector $j^k_{\text{in}} = \{\rho_{\text{in}}, \mathbf{j}_{\text{in}}\}$. Eq.(4.15) describes the motion of N drops each of which is described by the vector $j^k_A = \{\rho_A, \mathbf{j}_A\}$, $A = 1, 2, \ldots N$. The motion of the fluid described by the vector $j^k_{\text{in}} = \{\rho, \mathbf{j}\}$, coincides with the motion of N drops of the fluid described by the vector $j^k_{\text{ex}} = \sum_{A=1}^{N} j^k_A$ in those space-time regions, where the drops do not overlap. Inside the region of the overlapping drops the fluid motion is different for formulations (4.14) and (4.15). Really, let some function ψ_A

$$\psi_A(t, \mathbf{x}) \simeq (\pi L^2)^{-3/2} \exp[-(\mathbf{x} - m\mathbf{v}_A t)^2/(2L)^2 + i\mathbf{kx}], \qquad mL \gg 1.$$

describe the wave packet of size L, moving with the velocity $\mathbf{v}_A$. Then estimations lead to the following result $\rho_A \cong L^{-3}$, $P^{\alpha\beta}_A \cong \rho_A (mL)^{-2}$, $mL \gg 1$. Both ρ_A and $P^{\alpha\beta}_A$ change very slightly inside the drop independently of overlapping. The same is also valid for ρ_{in} and $P^{\alpha\beta}_{\text{in}}$ inside non-overlapping wave packets. But inside overlapping wave packets ρ_{in} oscillates rather rapidly. $P^{\alpha\beta}_{\text{in}}$ oscillates between the limits of the order $m^{-2}L^{-5}(1 \pm v^2 m^2 L^2)$, $vmL \gg 1$. Although in the cases (4.14) and (4.15) the density ρ and the stress tensor are different inside the overlapping wave packets, nevertheless the motion of the wave packets as a whole is similar in both cases.

In the case (4.14) (inclusive description) the motion of N wave packets is considered as a motion of one fluid, whereas in the case (4.15) (exclusive description) the same is described as a motion of N non-interacting fluids.

Thus, one fluid can describe a motion of N non-interacting particles. Is it possible also in the case of interacting particles? Is it possible to describe a collision of two wave packets of interacting particles in terms of one fluid? Such a statement of the problem seems to be rather natural. But it is a rather complicated mathematical problem. It is hardly worth considering this problem for two colliding non-relativistic particles, because in this case there is a well developed theory, using exclusive description. At high energy collisions, when there is an intensive pair production, such a description in terms of one fluid could be useful.

As far as the problem of particle production is a principal problem of high energy physics, then the usage of the inclusive description (one fluid description) sets an important problem to be solved, whether there exists a fluid which could describe pair production. Hydrodynamic mode of the particle production description was presented by Fermi [12] and developed by Landau [13]. With this approach a collision of two high energy particles is considered as a collision of two drops of a dissipative "nuclear" fluid. The entropy of the fluid increases after the collision. The number of produced particles is supposed to be conditioned by this increase of entropy. It is a phenomenological way of the particle production description. It can be used only at a very high energy of colliding particles, when many pairs are produced.

Generally, there is another way, when the pair production is described by means of inviscid non-dissipative fluid. The description of pair production in terms of a fluid can be represented as follows. Let us construct lines tangent to the flux 4-vector j^i. These lines are treated as world lines of the fluid particles. Zigzags of these world lines in time directions of space-time are treated as a pair production effect. The existence of such a fluid is an interesting problem, because it is believed that the pair production is a specific quantum effect which cannot be described in terms of fluid dynamics. If the fluid can really describe pair production, then the collision of two fluid drops with $j^0 > 0$ is to lead to the appearance of small drops with $j^0 > 0$ and $j^0 < 0$. The last drops are antidrops describing antiparticles.

5 Summary

Let us summarize results of our consideration. There are two equivalent modes of statistical description (exclusive and inclusive) of a system with indefinite number of particles. Equivalency of both modes takes place, only if the information about the state of the system is comprehensive. Incomplete information can be valuable only with the inclusive description. With the exclusive description the incomplete information has no value. As a result the exclusive description realizes an "all or nothing" investigation strategy.

The conventional technique of the quantum field theory is based on the exclusive mode of description. In application to the high energy physics it means that the conventional technique cannot be effective, even if one succeeds in overcomeing difficulties of QFT dynamic problems.

One can hope that the QFT technique on the basis of the inclusive description will be effective in the high energy physics. I believe that such a technique is to be developed.

6 Appendix

Let us prove formulae (2.7), (2.8) for the case of the particle distribution over cells with finite volumes. The space V is devided into identical cubic cells of the volume $v = a_0^3$. Each cell is numerated by a vector $\mathbf{y} = \{y^1, y^2, y^3\}$ with integer components. The coordinates of the center of the cell $\mathbf{y}$ are $\mathbf{x} = a_0\mathbf{y}$. Let V_f be some fictitious volume, containing one cell y_∞.

The distribution of indefinite number l of particles inside the volume V is considered as a distribution of N numerated particles inside the volume $V_0 = V \cup V_f$. The number N is supposed to be very large, but fixed. At the end N is supposed to be equal to ∞.

Let us introduce designations

$$
{}^sY^l = \{\mathbf{y}_s, \mathbf{y}_{s+1}, \ldots \mathbf{y}_l\}, \qquad s \le l; \qquad Y^l = {}^0Y^l,
$$

$$
{}^sV^l = \bigotimes_{k=s+l}^{k=l} V, \qquad {}^sV_0^l = \bigotimes_{k=s+1}^{k=l} V_0, \qquad V^l = {}^0V^l \tag{A.1}
$$

Let $P(Y^N) = P(\mathbf{y}_1, \mathbf{y}_2, \ldots \mathbf{y}_N)$ be the probability of detecting the first particle in the cell $\mathbf{y}_1$, the second particle in the cell $\mathbf{y}_2, \ldots$ the Nth particle in the cell $\mathbf{y}_N$. The cells belonging to V and to V_f are numerated by the index $\mathbf{y}$. $P(Y^N)$ is a probability of the state Y^N of individual (numerated) particles. P satisfies the normalization condition

$$\sum_{Y^n \in V_0^N} P(Y^N) \equiv \sum_{\mathbf{y}_1 \in V_0} \sum_{\mathbf{y}_2 \in V_0} \dots \sum_{\mathbf{y}_N \in V_0} P(Y^N) = 1 \qquad (A.2)$$

$\mathbf{y}_s \in V_0$ means that the summation is made over all cells $\mathbf{y}_s$, belonging to the volume $V_0 = V \cup V_f$, $Y^N \in V_0^N$ means that the summation is made over all states Y^N, belonging to V_0^N. It is supposed that the function $P(Y^N) = P(\mathbf{y}_1, \mathbf{y}_2, \dots \mathbf{y}_N)$ is symmetric with respect to transposition of any pair of arguments.

Let $P_l(Y^l)$ be the probability of the state Y^l $(l \leq N)$ in V, i.e. $P_l(Y^l)$ is the probability of detecting l and only l some particles in V, one of them in the cell $\mathbf{y}_1$, one of them in the cell $\mathbf{y}_2, \dots$ one of them in the cell $\mathbf{y}_l$. $\mathbf{y}_s \in V$ $(s = 1, 2, \dots l)$. The rest of $N - l$ particles is found in V_f.

One obtains the following expression for $P_l(Y^l)$

$$P_l(Y^l) = \frac{N!}{(N-l)! Z_l(Y^l)} \sum_{{}^lY^N \in {}^lV_f^N} P(Y^N) \qquad (A.3)$$

where

$$Z_l(Y^l) = \prod_{i=1}^{i=l} \left[\sum_{s=1}^{s=i} \delta(\mathbf{y}_i, \mathbf{y}_s) \right],$$

$$\delta(\mathbf{y}_i, \mathbf{y}_s) = \begin{cases} 1, & \text{if} \quad \mathbf{y}_i = \mathbf{y}_s \\ 0, & \text{if} \quad \mathbf{y}_i \neq \mathbf{y}_s \end{cases} \qquad (A.4)$$

The factor before the sum in (A.3) takes into account that $P_l(Y^l)$ is a probability of a distribution of non-numerated particles. $N!/(N-l)!$ describes the number of ways to choose l some particles from N particles.

One can verify that

$$Z_l(Y^l) = \prod_{i=1}^{i=l} [N_0(\mathbf{y}_i)!] \qquad (A.5)$$

where $N_0(\mathbf{y}_s)$ is the number of particles in the cell $\mathbf{y}_s$ in the state Y^l, $\mathbf{y}_s \in Y^l$, ($N_0(\mathbf{y}_s)$ is the occupation number of the cell $\mathbf{y}_s$). The factor before the sum in (A.3) describes the number of states of the numerated particles which have the same set $\{N_0(\mathbf{y}_s)\}$ of occupation numbers of the cell $\{\mathbf{y}_s\}$ of the volume V.

The normalization condition of $P_l(Y^l)$ has the form

$$\sum_{l=0}^{N} \frac{Z_l(Y^l)}{l!} \sum_{Y^l \in V^l} P_l(Y^l) = 1 \qquad (A.6)$$

It can be verified by substituting Eq.(A.6) into Eq.(A.2) and using the relation

$$\sum_{l=0}^{N} \frac{N!}{(N-l)! l!} \sum_{Y^l \in V^l} \sum_{{}^lY^N \in {}^lV_f^N} P(Y^N) = \sum_{Y^N \in V_0^N} P(Y^N) \qquad (A.7)$$

which is valid for any symmetric function $P(Y^N)$.

Now let $W_l(Y^l)$ mean the probability of detecting some paticle in the cell $\mathbf{y}_1 \in V$, some particle in the cell $\mathbf{y}_2 \in V, \ldots$ some particle in the cell $\mathbf{y}_l \in V$ (the remaining particles can be found in any place). One obtains the following expression

$$W_l(Y^l) = \frac{N!}{(N-l)! Z_l(Y^l)} \sum_{^lY^N \in {}^lV_f^N} P(Y^N), \qquad l = 0, 1, \ldots \qquad (A.8)$$

Distinction between expressions (A.8) and (A.3) in the summation volume: $V_0 = V \cup V_f$ for Eq. (A.8) and V_f for Eq.(A.3).

The probabilities $W_l(Y^l)$ and $P_l(Y^l)$ are connected with each other by means of relations

$$W_l(Y^l) = \sum_{s=0}^{N-l} \frac{1}{s!} \sum^{^lY^{l+s} \in {}^lV^{l+s}} \frac{Z_{l+s}(Y^{l+s})}{Z_l(Y^l)} P_{l+s}(Y^{l+s}) \qquad (A.9)$$

$$P_l(Y^l) = \sum_{s=0}^{N-l} \frac{-1}{s!} \sum_{^lY^{l+s} \in {}^lV^{l+s}} \frac{Z_{l+s}(Y^{l+s})}{Z_l(Y^l)} W_{l+s}(Y^{l+s}) \qquad (A.9)$$

One obtains Eq.(A.9) from Eq.(A.8), replacing the summation over V_0 by the summation over V and over V_f by means of Eq.(A.7) and using Eq.(A.3) afterwards.

Let us express the distributions $W_l(Y^l)$ and $P_l(Y^l)$ through the mean values of occupation numbers, i.e. through quantities of the form $\langle N_0(\mathbf{y}_1), N_0(\mathbf{y}_2), \ldots N_0(\mathbf{y}_l)\rangle$, where $N_0(\mathbf{y}_l)$ is the number of particles in the cell $\mathbf{y}_l$. The angular brackets mean averaging over the distribution $P(Y^N)$. The set of all $\langle N_0(\mathbf{y}_1), N_0(\mathbf{y}_2), \ldots N_0(\mathbf{y}_l)\rangle$ determines the distribution W_l and P_l $(l = 0, 1, \ldots)$, and vice versa, any of distributions W_l or P_l determines $\langle N_0(\mathbf{y}_1), N_0(\mathbf{y}_2), \ldots N_0(\mathbf{y}_l)\rangle$.

The occupation number of the cell $\mathbf{y}_i$ in the state $Y'^N = \{\mathbf{y}_1^{/prime}, \mathbf{y}_2', \ldots \mathbf{y}_N'\}$ is defined as follows

$$N_0(\mathbf{y}_i) = \sum_{s=1}^{N} \delta(\mathbf{y}_i, \mathbf{y}_s') \qquad (A.11)$$

After averaging the relation (A.11) over Y'^N for $i = 1$, one obtains

$$\langle N_0(\mathbf{y}_1)\rangle = \sum_{Y'^N \in V_0^N} \sum_{s=1}^{N} \delta(\mathbf{y}_1, \mathbf{y}' - 1) P(Y'^N) = \sum_{^1Y'^N \in {}^1V_0^N} P(Y'^N) = W_1(\mathbf{y}_1) \quad (A.12)$$

In the general case one obtains

$$W_l(Y^l) = \frac{1}{Z_l(Y^l)} \left\langle \mathcal{N}_s \left\{ \prod_{i=1}^{l} N_0(\mathbf{y}_i) \right\} \right\rangle \equiv \frac{1}{Z_l(Y^l)} \left\langle \left\{ \prod_{i=1}^{l} N_0(\mathbf{y}_i) - \sum_{s=1}^{i-1} \delta(\mathbf{y}_s, \mathbf{y}_i) \right\} \right\rangle$$

$$= \frac{1}{Z_l(Y^l)} \left\langle \prod_{i=1}^{l} \left\{ \sum_{s=1}^{N} \delta(\mathbf{y}_i, \mathbf{y}_s') - \sum_{s=1}^{i-1} \delta(\mathbf{y}_s, \mathbf{y}_i) \right\} \right\rangle \qquad (A.13)$$

In order to prove the relation (A.13), one transforms the quantity

$$B(Y^l, Y'^N) = \prod_{i=1}^{l} \left\{ \sum_{s=1}^{N} \delta(\mathbf{y}_i, \mathbf{y}'_s) - \sum_{s=1}^{i-1} \delta(\mathbf{y}_s, \mathbf{y}_i) \right\}, \qquad (A.14)$$

using identity

$$\sum_{s=1}^{N} \delta(\mathbf{y}_i, \mathbf{y}'_i) - \sum_{s=1}^{i-1} \delta(\mathbf{y}_s, \mathbf{y}_i) \equiv \sum_{r_i=1}^{N} \left\{ \delta(y_i, \mathbf{y}'_i) - \sum_{s=1}^{i-1} \delta(r_i, r_s) \delta(\mathbf{y}_s, \mathbf{y}_i) \right\} \qquad (A.15)$$

$$1 \leq r_s \leq N$$

One obtains

$$B(Y^l, Y'^N) = \sum_{r_1, r_2 \ldots r_l = 1}^{N} \prod_{i} = 1^l \left\{ \delta(\mathbf{y}_i, \mathbf{y}'_i) - \sum_{s=1}^{i-1} \delta(r_i, r_s) \delta(\mathbf{y}_s, \mathbf{y}_i) \right\} \qquad (A.16)$$

Let us prove that the product in Eq.(A.16) vanishes, if some of indices r_s ($s = 1, 2, \ldots l$) coincide. For instance, let $r_1 = r_2 = r$, then the first two factors take the form

$$\delta(\mathbf{y}_1, \mathbf{y}'_1)[\delta(\mathbf{y}_2, \mathbf{y}'_r) - \delta(\mathbf{y}_1, \mathbf{y}_2)] = 0 \qquad (A.17)$$

Thus, the statement is proved for $l = 2$. Now let us assume that it is proved for some $l - 1 \geq 2$. Let us prove that it will be valid for l. According to this supposition $r_1, r_2, \ldots r_{l-1}$ are all different, but $r_l = r_k$, $k < l$. If the numbers $r_1, r_2, \ldots r_{l-1}$ are different, the product in Eq.(A.16) can be written in the form

$$\delta(\mathbf{y}_1, \mathbf{y}'_{r_1}) \delta(\mathbf{y}_2, \mathbf{y}'_{r_2}) \ldots \delta(\mathbf{y}_{l-1}, \mathbf{y}'_{r_{l-1}})[\delta(\mathbf{y}_l, \mathbf{y}'_{r_l}) - \delta(r_k, r_l) \delta(\mathbf{y}_k, \mathbf{y}_l)]$$
$$= \delta(\mathbf{y}_1, \mathbf{y}'_{r_1}) \delta(\mathbf{y}_2, \mathbf{y}'_{r_2}) \ldots \delta(\mathbf{y}_{l-1}, \mathbf{y}'_{r_{l-1}})[(\mathbf{y}_l, \mathbf{y}_k) - \delta(r_k, r_l) \delta(\mathbf{y}_k, \mathbf{y}_l)] = 0$$

Thus, if the statement is valid for $l - 1$, then it is valid for l, and the product in Eq.(A.16) vanishes, if among indices $r_1, r_2, \ldots r_{l-1}$ there are at least two indices, having the same value. Now the expression (A.16) reduces to the form

$$B(Y^l, Y'^N) = \sum_{r_1, r_2 \ldots r_l = 1}^{N} \prod_{i=1}^{l} \delta(\mathbf{y}_i, \mathbf{y}'_{r_i})$$

Using the symmetry of $P(Y^N)$ and Eq. (A.8), one obtains

$$\frac{1}{Z_l(Y^l)} \langle B(Y^l, Y'^N) \rangle =$$

$$\frac{N!}{(N-l)! Z_l(Y^l)} \sum_{Y'^N \in V_0^N} \prod_{i=1} \delta(\mathbf{y}_i, \mathbf{y}'_i) P(Y'N) = W_l(Y^l), \qquad l = 0, 1, \ldots N \qquad (A.20)$$

Let us substitute Eq.(A.13) into (A.10). One obtains

$$P_l(Y^l) = \sum_{s=0}^{N-l} \frac{(-1)^s}{s! Z_l(Y^l)} \sum_{{}^lY^{l+s} \in {}^lV^{l+s}} \left\langle \mathcal{N}_s \left\{ \prod_{i=1}^{i=l} N_0(\mathbf{y}_i) \right\} \right\rangle \qquad (A.21)$$

Tending $N \to \infty$, one obtains instead of (A.21)

$$P_l(Y^l) = \frac{1}{Z_l(Y^l)} \left\langle \mathcal{N}_s \left\{ \exp \left[- \sum_{y \in V} N_0(\mathbf{y}) \right] \prod_{i=1}^{l+s} N_0(\mathbf{y}_i) \right\} \right\rangle, \qquad l = 0, 1, \ldots \quad (A.22)$$

Finally, let us turn to the continuous distribution by substituting

$$P_l(Y^l) \Rightarrow p_l(X^l) v^l, \qquad W_l(Y^l) \Rightarrow w_l(X^l) v^l,$$
$$N_0(\mathbf{y}) \Rightarrow n(\mathbf{x}) v, \qquad \sum_{Y^l \in V^l} \Rightarrow \int_{V^l}$$

where v is the volume of the cell. If the mean number of particles inside V is finite, then $Z_l(Y^l) = \prod_{i=1}^{i=l} \{[n(x_i)v]!\}$ tends to 1, because $n(x_i)v \to 0$ when $v \to 0$ and $n(x_i)$ remain finite. In other words, the probability of finding more than one particle in a cell tends to zero.

REFERENCES

1. Schweber S.S., An Introduction to Relativistic Field Theory. (Row, Peterson and Co. New York, 1961).
2. Feynman R.P. Phys. Rev. Lett. **23**, 1415, (1969).
3. Logunov A.A., Mestvirishvili M.A., Nguen Van Hiew. Phys. Lett. **25**, 611, (1967).
4. Feinberg E.L. Phys. Rept. C. **5**, 237, (1972).
5. Grishin V.G. *Inclusive Processes at Hadron Interactions at High Energies.* Moscow, Atomizdat, 1982. (in Russian).
6. Grishin V.G. *Quarks and Hadrons in Interactions of High Energy Particles.* Moscow, Energoatomizdat, 1988. (in Russian).
7. Taylor R.E. Rev. Mod. Phys. **63,** 573, (1991).
8. Kendall H.W. Rev. Mod. Phys. **63**, 597, (1991).
9. Friedman J.I. Rev. Mod. Phys. **63**, 615, (1991).
10. Day D.B., McCarthy J.S., Meziani Z.B., Minchart R., Sealock R., Thornton S.T.,Jourdan J., Sick I., Filippone B.W., McKelown R.D., Milner R.G., Potterveld D.H., Szalata Z. Phys. Rev.C **48**, 1849, (1993).
11. Madelung E., Z. Phys. **40**, 322, (1926).
12. Fermi E., Progr. Theor. Phys. **5**, 570, (1950).
13. Landau L.D. Izvestiya Akademii Nauk USSR. **17**, 5, (1953).