

What is the wave function and why is it used in quantum mechanics?

Yuri A. Rylov

Institute for Problems in Mechanics, Russian Academy of Sciences,
101-1, Vernadskii Ave., Moscow, 119526, Russia.

e-mail: rylov@ipmnet.ru

Web site: *http://gasdyn – ipm.ipmnet.ru/~rylov/yrylov.htm*

Abstract

It is shown that quantum mechanics can be presented as a hydrodynamic of some quantum fluid. In this case the quantum mechanics ceases to be an axiomatic conception, because the axiomatic object of QM (wave function) ceases to be an axiomatic object. In the hydrodynamics the wave function is a method of an ideal fluid description. The quantum mechanics becomes to be a classical dynamics of the stochastic particles. Problems of the quantum mechanics interpretation disappear, because the interpretation is determined by the mathematical formalism of classical dynamics.

Key words: classical dynamics of stochastic particles; κ -field; classical gas dynamics; wave function; .

1 Introduction

The problem of the quantum mechanics interpretation exists almost a century. There are numerous versions of interpretations. It is connected with the fact, that quantum mechanics is an axiomatic conception. It means that the quantum mechanics conception contains an axiomatic object, defined only by its properties. This axiomatic object is the wave function. Nobody knows, what is the wave function. One cannot interpret an axiomatic conception exactly, because of indefinite axiomatic object. In the classical mechanics there are no problems of interpretation, because the mathematical formalism of classical mechanics admits one to interpret any physical phenomenon, described in terms of classical mechanics.

It was shown that the Schrödinger equation describes a nonrotational flow of some "quantum" fluid [1]. D.Bohm used this circumstance in his hydrodynamic interpretation of quantum mechanics [2]. Unfortunately, the connection between the quantum mechanics and hydrodynamics has been one-sided all the twentieth

century. One could obtain hydrodynamic description from the quantum mechanics, but one was not able to obtain the Schrödinger equation from hydrodynamical equations. The reason of such a situation was the axiomatic object - wave function. The wave function is considered as a natural attribute of quantum mechanics, whereas connection between the wave function and hydrodynamics was not clear. Situation changed, when it has been shown, that the wave function appears to be a method of description of any ideal (nondissipative) fluid [3]. Description in terms of wave function is connected with conventional hydrodynamic description in terms of hydrodynamic variables: density and velocity.

Due to this connection the wave function ceased to be an axiomatic object, Wave function and spin turned to natural attributes of a classical dynamics of continuous medium. The quantum mechanics, considered as a hydrodynamics, became to be a classical dynamics, where there is no problem of interpretation. Besides, some force field appears. It is responsible for quantum effects. In the quantum mechanics there is the problem of uniting of nonrelativistic quantum principles with the principles of relativity theory. In the quantum mechanics, considered as a hydrodynamics, such a problem is absent, because the principles of quantum mechanics are absent. Instead there is a force field κ_l , $l = 0, 1, 2, 3$, which is responsible for quantum effects.

A use of a fluid is a natural method of a stochastical particle description. There are no dynamic equations for a single stochastic particle description. One can describe only a mean motion of a stochastic particle. A use of the distribution function for description of a stochastic particle motion is possible only for a nonrelativistic stochastic particle, because the distribution function in the phase space of coordinates and momenta is a nonrelativistic construction. However, the nonrelativistic quantum mechanics should be considered as a relativistic conception, because random component of the particle velocity may be relativistic, even if the regular component of the particle velocity is nonrelativistic.

For description of relativistic particles one needs to use a statistical ensemble. Statistical ensemble $\mathcal{E}[S_d]$ of deterministic particles S_d is a set of N ($N \rightarrow \infty$) independent particles S_d . $\mathcal{E}[S_d]$ is a fluidlike dynamical system. One can obtain dynamic equations for S_d from dynamic equations for $\mathcal{E}[S_d]$. On the contrary, one can obtain dynamic equations for $\mathcal{E}[S_d]$ from dynamic equations for S_d . Thus, descriptions in terms of $\mathcal{E}[S_d]$ and in terms of S_d are equivalent, if S_d is a deterministic particle and there exist dynamic equations for S_d .

Let now particles S_d interact between themselves via some force field κ . Then the statistical ensemble $\mathcal{E}[S_d]$ ceases to be a statistical ensemble, because its elements S are not independent particles. The particles S_d turn to interacting particles S_{st} . The set $\mathcal{E}[S_{st}]$ ceases to be a statistical ensemble, but it remains to be a fluidlike dynamic system. We shall use the term int-ensemble for the dynamic system $\mathcal{E}[S_{st}]$. One cannot obtain dynamic equations for S_{st} from dynamic equations for $\mathcal{E}[S_{st}]$. It means that elements S_{st} of int-ensemble $\mathcal{E}[S_{st}]$ are not deterministic particles. They are stochastic particles. Dynamic equations for $\mathcal{E}[S_{st}]$ describe a mean motion of stochastic particles S_{st} .

A simple example of such a situation is an ideal gas. Molecules of ideal gas are

stochastic particles, and there are no dynamic equations for a single molecule inside a gas. Motion of the "gas particles" is described by equations of classical gas dynamics. Any gas particle contains many molecules, and motion of a gas particle describes a mean motion of the gas molecule. Interaction between the gas molecules is realized by the molecular collisions. A single molecule (outside the gas) is a deterministic particle. There are dynamic equations for a single molecule outside the gas. As to stochastic particles, they are not deterministic particles, and there are no dynamic equations for a single particle S_{st} . A use of int-ensemble $\mathcal{E}[S_{st}]$, where the stochastic particles S_{st} interact via some force field κ is only a mathematical method, which enables to describe the mean motion of S_{st} by means a int-ensemble $\mathcal{E}[S_{st}]$, which is a fluidlike dynamic system. Of course, the kind of the particle stochastics depends on the force field κ . This circumstance admits one to classify stochastic particles by the force field κ , which appears in dynamic equations for int-ensemble $\mathcal{E}[S_{st}]$. It admits one also to consider the force field κ as a source of the particle stochasticity, although the κ -field appears as a mathematical means of the stochastical particle description.

2 Clebsch potentials

The complete system of hydrodynamic equations contains seven equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \mathbf{v} = -\frac{\nabla p(\rho)}{\rho} \quad (2.1)$$

$$\frac{\partial \boldsymbol{\xi}}{\partial t} + (\mathbf{v} \nabla) \boldsymbol{\xi} = 0 \quad (2.2)$$

Three equations (2.2) describe the motion of the fluid particle in the given velocity field $\mathbf{v} = \mathbf{v}(t, \mathbf{x})$. They can be presented in the form

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(t, \mathbf{x}) \quad (2.3)$$

Solutions $\boldsymbol{\xi} = \{\xi_1(t, \mathbf{x}), \xi_2(t, \mathbf{x}), \xi_3(t, \mathbf{x})\} = \text{const}$ of (2.2) are three independent integrals of (2.3). Indeed, due to (2.3) and (2.2) the expression

$$\frac{d}{dt} \boldsymbol{\xi}(t, \mathbf{x}(t)) = \frac{\partial \boldsymbol{\xi}(t, \mathbf{x}(t))}{\partial t} + \left(\frac{d\mathbf{x}}{dt} \nabla \right) \boldsymbol{\xi}(t, \mathbf{x}(t)) = 0 \quad (2.4)$$

vanishes. It means that $\boldsymbol{\xi}(t, \mathbf{x}(t)) = \text{const}$ are integrals of (2.3). If $\xi_1(t, \mathbf{x}), \xi_2(t, \mathbf{x}), \xi_3(t, \mathbf{x})$ are three independent solutions of (2.2), equations (2.2) and (2.3) are equivalent. Variables $\boldsymbol{\xi}$ are known as Lagrangian coordinates, which are constant along the fluid particle trajectories according to their definition.

Four equations (2.1) form a closed subsystem of the system of dynamic equations (2.1), (2.2). It is a reason, why usually one considers only four equations (2.1) as a system of hydrodynamic equations, ignoring (2.2). One keeps in mind, that the

ordinary equations (2.3) can be solved relatively easy, if the velocity field $\mathbf{v}(t, \mathbf{x})$ is determined from (2.1). Besides, in many hydrodynamic problems the trajectories of the fluid particles are not interesting.

The complete hydrodynamic system (2.1), (2.2) may be integrated partly in the form

$$\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{v}) = 0, \quad \mathbf{v} = b_0 (\nabla \varphi + g^\alpha(\boldsymbol{\xi}) \nabla \xi_\alpha) \quad (2.5)$$

$$\frac{\partial \boldsymbol{\xi}}{\partial t} + (\mathbf{v} \nabla) \boldsymbol{\xi} = 0 \quad (2.6)$$

where $g^\alpha(\boldsymbol{\xi})$, $\alpha = 1, 2, 3$ are arbitrary functions, which are determined from the initial conditions for \mathbf{v} . The variable φ is a new variable, introduced instead of \mathbf{v} . The quantity b_0 is an arbitrary constant. The second relation (2.5) is known as Clebsch potentials. Clebsch obtained them for incompressible fluid [4, 5].

The wave function $\psi = \{\psi_\alpha\}$, $\alpha = 1, 2, \dots, n$ is a n -component complex function. It is constructed from Clebsch potentials by means of relations,

$$\psi_\alpha = \sqrt{\rho} e^{i\varphi} w_\alpha(\boldsymbol{\xi}), \quad \psi_\alpha^* = \sqrt{\rho} e^{-i\varphi} w_\alpha^*(\boldsymbol{\xi}), \quad \alpha = 1, 2, \dots, n, \quad (2.7)$$

$$\psi^* \psi \equiv \sum_{\alpha=1}^n \psi_\alpha^* \psi_\alpha, \quad (2.8)$$

where (*) means the complex conjugate, $w_\alpha(\boldsymbol{\xi})$, $\alpha = 1, 2, \dots, n$ are functions of only variables $\boldsymbol{\xi}$. They satisfy the relations

$$-\frac{i}{2} \sum_{\alpha=1}^n \left(w_\alpha^* \frac{\partial w_\alpha}{\partial \xi_\beta} - \frac{\partial w_\alpha^*}{\partial \xi_\beta} w_\alpha \right) = g^\beta(\boldsymbol{\xi}), \quad \beta = 1, 2, 3, \quad \sum_{\alpha=1}^n w_\alpha^* w_\alpha = 1. \quad (2.9)$$

The number n is such a natural number, that equations (2.9) admit a solution. In general n may depend on the form of the arbitrary integration functions $\mathbf{g} = \{g^\beta(\boldsymbol{\xi})\}$, $\beta = 1, 2, 3$.

Hydrodynamic equations, described in terms of Clebsch potentials or in terms of the wave function, are rather bulky. Hydrodynamists do not use them, and such a presentation of hydrodynamic equations is known slightly. The only known exclusion takes place, when the internal energy E of a fluid has the form

$$E = \frac{1}{2} \rho \mathbf{v}_{\text{dif}}^2, \quad \mathbf{v}_{\text{dif}} = -\frac{\hbar}{2m} \nabla \log \rho, \quad E = \frac{\hbar^2}{8m^2} \frac{(\nabla \rho)^2}{\rho} \quad (2.10)$$

If besides the fluid flow is nonrotational, one can set in (2.5) $g^\alpha(\boldsymbol{\xi}) = 0$. In this case the wave function is one-component, and choosing $b_0 = \hbar$ in (2.5), hydrodynamic equations in terms of the wave function appear to be linear. They coincide with the Schrödinger equation.

Thus, we see, that the wave function is a method of description of any nondissipative fluid, but this fact was discovered only in 1999 [3], and this fact is not interesting for hydrodynamists, because the internal energy depending on $(\nabla \rho)^2$

is not used at a description of usual fluids. As concerns physicists, dealing with quantum mechanics, a linear dynamic equation in terms of the wave function is very attractive, even if it is not known, what is the wave function.

But why is the quantum particle described as a continuous medium, which has infinite number of the freedom degrees? The quantum particle is a nondeterministic (stochastic) particle. Dynamic equations do not exist for a single stochastic particle. One can describe only a mean motion of a stochastic particle. To describe the mean motion of a stochastic particle, one considers an int-ensemble (gas) of stochastic particles, i.e. a set $\mathcal{E}[S_{\text{st}}]$ of N ($N \rightarrow \infty$) identical stochastic particles S_{st} . The int-ensemble $\mathcal{E}[S_{\text{st}}]$ is a continuous medium at $N \rightarrow \infty$, and there are dynamic equations for $\mathcal{E}[S_{\text{st}}]$.

For instance, a gas as a continuous medium is a dynamic system. Molecules of the gas move stochastically, because of collisions. Equations of the gas dynamics describe only a mean motion of molecules. Exact motion of molecules remains unknown at such a description. To obtain a description of the mean motion of molecules, it is sufficient to know only mean energy of molecules. In the case of ideal gas the mean energy of the gas is $E = \frac{3}{2}kT\rho$, where T is the gas temperature, k is the Boltzmann constant, and ρ is the gas density. In the case of quantum particle the mean energy is defined by the relation (2.10), where \mathbf{v}_{dif} is the mean diffusion velocity. It means that the motion of a quantum particle deflects from the rectilinear motion uniformly in all directions. In the case of a gas a more detailed information on the velocity distribution (Maxwell distribution) is not necessary for description of the mean motion of the gas molecules. The same is valid for description of the mean motion of stochastic particles.

Note, that solving equations of the gas dynamics, one cannot determine the mean value $\langle F(\mathbf{x}, \mathbf{v}) \rangle$ of arbitrary function $F(\mathbf{x}, \mathbf{v})$ of coordinates and velocity of gas molecules. One can determine only the mean value $\langle f(\mathbf{x}) \rangle$ of the arbitrary function $f(\mathbf{x})$ of coordinates, and mean values of additive variables such as momentum $\mathbf{p} = m\mathbf{v}$, energy $E = m\mathbf{v}^2/2$, and angular momentum $\mathbf{L} = \mathbf{p} \times \mathbf{x}$. For determination of other mean values, one needs to know the distribution function $f(\mathbf{x}, \mathbf{v})$, which cannot be determined from the classical equations of the gas dynamics. The same is valid in the case of quantum mechanics, when one has the Schrödinger equation instead of the gas dynamics equations. The formula for calculation of mean values in quantum mechanics

$$\langle F(\mathbf{x}, \mathbf{p}) \rangle = \int \psi^*(\mathbf{x}) F(\mathbf{x}, -i\hbar\nabla) \psi(\mathbf{x}) d\mathbf{x} \quad (2.11)$$

is valid only for $f(\mathbf{x})$, \mathbf{p} , $E = \mathbf{p}^2/2m$, and $\mathbf{L} = \mathbf{p} \times \mathbf{x}$. However, according to principles of quantum mechanics the formula (2.11) is considered to be valid for arbitrary function $F(\mathbf{x}, \mathbf{p})$. The von Neumann's theorem [6] on incompatibility of hidden variables with principles of quantum mechanics is founded on the formula (2.11), which is considered to be valid for arbitrary functions F .

3 Statistical ensemble and int-ensemble of charged particles

Let us consider statistical ensemble $\mathcal{E} [S_d]$ of deterministic charged particles S_d . The action for $\mathcal{E} [S_d]$ has the form

$$\mathcal{E} [S_d] : \quad \mathcal{A} [x] = \int_{\xi_0} \int_{V_\xi} \left(-mc\sqrt{g_{lk}\dot{x}^l\dot{x}^k} - \frac{e}{c}A_l\dot{x}^l \right) d^4\xi, \quad \dot{x}^i = \frac{\partial x^i}{\partial \xi_0} \quad (3.1)$$

where $\xi = \{\xi_0, \xi_1, \xi_2, \xi_3\}$ are independent variables, and $x = \{x^0(\xi), x^1(\xi), x^2(\xi), x^3(\xi)\}$, $x = x(\xi)$ are dependent variables. The quantity ξ_0 is an evolutionary parameter along the world line of a particle. The quantity A_l is the 4-potential of the electromagnetic field. The electromagnetic field A_l is an external field, and via electromagnetic field there are no interaction between the particles.

As far as the particles are deterministic, there are dynamic equations for each single particle. They are obtained as a result of variation with respect to x^l

$$mc \frac{d}{d\xi_0} \frac{g_{lk}\dot{x}^k(\xi)}{\sqrt{\dot{x}_s(\xi)\dot{x}^s(\xi)}} + \frac{e}{c} (\partial_l A_k(x) - \partial_k A_l(x)) \dot{x}^k(\xi) = 0, \quad \xi = \text{const} \quad (3.2)$$

Let us imagine that, particles interact via some force field κ^l , $l = 0, 1, 2, 3$, which changes the particle mass m

$$m^2 \rightarrow M^2(x) = m^2 + \frac{\hbar^2}{c^2} (g_{kl}\kappa^k\kappa^l + \partial_l\kappa^l), \quad \partial_l \equiv \frac{\partial}{\partial x^l} \quad (3.3)$$

The force field κ^l , $l = 0, 1, 2, 3$ acts on the particle mass m , transforming it into effective mass M . Here the κ -field $\kappa^l = \kappa^l(x) = \{\kappa^0(x), \kappa^1(x), \kappa^2(x), \kappa^3(x)\}$.

Introduction of the κ -field in the action (3.1) turns the deterministic particle S_d into a stochastic particle S_{st} . The action (3.1) takes the form

$$\mathcal{E} [S_{st}] : \quad \mathcal{A} [x, \kappa] = \int_{\xi_0} \int_{V_\xi} \left(-mcK\sqrt{g_{lk}\dot{x}^l\dot{x}^k} - \frac{e}{c}A_l\dot{x}^l \right) d^4\xi, \quad \dot{x}^i = \frac{\partial x^i}{\partial \xi_0} \quad (3.4)$$

$$K = \frac{M}{m} = \sqrt{1 + \lambda^2 (\kappa_l\kappa^l + \partial_l\kappa^l)}, \quad \lambda = \frac{\hbar}{mc}, \quad \partial_l \equiv \frac{\partial}{\partial x^l} \quad (3.5)$$

Here $\lambda = \frac{\hbar}{mc}$ is the Compton wave length. After introduction of interaction between particles the statistical ensemble $\mathcal{E} [S_d]$ ceases to be a statistical ensemble, because elements (particles) of the statistical ensemble are to be independent by definition. Interaction of particles violates their independence. The statistical ensemble turns to int-ensemble, i.e. a set of identical interacting particles.

If one tries to obtain the action for a single particle, removing integration over $d^3\xi$ one obtains

$$S_d : \quad \mathcal{A} [x, \kappa] = \int \left(-mcK\sqrt{g_{lk}\dot{x}^l\dot{x}^k} - \frac{e}{c}A_l\dot{x}^l \right) d\xi_0, \quad \dot{x}^i = \frac{\partial x^i}{\partial \xi_0} \quad (3.6)$$

The action (3.6) appears to be not well defined, because the K-factor (3.5) contains derivatives of κ -field $\kappa^l(x)$ in all directions of the space-time, whereas the action (3.6) admits only derivatives along the world line. It means that the action (3.4) cannot describe a motion of a single stochastic particle (if $\kappa^l(x) \neq 0$). It can describe only an int-ensemble of stochastic particles. However, if $\kappa^l(x) \equiv 0$, K-factor $K \equiv 1$ in (3.6), and the action (3.6) becomes to be well defined. It can describe a single deterministic particle. In fact the particles, described by the action (3.4), interact between themselves via the κ -field.

After introduction of interaction between the particles of $\mathcal{E}[S_d]$ the statistical ensemble $\mathcal{E}[S_d]$ turns to int-ensemble. As we have mentioned, $\mathcal{E}[S_{st}]$ is not a statistical ensemble, because particles S_{st} of the int-ensemble $\mathcal{E}[S_{st}]$ interact between themselves. Introduction of interaction between independent particles of the statistical ensemble is a *method of description of stochastic particles*. Different force fields of interaction correspond to different internal energy of continuous medium of the int-ensemble $\mathcal{E}[S_{st}]$.

In the ideal gas this interaction is described by the collision integral, describing collisions between molecules. This integral describes a real interaction between real molecules. If there is only one molecule, its motion is deterministic. In the case of stochastic particles the particle interaction is fictitious in the sense, that the particle motion is stochastic even in the case of one particle. The reason of stochasticity may be internal reason of a single particle, but not an interaction with other particles. Nevertheless, the stochasticity is described as an interaction of a deterministic particle with other deterministic particles of the int-ensemble. The fact is that the classical dynamics is a dynamics of deterministic particles. *Considering stochastic particles as interacting deterministic particles, one can describe a motion of stochastic particles by methods of the classical dynamics*. Reducing the stochastic particle motion to a motion of interacting deterministic particles, one uses the form of the κ -field for classification of forms of the particle stochasticity. In the classical gas dynamics the stochasticity is presented by the internal energy E of the gas .

At first, we consider the nonrelativistic case of the action (3.4), (3.5), when component $\kappa^0 \ll |\boldsymbol{\kappa}|$. In this case, expanding radical in (3.4), and setting $\xi_0 = t$, one obtains

$$\mathcal{A}_{\mathcal{E}[S_{st}]}[\mathbf{x}, \boldsymbol{\kappa}] = \int_t \int_{V_{\xi}} \left(-mc^2 + \frac{m}{2} \left(\frac{d\mathbf{x}}{dt} \right)^2 + \frac{\hbar^2}{2m} \boldsymbol{\kappa}^2 + \frac{\hbar^2}{2m} \boldsymbol{\nabla} \boldsymbol{\kappa} - \frac{e}{c} A_0 - \frac{e}{c} \mathbf{A} \frac{d\mathbf{x}}{dt} \right) dt d^3 \boldsymbol{\xi} \quad (3.7)$$

where $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi}) = \{x^1, x^2, x^3\}$ and $\boldsymbol{\kappa} = \boldsymbol{\kappa}(t, \mathbf{x}) = \{\kappa^1, \kappa^2, \kappa^3\}$, $A_0 = A_0(t, \mathbf{x})$, $\mathbf{A} = \mathbf{A}(t, \mathbf{x})$.

Let us introduce designation

$$\boldsymbol{\kappa}(t, \mathbf{x}) = -\frac{\hbar}{m} \mathbf{u}(t, \mathbf{x}) \quad (3.8)$$

where \mathbf{u} is the mean velocity of a particle in the int-ensemble $\mathcal{E}[S_{st}]$. Let us set

$eA_0/c = V(t, \mathbf{x})$, $A_\alpha = 0$, $\alpha = 1, 2, 3$. The action (3.7) takes the form.

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\text{st}}]}[\mathbf{x}, \mathbf{u}] = \int_t \int_{V_\xi} \left\{ \frac{m}{2} \left(\frac{d\mathbf{x}}{dt} \right)^2 + \frac{m}{2} \mathbf{u}^2 - \frac{\hbar}{2} \nabla \mathbf{u} - V \right\} dt d\xi, \quad (3.9)$$

The first term of (3.7) is omitted, because it does not contribute to dynamic equations. The variable $\mathbf{x} = \mathbf{x}(t, \xi)$ describes the regular component of the particle velocity. The variable $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$ describes the mean value of the stochastic velocity component. The first term of (3.9) describes the kinetic energy of regular motion. The second term in (3.9) describes the kinetic energy of the stochastic velocity component. The third term describes interaction between the stochastic component $\mathbf{u}(t, \mathbf{x})$ and the regular component $d\mathbf{x}/dt$. The operator

$$\nabla = \left\{ \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3} \right\} \quad (3.10)$$

is defined in the space of coordinates \mathbf{x} . Dynamic equations for the dynamic system $\mathcal{E}[\mathcal{S}_{\text{st}}]$ are obtained as a result of variation of the action (3.9) with respect to dynamic variables \mathbf{x} and \mathbf{u} .

Variation of (3.9) with respect to \mathbf{u} gives

$$\begin{aligned} \delta \mathcal{A}_{\mathcal{E}[\mathcal{S}_{\text{st}}]}[\mathbf{x}, \mathbf{u}] &= \int \int_{V_\xi} \left\{ m \mathbf{u} \delta \mathbf{u} - \frac{\hbar}{2} \nabla \delta \mathbf{u} \right\} dt d\xi \\ &= \int \int_{V_x} \left\{ m \mathbf{u} \delta \mathbf{u} - \frac{\hbar}{2} \nabla \delta \mathbf{u} \right\} \frac{\partial(\xi_1, \xi_2, \xi_3)}{\partial(x^1, x^2, x^3)} dt d\mathbf{x} \\ &= \int \int_{V_x} \delta \mathbf{u} \left\{ m \mathbf{u} \rho + \frac{\hbar}{2} \nabla \rho \right\} dt d\mathbf{x} - \int \oint \frac{\hbar}{2} \rho \delta \mathbf{u} dt d\mathbf{S} \end{aligned} \quad (3.11)$$

where

$$\rho = \frac{\partial(\xi_1, \xi_2, \xi_3)}{\partial(x^1, x^2, x^3)} = \left(\frac{\partial(x^1, x^2, x^3)}{\partial(\xi_1, \xi_2, \xi_3)} \right)^{-1} \quad (3.12)$$

We obtain the following dynamic equation

$$m \rho \mathbf{u} + \frac{\hbar}{2} \nabla \rho = 0, \quad (3.13)$$

Variation of (3.9) with respect to \mathbf{x} gives

$$m \frac{d^2 \mathbf{x}}{dt^2} = \nabla \left(\frac{m}{2} \mathbf{u}^2 - \frac{\hbar}{2} \nabla \mathbf{u} \right) \quad (3.14)$$

Here d/dt means the substantial derivative with respect to time t

$$\frac{dF}{dt} \equiv \frac{\partial(F, \xi_1, \xi_2, \xi_3)}{\partial(t, \xi_1, \xi_2, \xi_3)} \quad (3.15)$$

Resolving (3.13) with respect to \mathbf{u} , we obtain the equation

$$\mathbf{u} = -\frac{\hbar}{2m} \nabla \ln \rho, \quad (3.16)$$

which reminds the expression for the mean velocity of the Brownian particle with the diffusion coefficient $D = \hbar/2m$.

Eliminating the velocity \mathbf{u} from dynamic equations (3.14) by means of (3.16), we obtain the dynamic equations for the mean motion of the stochastic particle \mathcal{S}_{st}

$$m \frac{d^2 \mathbf{x}}{dt^2} = -\nabla V - \nabla U_{\text{B}}, \quad U_{\text{B}} = U(\rho, \nabla \rho, \nabla^2 \rho) = \frac{\hbar^2}{8m} \frac{(\nabla \rho)^2}{\rho^2} - \frac{\hbar^2}{4m} \frac{\nabla^2 \rho}{\rho} \quad (3.17)$$

Here ρ is considered to be function of t, \mathbf{x} , and ∇ is the gradient in the space of coordinates \mathbf{x} . The density ρ is defined by (3.12). U_{B} is so called Bohm potential [2]. Equations (3.17) are dynamic equations in the Lagrangian representations, where $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$, $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$

To transform dynamic equations to the Euler representation, where dependent dynamic variables $\dot{\mathbf{x}} \equiv \mathbf{v}(t, \mathbf{x})$, $\boldsymbol{\xi} = \boldsymbol{\xi}(t, \mathbf{x})$, $\rho = \rho(t, \mathbf{x})$, one should consider the transformation Jacobian

$$J = J(\xi_{i,k}) = \frac{\partial(\xi_0, \xi_1, \xi_2, \xi_3)}{\partial(x^0, x^1, x^2, x^3)} = \det \|\xi_{i,k}\|, \quad i, k = 0, 1, 2, 3, \quad \xi_{i,k} \equiv \frac{\partial \xi_i}{\partial x^k} \quad (3.18)$$

After transformation one should set $\xi_0 = t$, $x^0 = t$. One obtains

$$\frac{\partial J}{\partial \xi_{0,\alpha}} = \frac{\partial(x^\alpha, \xi_1, \xi_2, \xi_3)}{\partial(x^0, x^1, x^2, x^3)} = \frac{\partial(x^\alpha, \xi_1, \xi_2, \xi_3)}{\partial(\xi_0, \xi_1, \xi_2, \xi_3)} \frac{\partial(\xi_0, \xi_1, \xi_2, \xi_3)}{\partial(x^0, x^1, x^2, x^3)} \quad (3.19)$$

Setting in (3.19) $\xi_0 = t$, $x^0 = t$, one obtains

$$\frac{\partial J}{\partial \xi_{0,\alpha}} = \frac{\partial(x^\alpha, \xi_1, \xi_2, \xi_3)}{\partial(t, \xi_1, \xi_2, \xi_3)} \frac{\partial(t, \xi_1, \xi_2, \xi_3)}{\partial(t, x^1, x^2, x^3)} \quad (3.20)$$

According to (3.15) and (3.12)

$$\mathbf{v} = \frac{d\mathbf{x}}{dt} = \frac{\partial(x^\alpha, \xi_1, \xi_2, \xi_3)}{\partial(t, \xi_1, \xi_2, \xi_3)}, \quad \rho = \frac{\partial(t, \xi_1, \xi_2, \xi_3)}{\partial(t, x^1, x^2, x^3)} = \frac{\partial(\xi_1, \xi_2, \xi_3)}{\partial(x^1, x^2, x^3)} \quad (3.21)$$

It follows from (3.20), (3.21), that According to (3.12)

$$\frac{\partial J}{\partial \xi_{0,0}} = \rho, \quad \frac{\partial J}{\partial \xi_{0,\alpha}} = \rho v^\alpha, \quad \alpha = 1, 2, 3 \quad (3.22)$$

Using identity

$$\frac{\partial}{\partial x^k} \frac{\partial J}{\partial \xi_{0,k}} = \frac{\partial}{\partial x^0} \frac{\partial J}{\partial \xi_{0,0}} + \frac{\partial}{\partial x^\alpha} \frac{\partial J}{\partial \xi_{0,\alpha}} \equiv 0 \quad (3.23)$$

and relations (3.22) one obtains the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{v}) = 0 \quad (3.24)$$

The equation (3.17) takes the form

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \mathbf{v} = -\nabla V - \nabla U_B \quad (3.25)$$

Hydrodynamic form (3.24), (3.25) of the Schrödinger equation has been obtained by Madelung [1]. It has been used by Bohm [2] for the case of nonrotational flow, when $\mathbf{v} = \nabla \varphi$, and equation (3.25) can be written in the form

$$\frac{\partial \nabla \varphi}{\partial t} + \nabla \frac{(\nabla \varphi)^2}{2} = -\nabla V - \nabla U_B \quad (3.26)$$

In the case of a rotational flow the equation (3.25) is not equivalent to the Schrödinger equation.

4 Relativistic case

Let us return to the action (3.4), (3.5) and show, that int-ensemble $\mathcal{E}[S_{st}]$ can be described in terms of wave function. We shall consider variables $\xi = \xi(x)$ in (3.4) as dependent variables and variables x as independent variables. After manipulations with the transformation Jacobian (3.18) and introduction of the wave function (2.7), (2.9) one obtains the action (3.4), (3.5) in the form (See details in Appendix)

$$\mathcal{A}[\psi, \psi^*] = \int \left\{ \left(i\hbar \partial_k + \frac{e}{c} A_k \right) \psi^* \left(-i\hbar \partial^k + \frac{e}{c} A^k \right) \psi - m^2 c^2 \rho - \frac{\hbar^2}{4} (\partial_l s_\alpha) (\partial^l s_\alpha) \rho \right\} d^4 x \quad (4.1)$$

where

$$\rho = \psi^* \psi, \quad s_\alpha = \frac{\psi^* \sigma_\alpha \psi}{\rho}, \quad \alpha = 1, 2, 3 \quad (4.2)$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \psi^* = (\psi_1^*, \psi_2^*), \quad (4.3)$$

σ_α are 2×2 Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (4.4)$$

Variations with respect to ψ^* leads to dynamic equation

$$\begin{aligned} & \left(-i\hbar \partial_k + \frac{e}{c} A_k \right) \left(-i\hbar \partial^k + \frac{e}{c} A^k \right) \psi - \left(m^2 c^2 + \frac{\hbar^2}{4} (\partial_l s_\alpha) (\partial^l s_\alpha) \right) \psi \\ &= -\hbar^2 \frac{\partial_l (\rho \partial^l s_\alpha)}{2\rho} (\sigma_\alpha - s_\alpha) \psi \end{aligned} \quad (4.5)$$

It is nonlinear, generally speaking. However, let the wave function be one-component, or components ψ_1 and ψ_2 be linear dependent $\psi_1 = a\psi_2$, $a = \text{const}$. Then $\mathbf{s} = \text{const}$ and $\partial_i s_\alpha = 0$. In this case nonlinear terms in (4.5) vanish, and dynamic equation (4.5) turns to the Klein-Gordon equation

$$\left(-i\hbar\partial_k + \frac{e}{c}A_k\right)\left(-i\hbar\partial^k + \frac{e}{c}A^k\right)\psi - m^2c^2\psi = 0 \quad (4.6)$$

5 Discussion

Considering dynamics of continuous medium as a mathematical instrument for description of stochastic particles, one can find the quantum mechanics and explain the origin of the wave function as a natural means of a fluid description.

Describing a gas by means of classical gas dynamics, one cannot say anything on structure and arrangement of the gas, because the gas dynamic equations are simply conservation laws of the matter and of its energy-momentum. However, adding the collision integral and describing the gas motion by means of a kinetic equation, one may determine the distribution function and learn the gas motion mechanism. Analogously, if one determines the nature of the κ -field, one may obtain information on arrangement of elementary particles. Axiomatic conception of quantum mechanics does not admit one to obtain any information on the elementary particles arrangement. The contemporary quantum theory describes elementary particles as *pointlike objects* provided by a set of quantum numbers. According to quantum theory elementary particles have no internal structure. When internal structure of hardrons has been discovered experimentally, the quantum theory explained this fact by existence of pointlike particles (quarks), which cannot exist singly outside the hardron. However, existence of quarks only inside hardrons shows that quarks are elements of the hardron structure, but the quantum theory cannot accept such a supposition. It considers quarks as single particles.

This reminds situation with investigations of chemical elements, where there are two approaches: (1) empirical approach and (2) structural approach. The empirical approach is used by chemists. They are not interested in the atom arrangement. They are interested only in systematization of chemical elements. Chemists ascribe some characteristic numbers (atomic weight, valency, etc...) to any chemical element and systematize the chemical elements according to these numbers. Physicists use the structural approach, which admits one to determine arrangement of atoms (nucleus, electronic envelope, etc.). Using empirical approach of chemists, one could not create atomic energetics and atomic weapon.

The quantum theory could explain arrangement of the atom, but it cannot explain arrangement of elementary particles, because of its empirical approach. For instance, the quantum mechanics describes the electron by the Dirac equation. In this presentation the Dirac particle (electron) is a pointlike particle, having the mass m , charge e , spin $\hbar/2$, and magnetic moment $\mu = e\hbar/2mc$. These quantum numbers contain the quantum constant \hbar , even at the classical approach. The world line of a free Dirac particle is a straight line.

The Dirac equation can be considered as an equation [7], describing some fluid. In this case the world line of a free Dirac particle is a helix with timelike axis. The particle rotation along the circles of the helix is a source of spin and of the magnetic moment, which are not simple quantum numbers now. Such a connection between the quantum numbers and the structure of the particle world line cannot be obtained in the framework of the axiomatic conception of quantum theory.

In the fluid dynamics the formula of type of (2.11) for calculation of mean values is valid only for some physical quantities. The axiomatic quantum mechanics expands action of this formula on all physical quantities [6]. As a result one obtains the Neumann's theorem on hidden variables, which is not true, because the formula (2.11) is valid not for all quantities, as it is supposed in the conditions of the theorem.

Linearity of dynamic equation for a certain kind of fluid is expanded to all dynamic equations of quantum mechanics without sufficient foundation. As a result one obtains the linearity principle instead of a special case of the fluid flow.

The force field κ determines properties of the fluid, in terms of which the motion of stochastic particles is described. One may say, that the κ -field describes properties of the stochastic particle. Thus, quantum effects are described by the some force field κ , but not by quantum principles and not by a change of physical quantities by some operators or matrices.

Changing the effective particle mass (3.3), the κ -field may make M^2 to be negative. It is a necessary condition of pair production (change of the world line direction in time). This change of M^2 needs a strong κ -field. The κ -field can be strong enough, if it is an external force field, because the internal κ -field of a particle, which is responsible for quantum effects, is too weak for such a change of the mass [8]. In the axiomatic quantum mechanics the internal κ -field is included in the wave function, whereas the external κ -field is not used. As a result the pair production effect in the quantum field theory is a corollary of inconsistency of the second quantization procedure in the relativistic case. [9, 10]. Situation with the second quantization of the nonlinear Klein-Gordon equation looks as follows. At the conventional second quantization in the relativistic case the wave function contains both annihilation operators and creation operators. As a result the Hamiltonian H coincides with the energy E of the system for free particles. Such a coincidence takes place in the nonrelativistic case (for instance, in the case of the second quantization of the Schrödinger equation). In the relativistic case such a coincidence takes place only in the absence of the pair generation. In the case, when the pair production is possible the conventional method of the second quantization leads to nonstationary vacuum state. This fact is explained usually in the sense, that vacuum state does not contain particles, but it contains virtual particles. In reality nonstationary vacuum is a corollary of inconsistent statement of the the second quantization problem. At a inconsistent statement of the problem one may obtain any results, which one wants. It is necessary only to have sufficient ingenuity.

Introducing the distribution function and kinetic equation for it, one obtains a more detailed information on the gas motion mechanism. In a like way considering a

source of the κ -field, one can obtain a more detailed information on the elementary particle arrangement [11].

6 Appendix. Transformation of the action to representation in terms of wave function

Let us consider variables $\xi = \xi(x)$ in (3.4) as dependent variables and variables x as independent variables. Let the Jacobian (3.18)

$$J = \frac{\partial(\xi_0, \xi_1, \xi_2, \xi_3)}{\partial(x^0, x^1, x^2, x^3)} = \det \|\xi_{i,k}\|, \quad \xi_{i,k} \equiv \partial_k \xi_i \equiv \frac{\partial \xi_i}{\partial x^k}, \quad i, k = 0, 1, 2, 3 \quad (6.1)$$

be considered to be a multilinear function of $\xi_{i,k}$. Then

$$d^4 \xi = J d^4 x, \quad \dot{x}^i \equiv \frac{dx^i}{d\xi_0} \equiv \frac{\partial(x^i, \xi_1, \xi_2, \xi_3)}{\partial(\xi_0, \xi_1, \xi_2, \xi_3)} = J^{-1} \frac{\partial J}{\partial \xi_{0,i}} \quad (6.2)$$

After transformation to dependent variables ξ the action (3.4) takes the form

$$\mathcal{A}[\xi, \kappa] = \int \left\{ -mcK \sqrt{g_{ik} \frac{\partial J}{\partial \xi_{0,i}} \frac{\partial J}{\partial \xi_{0,k}}} - \frac{e}{c} A_k \frac{\partial J}{\partial \xi_{0,k}} \right\} d^4 x, \quad (6.3)$$

$$K = \sqrt{1 + \lambda^2 (\kappa_l \kappa^l + \partial_l \kappa^l)}, \quad \lambda = \frac{\hbar}{mc}, \quad (6.4)$$

Now variables ξ and κ are considered as functions of independent variables x .

Let us introduce new variables

$$j^k = \frac{\partial J}{\partial \xi_{0,k}}, \quad k = 0, 1, 2, 3 \quad (6.5)$$

by means of Lagrange multipliers p_k

$$\mathcal{A}[\xi, \kappa, j, p] = \int \left\{ -mcK \sqrt{g_{ik} j^i j^k} - \frac{e}{c} A_k j^k + p_k \left(\frac{\partial J}{\partial \xi_{0,k}} - j^k \right) \right\} d^4 x, \quad (6.6)$$

Variation with respect to ξ_i gives

$$\frac{\delta \mathcal{A}}{\delta \xi_i} = -\partial_l \left(p_k \frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{i,l}} \right) = 0, \quad i = 0, 1, 2, 3 \quad (6.7)$$

Using identities

$$\frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{i,l}} \equiv J^{-1} \left(\frac{\partial J}{\partial \xi_{0,k}} \frac{\partial J}{\partial \xi_{i,l}} - \frac{\partial J}{\partial \xi_{0,l}} \frac{\partial J}{\partial \xi_{i,k}} \right) \quad (6.8)$$

$$\frac{\partial J}{\partial \xi_{i,l}} \xi_{k,l} \equiv J \delta_k^i, \quad \partial_l \frac{\partial J}{\partial \xi_{i,l}} \equiv 0, \quad \partial_l \frac{\partial^2 J}{\partial \xi_{0,k} \partial \xi_{i,l}} \equiv 0 \quad (6.9)$$

one can test by direct substitution that the general solution of linear equations (6.7) has the form (2.5)

$$p_k = b_0 (\partial_k \varphi + g^\alpha(\boldsymbol{\xi}) \partial_k \xi_\alpha), \quad k = 0, 1, 2, 3 \quad (6.10)$$

where $b_0 \neq 0$ is a constant, $g^\alpha(\boldsymbol{\xi})$, $\alpha = 1, 2, 3$ are arbitrary functions of $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, and φ is the dynamic variable ξ_0 , which ceases to be fictitious. Let us substitute (6.10) in (6.6). The term of the form $\partial J / \partial \xi_{0,k} \partial_k \varphi$ is reduced to Jacobian and does not contribute to dynamic equations. The terms of the form $\xi_{\alpha,k} \partial J / \partial \xi_{0,k}$ vanish due to identities (6.9). We obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa, j] = \int \left\{ -mcK \sqrt{g_{ik} j^i j^k} - j^k \pi_k \right\} d^4x, \quad (6.11)$$

where quantities π_k are determined by the relations

$$\pi_k = b_0 (\partial_k \varphi + g^\alpha(\boldsymbol{\xi}) \partial_k \xi_\alpha) + \frac{e}{c} A_k, \quad k = 0, 1, 2, 3 \quad (6.12)$$

Integration of (6.7) in the form (6.10) is that integration which admits to introduce a wave function. Note that coefficients in the system of equations (6.7) at derivatives of p_k are constructed of minors of the Jacobian (6.1). It is the circumstance that admits one to produce a formal general integration.

Variation of (6.11) with respect to κ^l gives

$$\frac{\delta \mathcal{A}}{\delta \kappa^l} = -\frac{\lambda^2 mc \sqrt{g_{ik} j^i j^k}}{K} \kappa_l + \partial_l \frac{\lambda^2 mc \sqrt{g_{ik} j^i j^k}}{2K} = 0, \quad \lambda = \frac{\hbar}{mc} \quad (6.13)$$

It can be written in the form

$$\kappa_l = \partial_l \kappa = \frac{1}{2} \partial_l \ln \rho, \quad e^{2\kappa} = \frac{\rho}{\rho_0} \equiv \frac{\sqrt{j_s j^s}}{\rho_0 K}, \quad \rho = \frac{\sqrt{j_s j^s}}{K} \quad (6.14)$$

where the variable κ is potential of the κ -field κ_i and $\rho_0 = \text{const}$ is the integration constant. Substituting (6.4) in (6.14), we obtain dynamic equation for κ

$$\hbar^2 (\partial_l \kappa \cdot \partial^l \kappa + \partial_l \partial^l \kappa) = m^2 c^2 \frac{e^{-4\kappa} j_s j^s}{\rho_0^2} - m^2 c^2 \quad (6.15)$$

Variation of (6.11) with respect to j^k gives

$$\pi_k = -\frac{mcK j_k}{\sqrt{g_{ls} j^l j^s}} \quad (6.16)$$

or

$$\pi_k g^{kl} \pi_l = m^2 c^2 K^2 \quad (6.17)$$

Substituting $\sqrt{j_s j^s} / K$ from the second equation (6.14) in (6.16), we obtain

$$j_k = -\frac{\rho_0}{mc} e^{2\kappa} \pi_k, \quad (6.18)$$

Now we eliminate the variables j^k from the action (6.11), using relation (6.18) and (6.14). We obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa] = \int \rho_0 e^{2\kappa} \{ -m^2 c^2 K^2 + \pi^k \pi_k \} d^4 x, \quad (6.19)$$

where π_k is determined by the relation (6.12). Using expression (3.5) for K , the first term of the action (6.19) can be transformed as follows.

$$\begin{aligned} -m^2 c^2 e^{2\kappa} K^2 &= -m^2 c^2 e^{2\kappa} (1 + \lambda^2 (\partial_l \kappa \partial^l \kappa + \partial_l \partial^l \kappa)) \\ &= -m^2 c^2 e^{2\kappa} + \hbar^2 e^{2\kappa} \partial_l \kappa \partial^l \kappa - \frac{\hbar^2}{2} \partial_l \partial^l e^{2\kappa} \end{aligned}$$

Let us take into account that the last term has the form of divergence. It does not contribute to dynamic equations and can be omitted. Omitting this term, we obtain

$$\mathcal{A}[\varphi, \boldsymbol{\xi}, \kappa] = \int \rho_0 e^{2\kappa} \{ -m^2 c^2 + \hbar^2 \partial_l \kappa \partial^l \kappa + \pi^k \pi_k \} d^4 x, \quad (6.20)$$

Here π_k is defined by the relation (6.12), where the integration constant b_0 is chosen in the form $b_0 = \hbar$

$$\pi_k = \hbar (\partial_k \varphi + g^\alpha(\boldsymbol{\xi}) \partial_k \xi_\alpha) + \frac{e}{c} A_k, \quad k = 0, 1, 2, 3 \quad (6.21)$$

Instead of dynamic variables $\varphi, \boldsymbol{\xi}, \kappa$ we introduce n -component complex function (2.7), (2.9)

$$\psi = \{\psi_\alpha\} = \{\sqrt{\rho} e^{i\varphi} w_\alpha(\boldsymbol{\xi})\} = \{\sqrt{\rho_0} e^{\kappa+i\varphi} w_\alpha(\boldsymbol{\xi})\}, \quad \alpha = 1, 2, \dots, n \quad (6.22)$$

Here w_α are functions of only $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$, having the following properties

$$\sum_{\alpha=1}^{\alpha=n} w_\alpha^* w_\alpha = 1, \quad -\frac{i}{2} \sum_{\alpha=1}^{\alpha=n} \left(w_\alpha^* \frac{\partial w_\alpha}{\partial \xi_\beta} - \frac{\partial w_\alpha^*}{\partial \xi_\beta} w_\alpha \right) = g^\beta(\boldsymbol{\xi}) \quad (6.23)$$

where (*) denotes the complex conjugation. The number n of components of the wave function ψ depends on the functions $g^\beta(\boldsymbol{\xi})$. The number n is chosen in such a way, that equations (6.23) have a solution. Then we obtain

$$\psi^* \psi \equiv \sum_{\alpha=1}^{\alpha=n} \psi_\alpha^* \psi_\alpha = \rho = \rho_0 e^{2\kappa}, \quad \partial_l \kappa = \frac{\partial_l (\psi^* \psi)}{2\psi^* \psi} \quad (6.24)$$

$$\pi_k = -\frac{i\hbar (\psi^* \partial_k \psi - \partial_k \psi^* \cdot \psi)}{2\psi^* \psi} + \frac{e}{c} A_k, \quad k = 0, 1, 2, 3 \quad (6.25)$$

Substituting relations (6.24), (6.25) in (6.20), we obtain the action, written in terms of the wave function ψ

$$\begin{aligned} \mathcal{A}[\psi, \psi^*] &= \int \left\{ \left[\frac{i\hbar (\psi^* \partial_k \psi - \partial_k \psi^* \cdot \psi)}{2\psi^* \psi} - \frac{e}{c} A_k \right] \left[\frac{i\hbar (\psi^* \partial^k \psi - \partial^k \psi^* \cdot \psi)}{2\psi^* \psi} - \frac{e}{c} A^k \right] \right. \\ &\quad \left. + \hbar^2 \frac{\partial_l (\psi^* \psi) \partial^l (\psi^* \psi)}{4(\psi^* \psi)^2} - m^2 c^2 \right\} \psi^* \psi d^4 x \quad (6.26) \end{aligned}$$

Let us use the identity

$$\begin{aligned} & \frac{(\psi^* \partial_l \psi - \partial_l \psi^* \cdot \psi) (\psi^* \partial^l \psi - \partial^l \psi^* \cdot \psi)}{4\psi^* \psi} + \partial_l \psi^* \partial^l \psi \\ \equiv & \frac{\partial_l (\psi^* \psi) \partial^l (\psi^* \psi)}{4\psi^* \psi} + \frac{g^{ls}}{2} \psi^* \psi \sum_{\alpha, \beta=1}^{\alpha, \beta=n} Q_{\alpha\beta, l}^* Q_{\alpha\beta, s} \end{aligned} \quad (6.27)$$

where

$$Q_{\alpha\beta, l} = \frac{1}{\psi^* \psi} \begin{vmatrix} \psi_\alpha & \psi_\beta \\ \partial_l \psi_\alpha & \partial_l \psi_\beta \end{vmatrix}, \quad Q_{\alpha\beta, l}^* = \frac{1}{\psi^* \psi} \begin{vmatrix} \psi_\alpha^* & \psi_\beta^* \\ \partial_l \psi_\alpha^* & \partial_l \psi_\beta^* \end{vmatrix} \quad (6.28)$$

Then we obtain

$$\begin{aligned} \mathcal{A}[\psi, \psi^*] = & \int \left\{ \left(i\hbar \partial_k + \frac{e}{c} A_k \right) \psi^* \left(-i\hbar \partial^k + \frac{e}{c} A^k \right) \psi - m^2 c^2 \psi^* \psi \right. \\ & \left. + \frac{\hbar^2}{2} \sum_{\alpha, \beta=1}^{\alpha, \beta=n} g^{ls} Q_{\alpha\beta, l} Q_{\alpha\beta, s}^* \psi^* \psi \right\} d^4 x \end{aligned} \quad (6.29)$$

Let us consider the case of nonrotational flow, when $g^\alpha(\boldsymbol{\xi}) = 0$. In this case $w_1 = 1$, $w_2 = 0$, and the function ψ has only one component. It follows from (6.28), that $Q_{\alpha\beta, l} = 0$. Then we obtain instead of (6.29)

$$\mathcal{A}[\psi, \psi^*] = \int \left\{ \left(i\hbar \partial_k + \frac{e}{c} A_k \right) \psi^* \left(-i\hbar \partial^k + \frac{e}{c} A^k \right) \psi - m^2 c^2 \psi^* \psi \right\} d^4 x \quad (6.30)$$

Variation of the action (6.30) with respect to ψ^* generates the Klein-Gordon equation

$$\left(-i\hbar \partial_k + \frac{e}{c} A_k \right) \left(-i\hbar \partial^k + \frac{e}{c} A^k \right) \psi - m^2 c^2 \psi = 0 \quad (6.31)$$

Thus, description in terms of the Klein-Gordon equation is a special case of the stochastic particles description by means of the action (3.4), (3.5).

In the case, when the fluid flow is rotational, and the wave function ψ is two-component, the identity (6.27) takes the form

$$\begin{aligned} & \frac{(\psi^* \partial_l \psi - \partial_l \psi^* \cdot \psi) (\psi^* \partial^l \psi - \partial^l \psi^* \cdot \psi)}{4\rho} - \frac{(\partial_l \rho) (\partial^l \rho)}{4\rho} \\ \equiv & -\partial_l \psi^* \partial^l \psi + \frac{1}{4} (\partial_l s_\alpha) (\partial^l s_\alpha) \rho \end{aligned} \quad (6.32)$$

where 3-vector $\mathbf{s} = \{s_1, s_2, s_3, \}$ is defined by the relations

$$\rho = \psi^* \psi, \quad s_\alpha = \frac{\psi^* \sigma_\alpha \psi}{\rho}, \quad \alpha = 1, 2, 3 \quad (6.33)$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \psi^* = (\psi_1^*, \psi_2^*), \quad (6.34)$$

and Pauli matrices $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ have the form (4.4). Note that 3-vectors \mathbf{s} and $\boldsymbol{\sigma}$ are vectors in the space V_ξ of the Clebsch potentials $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$. They transform as vectors at the transformations

$$\xi_\alpha \rightarrow \tilde{\xi}_\alpha = \tilde{\xi}_\alpha(\boldsymbol{\xi}), \quad \alpha = 1, 2, 3, \quad \frac{\partial(\tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3)}{\partial(\xi_1, \xi_2, \xi_3)} \neq 0 \quad (6.35)$$

In general, transformations of Clebsch potentials $\boldsymbol{\xi}$ and those of coordinates \mathbf{x} are independent. However, the action (6.26) does not contain any reference to the Clebsch potentials $\boldsymbol{\xi}$ and transformations (6.35) of $\boldsymbol{\xi}$. If we consider only linear transformations of space coordinates \mathbf{x}

$$x^\alpha \rightarrow \tilde{x}^\alpha = b^\alpha + \omega_{\beta}^{\alpha} x^\beta, \quad \alpha = 1, 2, 3 \quad (6.36)$$

nothing prevents from accompanying any transformation (6.36) with the similar transformation

$$\xi_\alpha \rightarrow \tilde{\xi}_\alpha = b^\alpha + \omega_{\beta}^{\alpha} \xi_\beta, \quad \alpha = 1, 2, 3 \quad (6.37)$$

of Clebsch potentials $\boldsymbol{\xi}$. The formulas for linear transformation of vectors and spinors in V_x do not contain the coordinates \mathbf{x} explicitly, and one can consider vectors and spinors in V_ξ as vectors and spinors in V_x , provided we consider linear transformations (6.36), (6.37) always together.

Using identity (6.32), we obtain from (6.26)

$$\mathcal{A}[\psi, \psi^*] = \int \left\{ \left(i\hbar\partial_k + \frac{e}{c}A_k \right) \psi^* \left(-i\hbar\partial^k + \frac{e}{c}A^k \right) \psi - m^2c^2\rho - \frac{\hbar^2}{4} (\partial_l s_\alpha) (\partial^l s_\alpha) \rho \right\} d^4x \quad (6.38)$$

Dynamic equation, generated by the action (6.38), has the form

$$\begin{aligned} & \left(-i\hbar\partial_k + \frac{e}{c}A_k \right) \left(-i\hbar\partial^k + \frac{e}{c}A^k \right) \psi - \left(m^2c^2 + \frac{\hbar^2}{4} (\partial_l s_\alpha) (\partial^l s_\alpha) \right) \psi \\ &= -\hbar^2 \frac{\partial_l (\rho \partial^l s_\alpha)}{2\rho} (\sigma_\alpha - s_\alpha) \psi \end{aligned} \quad (6.39)$$

The gradient of the unit 3-vector $\mathbf{s} = \{s_1, s_2, s_3\}$ describes rotational component of the fluid flow. If $\mathbf{s} = \text{const}$, the dynamic equation (6.39) turns to the conventional Klein-Gordon equation (6.31).

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