Unification of classical mechanics and quantum mechanics in unique conception of particle dynamics

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Abstract

It is shown that motion of quantum particles and classical particles can be described in the framework of the same formalism. Stochasticity of particle motion depends on the form of the space-time geometry, which is to be described as a physical geometry, i.e. a geometry obtained as a result of deformation of the proper Euclidean geometry. The new method of the particle motion description does not use quantum principles. It admits one to use the structural approach to theory of elementary particles. The structural approach admits one to consider structure and arrangement of elementary particles, that cannot been obtained at conventional approach, using quantum principles.

Key words: physical geometry; stochastic particles; dynamical disquantization; discrete geometry

1 Introduction

The most fast progress of mechanics arises, when two different conceptions of mechanics are united into one conception. In this paper one unites classical mechanics and quantum mechanics into united conception of particle dynamics. Such a unification admits one to investigate structure and arrangement of elementary particles. It is impossible in framework of quantum mechanics.

The Aristotelian mechanics contains the Earth mechanics and celestial mechanics [1]. These mechanics contain different objects and different concepts. Sir Isaac Newton united the two mechanics into the classical mechanics, which contained another concepts and another objects. For instance, in the classical mechanics such concepts as velocity and acceleration appeared. In Aristotelian mechanics these concepts were absent, because in the Aristotelian mechanics only ratios of like quantities were used. For instance, ratio of two lengths, or ratio of two periods of time. Concept of inertia was absent in Aristotelian mechanics. Construction of new concepts is a very difficult thing. An use of two alternative conceptions lightens this process.

Stochastic (nondeterministic) particles were not known in the time of Newton, and existence of stochastic particles was not taken into account in classical mechanics. Stochastic (quantum) particles were discovered in the beginning of the XX century One failed to apply classical mechanics for description of quantum particles. As a result the quantum mechanics has been created for description of quantum particles. Can the classical mechanics (CM) and the quantum mechanics (QM) be united into one mechanics? It is a very interesting question. Most researcher believe, that such a unification is impossible.

Unification of quantum mechanics (QM) and classical mechanics (CM) is produced on the basis of inverse problem of Madelung [2], who showed,that Schrödinger equation can be presented as gas dynamic equation, describing some nonrotational flow of gas. The inverse problem cannot be solved in XX century, because one did not know that wave function is a natural attribute of gas dynamics [3]

Let us consider a gas, whose molecules interact via classical force field κ^l, $l = 0,1,2,3$ which changes the particle mass m

$$m^2 \rightarrow M^2 = m^2 + \frac{\hbar^2}{c^2}\left(\kappa_l \kappa^l + \partial_l \kappa^l\right) \tag{1.1}$$

where $\hbar$ is the quantum constant. Variational principle for such a gas has the form

$$\mathcal{A}[x,\kappa] = \int_{\xi_0} \int_{V_\xi} \left(-mcK\sqrt{g_{lk}\dot{x}^l \dot{x}^k} - \frac{e}{c}A_l \dot{x}^l\right) d^4\xi, \quad \dot{x}^i = \frac{\partial x^i}{\partial \xi_0} \tag{1.2}$$

$$K = \frac{M}{m} = \sqrt{1 + \lambda^2\left(\kappa_l \kappa^l + \partial_l \kappa^l\right)}, \quad \lambda = \frac{\hbar}{mc}, \quad \partial_l \equiv \frac{\partial}{\partial x^l} \tag{1.3}$$

where κ^l, $\quad l = 0,1,2,3$ is a classical force field. Dynamic equations for irrotational flow of this gas is the Klein-Gordon equation [4]

$$\left(i\hbar\partial_k - \frac{e}{c}A_k\right)\left(i\hbar\partial^k - \frac{e}{c}A^k\right)\psi - m^2c^2\psi = 0 \tag{1.4}$$

Thus, classical force field $\kappa_l = \partial_l \kappa$ is responsible for quantum effects and pair production It means that quantum mechanics can be founded by means of classical gas dynamics.

However, to show, that QM and CM are different branches of the same conception, it is necessary to show, that dynamic equations of QM and those of CM are defined in the same way. But usually a classical particle is described as a single particle, whereas a quantum particle is a stochastic particle. It is described as a statistical ensemble of stochastic particles. A stochastic particle cannot be described as a single particle.

Quantum mechanics and classical mechanics can be united on the basis of logical reloading [6]. The logical reloading means that basic objects of classical dynamics (a single dynamic system S) is substituted by statistical ensemble $\mathcal{E}[S]$ of single dynamic systems S. Such a substitution does not change classical dynamics, because one can obtain dynamic equations for $\mathcal{E}[S]$ from dynamic equations for single S and vice versa. For a stochastic system one can obtain dynamic equations for statistical ensemble $\mathcal{E}[S_{\mathrm{st}}]$ of stochastic systems S_{st}. However, dynamic equations for a single stochastic system S_{st} cannot be obtained. Such a unification on the basis of logical reloading is possible, but it is less effective, than unification on the basis of physical geometry of space-time. Mathematical formalism on the basis of physical geometry admits one to investigate structure of elementary particles.

World lines of quantum particles are stochastic. They wobble. This wobbling can be described in the discrete space-time geometry. The discrete space-time geometry is such a geometry, where there is a minimal length λ_0. This condition can be written in the form

$$|\rho(P,Q)| \notin (0,\lambda_0), \quad \forall P,Q \in \Omega \tag{1.5}$$

where Ω is the set of points, where space-time geometry is given. $\rho(P,Q)$ is the distance between points P and Q. Usually one considers condition (1.5) as a constraint on Ω. Remaining only countable number of points in Ω, one obtains so called geometry on a lattice. It is impossible to work with the space-time geometry, which is a geometry on a lattice. For instance, one cannot define world line of a particle in the space-time geometry given on a lattice.

It is more effective to consider condition (1.5) as a constraint on the metric ρ or on the world function $\sigma = \frac{1}{2}\rho^2$. The world function σ_{d} of the simplest discrete geometry $\mathcal{G}_{\mathrm{d}}$ is described by the relation

$$\sigma_{\mathrm{d}}(P,Q) = \sigma_{\mathrm{M}}(P,Q) + \frac{\lambda_0^2}{2}\mathrm{sgn}(\sigma_{\mathrm{M}}(P,Q)), \quad \forall P,Q \in \Omega = \Omega_{\mathrm{M}} \tag{1.6}$$

where σ_{M} is world function of the geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$. Ω_{M} is the set of points, where $\mathcal{G}_{\mathrm{M}}$ is given. It easy to verify, that $\rho_{\mathrm{d}} = \sqrt{2\sigma_{\mathrm{d}}}$ satisfies the relation (1.5). World line of a free particle in $\mathcal{G}_{\mathrm{d}}$ wobbles [5]. It means that a discreteness of the space-time geometry may be a reason of the world lines wobbling. Maybe, one succeeds to replace the quantum wobbling of world lines by a use of a discrete space-time geometry. Then one succeeds to unite classical mechanics and quantum mechanics in united conception of particle dynamics.

If such a unification is possible, then deterministic particles and quantum particles will be described by the same mathematical formalism. But, in the conventional dynamics the dynamic equations describe any individual deterministic particle, whereas they describe some statistical average characteristics of quantum particles. There are conceptual mistakes in foundations of contemporary physics [7]. One of such mistakes consists in the statement, that space-time geometry is a Riemannian one. In reality the real space-time geometry is not a Riemannian geometry. General type of space-time geometry is a physical geometry, obtained as the Euclidean geometry deformation. In particular, the discrete geometry is a special type of a physical geometry. World line of a particle in physical space-time geometry is described as a broken line $\mathcal{L}_{\mathrm{br}}$ with vortices P_s, $s = ...0, 1...$ For free particle the adjacent vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$ are equal. Then dynamic equations look as follows:

$$\sigma(P_s, P_{s+1}) = \sigma(P_{s+1}, P_{s+2}), \quad s = ... - 1, 0, 1, ...$$

$$(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2}) = 2\sqrt{\sigma(P_s, P_{s+1})\,\sigma(P_{s+1}, P_{s+2})}, \quad s = ... - 1, 0, 1, ...$$

Here $(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2})$ is the scalar product of two vectors.

$$(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2}) = \sigma(P_s, P_{s+2}) + \sigma(P_{s+1}, P_{s+1}) - \sigma(P_s, P_{s+1}) - \sigma(P_{s+1}, P_{s+2})$$

In general, two dynamic equations are insufficient for unique determination of four dynamic variables (t, x, y, z). As a result the broken line $\mathcal{L}_{\rm br}$ appears to be stochastic, generally speaking. But in the geometry of Minkowski the timelike world line $\mathcal{L}_{\rm br}$ appears to be deterministic. As far as deterministic world lines and stochastic world lines can be described by the same dynamic formalism, it means that classical mechanics and quantum mechanics can be united into one mechanics.

Dynamic equations describing a single quantum particle must have many solutions, in order that world line of a quantum particle wobbles. Statistical averaging over this wobbling must lead to conventional quantum dynamic equations. Unfortunately, we cannot obtain description of a single quantum particle. Why? The answer is rather unexpected. We cannot obtain dynamic equation for a single stochastic particle, because we know space-time geometry incomplete (we use only Riemannian space-time geometry instead of a physical geometry.)

A negative relation of mathematicians to the physical geometry hinders to development and application of physical geometry. When mathematicians had understood, that the physical geometry is not a logical construction, because the equivalence relation is intransitive, they stated, that such a geometry cannot exist, because a geometry is a logical construction with a necessity. When I submitted a report on physical geometry to a seminar in Steklov Mathematical Institute, secretary of the seminar said me: "How strange geometry. There are no theorems in it. Only definitions." He was right. In the physical geometry there are no theorems. All definitions are taken from the proper Euclidean geometry, where they are obtained by means of Euclidean theorems. At deformation of Euclidean geometry, which is a logical construction, logical connections between different geometrical statements were violated. It is possible that the physical geometry is not interesting for mathematicians, because it does not contain theorems. (Proves of theorems is a favorite business of mathematicians.)

A use of the physical space-time geometry admits one to use a structural approach to the elementary particle theory [8, 9, 10], when one describes structure and arrangement of elementary particles. Usually one uses experimentally analytical approach, when any elementary particle is considered as a pointlike physical object, whose properties are described by means of a set of quantum numbers assigned to the elementary particle. These quantum numbers were obtained from experiments. Mathematical formalism of quantum theory does not admit to consider arrangement of an elementary particle. When one discover experimentally a complex structure of nucleons, quarks were considered as single particles, which cannot leave nucleon, although it would be more reasonable to consider quarks as elements of the nucleon structure.

The same picture we had at investigations of chemical elements. In the nineteenth century only experimentally logical approach existed, when atom was considered as an object without structure. Its properties were described by a set of numbers, obtained from experiments. Chemical elements were classified according to these numbers. They formed periodic system of chemical elements. Nature of this periodical system became to be clear only in twentieth century, when arrangement of atoms of chemical elements became clear. Arrangement of atoms had been established on the basis of quantum theory. But the quantum theory is insufficient to establish arrangement of elementary particles.

Now we find ourselves on that stage, when we have a classification of elementary particles (standard model), which are considered as objects without a structure. The structural approach to elementary particles is not yet sufficiently developed.

2 Space-time geometry

It is used to think, that the most general type of space-time geometry is a Riemannian geometry. It is not valid. The most general type of space-time geometry is the physical geometry, i.e. the geometry described completely in terms of world function. The physical geometry is obtained from the proper Euclidean geometry by means of a deformation [11], [12], [13], [15], [16], [17].

Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ can be described completely in terms of metric ρ and only in terms of metric. It is convenient to use world function $\sigma = \frac{1}{2}\rho^2$ instead of the metric ρ. All objects and statements of $\mathcal{G}_{\mathrm{E}}$ are expressed via world function σ_{E} of $\mathcal{G}_{\mathrm{E}}$. Substituting σ_{E} in all statements of $\mathcal{G}_{\mathrm{E}}$ by the world function σ of a physical geometry $\mathcal{G}$, one obtains all statements of $\mathcal{G}$, i.e. the geometry $\mathcal{G}$. Operation of substitution σ_{E} by σ is a deformation of $\mathcal{G}_{\mathrm{E}}$. Such a deformation of $\mathcal{G}_{\mathrm{E}}$ is very simple. It does not need a consideration of the basic axioms compatibility, because $\mathcal{G}_{\mathrm{E}}$ is used in monistic representation, when there is essentially only one quantity, which defines $\mathcal{G}_{\mathrm{E}}$.

The physical geometry can describe discrete geometry and geometry, which is not continuous. There is no necessity to consider topology as a special science, because all topological properties are included in the form of world function. For instance, if five dimensional space-time of Kaluza-Klein is considered as a Riemannian geometry, one obtains,that electric charge $q = ne_0$, where e_0 is the elementary charge, and n is an integer number. If the space-time of Kaluza-Klein is considered as a physical geometry [18], the electric charge $q = \pm e_0$, or $q = 0$. Experiment shows, that electric charge of elementary particle satisfies the condition $|q| \leq e_0$. In other words, space-time geometry of Kaluza-Klein should be described by physical geometry.

A set of physical geometries is more powerful, than a set of Riemannian geometries. Let us consider uniform isotropic discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ [5]. It is not a Riemannian geometry. It is a physical geometry $\mathcal{G}_{\mathrm{d}}$, described by the world function σ_{d}

$$\sigma_{\mathrm{d}}(P,Q)=\sigma_{\mathrm{M}}(P,Q)+\frac{\lambda_0^2}{2}\mathrm{sgn}(\sigma_{\mathrm{M}}(P,Q)), \quad \forall P,Q\in\Omega=\Omega_{\mathrm{M}} \tag{2.1}$$

where Ω is the set of points, where the geometry is given. λ_0 =const is a minimal length in $\mathcal{G}_{\mathrm{d}}$. σ_{M} is a world function of geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$. The set Ω coincides with the set Ω_{M}, where the Minkowski geometry $\mathcal{G}_{\mathrm{M}}$ is given. It easy to verify, that σ_{d} satisfies the relation (1.5)

$$|\rho_{\mathrm{d}}(P,Q)|\notin(0,\lambda_0), \quad \forall P,Q\in\Omega_{\mathrm{M}}, \quad \rho_{\mathrm{d}}=\sqrt{2\sigma_{\mathrm{d}}} \tag{2.2}$$

i.e. λ_0 is the minimal length in $\mathcal{G}_{\mathrm{d}}$.

The discrete geometry $\mathcal{G}_{\mathrm{d}}$ is uniform and isotropic, because σ_{d} is a function of σ_{M}, as one can see from (2.1). However, there are no smooth world lines of particles in $\mathcal{G}_{\mathrm{d}}$, because of (2.2). In $\mathcal{G}_{\mathrm{d}}$ world line of a particle is given as a broken line $\mathcal{L}_{\mathrm{br}}$, consisting of rectilinear segments $\mathcal{T}_{\mathrm{d}[s.s+1]}$

$$\mathcal{L}_{\mathrm{br}}=\bigcup_s\mathcal{T}_{\mathrm{d}[s.s+1]}, \tag{2.3}$$

where $\mathcal{T}_{\mathrm{d}[s.s+1]}$ is a segment of straight line in $\mathcal{G}_{\mathrm{d}}$

$$\mathcal{T}_{\mathrm{d}[s.s+1]}=\left\{R|\sqrt{2\sigma_{\mathrm{d}}(P_s,P_{s+1})}=\sqrt{2\sigma_{\mathrm{d}}(P_s,R)}+\sqrt{2\sigma_{\mathrm{d}}(R,P_{s+1})}\right\} \tag{2.4}$$

Length $\left|\mathcal{T}_{\mathrm{d}[s.s+1]}\right|$ of all segments is the same

$$\left|\mathcal{T}_{\mathrm{d}[s.s+1]}\right|=\mu, \quad s=...0,1,.. \tag{2.5}$$

It is easy to verify that in the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ a rectilinear segment $\mathcal{T}_{\mathrm{E}[s.s+1]}$ between the points P_s and P_{s+1} is described by (2.4) with σ_{d} replaced by Euclidean world function σ_{E}. Segment $\mathcal{T}_{\mathrm{d}[s.s+1]}$ in $\mathcal{G}_{\mathrm{d}}$ is expressed via σ_{d} in the same way, as segment $\mathcal{T}_{\mathrm{E}[s.s+1]}$ in $\mathcal{G}_{\mathrm{E}}$ is expressed via σ_{E}. It is a reason, why (2.4) is a rectilinear segment in $\mathcal{G}_{\mathrm{d}.}$.

For a free particle the adjacent vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$, are equal. $(\mathbf{P}_s\mathbf{P}_{s+1}\mathrm{eqv}\mathbf{P}_{s+1}\mathbf{P}_{s+2})$. It means that

$$\sigma(P_s,P_{s+1})=\sigma(P_{s+1},P_{s+2}), \quad s=...-1,0,1,... \tag{2.6}$$

$$(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2})=2\sqrt{\sigma(P_s,P_{s+1})\,\sigma(P_{s+1},P_{s+2})}, \quad s=...-1,0,1,... \tag{2.7}$$

where $(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2})$ is the scalar product of vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$

$$(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2})=\sigma(P_s,P_{s+2})+\sigma(P_{s+1},P_{s+1})-\sigma(P_s,P_{s+1})-\sigma(P_{s+1},P_{s+2}) \tag{2.8}$$

The scalar product in $\mathcal{G}_{\mathrm{d}}$ has the form (2.8), because the scalar product $(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2})$ in $\mathcal{G}_{\mathrm{E}}$ has the same form (2.8) with σ substituted by σ_{E}.

Equations (2.6), (2.7) are dynamical equations for a free particle. They are to be fulfilled at any point P_{s+1}. In general, in equations (2.6), (2.7) in $\mathcal{G}_{\rm d}$, where $\sigma = \sigma_{\rm d}$, have many solutions, because the number of dynamic variables is equal to 4, whereas the number of dynamic equations is equal to 2. Solutions in different points P_{s+1} are independent. As a result the shape of $\mathcal{L}_{\rm br}$ will be indefinite in $\mathcal{G}_{\rm d}$. The world line of free particle will wobble [5].

However, in geometry of Minkowski, where $\sigma = \sigma_{\rm M}$ equations (2.6), (2.7) have unique solution [5], although the number of dynamical variables is larger, than the number of dynamic equations. In this case a world line of free particle is a straight line, which does not wobble.

Thus, if one uses the physical space-time geometry, the dynamic equations (2.6), (2.7), which can be written also in the form

$$\sigma\left(P_s, P_{s+1}\right) = \sigma\left(P_{s+1}, P_{s+2}\right), \quad s = ... - 1, 0, 1, ... \tag{2.9}$$

$$\sigma\left(P_s, P_{s+2}\right) = 4\sigma\left(P_s, P_{s+1}\right), \quad s = ... - 1, 0, 1, ... \tag{2.10}$$

may describe both deterministic and stochastic world lines of a free particle. Result depends on the space-time geometry.

It means that there exists such a presentation of dynamic equations, which is the same for deterministic and stochastic (quantum) particles. It may point out that classical mechanics and quantum mechanics can be united in one mechanics with mathematical formalism which is common for classical mechanics and for quantum mechanics.

Additional argument in behalf of physical space-time geometry is the fact, that in this case the relativity principle can be formulated in the coordinate free form. Indeed, it is rather strange, when important physical principle is formulated by means of a reference to transformations of coordinate system [20]

In the physical geometry the relativity principle is formulated as follows. *Space-time geometry is described by means of unique space-time structure (world function).* In the nonrelativistic case the space-time is described by two space-time structures: absolute space distance $S\left(P, Q\right)$ and absolute time interval $T\left(P, Q\right)$. Besides, the coordinate free formulation is valid in any space-time, but not only in the space-time of Minkowski, as it is used in usual formulation.

3 Methods of description

It seems to be rather unusual to describe particles by means of dynamic equations, which have no unique solution. Stochastic particles, which are described by equations (2.9), (2.10) are described usually by gas dynamic equations. These gas dynamic equations can be obtained from equations (2.9), (2.10) by means of statistical averaging. We shall refer to description in form of dynamic equations (2.9), (2.10) as a basic representation. Description in terms of classical gas dynamic equations will be referred to as a gas dynamic representation.

Any gas dynamic equations can be described in terms of the wave function [3]. If the obtained equation is linear in terms of wave function, such a representation is qualified as quantum representation. In gas dynamics the wave function is a natural attribute of gas dynamics, but in quantum mechanics the wave function is an axiomatic object. Its nature in quantum mechanics is unknown. Linearity of dynamic equations in terms of the wave function is considered as a mark of quantum mechanics. Linearity of quantum mechanics is considered as a principle of quantum mechanics.

If the gas molecules interact via some force field κ^l, and the action has the form [4]

$$\mathcal{A}[x,\kappa]=\int_{\xi_0}\int_{V_\xi}\left(-mcK\sqrt{g_{lk}\dot{x}^l\dot{x}^k}-\frac{e}{c}A_l\dot{x}^l\right)d^4\xi,\quad \dot{x}^i=\frac{\partial x^i}{\partial\xi_0} \tag{3.1}$$

$$K=\frac{M}{m}=\sqrt{1+\lambda^2\left(\kappa_l\kappa^l+\partial_l\kappa^l\right)},\quad \lambda=\frac{\hbar}{mc},\quad \partial_l\equiv\frac{\partial}{\partial x^l} \tag{3.2}$$

then dynamic equations in terms of the wave function are linear. Irrotational flows of such a gas coincide with the Klein-Gordon equation. Thus, the gas dynamic description may be treated as a quantum description.

Describing gas dynamical equation in terms of the wave function [3], one can obtain quantum description in terms of the wave function. Although obtaining of gas dynamic description from quantum description was known in the beginning of XX century [2], derivation of quantum description from gas dynamic description appears to be known only recently ([4]). Apparently, the problem of deriving the quantum description from the gas dynamic description was connected with inaptitude of introduction of the wave function into gas dynamics.

Any gas dynamic equations can be written in terms of the wave function. But equations in terms of a wave function are linear only for quantum particles. Although nature of wave function in gas dynamics is quite clear, in the quantum mechanics the nature of a wave function is not clear. Instead of nature of the wave function one consider linearity of quantum mechanics.

In quantum mechanics the elementary particles are considered as pointlike objects equipped by a series of quantum numbers. On the other side classical particles corresponding to elementary particles appear sometimes to be a composite particles. Let the Dirac particle S_{D}, satisfy the Dirac equation, and the Klein-Gordon particle S_{KG}, satisfy the Klein-Gordon equation. The classical Dirac particle S_{Dcl} which associates with S_{D}, has another structure and number of freedom degrees, than the classical Klein-Gordon particle S_{KGcl}, which associates with S_{KG}.

Basic representation is more informative, than gas dynamic representation and quantum representation. Basic representation cannot be obtained from quantum representation. Basic representation has more important properties. It can describe structure of elementary particle. The world line (2.3) may consist not only of rectilinear segments. It may consist of more complicated geometrical objects, for instance, from skeletons $\mathcal{S}^{(n)} = \{P_0^n, P_1^n ... P_n^n\}$

$$\mathcal{L}_{\mathrm{br}} = \bigcup_s \mathcal{S}_s^{(n)} \tag{3.3}$$

All skeletons $\mathcal{S}_s^{(n)}$ are supposed to be equal

$$\mathcal{S}_s^{(n)} = \mathcal{S}_{s+1}^{(n)}, \quad s = ...0, 1, ... \tag{3.4}$$

Equality of skeletons $\mathcal{S}_s^{(n)}$ and $\mathcal{S}_{s+1}^{(n)}$ means that vectors $(\mathbf{P}_i^n)_s\,(\mathbf{P}_k^n)_s$ and $(\mathbf{P}_i^n)_{s+1}\,(\mathbf{P}_k^n)_{s+1}$are equal

$$(\mathbf{P}_i^n)_s\,(\mathbf{P}_k^n)_s = (\mathbf{P}_i^n)_{s+1}\,(\mathbf{P}_k^n)_{s+1}\,, \quad i, k = 0, 1, ..n \tag{3.5}$$

Besides, adjacent skeletons are linked by relation

$$(P_n^n)_s = (P_0^n)_{s+1}\,, \quad s = ...0, 1, ... \tag{3.6}$$

Skeleton $\mathcal{S}^{(n)}$ describes position of a physical object. It does not depend on the shape of the physical object [8] sec 10. Using description (3.3) one can describe motion of physical objects which are not pointlike. Relation (3.3) together with (3.4) describes free motion of skeleton in the direction of the vector $\left(\mathbf{P}_n^{(\mathbf{n})}\right)_s \left(\mathbf{P}_0^{(\mathbf{n})}\right)_{s+1}$. As a result (3.3) describes a motion of a complicated physical object [10]. It admits one to describe structure of elementary particles. Conventional quantum mechanics does not admit to describe structure of elementary particles. Any elementary particle is considered as a pointlike object equipped by quantum numbers. Different elementary particles have different sets of quantum numbers, but all they are pointlike. When one discovered experimentally complex structure of nucleons, it has been decided that nucleons consist of quarks, which cannot exist outside nucleon. It would be more reasonable to consider quarks as elements of nucleon structure, but quantum theory does not admit this. It admits one to consider quarks only as single particles.

Numerical simulation showed, that the basic representation of a particle, described by the Dirac equation, contains three-point skeleton $\mathcal{S}^{(3)} = (P_0, P_1, P_2)$. In the classical approximation the classical Dirac particle is a spiral world line with timelike axis.

To obtain classical approximation one does not use cut-off of quantum interaction (for instance, $\hbar = 0$). Instead one uses quasi-balanced state, which cut-off all stochastic interactions, but not only quantum ones. For instance, a mean velocity of a Brownian particle is described by the relation

$$\mathbf{v} = -\boldsymbol{\nabla} f(\rho) \tag{3.7}$$

This velocity vanishes, if $f(\rho)$=const, and the state of Brownian particles is balanced. A balanced state can be created in dynamics of stochastic particles, when stochasticity does not influence on the mean motion of particles. Such a situation will be referred to as dynamic disquantization [21], [22], [14], [23].

Statistical ensemble $\mathcal{E}[S_{\rm st}]$ of stochastic systems $S_{\rm st}$ is a dynamical system. It is described by a system of partial differential equations. In the balanced state the terms containing transversal derivatives $\partial_{\rm tr}^{i} = \partial^{l} - \frac{j^l j^k}{j_s j^s}\partial_k$ are small, because they describe stochastic component of motion. Here j^k describes 4-current of particles. If in the dynamic equations one substitutes all derivatives ∂^l by $\frac{j^l j^k}{j_s j^s}\partial_k$,

$$\partial^l \to \frac{j^l j^k}{j_s j^s}\partial_k \qquad (3.8)$$

then all transversal derivatives $\partial_{\rm tr}^{i}$ will be suppressed, and system of partial equations turns into a system of ordinary equations, because all derivatives ∂^k will be along j^k. This system of ordinary equations describes a statistical ensemble $\mathcal{E}[S_{\rm dc}]$ of deterministic systems $S_{\rm dc}$. Any system $S_{\rm dc}$ has finite number of freedom degrees.

In the case of Schrödinger equation the system $S_{\rm dc}$ has 6 degrees of freedom. In the case of Dirac equation it has 10 degrees of freedom. Quantum constant may remain to be a parameter of the system of ordinary equations. The deterministic dynamic system $S_{\rm dc}$ associates with quasi-classical approximation of stochastic system $S_{\rm st}$. This procedure (dynamic disquantization) is *purely dynamic procedure*, which removes stochasticity of any nature, but not only quantum stochasticity.

4 Pecularities of skeleton approach

The new approach arose as a result of strategical conception, when many problems of contemporary physics are considered as results of mistakes in foundation. The main mistake was consideration of the space-time geometry as a kind of Riemannian geometry. The Riemannian geometry is not a general type of the space-time geometry. The space-time geometry is a kind of physical geometry, which is not a logical construction, generally speaking. In physical geometry the equivalence relation is intransitive in general. In particular, it means, that if

$$\mathbf{AB} = \mathbf{CD} \wedge \mathbf{CD} = \mathbf{EF} \qquad (4.1)$$

then relation

$$\mathbf{AB} \neq \mathbf{EF} \qquad (4.2)$$

may take place. This property of the space-time geometry admits that world lines of free particles may wobble. In any logical construction the equivalence relation is transitive, and situation (4.1), (4.2) is impossible. Indeed, if in some conception the relations (4.1), (4.2) take place, one cannot build any logical conclusions in this conception. Most mathematicians consider such a situation as illegal, because one cannot produce logical deductions in such a conception. Such an approach is natural, because a possibility of logical construction is important for mathematicians. The form of conception, but not the objects, which are described by the conception, is important for mathematicians.

Result of such an approach is a such one, that quantum phenomena are described by quantum mechanics, which is an additional conception to classical mechanics. A possible unification of classical mechanics and quantum mechanics in the framework of physical space-time geometry leads to another approach to the theory of elementary particles [24] [10].

It is important to compare the skeleton conception with the conventional theory of elementary particles. Interrelation between the two approaches reminds interrelation between the chemistry and physics of chemical elements. Chemistry investigates properties and reactions of different chemical elements, basing on experimental data without considerations of structure and arrangement of their atoms. Result of these considerations is the periodic system of chemical elements. On the contrary, physics is interested in arrangement and structure of atoms of chemical elements. As a result chemistry and physics investigate atoms from different points of view. The two approaches match well.

In the case of elementary particles the conventional approach is an analog of the chemical approach, because it is not interested in structure and arrangement of elementary particles. Result of this approach is the standard model of elementary particles, which is an analog of the periodic system of chemical elements. On the contrary, the skeleton conception is an analog of physical approach to chemical elements, because it is interested in structure and arrangement of elementary particles. Conventional approach to elementary particles and skeleton approach match well, considering elementary particles from different points of view.

5 Arguments in behalf of skeleton approach

The transition from dynamic equations (2.9), (2.10) for a pointlike particle to equations (3.3) for a volume particle is rather unexpected. But position of any volume physical body is described by its skeleton, and dynamic equations for motion of the skeleton are dynamic equations for motion of a physical body. One can test this fact on the example of a particle described by the Dirac equation [25].

According to [26, 27] γ-matrices in the Dirac equation can be considered as hypercomplex numbers. Let us use designations [25]

$$\gamma_5 = \gamma^{0123} \equiv \gamma^0\gamma^1\gamma^2\gamma^3, \tag{5.1}$$

$$\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3, \} = \{-i\gamma^2\gamma^3, -i\gamma^3\gamma^1, -i\gamma^1\gamma^2\} \tag{5.2}$$

where γ^k are 4×4 Dirac matrices, satisfying the commutation relations

$$\gamma^i\gamma^k + \gamma^k\gamma^i = 2g^{ik} \tag{5.3}$$

where g^{ik} is the metric tensor. Let us make the change of variables

$$\psi = Ae^{i\varphi + \frac{1}{2}\gamma_5\kappa} \exp\left(-\frac{i}{2}\gamma_5\boldsymbol{\sigma\eta}\right) \exp\left(\frac{i\pi}{2}\boldsymbol{\sigma}\mathbf{n}\right) \Pi \tag{5.4}$$

$$\psi^* = A\Pi \exp\left(-\frac{i\pi}{2}\boldsymbol{\sigma}\mathbf{n}\right)\exp\left(-\frac{i}{2}\gamma_5\boldsymbol{\sigma}\boldsymbol{\eta}\right)e^{-i\varphi-\frac{1}{2}\gamma_5\kappa} \tag{5.5}$$

where (*) means the Hermitian conjugation, and

$$\Pi = \frac{1}{4}(1+\gamma^0)(1+\mathbf{z}\boldsymbol{\sigma}), \qquad \mathbf{z} = \{z^\alpha\} = \text{const}, \qquad \alpha = 1,2,3; \qquad \mathbf{z}^2 = 1 \tag{5.6}$$

is a zero divisor. The quantities A, κ, φ, $\boldsymbol{\eta} = \{\eta^\alpha\}$, $\mathbf{n} = \{n^\alpha\}$, $\alpha = 1,2,3$, $\mathbf{n}^2 = 1$ are eight real parameters, determining the wave function ψ. These parameters may be considered as new dependent variables, describing the state of dynamic system $\mathcal{S}_{\mathrm{D}}$. The quantity φ is a scalar, and κ is a pseudoscalar. Six remaining variables A, $\boldsymbol{\eta} = \{\eta^\alpha\}$, $\mathbf{n} = \{n^\alpha\}$, $\alpha = 1,2,3$, $\mathbf{n}^2 = 1$ can be expressed through the flux 4-vector $j^l = \bar{\psi}\gamma^l\psi$ and spin 4-pseudovector

$$S^l = i\bar{\psi}\gamma_5\gamma^l\psi, \qquad l = 0,1,2,3 \tag{5.7}$$

Because of two identities

$$S^l S_l \equiv -j^l j_l, \qquad j^l S_l \equiv 0. \tag{5.8}$$

there are only six independent components among eight components of quantities j^l, and S^l.

Dynamic equations in terms of dynamic variables A, κ, φ, $\boldsymbol{\eta} = \{\eta^\alpha\}$, $\mathbf{n} = \{n^\alpha\}$, $\alpha = 1,2,3$, $\mathbf{n}^2 = 1$ do not contain γ-matrices. Producing dynamic disquantization [28], one obtains dynamic equations for statistical ensemble $\mathcal{E}\left[S_{\mathrm{Dcl}}\right]$ of deterministic dynamic systems S_{Dcl}. Action for S_{Dcl} has the form

$$\mathcal{S}_{\mathrm{Dcl}}: \qquad \mathcal{A}_{\mathrm{Dcl}}[x,\boldsymbol{\xi}] = \int \left\{-\kappa_0 m\sqrt{\dot{x}^i\dot{x}_i} + \hbar\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{2(1+\boldsymbol{\xi}\mathbf{z})} + \hbar\frac{(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}}{2\sqrt{\dot{x}^s\dot{x}_s}(\sqrt{\dot{x}^s\dot{x}_s}+\dot{x}^0)}\right\}d\tau_0 \tag{5.9}$$

Here $x^k = x^k(\tau_0,\boldsymbol{\xi})$ and $\mathbf{z}$, $\mathbf{z}^2 = 1$ is a constant 3-vector, $\kappa_0 = \pm 1$. The dynamic system S_{Dcl} will be referred to as classical Dirac particle. S_{Dcl} associates with the Dirac particle S_{D}. Dynamic equations for S_{Dcl} can be considered as classical approximation of S_{D}. S_{Dcl} can be interpreted as a rotator, consisting of two particles. World line of the particle S_{Dcl} is a helix with timelike axis. The first term in (5.9) describes progressive motion, whereas two others describe rotational part of motion.

Note, that two last terms of (5.9), describing rotation, are nonrelativistic, although dynamic disquantization (3.8), acting on relativistic dynamic system, remains it to be relativistic. In the given case appearance of nonrelativistic terms is connected with elimination of γ-matrices [29]. Nonrelativistic terms admit rotation with the velocity larger, than the speed of the light. Circular rotation in S_{Dcl} associates with spin and magnetic moment of Dirac particle S_{D}. The quantum constant remains in the action (5.9) also.

Investigation shows, that the Dirac equation can be presented in the form of (3.3) for three-point skeleton [30]. One vector of the skeleton is spacelike. Spacelike vectors admit large wobbling, and this vector is responsible for rotations. Two other vectors are timelike. They provide stabilization of the skeleton world line. Calculation in [30] was produced for a model of the space-time (not for real space-time). Nevertheless calculation demonstrates real properties of three-point skeleton and its role in description of Dirac particle.

6 Concluding remarks

The new approach to elementary particle theory, which admits one to describe arrangement of elementary particles, has been presented. One should stress, that unification of classical mechanics and quantum mechanics and the new approach to elementary particles is a result of correction of mistakes in our understanding of the space-time geometry, but not a result of some new ideas. Such a way seems to be more reliable. The way to unification of classical mechanics and quantum mechanics has been presented in the paper. It remains only to determine world function of real space-time geometry in microworld, which could generate the quantum stochasticity. I hope, that this problem will be solved, because the set of physical geometries is very powerful, and practically each physical space-time geometry can generate stochasticity.

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