Tachyon gas as a candidate for dark matter

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Abstract

In the physical geometry (i.e. in geometry, described completely by its world function) identical geometric objects have identical description in terms of the world function. As a result spacelike straight segment is a three-dimensional surface even in the space-time geometry of Minkowski. Tachyons have two unexpected properties: (1) a single tachyon cannot be detected and (2) the tachyon gas can be detected by its gravitational influence. Although molecules (tachyons) of the tachyon gas moves with superluninal velocities, the mean motion of these molecules appears to be underluminal. The tachyon gas properties differs from those of usual gas. The pressure of the tachyon gas depends on the gravitational potential and does not depend on temperature. As a result the tachyon gas may form huge halos around galaxies. These halos have almost constant density, and this circumstance can explain the law of star velocities at the periphery of a galaxy. Properties of the tachyon gas admit one to consider it as a dark matter.

Key words: discrete geometry, tachyon, dark matter, dark energy, rotation curves

1 Introduction

At the metric approach to geometry the space-time geometry is described in terms of the world function and only in terms of the world function. All geometrical objects and all geometrical quantities are expressed in terms of the world function σ. Such a representation of geometrical quantities will be referred as σ-immanent representation. At the metric approach two different regions $\mathcal{R}_1$ and $\mathcal{R}_2$ of the space-time may have different geometries described relatively be world functions σ_1 and σ_2. Let a physical body, having the shape $G_1 = g_1(\sigma_1)$ in the region $\mathcal{R}_1$

evolves as a free moving body and appears in the region $\mathcal{R}_2$ with other geometry. The shape of the body is described now as $G_2 = g_2(\sigma_2)$. How are functions g_1 and g_2 connected? As far as the physical geometry is a monistic construction, which is described completely by the only quantity (world function), the only possibility may take place

$$g_1(\sigma) = g_2(\sigma) = g(\sigma) \tag{1.1}$$

The conventional (Riemannian) space-time geometry is pluralistic. It is described by several basic geometrical quantities, whose properties are described by axioms. In the pluralistic conception of geometry it is very difficult to consider the problem of geometrical objects identification in different geometries. This problem is not considered in the general relativity, which uses different geometries for different regions of the space-time. The only geometric object which is considered in dynamics of the general relativity is the world line of a free pointlike body. It is supposed that the world line of a free body is a geodesic.

In the framework of Riemannian space-time geometry the shape of a geodesic is determined by the metric tensor. This conventional definition of the world line of a free body agrees with the definition (1.1) for timelike world lines. However it disagrees with (1.1) for spacelike world lines, because in the physical geometry a spacelike straight segment is not a one-dimensional line. It is a three-dimensional surface. It is easy to verify, using definition of the straight segment $\mathcal{T}_{[P_0P_1]}$ between points P_0 and P_1

$$\mathcal{T}_{[P_0P_1]} = \left\{R | \sqrt{2\sigma(P_0,R)} + \sqrt{2\sigma(R,P_1)} = \sqrt{2\sigma(P_0,P_1)}\right\} \tag{1.2}$$

Indeed, in the 4-dimensional space-time one equation (1.2) describes 3-dimensional surface, in general. For timelike distances this surface degenerates into one-dimensional line, because in this case the distance satisfies the anti-triangle axiom

$$\sqrt{2\sigma(P_0,P_2)} + \sqrt{2\sigma(P_2,P_1)} \leq \sqrt{2\sigma(P_0,P_1)}, \quad \sigma(P_i,P_k) > 0, \quad i,k = 0,1,2 \tag{1.3}$$

For spacelike distances the triangle axiom (1.3) is not fulfilled, and the set of points R satisfying equation (1.2) is 3-dimensional.

Of course, points of any segment of the "straight" line

$$\mathbf{x} = \mathbf{v}t + \mathbf{x}_0, \quad |\mathbf{v}|^2 > c^2$$

satisfy the relation (1.2), but it is only a small part of points R satisfying (1.2).

Our conceptual logical consideration disagrees with the general opinion that the segment of straight is a one-dimensional set in any geometry. For instance, Blumental constructed the distance geometry [1], where he used metric approach to geometry with distance which does not satisfy the triangle axiom. Blumental failed to construct a curve in the framework of the metric approach. He was forced to define a curve as continuous mapping of a segment of the numerical axis onto the space, where the geometry is given. According to this definition the straight line is a one-dimensional set, that cannot be formulated in terms of a distance. It is a remnant of the pluralistic geometric conception.

Ellipsoid $\mathcal{EL}_{P_0P_1P_3}$ is defined in terms of distance

$$\mathcal{EL}_{P_0P_1P_3} = \left\{R|\sqrt{2\sigma\left(P_0,R\right)} + \sqrt{2\sigma\left(P_1,R\right)} = \sqrt{2\sigma\left(P_0,P_3\right)} + \sqrt{2\sigma\left(P_1,P_3\right)}\right\} \tag{1.4}$$

where points P_0, P_1 are focuses of the ellipsoid, and P_3 is some point on the surface of the ellipsoid.

Degenerated ellipsoid, where the point P_3 on its surface coincides with one of focuses is by definition segment $\mathcal{T}_{[P_0P_1]} = \mathcal{EL}_{P_0P_1P_1}$ of straight between focuses P_0, P_1. In the geometry, where distance satisfies the triangle axiom the degenerated ellipsoid is a one-dimensional set. However, when triangle axiom is not satisfied the degenerated ellipsoid is a $(n-1)$-dimensional surface in n-dimensional space.

The straight segment is defined in the Euclidean geometry by the relation (1.2). In the same form it is defined in the space-time geometry of Minkowski. In the proper Euclidean geometry any smooth curve line is defined as a limit of a broken line, when lengths of its links (straight segments) tend to zero. In the physical geometry a curve is defined in the same form. If the curve describes a world line of a free particle, the vectors describing adjacent links of the broken line are equivalent. Equivalence of vectors means that vectors are in parallel and their lengths are equal. For timelike world line these conditions lead to one-dimensional straight line. For the spacelike world line (tachyon) these conditions lead to a world chain with wobbling links. Amplitude of this wobbling is infinite and any link is an infinite three-dimensional surface. A single tachyon described by such a world chain cannot be detected. However, the tachyon gas may be detected by its gravitational field.

Tachyon gas is considered here, because the tachyon gas has characteristic properties of so-called dark matter. On one hand, one failed to detect single particles of the dark matter. On the other hand, the dark matter form a huge halos around galaxies with almost constant mass distribution inside the halo. Existence of such halos is discovered by its gravitational influence on the star velocities in the galaxy periphery. Tachyon gas has similar properties. A single tachyon cannot be detected according to geometric properties. Besides, tachyon gas has almost constant mass density in the gravitational field of a galaxy.

Tachyon is a hypothetical faster-than-light particle. Its rest mass is imaginary. Such particles have not been detected. First such particles were considered by A.Sommerfeld [2]. Particles with negative and imaginary masses were investigated by Ya.P.Terletsky [3]. Tachyons were investigated also by other investigators [4, 5, 6, 7]. One considered not only tachyons, but also tachyonic fields which are results of the tachyon quantization.

Unfortunately, effective description of tachyons is possible only in a discrete space-time geometry. Conventional consideration of tachyons in the continuous Riemannian space-time geometry leads to conclusion that tachyons do not exist, whereas investigations of tachyons in the framework of a discrete space-time geometry leads only to the conclusion that a single tachyon cannot be detected. Impossibility of the tachyon detection does not mean that tachyons do not exist. Tachyons may exist, but one cannot detect a single tachyon, even it will appear that a tachyon may interact with some elementary particle. For instance, neutron decays spontaneously into proton, electron and neutrino. However, one cannot be sure that this decay is not a result of collision with tachyon, because the tachyon gas may fill the whole universe with almost constant density. In this relation the tachyon gas properties remind the vacuum properties.

Such unusual properties of tachyons are conditioned by the fact that in the discrete space-time geometry there are world chains instead of smooth world lines. Links of the tachyon world chain are spacelike segments. Two adjacent points of the tachyon world chain are divided by very large spatial distance. Discovering one point of this world chain, one cannot detect the another point of the world chain.

Crucial point of our investigation is a use of the discrete space-time geometry, whose properties differ strongly from properties of the Riemannian geometry and other continuous geometries. Conventional mathematical technique of differential geometry is inadequate in the discrete geometry. Linear vector space, which is a foundation of the differential geometry, cannot be introduced in the discrete geometry. Introducing the linear vector space formalism in the discrete geometry, one obtains multivalence of such operations as summation of vectors and decomposition of a vector into components. The only quantity which is common for continuous geometry and the discrete one is the distance d or the world function $\sigma = \frac{1}{2}d^2$.

In the discrete geometry, as well as in any physical geometry, the linear vector space cannot been introduced, in general. Mathematical formalism of discrete geometry differs essentially from formalism of differential geometry. This formalism is obtained from formalism of the proper Euclidean geometry, expressed in terms of the world function of the Euclidean geometry. It can be found in the paper [8]. Particle dynamics in the discrete space-time geometry is described in [9]. Formalism of the physical geometry based on the world function is rather unusual and unexpected. It is coordinateless, and it does not use concept of the linear vector space. Absence of linear vector space in the discrete (physical) geometry is not customary for mathematicians (and physicists). They cannot imagine a geometry without a linear vector space. Typical question of mathematicians looks as follows: "Why do you define scalar product in the form (2.4)? The scalar product is an operation in the linear vector space. The name of scalar product has been used already. Use another name for the operation (2.4), for instance, σ-scalar product." In reality the definition (2.4) is a more general definition, because it does not use concept of linear vector space and it coincides with the conventional definition via the linear vector space in the case, when it can be introduced. According to the rules of logic the name scalar product is to have the more general definition, i.e. (2.4) , whereas the conventional definition via linear vector space must have the name "linear scalar product", or scalar product with some additional epithet, because it is a more special definition, using concept of the linear vector space. It is of no importance the fact, that the conventional definition has been introduced then, when we did not know anything on discrete geometry.

If one considers a discrete space-time geometry, one may not use quantum principles, because for usual particles of positive rest mass (tardions) the quantum principles are corollaries of the space-time geometry discreteness. Consideration of quantum principles in the discrete space-time geometry reminds description of Brownian motion in terms of thermogen (in terms of axiomatic thermodynamics). If the elementary length λ_0 of discrete space-time geometry is connected with the quantum constant $\hbar$ by means of the relation $\lambda_0^2 = \hbar/bc$ (constants $\hbar$, b, c are universal constants), the quantum effects for tardions can be explained as geometrical effects of the discrete space-time geometry [10]. In such a situation it is useless to quantize tachyons and to consider tachyonic fields. One should consider tachyons as classical particles in the discrete space-time geometry.

Mathematical technique of differential (continuous) geometry cannot be applied to a discrete geometry. In the discrete geometry there are no continuous world lines, there are no differential equations and differential relations. One may not use the phase space of coordinates and momenta for description of the particle state, because the momentum is a result of differentiation along the continuous world line. But one cannot use differentiation in the discrete geometry. In the discrete space-time geometry the particle state is described by two points P_s, P_{s+1}. Vector $\mathbf{P}_s\mathbf{P}_{s+1}$ describes the geometric momentum of a particle, and its geometric mass $\mu = |\mathbf{P}_s\mathbf{P}_{s+1}|$ determines the usual particle mass m by the relation

$$m = b\mu \tag{1.5}$$

where b is an universal constant. The particle dynamics in the discrete space-time geometry is described by the skeleton conception [9], where instead of the continuous world line one uses the world chain $\mathcal{C}$ (broken line), whose links are vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ of the same length μ

$$\mathcal{C} = \bigcup_s \mathbf{P}_s\mathbf{P}_{s+1}, \quad |\mathbf{P}_s\mathbf{P}_{s+1}| = \mu = \text{const}, \quad s = ...0, 1, 2, ... \tag{1.6}$$

For free particle the adjacent vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$ are equivalent ($\mathbf{P}_s\mathbf{P}_{s+1}\text{eqv}\mathbf{P}_{s+1}\mathbf{P}_{s+2}$). It means that

$$((\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2}) = |\mathbf{P}_s\mathbf{P}_{s+1}| \cdot |\mathbf{P}_{s+1}\mathbf{P}_{s+2}|) \wedge (|\mathbf{P}_s\mathbf{P}_{s+1}| = |\mathbf{P}_{s+1}\mathbf{P}_{s+2}|) \tag{1.7}$$

Relations (1.5) - (1.7) are not special suppositions. They are corollaries of the dynamic conception in the discrete space-time geometry [9]. If the vector $\mathbf{P}_s\mathbf{P}_{s+1}$ is fixed the equivalence relation (1.7) determines the adjacent vector $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$ ambiguously, provided the space-time geometry is discrete. As a result the world chain wobbles. Amplitude of this wobbling is of the order of the elementary length λ_0 for tardions ($\mu^2 > 0$). This wobbling is a reason of quantum effects. For tachyons ($\mu^2 < 0$) amplitude of this wobbling is infinite .

For tachyons the spatial distance between adjacent points P_s and P_{s+1} is random, and it may be infinitely large. As a result one cannot detect a single tachyon. In other words, single tachyons were not discovered in experiments, because they are unobservable, but not because they do not exist.

However, if one cannot detect a single tachyon, it does not mean that one cannot observe the gravitational influence of the tachyon gas, consisting of many unobservable tachyons. Unobservable tachyons may form so-called dark matter, which form large spherical halo around some galaxies. Existence of such a halo is necessary for explanation of the rotational velocities of stars (rotation curves) in some galaxies [11]. In these galaxies the rotational velocities of stars do not depend practically on the distance r from the galaxy core. Sometimes the star velocities increase arises with increasing of the distance r. If the gravitating mass is concentrated in the galaxy core, then the Newtonian force of gravitation is proportional to r^{-2}, and rotational velocity is to be proportional $r^{-1/2}$. Inside the gravitating sphere with uniform distribution of the mass the Newtonian gravitation force is proportional to r, and the rotational velocity is proportional to r.

In this paper we try to calculate parameters of the tachyon gas in order to determine, whether the tachyon gas can fill the halo of galaxies with necessary density.

2 Discrete space-time geometry

Discrete geometry is obtained as a generalization of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$, which is constructed usually as a logical construction. Conventionally one uses the Euclidean method, when all statements of $\mathcal{G}_{\mathrm{E}}$ are deduced from a system of axioms, describing properties of simplest geometrical objects of $\mathcal{G}_{\mathrm{E}}$. The Euclidean method is inadequate for construction of the discrete geometry $\mathcal{G}_{\mathrm{d}}$. Inadequacy of the Euclidean method is connected with the fact, that one does not know, how the simplest geometrical objects of $\mathcal{G}_{\mathrm{E}}$ look in other geometries. For instance, the straight segment $\mathcal{T}_{[P_0P_1]}$ between the points P_0 and P_1 is one-dimensional line in $\mathcal{G}_{\mathrm{E}}$, whereas $\mathcal{T}_{[P_0P_1]}$ is a surface in $\mathcal{G}_{\mathrm{d}}$. There is only one quantity, which is common for $\mathcal{G}_{\mathrm{E}}$ and $\mathcal{G}_{\mathrm{d}}$. It is the distance $d\left(P_0, P_1\right)$ between two arbitrary points P_0 and P_1 of the point set Ω, where the geometry is given. It is more effective to use the world function $\sigma = \frac{1}{2}d^2$ instead of the distance d, because the world function σ is always real (even in the geometry of Minkowski, where d may be imaginary).

The world function σ is a real single-valued function. It is defined by the relation [8]

$$\sigma: \quad \Omega\times\Omega\rightarrow\mathbb{R},\quad \sigma(P,Q)=\sigma(Q,P),\quad \sigma(P,P)=0,\quad \forall P,Q\in\Omega \tag{2.1}$$

To generalize $\mathcal{G}_{\mathrm{E}}$ onto $\mathcal{G}_{\mathrm{d}}$, one needs to describe $\mathcal{G}_{\mathrm{E}}$ in terms of the Euclidean world function σ_{E}. Thereafter replacing σ_{E} by the world function σ_{d} of $\mathcal{G}_{\mathrm{d}}$ in all statements of $\mathcal{G}_{\mathrm{E}}$, one obtains all statements of $\mathcal{G}_{\mathrm{d}}$. The world function σ_{d} of $\mathcal{G}_{\mathrm{d}}$ may be taken in the form

$$\sigma_{\mathrm{d}}\left(P,Q\right)=\sigma_{\mathrm{M}}\left(P,Q\right)+\frac{\lambda_0^2}{2}\mathrm{sgn}\left(\sigma_{\mathrm{M}}\left(P,Q\right)\right),\quad \forall P,Q\in\Omega \tag{2.2}$$

where σ_{M} is the world function of the Minkowski geometry $\mathcal{G}_{\mathrm{M}}$, and λ_0 is the elementary length. Due to relation (2.2) in $\mathcal{G}_{\mathrm{d}}$ all distances satisfy the relation

$$|\rho_{\mathrm{d}}(P,Q)| = \left|\sqrt{2\sigma_{\mathrm{d}}(P,Q)}\right| \notin (0,\lambda_0), \quad \forall P,Q \in \Omega \tag{2.3}$$

which is definition of the discrete geometry.

Being presented in terms of the world function σ_{E}, the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ contains two kinds of relations: (1) general geometric relations, which contains only world function σ_{E}, and (2) special relations of the geometry $\mathcal{G}_{\mathrm{E}}$, which are constraints, imposed on the world function σ_{E}. The approach, when a geometry is described in terms and only in terms of the world function, will be referred to as metric approach. Any geometry described completely by the world function will be referred to as a physical geometry.

Let us adduce some general geometric definitions which are important in the particle dynamics:

Vector $\mathbf{PQ}$ is an ordered set $\{P,Q\}$ of two points P,Q (but not an element of the linear vector space as usually). Scalar product $(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1)$ of two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is defined by the relation [8]

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma(P_0,Q_1) + \sigma(P_1,Q_0) - \sigma(P_0,Q_0) - \sigma(P_1,Q_1) \tag{2.4}$$

The length $|\mathbf{PQ}|$ of the vector $\mathbf{PQ}$ is defined by the relation

$$|\mathbf{PQ}|^2 = (\mathbf{PQ}.\mathbf{PQ}) = 2\sigma(P,Q) \tag{2.5}$$

n vectors $\mathbf{P}_0\mathbf{P}_1, \mathbf{P}_0\mathbf{P}_2, ...\mathbf{P}_0\mathbf{P}_n$ are linear dependent, if and only if the Gram determinant

$$F_n(\mathcal{P}_n) = \det||(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)||, \quad i,k = 1,2,...n, \quad \mathcal{P}_n \equiv \{P_0, P_2, ...P_n\} \tag{2.6}$$

vanishes

$$F_n(\mathcal{P}_n) = 0 \tag{2.7}$$

Two vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent (equal) ($\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1$), if the vectors are in parallel [8]

$$(\mathbf{P}_0\mathbf{P}_1 \upuparrows \mathbf{Q}_0\mathbf{Q}_1): \quad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \tag{2.8}$$

and their lengths are equal

$$\sigma(P_0,P_1) = \sigma(Q_0,Q_1) \tag{2.9}$$

According to (2.8), (2.9) the equivalence definition has the form (1.7)

$$\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1: \quad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1|^2 \wedge |\mathbf{P}_0\mathbf{P}_1|^2 = |\mathbf{Q}_0\mathbf{Q}_1|^2 \tag{2.10}$$

All general geometric relations (2.4) - (2.10) are obtained as properties of the linear vector space. However, they do not contain any reference to the linear vector space. They are written in terms of the world function $\sigma_{\rm E}$ of the proper Euclidean geometry, and they may be used in any physical geometry even in the case, when one cannot introduce linear vector space in this geometry. To use the relations (2.4) - (2.10) in a discrete geometry, it is sufficient to use the world function $\sigma_{\rm d}$ of the discrete geometry $\mathcal{G}_{\rm d}$ in them.

Formally general geometric relations (2.4) - (2.10) realize some processing of information, contained in the world function. Such a processing is to be universal, i.e. it is uniform for all generalized geometries. This method of processing is known for the proper Euclidean geometry $\mathcal{G}_{\rm E}$. It may be applied for construction of general geometric relations for other generalized geometries. In the case, when one can introduce linear vector space, such a processing admits one to construct the particle dynamics in the space-time geometry, equipped by the linear vector space. As far as the general geometric relations (2.4) - (2.10) are universal in the sense that they do not refer to the linear vector space, they may be used for construction of the particle dynamics in those space-time geometries, where introduction of the linear vector space is impossible.

Such a construction of geometry is very effective, because it does not need proofs of numerous theorems and a test of the axioms compatibility. Besides, the geometry can be constructed in the coordinateless form. Monistic character of the geometry (description in terms of one basic quantity - world function) provides automatically a correct connection between all secondary quantities in all physical geometries. Ascertainment of a connection between different geometric quantities is the main problem of a pluralistic construction of a geometry, which is based on a use of several independent basic quantities.

The special relations of the proper Euclidean geometry have the form [8]:

I. Definition of the metric dimension:

$$\exists \mathcal{P}_n \equiv \{P_0, P_1, ...P_n\} \subset \Omega, \qquad F_n\left(\mathcal{P}_n\right) \neq 0, \qquad F_k\left(\Omega^{k+1}\right) = 0, \qquad k > n \quad (2.11)$$

where $F_n\left(\mathcal{P}_n\right)$ is the n-th order Gram's determinant (2.6). Vectors $\mathbf{P}_0\mathbf{P}_i$, $i = 1, 2, ...n$ are basic vectors of the rectilinear coordinate system K_n with the origin at the point P_0. The covariant coordinates of the point P in the coordinate system K_n are defined by the relation

$$x_i\left(P\right) = \left(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}\right), \qquad i = 1, 2, ...n \qquad (2.12)$$

The metric tensors $g_{ik}\left(\mathcal{P}_n\right)$ and $g^{ik}\left(\mathcal{P}_n\right)$, $i, k = 1, 2, ...n$ in K_n are defined by the relations

$$\sum_{k=1}^{k=n} g^{ik}\left(\mathcal{P}_n\right) g_{lk}\left(\mathcal{P}_n\right) = \delta_l^i, \qquad g_{il}\left(\mathcal{P}_n\right) = \left(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_l\right), \qquad i, l = 1, 2, ...n \quad (2.13)$$

II. Linear structure of the Euclidean space:

$$\sigma_{\mathrm{E}}(P,Q)=\frac{1}{2}\sum_{i,k=1}^{i,k=n}g^{ik}(\mathcal{P}_n)(x_i(P)-x_i(Q))(x_k(P)-x_k(Q)), \qquad \forall P,Q\in\Omega \tag{2.14}$$

where coordinates $x_i(P)$, $x_i(Q)$, $i=1,2,...n$ of the points P and Q are covariant coordinates of the vectors $\mathbf{P}_0\mathbf{P}$, $\mathbf{P}_0\mathbf{Q}$ respectively in the coordinate system K.

III: The metric tensor matrix $g_{lk}(\mathcal{P}^n)$ has only positive eigenvalues g_k

$$g_k>0, \qquad k=1,2,...,n \tag{2.15}$$

IV. The continuity condition: the system of equations

$$(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P})=y_i\in\mathbb{R}, \qquad i=1,2,...n \tag{2.16}$$

considered to be equations for determination of the point P as a function of coordinates $y=\{y_i\}$, $i=1,2,...n$ has always one and only one solution. Conditions I – IV contain a reference to the dimension n of the Euclidean space, which is defined by the relations (2.11).

Special relations of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ may be not valid for other physical geometries. In some cases these relations may used partly. For instance, the metric dimension may be defined locally. Instead of constraint (2.11) one uses the condition

$$\forall P_0\in\Omega, \quad \exists\mathcal{P}_n\equiv\{P_0,P_1,...P_n\}\subset\Omega, \quad F_n(\mathcal{P}_n)\neq 0, \quad F_k(\mathcal{P}_k)=0, \quad k>n \tag{2.17}$$

where all skeletons $\mathcal{P}_n$ contain only infinitely close points, that is possible only in a continuous geometry . The conditions (2.17) determine the metric dimension for locally flat (Riemannian) geometry.

All relations I – IV are written in terms of the world function. They are constraints on the form of the world function of the proper Euclidean geometry.

The proper Euclidean geometry looks in the σ-representation quite different, than in conventional representation on the basis of the linear vector space. For instance, such a quantity as dimension has two different meanings in the σ-representation. On one hand, the metrical dimension n_{m} is the maximal number of linear independent vectors, which is determined by the relations (2.11). On the other hand, the coordinate dimension n_{c}, is the number of coordinates, which is used at the description of the point set Ω. In the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ the coordinate dimension coincides with the metric dimension ($n_{\mathrm{c}}=n_{\mathrm{m}}$), and this fact is a corollary of special (not general geometric) relations (2.11), (2.12)

In general, the coordinate labelling of points of Ω has no relation to the geometry. In the proper Euclidean geometry the two dimensions coincide, because the coordinate dimension n_{c} is determined by the special conditions (2.11), (2.12), which are characteristic for the proper Euclidean geometry. In the geometry $\mathcal{G}_{\mathrm{d}}$ the number

n_m of linear independent vectors is more, than the number of coordinates n_c. For instance, for six points $\mathcal{P}_5 = \{P_0, P_1 ... P_5\}$

$$\begin{aligned} P_0 &= \{0,0,0,0\}, \quad P_1 = \{0,l,0,0\}, \quad P_2 = \{0,0,l,0\}, \\ P_3 &= \{0,0,l,0\}, \quad P_4 = \{0,0,0,l\}, \quad P_5 = \{a,0,0,0\} \end{aligned}$$

the Gram determinant $F_5(\mathcal{P}_5)$ vanishes in the geometry of Minkowski $\mathcal{G}_\mathrm{M}$ with the world function

$$\sigma_\mathrm{M}(x,x') = \frac{1}{2}\left(x^0 - x'^0\right) - \frac{1}{2}\left(\mathbf{x} - \mathbf{x}'\right)^2 \tag{2.18}$$

However, the Gram determinant $F_5(\mathcal{P}_5)$, calculated in the discrete geometry $\mathcal{G}_\mathrm{d}$ with the world function σ_d, given by (2.2) does not vanish.

$$F_5(\mathcal{P}_5) = d\left(-a^2 l^6 + 3al^7 - l^8\right) + O\left(d^2\right) \tag{2.19}$$

Here $d = \lambda_0^2/2 \ll a^2, l^2$. For five points $\mathcal{P}_4 = \{P_0, P_1 ... P_4\}$ one obtains in $\mathcal{G}_\mathrm{d}$

$$F_4(\mathcal{P}_4) = -l^8 - 4l^6 d + O\left(d^2\right) \tag{2.20}$$

Thus, in general, the metric dimension $n_\mathrm{m} \geq 5$ in $\mathcal{G}_\mathrm{d}$. In $\mathcal{G}_\mathrm{d}$ the metric dimension $n_\mathrm{m} \geq 5$ cannot coincide with the coordinate dimension $n_\mathrm{c} = 4$. It means essentially that one cannot introduce a finite number of linear independent basic vectors and expand space-time vectors over these basic vectors. It is very unexpected, because the conventional construction of a differential geometry (for instance, the Riemannian one) starts, giving n-dimensional manifold with a coordinate system on it. Of course, one assumes, that the maximal number of linear independent basic vectors at any point is equal to $n = n_\mathrm{m} = n_\mathrm{c}$. Only in this case one can expand vectors over basic vectors and use operations, defined in the linear vector space. In the case of a discrete space-time geometry, where $n_\mathrm{m} \neq n_\mathrm{c}$, the linear vector space cannot be introduced, although the coordinate system can be introduced, and the coordinate dimension $n_\mathrm{c} = 4$ as in the space-time geometry of Minkowski. Four coordinates $x = \{x^0, x^1, x^2, x^3\}$, $x^k \in \mathbb{R}$ are defined as usually.

Note, that the conditions (2.11), defining metric dimension n_m contain a lot of constraints, and all they are special conditions of $\mathcal{G}_\mathrm{E}$. It means that there is a lot of physical geometries, where $n_\mathrm{m} \neq n_\mathrm{c}$, and one cannot introduce a linear vector space there. In the limit $d \to 0$, $F_5(\mathcal{P}^5) = 0$ in (2.19), and $\mathcal{G}_\mathrm{d}$ transforms to $\mathcal{G}_\mathrm{M}$. In this case the metric dimension $n_\mathrm{m} = 4$ coincides with the coordinate dimension $n_\mathrm{c} = 4$. It means that one may use approximately the space-time geometry $\mathcal{G}_\mathrm{M}$ in the case, when typical lengths l of vectors are much greater, than the elementary length λ_0. In this case one may set approximately $\lambda_0 = 0$, and suppose that $n_\mathrm{m} = n_\mathrm{c}$.

The set of the Gram determinants values $F_n(\mathcal{P}_n)$, $n = 2, 3, ...$ may be such, that one cannot introduce the metric dimension n_m. Apparently, the discrete space-time geometries are geometries without a definite metric dimension. Such "dimensionless" geometries look especially exotic. Contemporary researchers deal only with the Euclidean method, which uses only space-time geometries of definite dimension.

They can hardly conceive properties of ”dimensionless” space-time geometries. On the other hand, the classical particle dynamics does not work in microcosm, described by the geometry of Minkowski. As far as the discrete (”dimensionless”) space-time geometries are not known for most researchers, they use quantum dynamics, which imitates the discrete geometry properties. This imitation is arbitrary and desultory. Besides, this imitation is not complete. There are such properties of real particle dynamics, which cannot be imitated by quantum dynamics in the space-time of Minkowski.

We see that coincidence of metric dimension n_{m} with the coordinate dimension n_{c} and a construction of a smooth manifold with the dimension $n = n_{\mathrm{m}} = n_{\mathrm{c}}$ is a special property of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$, which is not a general geometric property. The conventional Euclidean method of the differential geometry construction starts from the definition of a smooth manifold with fixed dimension. Such a method is not a general method of the generalized geometries construction, because it uses special properties of $\mathcal{G}_{\mathrm{E}}$, which are not characteristic for all generalized geometries, generally speaking. In general, a use of the coordinate description for the generalized geometries construction is a use of special properties of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ for such a construction. Such an approach cannot be a general method of the generalized geometries construction. Using special properties of $\mathcal{G}_{\mathrm{E}}$, one obtains only a part of possible generalized geometries. In particular, a use of the coordinate description does not admit one to construct geometries with indefinite metric dimension and with intransitive equality relation. However, the coordinate labelling of points of Ω has nothing to do with a construction of a manifold. The coordinate labelling of points may be used always, and it has no relation to a construction of generalized geometries. The coordinate labelling becomes to deal with the generalized geometry construction, when one imposes the condition $n_{\mathrm{c}} = n_{\mathrm{m}}$.

The relation $n_{\mathrm{c}} = n_{\mathrm{m}}$ is a special property of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$, and it may be wrong for many physical geometries, because physical geometries may have no definite metric dimension. Using the relation $n_{\mathrm{c}} = n_{\mathrm{m}}$ at the construction of a generalized geometry, one may meet such a situation, when the real space-time geometries appear beyond the scope.

3 Dynamics of particle with two-point skeleton

In the discrete space-time geometry the state of a particle (physical body) is described by its skeleton $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$, consisting of $n+1$ space-time points, connected rigidly [9], It is a corollary of mathematical formalism, based on a use of the world function. Phase space of coordinates and momenta cannot be used, because in the discrete geometry the operation of differentiation, which is necessary for the momentum definition, is not defined in the discrete geometry. The skeleton may be considered as a discrete analog of a frame connected rigidly with a physical body (particle). Tracing the motion of the skeleton one may trace the motion of the particle. The state of a pointlike particle is described by a two-point skeleton $\mathcal{P}_1 = \{P_0, P_1\}$. The vector $\mathbf{P}_0\mathbf{P}_1$ describes energy-momentum of the particle, and $\mu = |\mathbf{P}_0\mathbf{P}_1|$ is a geometric mass of the particle, connected with usual mass by the relation (1.5). Information on position of two skeleton points is sufficient for description of the state of a pointlike particle. Dynamics of the pointlike particle skeleton $\mathcal{P}_1$ is described by the world chain (1.6), (1.7). According to these relations and definition of the scalar product (2.4) the dynamic equations for the pointlike particle are written in the form

$$\sigma\left(P_{s-1}, P_s\right) = \sigma\left(P_s, P_{s+1}\right), \quad s = ...0, 1, 2... \tag{3.1}$$

$$\sigma\left(P_{s-1}, P_{s+1}\right) = 4\sigma\left(P_{s-1}, P_s\right), \quad s = ...0, 1, 2... \tag{3.2}$$

Solving dynamic equations (3.1), (3.2), one can determine set of point of the world chain.

In the inertial coordinate system of the Minkowski geometry, where $s = 1$, the points P_0, P_1, P_2 have coordinates

$$P_0 = \{x_0, \mathbf{x}\}, \quad P_1 = \{x_0 + p_0, \mathbf{x} + \mathbf{p}\}, \quad P_2 = \{x_0 + 2p_0 + \alpha_0, \mathbf{x} + 2\mathbf{p} + \boldsymbol{\alpha}\} \tag{3.3}$$

The 4-vector $\alpha = \{\alpha_0, \boldsymbol{\alpha}\}$ is a discrete analog of the acceleration vector.

Let us choose world function σ_{M} in the form, which it has in the extended general relativity [12, 13] with slight gravitational field described by the gravitational potential $V(\mathbf{x})$

$$\sigma_{\mathrm{M}}(x, x') = \frac{1}{2}\left(\left(c^2 - 2V(\mathbf{y})\right)(x_0 - x_0')^2 - (\mathbf{x} - \mathbf{x}')^2\right), \quad \mathbf{y} = \frac{\mathbf{x} + \mathbf{x}'}{2} \tag{3.4}$$

where $V = V(\mathbf{y})$ is a gravitational potential at the point $\mathbf{y}$, and the world function σ_{d} has the form (2.2). One obtains in $\mathcal{G}_{\mathrm{d}}$

$$\left(c^2 - 2V\right)(p_0 + \alpha_0)^2 - (\mathbf{p} + \boldsymbol{\alpha})^2 + \varepsilon\lambda_0^2 = \left(c^2 - 2V\right)p_0^2 - \mathbf{p}^2 + \varepsilon\lambda_0^2 = \mu^2, \quad \varepsilon = \operatorname{sgn}\left(\mu^2\right) \tag{3.5}$$

$$\left(c^2 - 2V\right)(2p_0 + \alpha_0)^2 - (2\mathbf{p} + \boldsymbol{\alpha})^2 + \varepsilon\lambda_0^2 = 4\left(\left(c^2 - 2V\right)p_0^2 - \mathbf{p}^2 + \varepsilon\lambda_0^2\right), \quad \varepsilon = \operatorname{sgn}\left(\mu^2\right) \tag{3.6}$$

Here quantities $x = \{x_0, \mathbf{x}\}$, $p = \{p_0, \mathbf{p}\}$ are supposed to be given and 4-vector $\alpha = \{\alpha_0, \boldsymbol{\alpha}\}$ is to be determined from dynamic equations (3.5), (3.6). It follows from (3.5) that

$$p_0 = \frac{\sqrt{\mathbf{p}^2 + \varepsilon|\mu|^2 - \varepsilon\lambda_0^2}}{\sqrt{c^2 - 2V}} \tag{3.7}$$

The dynamic equations have the same form for timelike ($\mu^2 > 0, \quad \varepsilon > 0$) and spacelike ($\mu^2 < 0, \quad \varepsilon < 0$) world chains. We have two equations for four components of 4-vector α. As a result the solution is not unique, in general. We consider separately two different cases: (1) $p_0 \neq 0$ and (2) $p_0 = 0$.

All quantities in the discrete geometry will be referred be names, which they have in the continuous (Riemannian) geometry at $\lambda_0 = 0$.

3.1 The case $p_0 \neq 0$

After transformation of equations (3.5), (3.6) one obtains two relations

$$\alpha_0 = \frac{2\boldsymbol{\alpha}\mathbf{p} + 3\varepsilon\lambda_0^2}{2p_0\left(c^2 - 2V\right)} \tag{3.8}$$

$$\frac{\left(v^2 - \left(c^2 - 2V\right)\right)}{\left(c^2 - 2V\right)}\left(\alpha_\parallel + \frac{\frac{3}{2}\varepsilon\lambda_0^2 p}{\left(p^2 - p_0^2\left(c^2 - 2V\right)\right)}\right)^2 - \boldsymbol{\alpha}_\perp^2 = r^2, \quad v = \frac{p}{p_0} \tag{3.9}$$

where the quantity r is defined by the relation

$$r^2 = -3\varepsilon\lambda_0^2 - \frac{9}{4}\frac{\lambda_0^4}{p_0^2\left(v^2 - \left(c^2 - 2V\right)\right)}, \quad v = \frac{p}{p_0} \tag{3.10}$$

$$\boldsymbol{\alpha}_\parallel = \mathbf{p}\frac{(\boldsymbol{\alpha}\mathbf{p})}{\mathbf{p}^2}, \quad \boldsymbol{\alpha}_\perp = \boldsymbol{\alpha} - \boldsymbol{\alpha}_\parallel, \quad \alpha_\parallel^2 = \frac{(\boldsymbol{\alpha}\mathbf{p})^2}{\mathbf{p}^2}, \quad \alpha_\parallel = \frac{\boldsymbol{\alpha}\mathbf{p}}{p}, \quad \mathbf{p}^2 = p^2 \tag{3.11}$$

Here $\boldsymbol{\alpha}_\parallel$ is the component of 3-vector $\boldsymbol{\alpha}$ which is in parallel with the vector $\mathbf{p}$, whereas $\boldsymbol{\alpha}_\perp$ is the components of 3-vector $\boldsymbol{\alpha}$, which are perpendicular to the vector $\mathbf{p}$.

Vector $\mathbf{v} = \mathbf{p}/p_0$ may be interpreted as 3-velocity of a particle described by world chain (1.6), (1.7). In the case of continuous Riemannian geometry ($\lambda_0 \rightarrow 0$) $\mathbf{v}$ is the usual 3-velocity.

In the case of timelike vector $\mathbf{P}_0\mathbf{P}_1$ (tardion) $\varepsilon = 1$, $v^2 < c^2$and according to (3.7) $\mu = |\mu|$, if $\lambda_0 \ll \mu$, $V \ll c^2$ and $p \ll cp_0$ (nonrelativistic case). In this case equation (3.9) has the form

$$\left(\alpha_\parallel - \frac{3\lambda_0^2}{2\mu}v\right)^2 + \boldsymbol{\alpha}_\perp^2 = r_1^2, \quad r_1^2 = -r^2 \simeq 3\lambda_0^2 + \mathcal{O}\left(\lambda_0^2\right) \tag{3.12}$$

Solution of this equation has the form

$$\alpha_\parallel = \frac{3\lambda_0^2}{2\mu}v + \sqrt{3}\lambda_0\cos\theta, \quad \alpha_{\perp 1} = \sqrt{3}\lambda_0\sin\theta\cos\phi, \quad \alpha_{\perp 2} = \sqrt{3}\lambda_0\sin\theta\sin\phi \tag{3.13}$$

$$\alpha_0 = \frac{3\lambda_0^2}{2\mu}v^2 + \sqrt{3}\lambda_0 v\cos\theta + \frac{3}{2}\frac{\lambda_0^2}{\mu}, \tag{3.14}$$

Here θ, ϕ are arbitrary real numbers. It means that the difference between adjacent vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$, described by the 4-vector α, is determined nonuniquely. The particle world chain wobbles with amplitude of the order of λ_0. Statistical description of this wobbling leads to the Schrödinger equation [10], provided $\lambda_0^2 = \hbar/\left(bc\right)$.

In the case of tachyon, when vector $\mathbf{P}_0\mathbf{P}_1$ is spacelike, $\varepsilon = -1$ and $v^2 > c^2$. Equation (3.9) takes the form

$$\frac{\left(v^2 - \left(c^2 - 2V\right)\right)}{\left(c^2 - 2V\right)}\left(\alpha_\parallel - \frac{\frac{3}{2}\lambda_0^2 v}{p_0\left(v^2 - \left(c^2 - 2V\right)\right)}\right)^2 - \boldsymbol{\alpha}_\perp^2 = r^2, \quad v = \frac{p}{p_0} \tag{3.15}$$

where

$$r^2 = 3\lambda_0^2 - \frac{9}{4}\frac{\lambda_0^4}{p_0^2\left(v^2 - \left(c^2 - 2V\right)\right)} \tag{3.16}$$

and $r^2 > 0$, if λ_0 is small enough ($\lambda_0^2 < p_0^2 c^2$). Solution of equation (3.15) is also nonunique

$$\alpha_\parallel = \frac{3\lambda_0^2}{2p_0\left(v^2 - \left(c^2 - 2V\right)\right)}v + \frac{r\sqrt{c^2 - 2V}}{\sqrt{v^2 - \left(c^2 - 2V\right)}}\cosh\theta \tag{3.17}$$

$$\alpha_{\perp 1} = r\sinh\theta\cos\phi, \quad \alpha_{\perp 2} = r\sinh\theta\sin\phi \quad v = \frac{p}{p_0} = \frac{p\sqrt{\left(c^2 - 2V\right)}}{\sqrt{\mathbf{p}^2 - |\mu|^2 + \lambda_0^2}} \tag{3.18}$$

$$\alpha_0 = \frac{2\boldsymbol{\alpha}\mathbf{p} - 3\lambda_0^2}{2p_0\left(c^2 - 2V\right)} = \frac{p\left(\frac{3\lambda_0^2}{2p_0\left(v^2 - \left(c^2 - 2V\right)\right)}v + \frac{r\sqrt{c^2 - 2V}}{\sqrt{v^2 - \left(c^2 - 2V\right)}}\cosh\theta\right) - \frac{3}{2}\lambda_0^2}{p_0\left(c^2 - 2V\right)} \tag{3.19}$$

Here θ, ϕ are arbitrary real numbers. But now the wobbling amplitude is infinite because of functions cosh and sinh. The wobbling amplitude is infinite even in the case of space-time geometry of Minkowski, when $\lambda_0 = 0$. Components of the tachyon velocity $\mathbf{u}$ are defined by relations

$$\mathbf{u} = \frac{\mathbf{p} + \boldsymbol{\alpha}}{p_0 + \alpha_0} \tag{3.20}$$

One obtains the following expressions

$$\begin{aligned} u_\parallel &= \frac{p + \alpha_\parallel}{p_0 + \alpha_0} = \frac{\left(p + \frac{3\lambda_0^2}{2p_0\left(v^2 - \left(c^2 - 2V\right)\right)}v + \frac{r\sqrt{c^2 - 2V}}{\sqrt{v^2 - \left(c^2 - 2V\right)}}\cosh\theta\right)}{p_0 + \frac{p\left(\frac{3\lambda_0^2}{2p_0\left(v^2 - \left(c^2 - 2V\right)\right)}v + \frac{r\sqrt{c^2 - 2V}}{\sqrt{v^2 - \left(c^2 - 2V\right)}}\cosh\theta\right) - \frac{3}{2}\lambda_0^2}{p_0\left(c^2 - 2V\right)}} \\ &= \frac{p_0\left(c^2 - 2V\right)}{p} + \mathcal{O}\left(\cosh^{-1}\theta\right) \end{aligned} \tag{3.21}$$

$$\begin{aligned} u_{\perp 1} &= \frac{\alpha_{\perp 1}}{p_0 + \alpha_0} = \frac{r\sinh\theta\cos\phi}{p_0 + \frac{p\left(\frac{3\lambda_0^2}{2p_0\left(v^2 - \left(c^2 - 2V\right)\right)}v + \frac{r\sqrt{c^2 - 2V}}{\sqrt{v^2 - \left(c^2 - 2V\right)}}\cosh\theta\right) - \frac{3}{2}\lambda_0^2}{p_0\left(c^2 - 2V\right)}} \\ &= \sqrt{1 - \frac{\left(c^2 - 2V\right)}{v^2}}\sqrt{\left(c^2 - 2V\right)}\frac{\sinh\theta}{\cosh\theta}\cos\phi + \mathcal{O}\left(\cosh^{-1}\theta\right) \end{aligned} \tag{3.22}$$

Averaging components of $\mathbf{u}$, one assumes, that all directions are equiprobable. The quantities p_0, p are fixed at the averaging. Then one uses the formula

$$\langle\mathbf{u}\rangle = \lim_{\Theta\to\infty}\frac{1}{N}\int_{-\Theta}^{\Theta}\sin\theta d\theta\int_0^{2\pi}\mathbf{u}\cosh\theta d\phi, \quad N = 4\pi\sinh\Theta \tag{3.23}$$

where symbol $\langle ... \rangle$ means averaging. One obtains as a result of averaging

$$\langle u_{\perp 1} \rangle = \langle u_{\perp 2} \rangle = 0 \tag{3.24}$$

$$\left\langle u_{\parallel} \right\rangle = \frac{p_0 \left(c^2 - 2V\right)}{p} + \mathcal{O}\left(\cosh^{-1}\Theta\right) = \frac{\left(c^2 - 2V\right)}{v} < c \tag{3.25}$$

$$\left\langle u_{\parallel}^2 \right\rangle = \left\langle u_{\parallel} \right\rangle^2 = \langle u \rangle^2 = \frac{\left(c^2 - 2V\right)^2}{v^2} \tag{3.26}$$

$$\begin{aligned}\left\langle u_{\perp}^2 \right\rangle &= 2\pi \frac{p_0^2 \left(c^2 - 2V\right)\left(v^2 - c^2 + 2V\right)}{p^2 4\pi \sinh\Theta} \int_{-\Theta}^{\Theta} \frac{\sinh^2\theta}{\sinh^2\theta - 1} \cosh\theta d\theta + \mathcal{O}\left(\cosh^{-1}\Theta\right) \\ &= \frac{p_0^2}{p^2}\left(c^2 - 2V\right)\left(v^2 - c^2 + 2V\right) + \mathcal{O}\left(\cosh^{-1}\Theta\right) \end{aligned} \tag{3.27}$$

In the limit $\Theta \to \infty$

$$\left\langle u_{\perp}^2 \right\rangle = \frac{\left(c^2 - 2V\right)\left(v^2 - c^2 + 2V\right)}{v^2} < c^2 \tag{3.28}$$

According to (3.24), (3.26)

$$\left\langle \mathbf{u}^2 \right\rangle - \langle \mathbf{u} \rangle^2 = \left\langle \mathbf{u}_{\perp}^2 \right\rangle = \left(c^2 - 2V\right)\left(1 - \frac{c^2 - 2V}{v^2}\right) < c^2 \tag{3.29}$$

If $p_0 \to 0$, then $v = p/p_0 \to \infty$ and

$$\left\langle \mathbf{u}^2 \right\rangle - \langle \mathbf{u} \rangle^2 = \left(c^2 - 2V\right) \tag{3.30}$$

The tachyon gas velocity dispersion does not depend on λ_0 and on parameters p_0, p. This circumstance explains averaging (3.23) at fixed parameters p_0, p. The obtained results are valid in the continuous space-time geometry, when $\lambda_0 = 0$. The results obtained above cannot be obtained formally for the case, when $p_0 = 0$. This case is considered separately.

3.2 The case $p_0 = 0$

In this case equations (3.5), (3.6) take the form

$$\left(c^2 - 2V\right)\alpha_0^2 - (\mathbf{p} + \boldsymbol{\alpha})^2 - \lambda_0^2 = -\mathbf{p}^2 - \lambda_0^2 = -\left|\mu\right|^2, \tag{3.31}$$

$$\left(c^2 - 2V\right)\alpha_0^2 - (2\mathbf{p} + \boldsymbol{\alpha})^2 - \lambda_0^2 = 4\left(-\mathbf{p}^2 - \lambda_0^2\right), \tag{3.32}$$

These equations are reduced to the form

$$\boldsymbol{\alpha}\mathbf{p} = \alpha_{\parallel} p = \frac{3}{2}\lambda_0^2, \quad \left(c^2 - 2V\right)\alpha_0^2 - \boldsymbol{\alpha}_{\perp}^2 - \alpha_{\parallel}^2 = 3\lambda_0^2 + \left(\frac{3\lambda_0^2}{2p}\right) \tag{3.33}$$

Solution of equations (3.33) has the form

$$\alpha_0 = \frac{r_2 \cosh\theta}{\sqrt{c^2-2V}}, \quad \alpha_{\|} = \frac{3\lambda_0^2}{2p}, \quad \alpha_{\perp 1} = r_2 \sinh\theta\cos\phi, \quad \alpha_{\perp 2} = r_2 \sinh\theta\sin\phi, \tag{3.34}$$

$$r_2^2 = 3\lambda_0^2 + \frac{9\lambda_0^4}{4p^2} = 3\lambda_0^2 + \frac{9\lambda_0^4}{4\left(|\mu|^2 - \lambda_0^2\right)} \tag{3.35}$$

Here θ, ϕ are arbitrary real numbers, as in the case $p_0 \neq 0$.

Let us average $\mathbf{u}$ and $\mathbf{u}^2$, defined by relations (3.20) (3.34). Using relations of the type (3.23), one obtains

$$\left\langle u_{\|}\right\rangle = \left\langle \frac{p+\alpha_{\|}}{p_0+\alpha_0} \right\rangle = \left\langle \frac{\left(p + \frac{3\lambda_0^2}{p}\right)\sqrt{c^2-2V}}{r_1\cosh\theta} \right\rangle = 0, \quad \left\langle \mathbf{u}_\perp \right\rangle = 0 \tag{3.36}$$

$$\left\langle u_{\|}^2\right\rangle = \left\langle u_{\|}\right\rangle^2 = 0 \tag{3.37}$$

$$\begin{aligned}\left\langle \mathbf{u}_\perp^2 \right\rangle &= \left\langle \left|\frac{\boldsymbol{\alpha}_\perp}{\alpha_0}\right|^2 \right\rangle = \left\langle \left(\frac{\sinh\theta}{\cosh\theta}\sqrt{c^2-2V}\right)^2 \right\rangle = \frac{2\pi\left(c^2-2V\right)}{4\pi\sinh\Theta}\int_{-\Theta}^{\Theta}\frac{\sinh^2\theta}{\cosh\theta}d\theta \\ &= \left(c^2-2V\right) + \mathcal{O}\left(\cosh^{-1}\Theta\right)\end{aligned} \tag{3.38}$$

As a result one obtains in the limit $\Theta \to \infty$

$$\left\langle \mathbf{u}^2\right\rangle = \lim_{\Theta\to\infty}\left(\left\langle u_{\|}^2\right\rangle + \left\langle \mathbf{u}_\perp^2\right\rangle\right) = \left(c^2-2V\right) \tag{3.39}$$

Results (3.36) – (3.39) agree with relations (3.24), (3.26), (3.30) in the case, when $p_0 \ll p$, and the velocity $v = p/p_0 \to \infty$.

In general, the tachyon gas may be described by some distribution of its parameters $p_0, \mathbf{p}$ (but not only by fixed values of these parameters). It influences slightly on the velocity distribution in the tachyon gas.

The pressure P of the tachyon gas is defined by the relation

$$P = \frac{1}{3}\rho\left(\left\langle \mathbf{u}^2\right\rangle - \left\langle |\mathbf{u}|\right\rangle^2\right) \tag{3.40}$$

Here $\rho = \rho(x)$ is the tachyon gas mass density. It follows from (3.37), (3.39) that

$$P(x) = \frac{1}{3}\rho(x)\left(c^2 - 2V(\mathbf{x})\right) \tag{3.41}$$

All tachyon gas parameters $\rho, \mathbf{u}, P$ are considered as functions of the space-time points $x = \{x^0, \mathbf{x}\}$. But gravitational potential $V(\mathbf{x})$ is considered as a function of the spatial coordinates $\mathbf{x}$.

4 Balanced state of the tachyon gas in the gravitational field

In the gravitational field of a galaxy the tachyon gas may be at rest, if the balance condition is fulfilled

$$\nabla P = \rho \nabla V \tag{4.1}$$

According (3.41) this condition is written in the form

$$\frac{1}{3}\left(c^2 - 2V(\mathbf{x})\right)\nabla\rho = \frac{5}{3}\rho\nabla V(\mathbf{x}) \tag{4.2}$$

Equation (4.2) is integrated in the form

$$\rho = \frac{\rho_0 c^5}{\sqrt{\left|c^2 - 2V(\mathbf{x})\right|^5}} \tag{4.3}$$

Here $\rho_0 = \text{const}$.

In the case of spherically symmetric gravitational field of a galaxy one obtains instead of (4.3)

$$\rho(r) = \frac{\rho_0 c^5}{\sqrt{\left|c^2 - 2V(r)\right|^5}} \tag{4.4}$$

If the gravitational field is not strong and $V(r) \ll c^2$, the potential $V(r)$ may be approximated by the expression

$$V(r) = \frac{GM}{r} + \frac{4\pi G}{3}\rho_0 r^2 \tag{4.5}$$

Here G is the gravitational constant and M is the mass of the galaxy. The expression (4.4) takes the form

$$\rho(r) = \frac{\rho_0 c^5}{\sqrt{\left|c^2 - \frac{2GM}{r} - \frac{8\pi}{3}G\rho_0 r^2\right|^5}} \approx \rho_0\left(1 + \frac{5GM}{rc^2} + \frac{20\pi G}{3c^2}\rho_0 r^2\right) \tag{4.6}$$

If ρ_0 is large enough and $20\pi\rho_0 r^2 \geq 15Mr^{-1}$, the density $\rho(r)$ may even increase with increase of r. At any rate the second term in (4.6) slacks the decrease of density $\rho(r)$ with increase of r.

It follows from (4.3) that the tachyon gas density is larger in regions with larger gravitational potential. It means that the tachyon gas is attracted to massive bodies as usual tardion gas. Besides, the tachyon gas density changes rather slowly with the change of the gravitational potential, whereas in isothermal atmosphere this dependence is exponential. Slowly dependence of the tachyon gas density on the gravitational potential facilitates formation of halo with the almost constant tachyon gas density.

Remark. Averaging solutions of the dynamic equations, one supposed, that the gravitational potential V was constant. In general, one should take into account the fact that potential V depends on coordinates and, hence, on the 4-vector α. We hope that our approximation does not change the tachyon gas properties essentially. Two main properties of the tachyon gas (its strong mobility and very high pressure) depend slightly on the form of gravitational potential.

5 Dark energy

There is an impression that many cosmological problems are connected with a use of Riemannian space-time geometry, which is inadequate in application to general relativity, because the methods of differential geometry describe only a small part of possible space-time geometries. Observation of accelerated expansion of universe is explained usually by so-called dark energy. There are different version of the dark energy nature [14, 15], but all these versions try to explain cosmic antigravitation which is a reason of accelerated expansion of universe. Conventional general relativity, based on the Riemannian space-time geometry can explain antigravitation only by means of negative mass, by negative pressure or by so-called Λ-term, taken with a proper sign.

The expanded general relativity (EGR) uses more general class of possible space-time geometries. In the physical geometries of EGR [12, 13] a spherical dust cloud of radius R and of the mass M cannot collapse and form a black hole. Decreasing radius R, the parameter $\varepsilon_0 = 2GM/\left(c^2 R\right)$ becomes large enough, a region of antigravitation arises in the center of the cloud. The antigravitation prevents from appearance of the dark hole. Impossibility of collapse prescribes another scenario for gravitational contraction of the dust cloud, than the conventional scenario. When the radius R of the cloud decreases as a result of gravitational contraction, the parameter ε_0 increases, and inside the cloud the region of antigravitation appears which prevents from the further contraction. However, the cloud contraction continue by inertia. When parameter ε_0 becomes large enough, the contraction stops, and the opposite process begins. The central region of the cloud begin to expand. There are different stages of expansion. At some stages this expansion may be accelerated. At other stages the speed of expansion may be decreased. It is possible that different parts of the central region of the cloud may be at different stages of expansion. It is important that there is no necessity to invent mythical essences like negative pressure and quintessence. One needs only to construct a true model of the universe expansion, based on a correct conception of the space-time geometry.

6 Concluding remarks

Our conclusions depend on existence and properties of tachyons, and these properties seem to be rather unexpected. This surprise is conditioned by a fundamental change of approach to geometry. Here one uses the metric approach to geometry, when geometry is considered as a science on the shape and dispositions of geometric objects. At such an approach any geometry is described completely by its world function and only by its world function. Although nobody deny the metric approach, the mathematical formalism of differential geometry is based on the idea that any geometry is a logical construction, and all statements of a geometry can be deduced from several geometric axioms. The logical structure of a geometry is considered as a principal property of geometry. So-called symplectic geometry is considered as a geometry, because its logical structure reminds the logical structure of the Euclidean geometry, although the symplectic geometry has no relation to a description of geometric objects.

Mathematical technique adequate to metric approach was unknown. Attempts of constructing such a technique failed [16, 1]. Formalism of world function was suggested by J.L.Synge, who used it for description of the Riemannian space-time geometry [17]. But he failed to obtain coordinateless description of space-time geometry.

Tachyons and their properties can be effectively described only in the framework of a discrete space-time geometry. However, the discrete geometry can be constructed only in framework of the metric approach and in cannot be constructed by methods of differential geometry. As a result tachyons appeared outside the scope of the space-time geometry. One considered tachyons as hypothetical objects, and their properties were unknown.

Now tachyon gas is a real gas, whose gravitational influence can be identified with the gravitational influence of the mysterious dark matter. One succeeded to construct the tachyon gas statics only due to developed coordinateless technique of metric approach to space-time geometry. The tachyon gas dynamics is not yet constructed. Possibility of the tachyon existence follows from the mathematical formalism based on a use of the world function. No new hypotheses on properties of tachyons were used. Tachyon gas as a candidate for the role of the dark matter has such a property as a very high pressure, which is necessary for a composition of large halo. another candidates for the role of dark matter do not possess this property.

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