Termination of the physics relativization and logical reloading in the space-time geometry

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Abstract

In the beginning of the twentieth century the relativity theory had not been completed in the sense that dynamic equations were relativistic, but the particle state remained to be nonrelativistic. Consecutive relativistic approach admits one to construct unified formalism of the particle dynamics which can be applied for deterministic and stochastic motion of particles. This formalism admits one to found the quantum mechanics and to explain quantum phenomena without a use of quantum principles. Refusing from the constraint on continuity of the space-time geometry and using the metric approach to geometry, one explains stochastic motion of elementary particles and constructs the skeleton conception of particle dynamics. The skeleton conception admits one to investigate the elementary particle structure (but not only to systematize the elementary particles, ascribing quantum numbers to them).

Key words: elementary length; discrete space-time geometry; world function; logical reloading; geometrization of particle parameters; skeleton conception of particle dynamics;

1 Introduction

In the beginning of the twentieth century the relativity theory had not been completed in the sense that dynamic equations were relativistic, but the description of the particle state remained to be nonrelativistic. Nonrelativistic concept of the particle state is a point in the 3-dimensional space or in the phase space. The real relativistic definition of the particle state looks otherwise.

Conventionally the special relativity principle is formulated as the Lorentz-invariance of dynamical equations. On the other hand, a general physical principles can be hardly formulated as a statement connected with such details of description as a coordinate transformation. We formulated the relativity principle as follows. *The space-time is described by one space-time structure ST.* It means that the space-time geometry is described by the only quantity: space-time distance ρ, or only by the world function $\sigma = \frac{1}{2}\rho^2$. In the non-relativistic physics the space-time is described by means of two independent quantities (structures): spatial distance S and and temporal interval T. Among three structures: T, S, and ST only two of them are independent. Such a formulation of the relativity principle is more general, because it is valid not only for the space-time geometry of Minkowski. It is valid for any space-time geometry, including a discrete space-time geometry. Besides, this formulation is coordinateless.

One cannot be sure that the space-time geometry is continuous in microcosm. Restricting our consideration by the continuous space-time geometries, we are mistaken. This mistake is justified by the fact that the formalism of a discrete geometry has not been developed, and one believes that the space-time geometry cannot be discrete. In reality, a discrete geometry, as any geometry, is a generalization of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. But the Euclidean geometry is to be described in terms of distance ρ and only in terms of distance, because other concepts of $\mathcal{G}_{\mathrm{E}}$ contain a reference to continuity of $\mathcal{G}_{\mathrm{E}}$, and they cannot be used for a construction of a discrete geometry.

The simplest discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ is described by the world function

$$\sigma_{\mathrm{d}}\left(P,Q\right) = \sigma_{\mathrm{M}}\left(P,Q\right) + \frac{\lambda_0^2}{2}\operatorname{sgn}\left(\sigma_{\mathrm{M}}\left(P,Q\right)\right), \quad \forall P,Q \in \Omega \tag{1.1}$$

where Ω is a set of all points of the space-time, σ_{M} is the world function of the Minkowski space-time geometry $\mathcal{G}_{\mathrm{M}}$, and λ_0 is the elementary length. In the inertial coordinate system the world function σ_{M} has the form

$$\sigma_{\mathrm{M}}\left(x,x'\right) = \frac{1}{2}g_{ik}\left(x^i - x'^i\right)\left(x^k - x'^k\right), \quad g_{ik} = \operatorname{diag}\left(c^2,-1,-1,-1\right) \tag{1.2}$$

In the discrete space-time geometry a pointlike particle cannot be described by a world line, because any world line is a limit of the broken line, when lengths of its links tend to zero. But in the discrete geometry $\mathcal{G}_{\mathrm{d}}$ there are no infinitesimal lengths, because all lengths are longer, than λ_0. In $\mathcal{G}_{\mathrm{d}}$ a pointlike particle is described by a world chain (broken line) instead of a smooth world line. Description of a pointlike particle state by means of the particle position and its momentum becomes inadequate. The reason lies in the fact that in the continuous (differential) space-time geometry the particle 4-momentum p_k is described by the relation

$$p_k = g_{kl}\frac{dx^l}{d\tau} = g_{kl}\lim_{d\tau\rightarrow 0}\frac{x^l\left(\tau + d\tau\right) - x^l\left(\tau\right)}{d\tau} \tag{1.3}$$

where $x^l = x^l\left(\tau\right)$, $l = 0,1,2,3$ is an equation of the world line. The limit in the formula (1.3) does not exist in $\mathcal{G}_{\mathrm{d}}$, and the 4-momentum p_k is not defined (at any rate in such a form). In general, the mathematical formalism of a differential geometry, based on the infinitesimal calculus (differential dynamic equations), is inadequate in the discrete space-time geometry, where infinitesimal distances are absent.

In the case of arbitrary space-time geometry the particle state is described by two space-time points. The two points P, Q determine the vector $\mathbf{PQ} = \{P, Q\}$, which can be interpreted as the particle momentum. In the case of a discrete space-time geometry $\mathcal{G}_{\rm d}$ the vector $\mathbf{PQ}$ can be also interpreted as a momentum, but its presentation in the form (1.3) is impossible.

In the arbitrary space-time geometry the pointlike particle is described by a world chain $\mathcal{C}$

$$\mathcal{C} = \bigcup_s P_s, \quad |\mathbf{P}_s\mathbf{P}_{s+1}| = \mu = \text{const}, \quad s = ...0, 1, ... \tag{1.4}$$

Here μ is a geometric mass of the particle (length of the world chain link), which is connected with the particle mass m by the relation

$$m = b\mu \tag{1.5}$$

where b is an universal constant.

In $\mathcal{G}_{\rm d}$ only coordinateless description is possible [1], which is produced in terms and only in terms of the world function $\sigma_{\rm d}$, or in terms of the space-time distance $\rho_{\rm d}$, because a use of all geometric concepts of the Riemannian geometry (except for distance) contains a reference to continuity of the geometry. In the coordinateless description the scalar product $(\mathbf{PQ}.\mathbf{RS})$ of two vectors $\mathbf{PQ}$ and $\mathbf{RS}$ has the form

$$(\mathbf{PQ}.\mathbf{RS}) = \sigma(P, S) + \sigma(Q, R) - \sigma(P, R) - \sigma(Q, S) \tag{1.6}$$

and

$$|\mathbf{PQ}|^2 = (\mathbf{PQ}.\mathbf{PQ}) = 2\sigma(P, Q) \tag{1.7}$$

The coordinateless definitions of the scalar product $(\mathbf{PQ}.\mathbf{RS})$ and of the vector length $|\mathbf{PQ}|$ coincide with their conventional definitions in the proper Euclidean geometry. They can be used in any space-time geometry $\mathcal{G}$, which is completely described by its world function σ. Such a space-time geometry will be referred to as a physical geometry.

Equivalency $(\mathbf{PQ}\text{eqv}\mathbf{RS})$ of two vectors $\mathbf{PQ}$ and $\mathbf{RS}$ is defined by two coordinateless relations

$$(\mathbf{PQ}\text{eqv}\mathbf{RS}): \quad (\mathbf{PQ}.\mathbf{RS}) = |\mathbf{PQ}| \cdot |\mathbf{RS}| \wedge |\mathbf{PQ}| = |\mathbf{RS}| \tag{1.8}$$

The discrete space-time geometry is multivariant in the sense, that there are many vectors $\mathbf{PQ}, \mathbf{PQ}', \mathbf{PQ}''$,... at the point P which are equivalent to the vector $\mathbf{RS}$ at the point R, but vectors $\mathbf{PQ}, \mathbf{PQ}', \mathbf{PQ}''$,... are not equivalent between themselves.

In the proper Euclidean geometry $\mathcal{G}_{\rm d}$ the equivalence relation (1.8) is single-variant, and there is only one vector $\mathbf{PQ}$ at the point P which is equivalent to the vector $\mathbf{RS}$ at the point R.

If the world chain (1.4) describes a free particle, its links satisfy the relations

$$(\mathbf{P}_s\mathbf{P}_{s+1}\mathrm{eqv}\mathbf{P}_{s+1}\mathbf{P}_{s+2}), \quad s = ...0, 1, ... \tag{1.9}$$

These relations are multivariant in $\mathcal{G}_{\mathrm{d}}$. It leads to a wobbling of the world chain. This wobbling means that the particle motion is stochastic (random). Amplitude of wobbling is restricted by the elementary length λ_0 in $\mathcal{G}_{\mathrm{d}}$ for timelike vectors, But this amplitude is infinite for spacelike vectors. In the geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$ the wobbling is absent for timelike vectors ($\lambda_0 = 0$), and amplitude of this wobbling is infinite for spacelike vectors.

In the nonrelativistic approximation a statistical description of timelike world lines in $\mathcal{G}_{\mathrm{d}}$ leads to the Schrödinger equation [2], if the elementary length

$$\lambda_0^2 = \frac{\hbar}{bc} \tag{1.10}$$

where b is the universal constant defined by (1.5). A single particle with the spacelike world chain (tachyon) cannot be detected, because of the infinite amplitude of its wobbling. However, gravitational field of the tachyon gas can be detected (dark matter) [3, 4].

It is very important that the statistical description of wobbling world chains is produced relativistically, when the pointlike particle state is described by two points (but not by a point in the phase space). In this case the statistical ensemble is a dynamic system of the type of a continuous medium, and one may introduce the wave function as a method of the continuous medium description [5]. In the nonrelativistic description the particle state is a point in the phase space. In this case the statistical description is a probabilistic construction describing evolution of the particle state probability [6, 7, 8].

Let us stress that the statistical ensemble as a dynamic system (but not as a probabilistic construction) is a result of a correct (relativistic) definition of the particle state (but not a result of some new hypothesis). Description of the particle motion by means of the world chain (1.4) is a corollary of the consecutive application of the relativity principle.

2 Unified formalism of particle dynamics

Foundation of the quantum mechanics on the basis of the stochastic particle dynamics is obtained as corollary of unified formalism of the particle dynamics [9]. Stochastic particle $\mathcal{S}_{\mathrm{st}}$ is not a dynamic system, and there are no dynamic equations for $\mathcal{S}_{\mathrm{st}}$. However, statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$, i.e. a set of many independent stochastic particles $\mathcal{S}_{\mathrm{st}}$, is a dynamic system of the type of the continuous medium. Deterministic particle $\mathcal{S}_{\mathrm{det}}$ as well as statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{det}}]$ are dynamic systems, and there are dynamic equations for them. At the conventional approach to the particle dynamics, when the basic element of dynamics is a single particle, one cannot construct a unified dynamic conception for stochastic and deterministic particles, because there are no dynamic equations for a single stochastic particle. However, after the logical reloading, when the statistical ensemble becomes a basic object of the particle dynamics, one obtains dynamic equations for statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ of stochastic particles $\mathcal{S}_{\mathrm{st}}$ and for statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{det}}]$ of deterministic particles $\mathcal{S}_{\mathrm{det}}$ [9].

For instance, the action for the statistical ensemble of stochastic particles $\mathcal{S}_{\mathrm{st}}$ has the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}[\mathbf{x},\mathbf{u}] = \int\int_{V_\xi} \left\{ \frac{m}{2}\dot{\mathbf{x}}^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\nabla\mathbf{u} \right\} \rho_1(\boldsymbol{\xi})\, dt d\boldsymbol{\xi}, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \tag{2.1}$$

The variable $\mathbf{x} = \mathbf{x}(t,\boldsymbol{\xi})$ describes the regular component of the particle motion. The independent variables $\boldsymbol{\xi} = \{\xi_1,\xi_2,\xi_3\}$ label elements (particles) of the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$. The variable $\mathbf{u} = \mathbf{u}(t,\mathbf{x})$ describes the mean value of the stochastic velocity component, $\hbar$ is the quantum constant, $\rho_1(\boldsymbol{\xi})$ is a weight function. One may set $\rho_1 = 1$. The second term in (2.1) describes the kinetic energy of the stochastic velocity component. The third term describes interaction between the stochastic component $\mathbf{u}(t,\mathbf{x})$ and the regular component $\dot{\mathbf{x}}(t,\boldsymbol{\xi})$ of the particle velocity. The operator

$$\nabla = \left\{ \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3} \right\} \tag{2.2}$$

is defined in the space of coordinates $\mathbf{x}$.

Formally the action (2.1) describes a set of deterministic particles $\mathcal{S}_{\mathrm{d}}$, interacting via the force field $\mathbf{u}$. The particles $\mathcal{S}_{\mathrm{d}}$ form a gas (or a fluid), described by the variables $\dot{\mathbf{x}}(t,\boldsymbol{\xi}) = \mathbf{v}(t,\boldsymbol{\xi})$. Here this description is produced in the Lagrange representation. Hydrodynamic description is produced in terms of density ρ and velocity $\mathbf{v}$, where

$$\rho = \rho_1 J, \quad J \equiv \frac{\partial(\xi_1,\xi_2,\xi_3)}{\partial(x^1,x^2,x^3)}, \quad v^\alpha = \frac{\partial(x^\alpha,\xi_1,\xi_2,\xi_3)}{\partial(t,\xi_1,\xi_2,\xi_3)}, \quad \alpha = 1,2,3 \tag{2.3}$$

Nonrotational flow of this gas is described by the Schrödinger equation [9].

The dynamic equation for the force field $\mathbf{u}$ is obtained as a result of variation of (2.1) with respect to $\mathbf{u}$. It has the form

$$\mathbf{u} = \mathbf{u}(t,\mathbf{x}) = -\frac{\hbar}{2m}\nabla \ln\rho \tag{2.4}$$

The vector $\mathbf{u}$ describes the mean value of the stochastic velocity component of the stochastic particle $\mathcal{S}_{\mathrm{st}}$. In the nonrelativistic case the force field $\mathbf{u}$ is determined by its source: the fluid density ρ.

In terms of the wave function the action (2.1) takes the form [9]

$$\mathcal{A}[\psi,\psi^*] = \int \left\{ \frac{i\hbar}{2}(\psi^*\partial_0\psi - \partial_0\psi^*\cdot\psi) - \frac{\hbar^2}{2m}\nabla\psi^*\cdot\nabla\psi + \frac{\hbar^2}{8m}\rho\nabla s_\alpha\nabla s_\alpha \right\} \mathrm{d}^4x \tag{2.5}$$

where the wave function $\psi = \left\{ \begin{smallmatrix} \psi_1 \\ \psi_2 \end{smallmatrix} \right\}$ has two complex components.

$$\rho = \psi^* \psi, \qquad s_\alpha = \frac{\psi^* \sigma_\alpha \psi}{\rho}, \qquad \alpha = 1, 2, 3 \tag{2.6}$$

σ_α are 2×2 Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2.7}$$

Dynamic equation, generated by the action (2.5), has the form

$$i\hbar \partial_0 \psi + \frac{\hbar^2}{2m} \boldsymbol{\nabla}^2 \psi + \frac{\hbar^2}{8m} \boldsymbol{\nabla}^2 s_\alpha \cdot (s_\alpha - 2\sigma_\alpha) \psi - \frac{\hbar^2}{4m} \frac{\boldsymbol{\nabla}\rho}{\rho} \boldsymbol{\nabla} s_\alpha \sigma_\alpha \psi = 0 \tag{2.8}$$

In the case of one-component wave function ψ, when the flow is nonrotational and $\boldsymbol{\nabla} s_\alpha = 0$, the dynamic equation has the form of the Schrödinger equation

$$i\hbar \partial_0 \psi + \frac{\hbar^2}{2m} \boldsymbol{\nabla}^2 \psi = 0 \tag{2.9}$$

Thus, the Schrödinger equation is a special case of the dynamic equation, generated by the action (2.1) or (2.5). Thus, linearity of dynamic equation in terms of the wave function is a special case of dynamic equation for the statistical ensemble of stochastic particles, although it is considered usually as a principle of quantum mechanics.

3 Reason of the elementary particles stochasticity

Stochasticity of elementary particles and wobbling of their world chains are conditioned by discreteness of the space-time geometry, more exactly by its multivariance [1]. A discrete geometry is constructed as a generalization of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. But for such a generalization one needs to produce a logical reloading and to present $\mathcal{G}_{\mathrm{E}}$ in terms of the world function [10, 11]. A use of the discrete space-time geometry admits one to formulate the skeleton conception of elementary particles, where the particle state and all parameters of an elementary particle are described by the particle skeleton [12]. The skeleton is several space-time points, connected rigidly. Distances between the skeleton points determine parameters of the particle. World chain (1.4) with the two-point skeleton describes the simplest case of elementary particle. In this case there is only one parameter of the skeleton. It is the length $\mu = |\mathbf{P}_s \mathbf{P}_{s+1}|$ of the world chain link. According to (1.5) the particle mass is a geometrical quantity. In other words, description of a particle motion is geometrized completely.

A generalization of two-point skeleton of a pointlike particle arises at consideration of the Dirac equation [13, 14, 15]. Analyzing the Dirac equation from the viewpoint of quantum mechanics, one meets abstract dynamic variables (γ-matrices), whose meaning is unclear. Analyzing the Dirac equation and using the united formalism of particle dynamics (without a use of quantum principles), one concludes that world line of the Dirac particle is a helix with timelike axis. Helical motion of a free particle is possible, if its skeleton contains three (or more) points [16]. Helical motion of a particle explains the particle spin and magnetic moment, whereas at the quantum approach spin and magnetic moment are simply quantum numbers, whose nature is unknown. Thus, the skeleton conception of elementary particle dynamics admits one to investigate structure and arrangement of elementary particles.

The skeleton conception is obtained as a direct corollary of physical principles without a use of artificial principles and hypotheses alike the quantum mechanics principles. It is the main worth of the skeleton conception. The skeleton conception is obtained as a result of correction mistakes in the conventional theory: (1) nonrelativistic concept of the particle state and (2) unfounded restriction by the continuous space-time geometry. Correction of these mistakes leads to the skeleton conception without any additional suppositions.

A use of the skeleton conception admits one to explain the dark matter as a tachyon gas and to explain impossibility of a single tachyon detection [3, 4]. These phenomena cannot be explained from the point of view of quantum approach.

A use of the logical reloading is followed by essential change of a mathematical formalism. This change of formalism is perceived hardly by people, using conventional formalism. For instance, the discrete geometry $\mathcal{G}_{\mathrm{d}}$ described by the world function (1.1) is uniform and isotropic. Indeed, the world function σ_{M} (1.2) of the geometry of Minkowski is invariant with respect to Poincare group of transformations. The world function σ_{d} (1.1) is a function of σ_{M}. It is also invariant with respect to Poincare group of transformations. It means that the discrete geometry (1.1) is uniform and isotropic. This fact contradicts to conventional approach to the discrete geometry which is considered as a geometry on a lattice. Geometry on a lattice cannot be uniform and isotropic. Besides, in the discrete geometry (1.1) there is no definite dimension (maximal number of linear independent vectors). At the conventional approach to geometry such a situation is impossible, because any construction of a geometry begins from a fixation of the geometry dimension in the form of a natural number.

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