Incompatibility of the Copenhagen interpretation with quantum formalism and its reasons

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Abstract

It is proved the mathematical theorem, that the wave function describes the statistical ensemble of particles, but not a single particle. Supposition, that the wave function describes a single particle appears to be incompatible with formalism of quantum mechanics. One discusses the reasons, why this very simple statement has not been proved mathematically for many years. The reason lies in application of the trial and error methods for construction of the quantum mechanics. Application of this method as the main tool of investigation during eighty years generated "fitting mentality" of all microwold researchers.

In this paper we prove a very important theorem, which claims that the wave function may not describe an individual quantum particle. It describes always a statistical ensemble of quantum particles. We shall show, that the action $\mathcal{A}_{\rm S}$ for the Schrödinger particle $\mathcal{S}_{\rm S}$ (the dynamic system described by the Schrödinger equation) turns into the action $\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\rm cl}]}$ for the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\rm cl}\right]$ of free classical particles $\mathcal{S}_{\rm cl}$, when the quantum constant $\hbar\rightarrow 0$. Such a transition is possible only in the case, when the wave function ψ describes a statistical ensemble of quantum particles, but not a single particle.

For the free Schrödinger particle $\mathcal{S}_{\rm S}$ the action has the form

$$\mathcal{S}_{\rm S}: \qquad \mathcal{A}_{\rm S}\left[\psi,\psi^{*}\right]=\int\left\{\frac{i\hbar}{2}\left(\psi^{*}\partial_{0}\psi-\partial_{0}\psi^{*}\cdot\psi\right)-\frac{\hbar^{2}}{2m}\nabla\psi^{*}\nabla\psi\right\}dt d\mathbf{x} \qquad (1)$$

where $\psi=\psi\left(t,\mathbf{x}\right)$ is a complex one-component wave function, $\psi^{*}=\psi^{*}\left(t,\mathbf{x}\right)$ is the complex conjugate to ψ, and m is the particle mass. It is supposed that in the classical limit $\hbar \to 0$ the description of the dynamic system $\mathcal{S}_{\mathrm{S}}$ becomes to be a classical description of a free particle $\mathcal{S}_{\mathrm{cl}}$.

However, there are two different classical descriptions of the free classical particle $\mathcal{S}_{\mathrm{cl}}$. The individual classical particle $\mathcal{S}_{\mathrm{cl}}$ is described by the action

$$\mathcal{A}_{\mathcal{S}_{\mathrm{cl}}}[\mathbf{x}] = \int \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 dt \tag{2}$$

where $\mathbf{x} = \{x^1(t), x^2(t), x^3(t)\}$.

Statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{cl}}]$ of free classical particles $\mathcal{S}_{\mathrm{cl}}$ is described by the action

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{cl}}]}[\mathbf{x}] = \int \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 dt d\xi \tag{3}$$

where $\mathbf{x} = \{x^1(t,\xi), x^2(t,\xi), x^3(t,\xi)\}$. Parameters $\xi = \{\xi_1, \xi_2, \xi_3\}$ label elements (particles) of the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{cl}}]$. Both dynamic systems $\mathcal{S}_{\mathrm{cl}}$ and $\mathcal{E}[\mathcal{S}_{\mathrm{cl}}]$ are classical. However, $\mathcal{S}_{\mathrm{cl}}$ has six degrees of freedom (the order of the system of the first order ordinary differential equations), whereas the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{cl}}]$ has infinite number of the freedom degrees, because it consists of the infinite number of the particles $\mathcal{S}_{\mathrm{cl}}$. To what classical dynamic system ($\mathcal{S}_{\mathrm{cl}}$ or $\mathcal{E}[\mathcal{S}_{\mathrm{cl}}]$) turns the dynamic system $\mathcal{S}_{\mathrm{S}}$ in the limit $\hbar \to 0$?

The quantum constant $\hbar$ is a parameter of the dynamic system (1). As a rule, a change of a parameter of a dynamic system does not change the number and the character of dynamic equations. The number of the freedom degrees does not changes also. The dynamic system $\mathcal{S}_{\mathrm{S}}$ has infinite number of the freedom degrees, and we should expect that at $\hbar \to 0$ the dynamic system $\mathcal{S}_{\mathrm{S}}$ turns into $\mathcal{E}[\mathcal{S}_{\mathrm{cl}}]$, which also has infinite number of the freedom degrees, but not into $\mathcal{S}_{\mathrm{cl}}$, which has six degrees of freedom.

However, at $\hbar = 0$ the description by means of the action (1) degenerates, and one should consider the limit $\hbar \to 0$ of the description by means of the action (1). To obtain this limit, we make a change of variables

$$\psi \to \Psi_b = |\psi| \exp\left(\frac{\hbar}{b}\log\frac{\psi}{|\psi|}\right), \qquad \psi = |\Psi_b| \exp\left(\frac{b}{\hbar}\log\frac{\Psi_b}{|\Psi_b|}\right) \tag{4}$$

where $b \neq 0$ is some real constant. After this change of variables the action (1) turns into

$$\mathcal{S}_{\mathrm{q}}: \qquad \mathcal{A}_{\mathcal{S}_{\mathrm{q}}}[\Psi_b, \Psi_b^*] = \int \left\{ \frac{ib}{2}\left(\Psi_b^*\partial_0\Psi_b - \partial_0\Psi_b^* \cdot \Psi_b\right) - \frac{b^2}{2m}\nabla\Psi_b^*\nabla\Psi_b \right.$$
$$\left. - \frac{\hbar^2 - b^2}{2m}\left(\nabla|\Psi_b|\right)^2 \right\} dt d\mathbf{x} \tag{5}$$

The transformation (4) is analytical for any values of parameters b and $\hbar$, except for the case, when $\hbar = 0$ or $b = 0$. The constant b is arbitrary, and it always can be chosen $b \neq 0$. The value $\hbar = 0$ is not considered, because in this case the action

(1) is degenerate. For all values of $\hbar \neq 0$ the dynamic systems (1) and (5) are equivalent. At $\hbar \to 0$ the dynamic system (5) does not degenerate, it turns into the dynamic system $\mathcal{E}'[\mathcal{S}_{\text{cl}}]$

$$\mathcal{A}_{\mathcal{E}'[\mathcal{S}_{\text{cl}}]}[\Psi_b,\Psi_b^*] = \int \left\{ \frac{ib}{2}\left(\Psi_b^*\partial_0\Psi_b - \partial_0\Psi_b^*\cdot\Psi_b\right) - \frac{b^2}{2m}\nabla\Psi_b^*\nabla\Psi_b + \frac{b^2}{2m}\left(\nabla\left|\Psi_b\right|\right)^2 \right\} dt d\mathbf{x} \tag{6}$$

which is a partial case of the dynamic system (3). This statement will be proved.

But independently of, whether or not dynamic systems $\mathcal{E}'[\mathcal{S}_{\text{cl}}]$ and $\mathcal{E}[\mathcal{S}_{\text{cl}}]$ coincide, the dynamic system (6) cannot coincide with the dynamic system (2), because the dynamic system (2) has six degrees of freedom, whereas the dynamic system (6) has infinite number of the freedom degrees. It means that the *wave function may not describe a single particle*, and the Copenhagen interpretation and other QM interpretations, founded on the statement, that the wave function describes a single particle, may not be used. In particular, such phenomena as superluminal interaction in the EPR experiment and many-worlds interpretation [1, 2] appear to be impossible as founded on the statement, that the wave function describes an individual particle.

Discussion of paradoxes, connected with the quantum measurements began, as soon as the quantum mechanics had appeared, and it lasts up to now. Is it so difficult to find a mistake in the transition from the quantum description to the classical one and realize that the wave function describe the statistical ensemble, but not a single particle? Does one need eighty years, to find a simple mathematical mistake?

We shall try to understand reasons of such a situation, because understanding of these reasons may be useful for further development of the quantum mechanics. We are interested in logical (but not in historical) reason of this mistake.

The connection between the dynamic systems (6) and (3) follows directly from the conventional consideration of transition from the quantum description to the classical one. It may be found in any textbook on quantum mechanics. It is amazing, that nobody concluded from this consideration that the wave function describes the statistical ensemble. As an example, we consider presentation of this problem in the book by L.D. Landau and E.M. Lifshitz [3]. There one uses the representation of the wave function in the form

$$\psi = a\exp(iS/h) \tag{7}$$

which is close to the result, which is obtained from the transformation (4). The expression (7) is substituted in the Schrödinger equation

$$i\hbar\partial_0\psi + \frac{\hbar^2}{2m}\nabla^2\psi = 0$$

As a result one obtains

$$a\frac{\partial S}{\partial t} + \frac{a}{2m}\left(\nabla S\right)^2 - \frac{\hbar^2}{2m}\nabla^2 a - i\hbar\left(\frac{\partial a}{\partial t} + \frac{a}{2m}\nabla^2 S + \frac{1}{m}\nabla S\nabla a\right) = 0 \tag{8}$$

The complex equation (8) is divided into real and imaginary parts. Setting $\hbar = 0$, one obtains two equations

$$\frac{\partial S}{\partial t} + \frac{1}{2m}(\nabla S)^2 = 0, \qquad \frac{\partial a^2}{\partial t} + \nabla\left(a^2 \frac{\nabla S}{m}\right) = 0 \tag{9}$$

One pays attention that the first equation (9) is the Hamilton-Jacobi equation, whereas the second equation (9) describes displacement of the probability density a^2 according to the laws of the classical mechanics with the classical velocity $\nabla S/m$ at any point. However, one does not make the evident conclusion that the equations (9) are dynamic equations for some continuous dynamic system. One pays attention only to the fact that equations (9) are equations of classical mechanics, but it remains unclear, how many degrees of freedom (finite or infinite) the classical dynamic system has, although the result is quite clear from (9). It is supposed tacitly that the first equation (9) describes a single particle, having finite number of the freedom degrees. But why? Is this a result of earlier papers, where the limit transition $\hbar \to 0$ was realized directly in the equation (8)? Result of such a limit is the first equation (9), whereas the second equation disappears, because of the vanishing factor $i\hbar$. If so, it was a mistake.

To show that dynamic equations (9) are a partial case of dynamic equations generated by the action (3), we introduce the density ρ and the velocity $\mathbf{v}$ by means of designations

$$\rho = a^2, \qquad \mathbf{v} = \frac{1}{m}\nabla S \tag{10}$$

Taking gradient of the first equation (9), we obtain four hydrodynamic equations instead of two dynamic equations (9)

$$\partial_0 \mathbf{v} + (\mathbf{v}\nabla)\,\mathbf{v} = 0, \qquad \partial_0 \rho + \nabla(\rho \mathbf{v}) = 0 \tag{11}$$

Equations (11) is the Eulerian form of hydrodynamic equations for a fluid without a pressure. Variation of the action (3) with respect to $\mathbf{x}$ gives the Lagrangian form of hydrodynamic equations for the same fluid without a pressure

$$m\frac{d\mathbf{v}}{dt} = 0, \qquad \frac{d\mathbf{x}}{dt} = \mathbf{v} \tag{12}$$

Thus, the dynamic system (6) is a partial (irrotational) case of the action (3). It means that the wave function Ψ_b describes the statistical ensemble of free classical particles and, hence, *the wave function ψ describes statistical ensemble of free quantum particles, but not a single quantum particle.*

The statement that the wave function describes always a statistical ensemble $\mathcal{E}[\mathcal{S}_{\text{st}}]$ has very important corollaries: All predictions of quantum mechanics are statistical. They may be tested only by a massive measurement (M-measurement), consisting of many single measurements (S-measurements). S-measurement and M-measurement have different properties. The S-measurement of the quantity $\mathcal{R}$ leads to a definite value R' of the quantity $\mathcal{R}$ (see details in [4]). The S-measurement is irreproducible, in general, in other S-measurements in the same state ψ, whereas the M-measurement of the quantity $\mathcal{R}$ leads to a distribution $d(R')$ of values R' of the quantity $\mathcal{R}$, which is reproducible at other M-measurements at the same state ψ. Besides, any S-measurement does not influence on the state ψ of the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$, whereas the M-measurement may lead to a change of the state ψ (so-called reduction).

Let at the M-measurement of the quantity $\mathcal{R}$, we obtain some value R'. It is impossible at the M-measurement, because it gives a distribution $d(R')$, but not some value R' of the measured quantity $\mathcal{R}$. To obtain a definite value R' at the M-measurement, we need to redefine the M-measurement. We must to introduce the selective massive measurement (SM-measurement). It is the M-measurement accompanied by the selection of those S-measurements, constituting the M-measurement, which give at the measurement the value R' of the measured quantity $\mathcal{R}$. Collecting all these S-measurements, giving the value R', we may constitute a new statistical ensemble, whose state is described by the wave function $\psi_{R'}$. Transformation of the previous state ψ into the state $\psi_{R'}$ is the reduction of the wave function ψ. There is nothing enigmatic or paradoxical in this reduction. The reduction is a result of the statistical procedure (selection). Reduction becomes enigmatic or paradoxical, if we use one name (measurement) for three different measurements (S-measurement, M-measurement, SM-measurement). A use of one term for all different measurement and a confusion of them become unavoidable, if we suppose that the wave function describes the state of an individual particle. Note that all paradoxes are connected only with consideration of quantum measurements. They are absent at consideration of the quantum mechanics formalism, because the quantum formalism deals only with the wave function, and in this context it is of no importance, what does the wave function describe (a single particle, or a statistical ensemble).

From the beginning of the quantum mechanics construction, it was known that the Schrödinger equation may be considered as describing an irrotational flow of some fluid [5], which has infinite degrees of freedom. In the hydrodynamic representation it was easy to go to the limit $\hbar \to 0$ and to obtain that in the classical limit the wave function describes a dynamic system with infinite number of the freedom degrees. Such a dynamic system cannot be a classical particle, which has only six degrees of freedom. It means that *the wave function cannot describe an individual particle.*

Interest to interpretation of quantum mechanics was very large from the very beginning of the quantum mechanics creation up to now. Some scientific journals (Physics Today (1999), Uspechi Fizicheskich Nauk (2002)) organized discussions devoted to problems of quantum measurements. There are scientists [6, 7, 8], which believe that the wave function describes a statistical ensemble. There are scientists [9, 10, 11, 12], which believe that the wave function describes a single particle. There are scientists [13, 14, 15], whose position is intermediate. There is a lot of papers devoted to interpretation of quantum mechanics. Now discussion of different viewpoints is only of historical interest, because all discussions were produced on the verbal level. None of researchers had not set the problem mathematically:

Which of interpretation does follow from the quantum mechanics formalism? or in negative form: Which of interpretation is incompatible with the quantum mechanics formalism? Such a statement of the problem seems to be very reasonable. However, the question in such a form was not set. After mathematical solution of this problem any discussion on the verbal level seems to be useless. One may analyze and discuss initial statements of a mathematical theorem, but a verbal commentary to a proof of a mathematical theorem has no convincing force, it may be used only for clarification.

Beginning from the Heisenberg papers most scientists consider that the wave function describes an individual particle, and none of them sets the question on compatibility of such an interpretation with the quantum formalism. None of their opponents sets this question also, although such a statement of the problem is very reasonable.

We believe that such a situation is corollary of the general strategy of the quantum mechanics construction. The quantum mechanics was constructed by means of the trial and error method. This method is effective for description and explanation of concrete physical phenomena, but it is not adequate for construction of a fundamental theory, which tries to describe and to explain all physical phenomena from the united viewpoint. Using the trial and error method, one suggests some hypotheses, whose trueness is tested by an experiment. As a result of application of the trial and error method the quantum mechanics is a list of prescriptions (quantum principles), which stated what is the wave function and how this wave function should be used for description of physical phenomena. Agreement between the different prescriptions is not so important, as a possibility of the experimental data explanation. The conception, constructed by the trial and error method is not sensitive the trueness and logical agreement between different statements (prescriptions) of the conception. Some statements may be incorrect, or one statement my be incompatible with another. Nevertheless, the conception as a whole may explain some circle of experimental data. Incompatibility between some prescriptions may hinder from the expansion of the conception onto a wider circle of physical phenomena. It is the main defect of the compensating theory, i.e. a theory constructed by the trial and error method. In the framework of a compensating theory some mistakes in the foundation of the theory are not discovered and corrected. They are simply ignored, and their influence is compensated by newly invented hypotheses.

Quantum principles are the list of prescription, which admits one to describe and explain nonrelativistic quantum phenomena. Quantum principles are nonrelativistic, and this fact is an obstacle for construction of the relativistic quantum theory. Why do we fail to discover the evident fact, that the wave function describes the statistical ensemble of particles, but not an individual particle? Because the trial and error method does not provide for a test of compatibility of different points of the prescription list. All the more so, the given point concerns only interpretation but not the formalism of the quantum mechanics. Systematic use of the trial and error method as a main tool of theoretical research in quantum theory had led to appearance of a specific "fitting mentality".

Unfortunately, an influence of the fitting mentality appears not only in the interpretation of quantum mechanics. Let us consider another example of this mentality.

The Pauli particle $\mathcal{S}_{\rm P}$ is the dynamic system, described by the dynamic equation

$$i\hbar\partial_0\psi_{\rm P}=\frac{1}{2m}\left(i\hbar\partial_\mu+\frac{e}{c}A_\mu\right)\left(i\hbar\partial_\mu+\frac{e}{c}A_\mu\right)\psi_{\rm P}+\frac{ie\hbar}{2mc}\varepsilon_{\nu\mu\alpha}\partial_\nu A_\mu\sigma_\alpha\psi_{\rm P} \tag{13}$$

where $\psi_{\rm P}$ is the two-component complex wave function, A_μ is the electromagnetic potential and σ_α are 2×2 Pauli matrices.

The Dirac particle $\mathcal{S}_{\rm D}$ is the dynamic system, described by the Dirac equation

$$\gamma^l\left(-i\hbar\partial_l+\frac{e}{c}A_l\right)\psi_{\rm D}+mc\psi_{\rm D}=0 \tag{14}$$

where $\psi_{\rm D}$ is the four-component complex function, the quantities γ^i, $i=0,1,2,3$ are 4×4 complex constant matrices, satisfying the relation

$$\gamma^l\gamma^k+\gamma^k\gamma^l=2g^{kl}I,\qquad k,l=0,1,2,3. \tag{15}$$

Here I is the 4×4 identity matrix, and g^{kl} =diag$(c^{-2},-1,-1,-1)$ is the metric tensor. The quantity A_k, $k=0,1,2,3$ is the electromagnetic potential.

Conventionally the Pauli particle is considered to be a nonrelativistic approximation of the Dirac particle [16]. However, the Pauli equation (13) contains two complex first order differential equations for description of evolution of the two-component complex wave function $\psi_{\rm P}$, whereas the Dirac equation (14) contains four complex first order differential equations for description of evolution of the four-component complex wave function $\psi_{\rm D}$. How is it possible, that nonrelativistic approximation of the four-component wave function is a two-component wave function? Are another two components small? They are not small, in general. The additional two components have very high temporal frequencies. The energy of these high frequency components is radiated in the form of electromagnetic waves very rapidly. These components becomes small and unessential from experimental viewpoint, because they are unobservable at low frequencies.

Such a result is obtained, because one omits the temporal derivatives with small parameter c^{-2} before them and reduces the order of the system of differential equations. From mathematical viewpoint such a procedure is incorrect, because it leads to a loss of high frequency solutions (see details in [17]). Why this mathematical mistake was not discovered during eighty years of the quantum mechanics existence? Because of the fitting mentality of researchers, which are interested only in explanation of experimental data and ignore the logical consistency in the problems, which do not concerns experimental data directly. This example of application of the fitting mentality concerns already not only the QM interpretation. It concerns already the relativistic quantum effects, which are not observed now, but which may be observed in future.

Finally, we point out an application of the fitting mentality, which concerns conceptual problems of the relativistic quantum theory construction. From viewpoint of the conventional quantum theory the nonrelativistic character of quantum principles is the main problem of the relativistic quantum theory construction. Most researchers, working in the quantum field theory, believe that applying the conventional second quantization procedure to the classical fields, written in the relativistically covariant form, one can overcome the problem of the nonrelativistic character of quantum principles. Unfortunately, they are not right, because besides, one needs to use relativistic concept of the object of quantization. Particles and antiparticles are not an adequate object of quantization (PA-quantization). Relativistic object of the second quantization is the world line (WL-quantization). Practically nobody objects that the relativistic state of the particle is its world line, but in practice all researchers use a particle and an antiparticle as objects of the second quantization. It leads to an incompatibility of dynamic equations with the commutation relations (see details in [18]). Unfortunately, nobody tests compatibility of dynamic equation with the commutation relations, assuming that the commutation relations are something like initial conditions for solution of dynamic equations. The initial conditions may be given arbitrarily, and there is no necessity to test a their compatibility with the dynamic equations. However, there are no complete analogy between the initial condition and the commutation relations, one needs to test the compatibility of dynamic equations with the computation relations.

Such an approach, founded on the PA-scheme of the second quantization, is very complicated. It generates a use of the perturbation theory, the vacuum nonstationarity and other problems. But on the other hand, because of inconsistency such an approach admits one to obtain the particle production in such a situation, where the particle production is really absent. Practically the particle production appears at any nonlinear interaction, although the classical investigation shows, that the particle production is possible only for a very special form of interaction [19].

The WL-scheme of quantization, where dynamic equations are compatible with the commutation relations was applied [18] for the second quantization of the nonlinear complex scalar field φ, described by the dynamic equation

$$\hbar^2\partial_k\partial^k\varphi + m^2c^2\varphi = \lambda\varphi^+\varphi\varphi \tag{16}$$

where λ is the self-action constant and φ^+ is the Hermitian conjugate to φ. Application of the consistent WL-scheme of quantization admits one to eliminate the perturbational methods and use the exact methods of calculation, which contain, in particular, the stationary vacuum. One shows that the nonlinear interaction of the form $\lambda\varphi^+\varphi\varphi$ cannot produce particles. Besides, the simultaneous commutation relations appear to depend on the constant λ, which shows that the commutation relation may not be given arbitrarily. The corresponding paper [18] was not published, because the reviewer of the paper said: "The paper cannot be published, because the method, suggested by the author, does not permit to obtain the particle production". The fitting mentality has worked in the regular time, choosing inconsistent approach, which gives desirable results and removing the consistent approach which gives undesirable ones.

It is true, that the relativistic quantum theory, which cannot obtain the particle production, is not a satisfactory theory. However, it is true also that the inconsistent relativistic quantum theory, which explain the particle production, is not also a satisfactory theory, because it cannot predict new physical phenomena. It can only to explain them, using additional hypotheses. The author of the paper [18] knew in advance that a consistent use of quantum principles in the second quantization of the equation (16) cannot lead to a success. He hoped that the consistent method and negative result of its application to (16) may lead to hesitations in the validity of quantum principles and to a search of alternative approach (According to the logical Newtonian method, if one uses the consistent investigation methods and obtains incorrect result, one should look for a mistake in the foundation of the theory). But the author made an error, he did not take into account the fitting mentality of the microworld researchers.

Note, that in classical physics of 19th century the fitting mentality and a fitting in theoretical researches was absent. It was considered as something undignified in the investigation of a good researcher. However, the Newtonian logical method of investigation, which was used in 19th century is very sensitive to mistakes in the foundations of the theory. It cannot work effectively, if there is a mistake in the fundamental statements of a theory. On the contrary, the trail and error methods is insensitive to mistakes in the theory foundation. It uses additional hypotheses, and can work without any foundation, creating new foundation in the process of working.

The reasonable idea of describing quantum effects as a result of a statistical description of stochastically moving particle could not be realized mathematically, because of a simple mistake in the understanding of the statistical description. One believed that the statistical description and the probabilistic description is the same, and any statistical description is carried out in terms of the probability and probability density. In reality, the statistical description is a description of many independent identical objects, whereas the probabilistic description is a description, which uses the probability theory. Statistical description of pointlike objects may be produced in terms of the probability theory, whereas the statistical description of lengthy objects (for instance, world lines) in terms of probability is impossible. Statistical ensemble of stochastic world lines is described simply as a dynamic system, without a reference to the probability or to the probability density. This approach generates the dynamic conception of the statistical description (DCSD).

In particular, the statistical ensemble of nonrelativistic classical particles can be described either in the form (6) or in the form (3). In the same way, the action (1) for the Schrödinger particle can be written in the form of the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ of stochastic particles $\mathcal{S}_{\mathrm{st}}$. We have [20]

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}\left[\mathbf{x},\mathbf{u}\right]=\int\left\{\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^{2}+\frac{m}{2}\mathbf{u}^{2}-\frac{\hbar}{2}\nabla\mathbf{u}\right\}dtd\xi \tag{17}$$

where $\mathbf{u}=\mathbf{u}\left(t,\mathbf{x}\right)$ is a vector function of arguments $t,\mathbf{x}$ (not of t,ξ), and $\mathbf{x}=\mathbf{x}\left(t,\xi\right)$ is a vector function of independent variables t,ξ. The 3-vector $\mathbf{u}$ describes the mean value of the stochastic component of the particle velocity, which is a function of the variables $t, \mathbf{x}$. The first term $\frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2$ describes the energy of the regular component of the particle motion. The second term $m\mathbf{u}^2/2$ describes the energy of the random component of velocity. The components $\frac{d\mathbf{x}}{dt}$ and $\mathbf{u}$ of the total velocity are connected with different degrees of freedom, and their energies should be added in the expression for the Lagrangian density. The last term $-\hbar\nabla\mathbf{u}/2$ describes interaction between the regular component $\frac{d\mathbf{x}}{dt}$ and random one $\mathbf{u}$.

The particles $\mathcal{S}_{\text{st}}$ are stochastic, because there are no dynamic equations for them. Indeed, *formally* the action for a single stochastic particle $\mathcal{S}_{\text{st}}$ has the form

$$\mathcal{A}_{\mathcal{S}_{\text{st}}}[\mathbf{x}, \mathbf{u}] = \int \left\{ \frac{m}{2}\left(\frac{d\mathbf{x}}{dt}\right)^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\nabla\mathbf{u} \right\} dt \tag{18}$$

where $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$ is a vector function of arguments $t, \mathbf{x}$, and $\mathbf{x} = \mathbf{x}(t)$ is a vector function of independent variable t. The action functional (18) is not well defined, if $\hbar \neq 0$, because the operator $\nabla = \left\{\frac{\partial}{\partial x^\alpha}\right\}$, $\alpha = 1, 2, 3$ is defined in the three-dimensional vicinity of the point $\mathbf{x}$, but not in the one-dimensional vicinity of the point $\mathbf{x}$ on the one-dimensional curve. Variation of (17) with respect to dependent variables $\mathbf{x}$ and $\mathbf{u}$ generates dynamic equations which contain the Schrödinger equation as a partial case. Statistical description in the form of the action (17) does not contain any reference to a probability density.

Statistical description of stochastic world lines in terms of probability was impossible, the dynamic conception of statistical description was unknown. As a result the trial and error method of investigation appeared to be the only possible method of the quantum phenomena investigation. Application of this method admits one to construct the nonrelativistic quantum mechanics on the basis of quantum principles, which are perceived simply as a list of prescriptions for working with such an enigmatic object as the wave function. Three generations of scientists used the trial and error method as the *only possible method* of research. The old logical Newtonian method was forgotten. It did not work under such condition, when there are mistakes in the primary principles of the fundamental theory. The fitting mentality is a result of the long application of the trial and error method. Application of the trial and error method for construction of the relativistic quantum theory is very difficult, because the relativistic quantum theory is constructed on the basis of *non-relativistic* quantum principles, which prevent the created theory from introducing relativistic concepts in it.

Now when one has succeeded to correct some mistakes in the foundation of the microcosm phenomena theory and to construct DCSD, one needs only to apply correctly the fundamental principles to the relativistic phenomena. At this stage of investigation the fitting mentality is the main obstacle on the way of the fundamental theory development. The fact is that, the criteria of a true theory are different in the compensating theory (created by means of the trial and error method) and in the fundamental theory (created on the basis of primary principles). For instance, at the presentation of new theory the typical question looks as follows. What new phenomena does your theory explain? This question is crucial at the presentation of a new compensating theory, because the new compensating theory suggested some new hypotheses. The suggested question tests, to what extent the new hypotheses are satisfactory. At the same time this question is useless at the presentation of the fundamental theory, because there are no new hypotheses in the fundamental theory. The fundamental principles are not tested by a single experiments. They are verified by the totality of all experimental data. If the fundamental principles are true, any test of predictions of the theory in a single experiment verifies only correctness of application of fundamental principles in the given case, but not the trueness of the fundamental principles themselves. Correctness of application is determined by the qualification of the researcher, but not by the trueness of the fundamental principles. It is a reason, why predictions of the true fundamental theory for new physical phenomena appear to be more exact, than those of the compensating theory. In many cases the compensating theory cannot make prediction at all. It can only explain, using new hypotheses.

Finally, we shall point out, how one should construct relativistic quantum theory, using fundamental principles. These fundamental principles are the conventional classical principles. For instance, we want to quantize the free relativistic particle of the mass m. The quantization operation is a usual dynamic procedure. We construct statistical ensemble of classical relativistic particles described by the action

$$\mathcal{A}[x] = -\int mc\sqrt{g_{ik}\dot{x}^i\dot{x}^k}d^4\xi, \qquad d^4\xi = d\xi_0 d\boldsymbol{\xi}, \tag{19}$$

where $x = \{x^i(\xi_0, \boldsymbol{\xi})\}$, $i = 0,1,2,3$ are dependent variables. $\xi = \{\xi_0, \boldsymbol{\xi}\} = \{\xi_k\}$, $k = 0,1,2,3$ are independent variables, and $\dot{x}^i \equiv dx^i/d\xi_0$.

To quantize the relativistic particle, we should make the change

$$m \to mK, \qquad K = \sqrt{1 + \lambda^2(\kappa_l\kappa^l + \partial_l\kappa^l)}, \qquad \lambda = \frac{\hbar}{mc} \tag{20}$$

where the quantities $\kappa^l = \{\kappa^l(x)\}$, $l = 0,1,2,3$ are additional dependent variables, describing stochastic component of the particle motion. Then the action for the statistical ensemble of stochastic (quantum) particles takes the form

$$\mathcal{A}[x,\kappa] = \int \left\{-mc\sqrt{1 + \lambda^2(\kappa_l\kappa^l + \partial_l\kappa^l)}\sqrt{g_{ik}\dot{x}^i\dot{x}^k} - \frac{e}{c}A_k\dot{x}^k\right\} d^4\xi, \tag{21}$$

which does not contain neither wave function, nor operators. The action (21) does not contain any reference to the quantum principles, and interpretation of description is realized in terms of the particle position x^k. Nevertheless, using a proper change of variables one can introduce the wave function [19] and show, that in terms of the wave function the dynamic equation, generated by the action (21), is the Klein-Gordon equation. In the nonrelativistic approximation the action (21) turns into the action (17)

To explain the pair production effect, it is necessary to apply DCSD to the relativistic dynamic systems, consisting of several particles. There is no necessity to invent new hypotheses for this purpose.

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